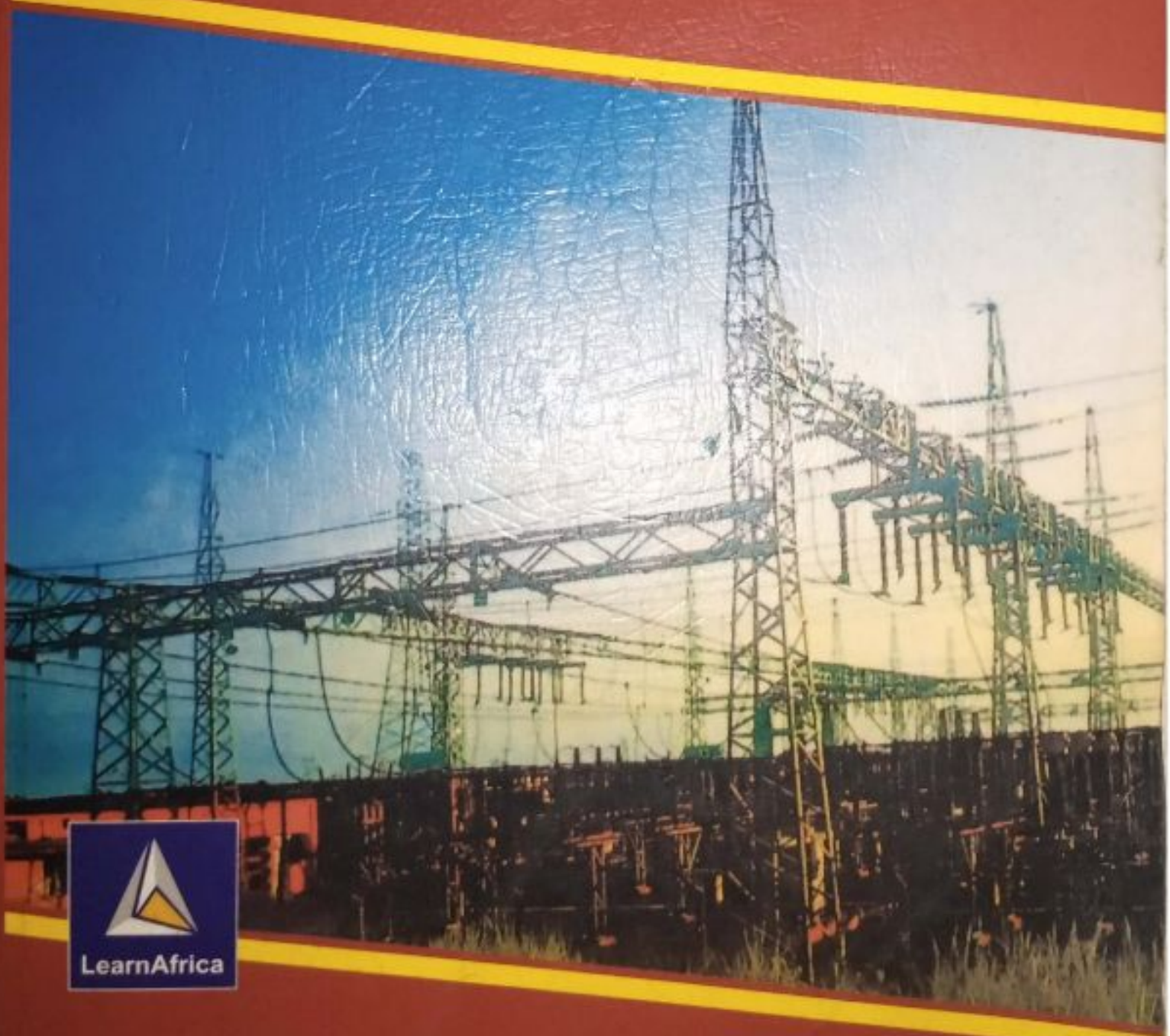
PRINCIPLES OF PHYSICS

for Senior Secondary Schools



LearnAfrica

M. Nelkon

Scanned with CamScanner

Israel Chinedu Ogubo

Principles of Physics

for Senior Secondary Schools

M. Nelkon M.Sc., F.Inst.P., A.K.C.
formerly Head of Science, William Ellis School, London

LearnAfrica

Scanned with CamScanner

Learn Africa Plc
Felix Iwerebon House
52, Oba Akran Avenue
P.M.B. 21036
Ikeja, Lagos State
Telephone : (01) 4393111, (01) 7403967
Fax : (01) 4964370
E-mail : learnafrica@learnafricaplc.com
Web : www.learnafricaplc.com

Area Offices and branches

Abuja, Ajegunle, Akure, Benin, Enugu, Ibadan, Ilorin, Jos, Kano, Onitsha, Owerri, Zaria, with representatives throughout Nigeria.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of Learn Africa Plc.

© Learn Africa Plc.

First Published 1981

Reprinted 1981, 1982, 1983, 1984, 1985, 1986, 1991, 2010, 2013.

ISBN 978 139 461 7

STUDENT HELP LINE 01 4545880

TEACHER HELP LINE 01 4545890

Printed by
BHS Book Printing Sdn. Bhd.
Kuala Lumpur • Malaysia
E-mail: bhsprint@tm.net.my Website: www.bhsprint.com

Scanned with CamScanner

Preface to the Senior Secondary Schools Edition

In this edition, I have taken the opportunity to add a section on Further Topics in (*a*) Mechanics—projectiles, simple harmonic motion, (*b*) Gravitational, electric and magnetic fields and capacitors, (*c*) Alternating current circuits, (*d*) Atom energy levels and quanta, (*e*) Photoelectricity, (*f*) Wave-particle duality. The topics cover the more advanced parts of the new Nigerian syllabus for Senior Secondary schools and with examples and exercises, it is hoped that students will find this helpful in their final years. The treatment is necessarily an introduction and more details of all the points will be found in *Advanced Level Physics* by Nelkon and Parker (Heinemann).

I have also taken the opportunity to add a selection of multiple choice questions to the exercises and parts of the text have been revised.

I am very grateful to B. L. N. Ndupu, Assistant Director of Education, Federal Ministry of Education, Nigeria, for his considerable advice and assistance with the new section on Further Topics. I am also indebted to teachers in Nigeria, Singapore, Hong Kong and Maldives for helpful comments.

The following points may be noted:

1. SI units are used. Where they are more convenient for calculations, allowed sub-multiples of mass and length, the gram and centimetre, have also been used;
2. in *Dynamics*, vectors and scalars, linear momentum and circular motion have been emphasised and modern apparatus introduced;
3. in *Heat*, electrical heating has been used for measuring heat capacities, other conversions to heat energy have been discussed, and the molecular kinetic theory has been applied to gases;
4. in *Waves*, the properties of waves are described, including diffraction and interference, and the difference between matter waves and electromagnetic waves is emphasised;
5. in *Electricity*, prominence has been given to electrons and ions as the charge carriers in metals and electrolytes, to the relation between potential difference and energy, and to the use of magnetic fields in electromagnetism;
6. the section on *Electronics* contains an account of the diode valve, the cathode ray tube and its uses, and an introduction to semiconductors, junction diodes and transistors;
7. the chapter on *Radioactivity* includes an introduction to atomic structure and nuclear energy.

Many worked examples have been given in the text in illustration of the subject matter and there are numerous exercises to assist comprehension. It is hoped that the book will provide a useful modern introduction to the principles of the subject.

The author is indebted to S. S. Alexander, The Mount School for Girls, London, and formerly senior science master, Woodhouse School, London, and to M. V. Detheridge, head of the physics department, William Ellis School, London, for their valuable assistance with the preparation of this new edition.

For their generous assistance with past editions, the author is also grateful to the following: Chief Olu Ibukun, UNESCO Regional Office of Science and Technology for Africa; F. Anstis, Reed's School, Surrey; Rev. M. D. Phillips, O.S.B., Ampleforth College, York; J. Lister, Wheelwright School for Girls; W. G. Sale, Nairobi, Kenya; A. Loizou, The English School, Cyprus; Dr R. A.

Scanned with CamScanner

McCurrie, University of Bradford; Dr R. P. T. Hills, formerly St John's College, Cambridge; and T. E. Walton, William Ellis School, London. I am indebted for valuable advice in earlier considerations to F. C. Brown, formerly head of the physical sciences department, Institute of Education, London University.

I am also much indebted to Sue Justice of the publishers for her valuable advice and diligence in the preparation of the new edition.

The author and publishers are grateful for permission to reprint questions set in past examinations by the following boards:

Cambridge Local Examinations Syndicate (C)
Joint Matriculation Board (N)
London University School Examinations Council (L)
Oxford and Cambridge Schools Examinations Board (O and C)
Oxford Local Examinations Syndicate (O)
Associated Examining Board (AEB)

Scanned with CamScanner

Contents

Preface iii
Tables of Formulae and Units x

1 Introduction · Some Basic Points 2
Theory and experiment. Forms of energy. Units. Measurements. Graphs.

MECHANICS AND FLUIDS 12

2 Dynamics · Speed, Velocity, Acceleration 13
Speed. Velocity. Acceleration. Deceleration. Equations of linear motion. Measurement of g by centisecond clock. Pendulum method. Displacement–time and velocity–time graphs. Scalars and vectors. Components of g.

3 Force, Momentum, Energy, Power 36
Force and acceleration. Forces of gravity and friction. Inertial mass. Investigations of $F = ma$. Units. Mass and weight. Friction and motion. Terminal velocity. Circular motion and acceleration. Force and momentum. Conservation of linear momentum. Work. Energy. Potential and kinetic energy. Conservation of energy. Power.

4 Forces and Moments 64
Moments. Applications. Equilibrium and moments. Equilibrium of parallel forces. Classes of levers. Centre of gravity. Stable, unstable, neutral equilibrium.

5 Machines · Resultant and Component of Forces 81
Mechanical advantage, velocity ratio, efficiency. Levers. Pulleys. Inclined plane. Screw-jack. Parallelogram of forces. Resultant. Components.

6 Density · Forces due to Fluids 100
Density. Upthrust. Investigations. Archimedes' Principle. Measurement of density and relative density. Floating objects. Principle of Flotation. Balloons, ships, floating docks, submarines, Plimsoll line. Hydrometers.

7 Pressure in Fluids 119
Pressure: meaning and units. Variation with depth and density. Some laws of pressure. Hydraulic press. Atmospheric pressure. Simple barometer. Units and magnitude of air pressure. Aneroid barometer. Siphon. Gas pressure. Boyle's law. Bicycle pump.

Scanned with CamScanner

MOLECULES AND MATTER 141

8 Molecules and Matter 142
Solid, liquid and gaseous states. Size of molecules. Oil film. Diffusion of molecules. Osmosis. Brownian motion. Elasticity. Hooke's law. Friction: static and kinetic, advantages and disadvantages. Surface tension. Capillarity.

HEAT 167

9 Temperature and Thermometers 168
Thermometers. Fixed points. Thermometer manufacture and calibration. Celsius and Kelvin scales. Alcohol, maximum and minimum, gas and thermoelectric thermometers.

10 Thermal Expansion of Solids and Liquids 175
Solids. Effects of expansion. Bimetal applications, thermostat. Linear expansivity. Determination. Formulae. Liquid and cubic expansivity. Hope's experiment. Volume and density variation of water.

11 Thermal Expansion of Gases 188
Gas laws. Volume expansion at constant pressure: experiment and coefficient. Absolute temperature. Pressure change at constant volume: experiment and coefficient. Constant-volume gas thermometer. General gas laws, $pV/T = \text{constant}$. Conversion to s.t.p. Kinetic theory explanation of gas laws.

12 Heat Capacity · Heat Energy 201
Joule as unit of heat. Electrical heating. Heat capacity, specific heat capacity. $Q = mct$. Calorimeters. Measurements. Molecular explanation of heat capacity. Transformation of energy and work into heat. Engines. Sun and energy transformation.

13 Latent Heat · Vapours 214
Latent heat of fusion. Molecular explanation. Melting point. Freezing and melting of water and ice. Regelation. Latent heat of vaporisation. Evaporation. Factors affecting evaporation. Molecular explanation. Refrigerator. Saturation vapour pressure. Molecular explanation. Boiling and s.v.p. Effect of pressure changes. Humidity. Wet-and-dry bulb hygrometer. Pressure cooker.

14 Transmission of Heat 234
Conduction. Molecular explanation. Metals and non-conductors. Conductivity of water. Ignition temperature. Convection currents in liquids. Hot-water system. Convection in gases. Land and sea breezes. Radiation. Emission and absorption. Infra-red radiation.

OPTICS 250

15 Straight-line Propagation of Light 251
Luminous and non-luminous objects. Rays and beams of light. Shadows. Eclipses. Pin-hole camera.

Scanned with CamScanner

16 Reflection at Plane Mirrors 259
Reflection of rays. Diffuse reflection. Laws of reflection. Periscopes. Formation of images: virtual and real images. Location of image. Lateral inversion. Inclined mirrors.

17 Curved Spherical Mirrors 268
Reflection of rays. Parabolic mirror. Centre and radius of curvature. Focal length. Images in curved mirrors. Real and virtual images. Ray drawings. Magnification. Convex mirror. Formulae.

18 Refraction at Plane Surfaces 280
Refraction in solids and liquids. Apparent depth. Partial immersion in water. Glass prism refraction. Laws. Refractive index. Total internal reflection. Critical angle. Total reflecting prisms. Mirages.

19 Refraction through Lenses 300
Converging (convex) and diverging (concave) lenses. Action on rays. Focal length of converging lens. Images in lenses. Magnifying lens. Magnification. Lens formulae.

20 Applications of Lenses 313
The eye. Binocular vision. Accommodation. Normal vision: near and far point. Defects of vision. Lens camera. Slide projector. Simple and compound microscope. Telescopes. Prism binoculars.

21 Colours of Light · The Spectrum 328
Colours in white light. Dispersion and recombination. Newton's colour disc. Formation of pure spectrum. Colours of objects. Additive and subtractive colour mixing. Ultra-violet and infra-red rays.

WAVES 340

22 Wave Propagation · Wave Effects 341
Ripple tank. Water waves. Wavelength, frequency, velocity. Transverse and longitudinal waves. Sound, light, electromagnetic waves. Resonance. Reflection, refraction. Interference. Young's experiment. Stationary waves. Diffraction, diffraction grating.

SOUND 367

23 Basic Principles of Sound · Strings and Pipes 368
Sound waves: rarefactions, compressions. Pitch, intensity, timbre. Echoes. Echo-sounding. Vibrations in strings. Sonometer. Closed pipe. Resonance tube experiment, velocity of sound.

ELECTRICITY 387

24 Current, Potential Difference, Resistance 388
Electric current. Charge carriers. Conductors. Insulators. Semiconductors. Ammeter. Potential difference. Voltmeter. Ohm's law. Fundamental formulae. Measurement of resistance. Effect of temperature. Resistances in series and parallel. Resistivity.

Scanned with CamScanner

25 Batteries, E.M.F. and Circuits 408
Voltaic cell. Simple cell. Leclanché dry cell. Accumulator: discharging and charging. E.m.f. and terminal p.d. Internal resistance. Series and parallel cells. Potentiometer. Circuit calculations.

26 Electrical Energy and Power 424
Electrical energy. *IVt*. Machines. Power. E.m.f. and energy. Electrical lamps. Kilowatt-hour. Heating effect-Joule's laws. Hot wire ammeter. High-voltage transmission. Domestic installations. Fuses.

27 Magnetism 439
Fundamental laws. Methods of making magnets. Demagnetisation. Materials. Ferrites. Magnetic fields. Neutral points. Induction. Shielding. Domain theory.

28 Magnetic Effect of a Current 454
Oersted's experiment. Forces due to magnetic fields. Straight wire, circular coil, solenoid. Electromagnets. Magnetic relay. Telecommunications. Moving iron ammeter.

29 Forces on Conductors · The Electric Motor 465
Interaction with magnetic fields. Rules for direction. Faraday motor. Rectangular coil and moving-coil instrument. Extension of range. Conversion to voltmeter. Motor and commutator. Loudspeaker. Forces between currents. Current balance.

30 Electromagnetic Induction 478
Induced e.m.f., current. Bicycle dynamo. Primary and secondary coils. Induction coil. Faraday. Lenz. Speedometer. Fleming's rule. Simple dynamo. A.c. and d.c. Transformers. High-voltage transmission.

31 Chemical Effect of a Current 496
Electrolytes. Electrolysis. Hoffman voltameter. Applications. Faraday's first law. Electrochemical equivalent. Ammeter calibration. Faraday's second law. Faraday constant. Ionic explanations.

32 Static Electricity · Capacitors 509
Positive and negative charges. Fundamental laws. Gold-leaf electroscope. Testing charges. Induction. Action at points. Electrophorus. Van de Graaff generator. Electric fields. Screening. Positive and negative potential. Earth potential. Effects of static electricity. Capacitance. Factors affecting capacitance. Units. Practical capacitors.

ATOMIC PHYSICS 531

33 Electrons and Electronics 532
Cathode rays. Thermionic emission. Hot cathodes. Diode valve. Rectification. Perrin tube. Fine beam tube. Electric and magnetic field effects. Cathode-ray tube. X-ray tube. Properties and nature of X-rays. Semiconductors. Electrons and holes. P- and n-type semiconductors. Junction diode. Transistor amplifier. Simple receiver.

Scanned with CamScanner

34 Radioactivity · Atomic Structure · Nuclear Energy 556
Geiger–Müller tube and scaler. Alpha- and beta-particles, gamma-rays. Ionisation and penetrating powers. Diffusion cloud chamber. Random decay. Half-life period. Geiger and Marsden experiment. Protons and neutrons. Atomic structure. Transmutations. Isotopes. Einstein mass-energy relation. Nuclear fission. Nuclear fusion.

FURTHER TOPICS

35 Further Mechanics: Projectiles · Simple Harmonic Motion 582
Projectiles Maximum height. Range. Time of Flight.
Simple Harmonic Motion Circular and simple harmonic motion. Equations of s.h.m. Kinetic and potential energy. Period of mass-spring system, loaded tube, simple pendulum.

36 Gravitational and Electrical Fields · Capacitors · Magnetic Field 595
Gravitational Field Newton's law of gravitation. Forces between planets, protons, proton aad electron. Relation between g and G. Gravitational field strength (intensity). Gravitational potential. Velocity of escape. Earth and Moon atmosphere.
Electric Field Force between charges. Force between protons, proton and electron. Comparison with gravitational force. Electric field strength or intensity, E. E for point charge. Deflection of charged particle in field. Electric potential, V. V for point charge. Potential gradient. $E = V/d$. Energy gain in field. 600
Capacitors Capacitors in parallel and series. Calculations. Energy in capacitor. 607
Magnetic Field Force $= Bqv$. Circular path and radius. Deflection and energy. 611

37 Alternating Current Circuits 616
Peak and root-mean-square values. A.c. voltage and resistance. Phase. Inductor. Reactance of inductor. Phase difference of V and I. Capacitor and reactance. Impedance of L,R and C,R and L,C,R in series. Resonance. Power in a.c. circuits.

38 Energy Levels in Atom · Quanta · Photoelectricity · Wave-Particle Duality 629
Energy Levels Bohr theory of hydrogen atom. Energy levels. Ground state. $E = hf$. Emission spectra and wavelengths. Absorption spectra.
Photoelectricity Threshold wavelength. Work function. Wave theory failure. The photon and particle theory. Einstein photoelectric equation. Calculations. Stopping potential. Confirmation of Einstein equation. 633
Wave-Particle De Broglie formula. Wave-particle duality. Heisenberg Uncertainty Principle. 637

Answers to Exercises 640
Multiple-choice Questions 645
Revision Questions 655
Mathematical Tables 660
Index 666

Scanned with CamScanner

MECHANICS

QUANTITY	DEFINITION	FORMULA	UNITS
Velocity (v)	$\frac{\text{displacement}}{\text{time}}$	$\frac{s}{t}$	metre per second (m/s)
Acceleration (a)	$\frac{\text{velocity change}}{\text{time}}$	$\frac{v-u}{t}$	m/s^2 $g = 9{\cdot}8\ m/s^2$ or $10\ m/s^2$ approx.
Force (F)	mass × acceleration	ma	newton (N)
Momentum	mass × velocity	mu	newton second
Work (W)	force × distance	$F \times s$	joule (J) 1 newton × 1 metre = 1 J
Kinetic energy	energy due to motion	$\frac{1}{2}mu^2$	joule (J)
Potential energy	energy due to level or position	mgh	joule (J)
Power	energy (work) per second	$\frac{\text{work}}{\text{time}}$	Watt (W)
Resolved component of force	force × cosine of angle concerned	$F \cos \theta$	newton (N)
Moment of force	force × perpendicular distance		newton metre (N m)
Mechanical advantage (M.A.)	$\frac{\text{load}}{\text{effort}}$	$\frac{W}{P}$	
Velocity ratio (V.R.)	$\frac{\text{distance/s moved by effort}}{\text{distance/s moved by load}}$		
Efficiency	$\frac{\text{work (energy) obtained}}{\text{work (energy) supplied}} \times 100\%$	$\frac{\text{M.A.}}{\text{V.R.}} \times 100\%$	
Density (ρ)	$\frac{\text{mass}}{\text{volume}}$	$\frac{m}{V}$	g/cm^3 or kg/m^3
Relative density	$\frac{\text{mass of substance}}{\text{mass of equal vol of water}}$		
Pressure (p)	force per unit area	$h\rho g$	newton per metre^2 (N/m^2) or Pa

Scanned with CamScanner

HEAT

QUANTITY	DEFINITION	FORMULA	UNITS
Joule (J)	work done when 1 newton moves through 1 metre		1 kJ = 10^3 J 1 MJ = 10^6 J
Heat capacity (C)	heat to raise temperature of substance by 1°C	mc	J/K or J/°C (1 K = 1 °C)
Specific heat capacity (c)	heat to raise temperature of unit mass of substance by 1°C	C/m	J/kg K or J/g K (for water, c = 4200 J/kg K or 4·2 J/g K)
Specific latent heat (l)	heat to change unit mass from solid to liquid, or from liquid to vapour, without temperature change		J/kg or J/g
Linear expansivity (α)	increase in length per unit length per °C temperature rise	$\alpha = \frac{l_2 - l_1}{l_1 \times t}$	per K or per °C
Cubic expansivity (γ)	increase in volume per unit volume per °C temperature rise	$\gamma = \frac{V_2 - V_1}{V_1 \times t}$	per K or per °C
Volume coefficient of gas	increase in volume per unit volume at 0°C per °C temperature rise, pressure being constant	$\gamma = \frac{V_1 - V_0}{V_0 \times t}$	per K or per °C (γ = 1/273 approx. for most gases)
Pressure coefficient of gas	increase in pressure per unit pressure at 0°C per °C temperature rise, volume being constant	$\gamma = \frac{p_t - p_0}{p_0 \times t}$	per K or per °C (γ = 1/273 approx. for most gases)
Absolute temperature (T)	temperature measured from absolute zero	$T = 273 + t$ (°C)	K (0 K = absolute zero 273 K = 0°C)

Scanned with CamScanner

OPTICS

QUANTITY	DEFINITION	FORMULA	UNITS
Refractive index (n)	$\frac{\text{sine of angle of incidence}}{\text{sine of angle of refraction}}$	$\frac{\sin i}{\sin r}$	
Critical angle (c)	angle of incidence when angle of refraction is 90°	$n = \frac{1}{\sin c}$	degrees
Focal length (f)	distance from principal focus to mirror or lens		m
Mirror/lens formula		$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$	
Magnification (m)	$\frac{\text{image length}}{\text{object length}}$	$\frac{v}{u}$	
Wavelength (λ)	distance between successive crests of waves	$\lambda = \frac{ay}{D}$ (Young)	m
		$\lambda = d \sin \theta$ (diffraction grating, first order)	

WAVES and SOUND

Frequency (f)	number of vibrations per second		Hz
Wavelength (λ)	distance between successive crests or troughs		metre
Velocity (v)	distance travelled per second	$v = f\lambda$	metre/second
Amplitude	Maximum displacement		metre
Node	point permanently at rest in stationary wave	$\text{NA} = \frac{\lambda}{4}$ $\text{NN} = \frac{\lambda}{2}$	
Antinode	point of maximum amplitude in stationary wave	$\text{AA} = \frac{\lambda}{2}$	
Fundamental frequency — stretched string	lowest frequency	$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$	Hz
Fundamental frequency of closed pipe	lowest frequency	$\frac{v}{4l}$	Hz

Scanned with CamScanner

ELECTRICITY

QUANTITY	DEFINITION	FORMULA	UNITS
Current (I)	Quantity of charge per second	$I = \frac{Q}{t}$	ampere (A)
Potential difference (V)	Energy per coulomb	$V = \frac{W}{Q}$	volt (V)
Resistance (R)	$\frac{\text{potential difference}}{\text{current}}$	$R = \frac{V}{I}$ ($I = V/R$, $V = IR$)	ohm (Ω)
Charge (Quantity) (Q)		$Q = It$	coulomb (C)
Resistances in series		$R = R_1 + R_2$	
Resistances in parallel		$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$	
Resistivity (ρ)	resistance of unit length with unit cross-section area	$R = \frac{\rho l}{A}$	ohm metre (Ω m)
Electrical energy (W)		$W = QV = IVt$	joule (J)
Electrical power (P)		$P = IV$	watt (W)
Heat in resistor		I^2Rt or IVt or V^2t/R	joule (J)
Electromotive force (E)	total energy per coulomb (= p.d. at terminals when no current flows)	$I = \frac{E}{R + r}$	volt (V)
Electrochemical equivalent (z)	mass deposited per coulomb	$m = zIt$	kg/C
Capacitance (C)	$\frac{\text{charge on plates}}{\text{p.d. between plates}}$	$C = \frac{Q}{V}$	farad (F)

ATOMIC PHYSICS

			charge	rel. mass to m_H*
α-particle	helium nucleus	$^4_2$He	$+2e$	4
β-particle	electron	e^-	$-e$	1/1830
γ-ray	electromagnetic wave			
Electron (e^-)	lightest particle with -ve charge	e^-	$-e$	1/1830
Proton (p)	hydrogen nucleus	$^1_1$H	$+e$	1
Neutron (n)	nuclear particle with no charge	$^1_0$n	0	1
Mass number (A)	number of nucleons			
Atomic number (Z)	number of protons			

*m_H = mass of hydrogen atom

Scanned with CamScanner

Natural energy

Oil pumping station in Eastern Nigeria (*Alan Hutchison Library*)

Human power – Tessa Sanderson, a world class athlete, about to throw the javelin (*Sporting Pictures (U.K.) Ltd*)

Volta Dam at Akosombo, north of Ghana's capital, Accra, built for hydroelectric power with an enormous lake behind it (*Alan Hutchison Library*)

Active volcano, Mount Ngaruahue, New Zealand, during eruption (*High Commissioner for New Zealand*)

Scanned with CamScanner

Introduction · Some Basic Points

Physics is a science which deals with the behaviour of matter. Some of its branches are electricity, optics, heat, sound, properties of matter and atomic physics.

Scientists of many different nationalities are today doing research in physics. As a result, many useful inventions and machines have been produced. For example, radar control at airports; computers in banks; colour television transmission; high-power microscopes for use in medical research; and anti-skid tyres for cars.

Theory and experiment

In ancient times people believed something simply because a famous person said it. A good example was the case of falling objects. A famous Greek philosopher called Aristotle said that heavy objects always fell to the ground faster than light objects. This was believed for nearly 2000 years.

An experiment showed that this statement was incorrect. A heavy and a light object were dropped together from the top of a tall building. (Legend says that the building was the Leaning Tower of Pisa in Italy, which still exists.) It was seen that the heavy and the light object both reached the ground at the same time. Aristotle's theory was wrong.

Today, scientists will not accept a theory unless there is experimental evidence to support it. If an experiment gives results which do not agree with the theory, the theory is given up or modified. Sometimes the result of an experiment suggests a new theory.

About 1910, for example, Lord Rutherford examined the results of an experiment by two of his research students at Cambridge. They were firing tiny particles at atoms. Some of the particles bounced off at large angles on making collisions. Some even bounced right back. He came to the conclusion that the atom contained a very tiny concentrated mass in the middle which repelled the particles violently. He called it the nucleus of the atom. And this led years later to the discovery of nuclear energy and then to the development of the large nuclear power stations throughout the world today.

Forms of energy

'Work' and 'energy' are two ideas which are widely used in all branches of physics.

A boy pulling a cart or a girl pulling a sledge is said to do *work*. An object which produces movement is said to do work. On climbing the stairs, for example, we do work in moving our bodies upward.

Scanned with CamScanner

If an object is able to do work, it is said to have energy. The spring of a watch has energy when wound up because it moves gear wheels as it slowly unwinds. A cricket ball thrown at the wicket has energy because it can knock down the stumps.

The wound spring and the fast-moving cricket ball are examples of objects having *mechanical energy*. Over the past centuries, scientists gradually realised that there are many different forms of energy. An electric motor uses *electrical energy* to drive an electric train. *Light energy*, falling on a light meter used in photography, causes a pointer to move across a scale. *Sound energy* makes a microphone plate vibrate. *Chemical energy* is the source of energy in our food which makes us grow and also gives us muscular energy to move objects. *Nuclear energy*, the energy in the nucleus of atoms, produces heat energy, which in turn is used to generate electrical power in nuclear power stations.

Energy changes

By using suitable machines or apparatus, energy can be changed or transformed from one form to another. This is illustrated in Fig. 1.1. A steam engine changes

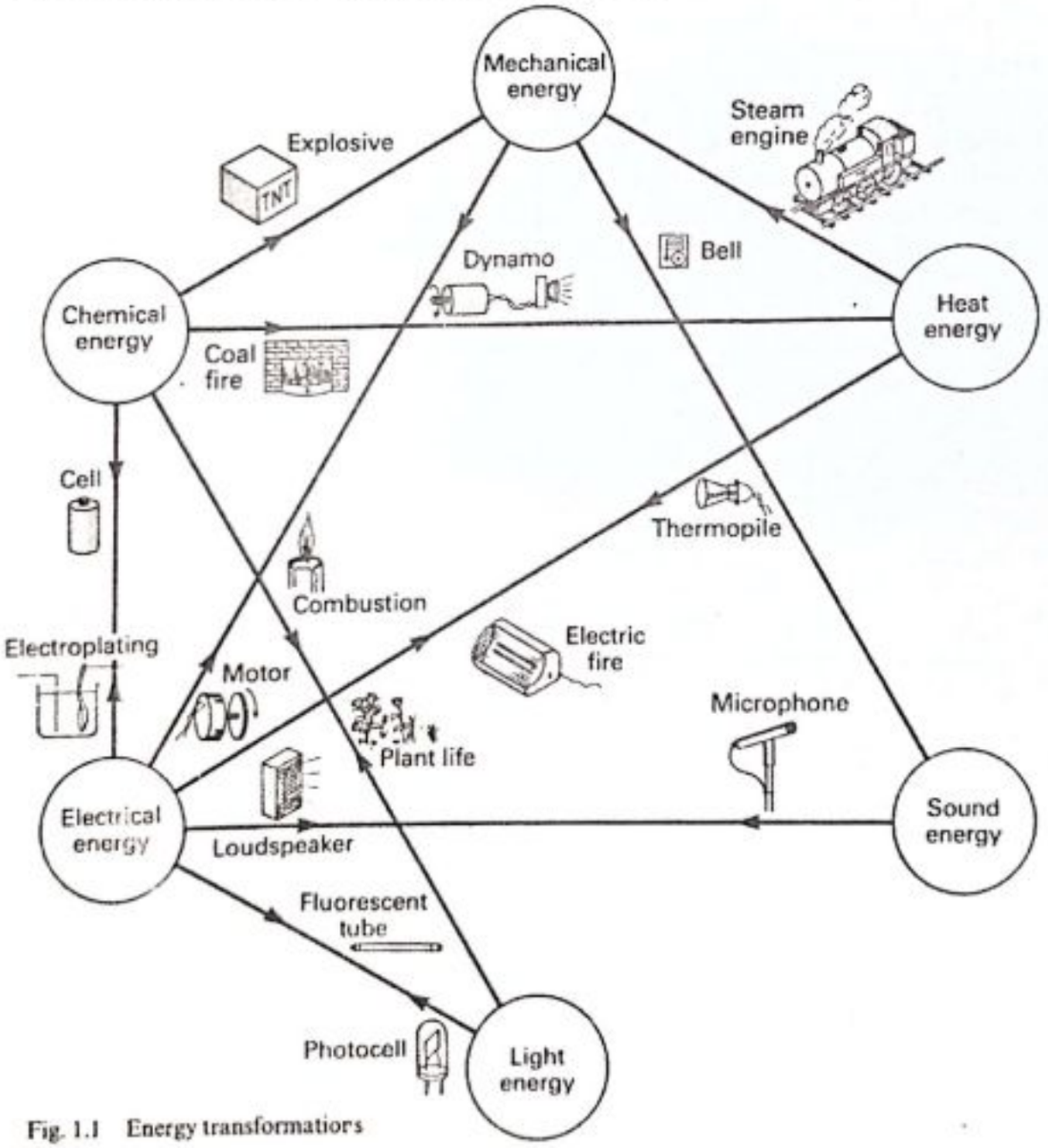


Fig. 1.1 Energy transformations

Scanned with CamScanner

heat energy to mechanical energy. Mechanical energy is changed to heat energy when a match is struck. An electric lamp changes electrical energy to light energy. A solar cell changes the heat of the sun to electrical energy to power space ships. An electric fire changes electrical energy to heat energy. A microphone changes sound energy to electrical energy. A telephone earpiece changes electrical energy to sound energy. A battery changes chemical to electrical energy; a reverse change occurs in electroplating. The energy from the sun produces chemical changes which make plants and trees grow, and the energy is stored underground in coal centuries later, after the wood is absorbed by the soil and sinks.

An electric power station illustrates how energy can be changed from one form to another to produce heat and light energy in your home. Coal is first burned, so that heat energy is produced from chemical energy. By means of a steam engine or turbine, the heat energy is converted into mechanical energy, which turns the coils of an electric generator. Electrical energy is then produced. Electric lamps and heaters in homes and buildings now convert electrical energy to light and heat energy.

Principle of conservation of energy

The heat energy received by the steam turbine is not all changed into mechanical energy. Some of the energy is wasted in overcoming the frictional forces in the wheels of the turbine. Sound energy is also produced by the spinning wheels owing to air disturbance. However, if the whole generating plant receives 100 units of energy, initially in the form of heat from the coal used, the total energy produced (calculated by adding together all the different forms of energy) will still be 100 units.

This leads to a statement known as the *Principle of the Conservation of Energy*. It was reached after many years of experience, and it is recognised today as one of the most important principles in science. It states that, *in a given or closed system, the total amount of energy is always constant, although energy may be changed from one form to another.*

Système International (SI) units

A system of units, known as the Système International (SI) units, is now used for all branches of physics. It is based on the *metre* as the unit of length, the *kilogram* as the unit of mass, the *second* as the unit of time, the *ampere* as the unit of electric current and the *kelvin* as the unit of temperature. The unit of force in this system is the *newton* and the unit of energy is the *joule*.

For many years the metre (m) was taken as the distance between two lines on a particular platinum-iridium rod at 0°C kept near Paris. It is now defined as the length of a certain number of wavelengths in a vacuum of a particular orange radiation of the krypton-86 atom. Unlike the distance between the marks on the rod, the wavelength of the radiation from an atom is a constant. By definition,

$$1\text{ m} = 1\,650\,763{\cdot}73 \text{ wavelengths of the above radiation.}$$

The *kilogram* (kg) is the mass of a particular solid cylinder made of platinum-iridium alloy kept in Paris, known as the International Prototype Kilogram.

Scanned with CamScanner



Standards for measurement
Plate 1A The International Prototype Kilogram is kept at the International Bureau of Weights and Measures near Paris. This is the British national copy (number 18). (*Crown copyright/National Physical Laboratory*)

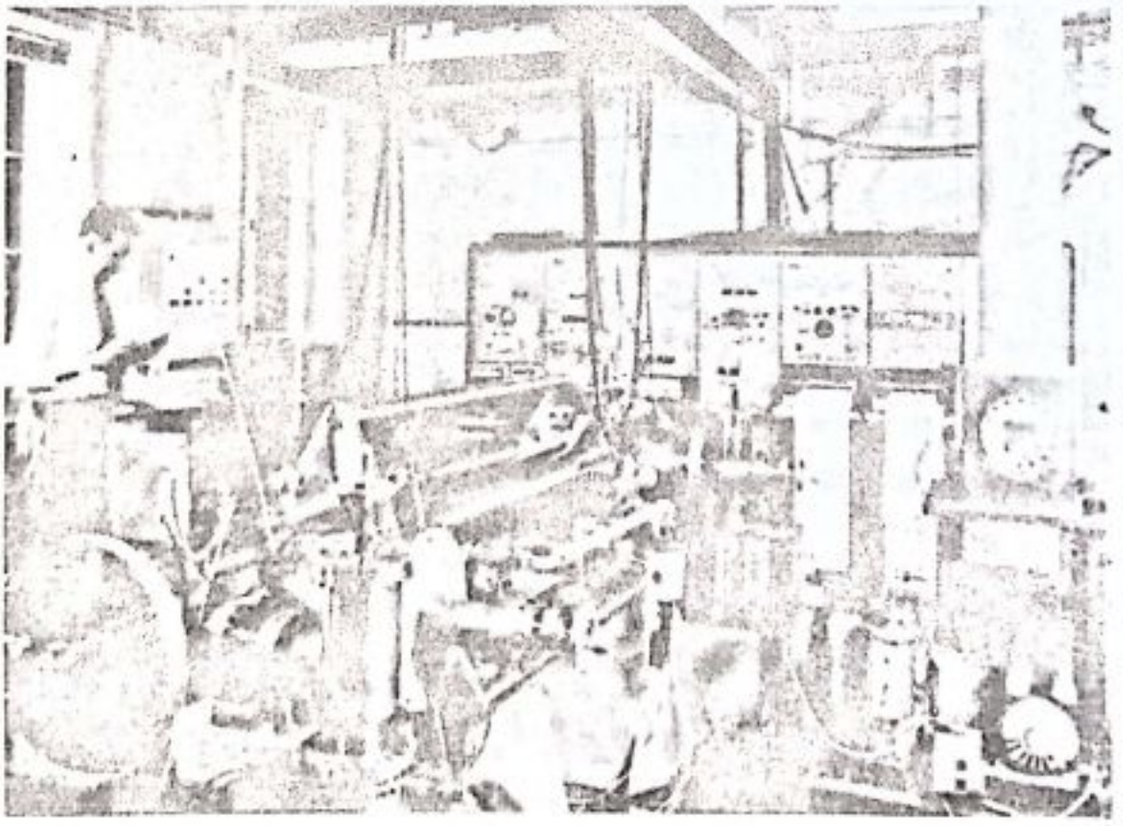


Plate 1B The caesium atomic clock. The large horizontal tube in the foreground is a new caesium beam machine. The vertical tube on the right is the previous model, which is bigger but less accurate. The electronics of the clock can be seen against the wall at the back, with two rolls of paper (stretching down to the floor) giving the read-out. (*Crown copyright/National Physical Laboratory*)

Scanned with CamScanner

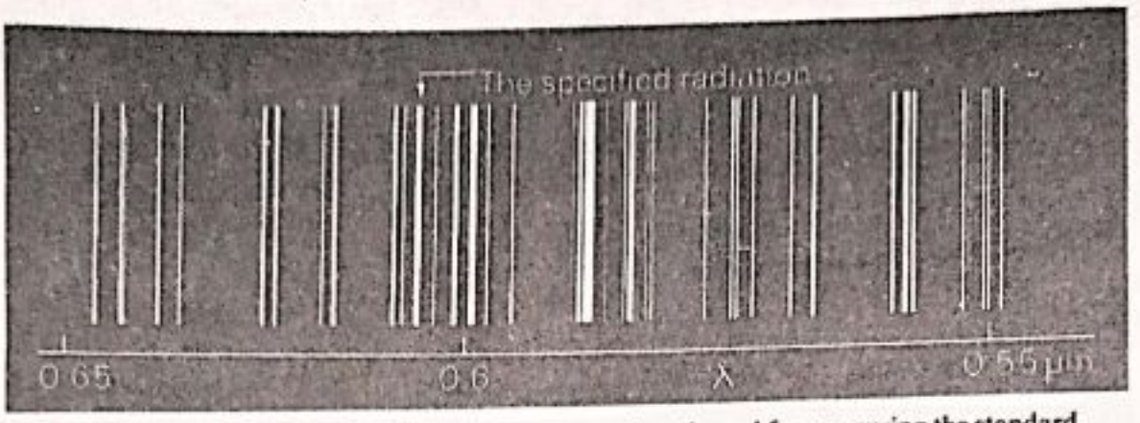


Plate 1C Part of the spectrum of krypton-86. The wavelength used for measuring the standard metre is indicated. (*Crown copyright/National Physical Laboratory*)

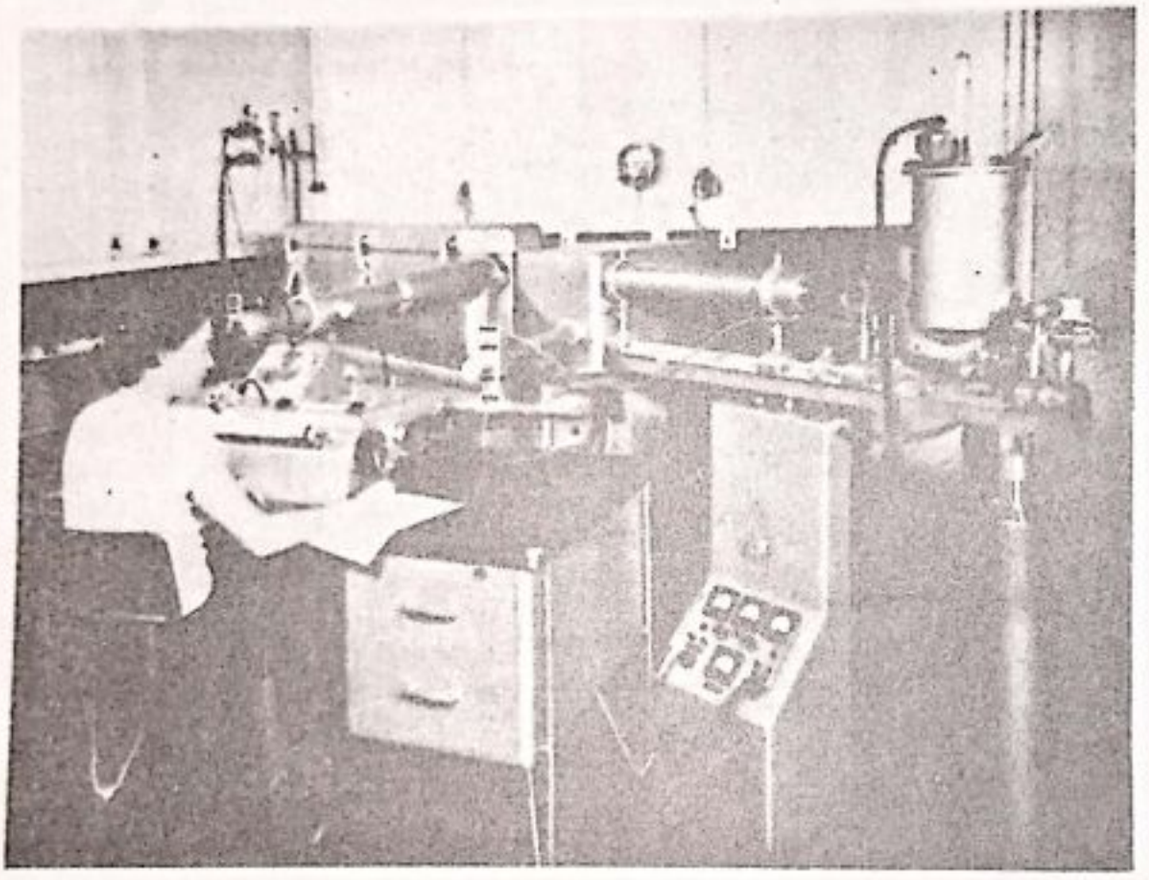


Plate 1D Using an interferometer for measuring bars to a precise length, based on the radiation from krypton-86 (*Crown copyright/National Physical Laboratory*)

In practice, the following smaller units may also be used.

The *millimetre* (mm), which is $\frac{1}{1000}$ m

The *centimetre* (cm), which is $\frac{1}{100}$ m

The *gram* (g), which is $\frac{1}{1000}$ kg

The *second* (s) was formerly 1/86400th part of a mean solar day*. This unit, used by astronomers, has now been replaced by an atomic unit. Dr L. Essen of the National Physical Laboratory at Teddington in England designed a caesium

*The *mean solar day* is the average period between successive transits of the sun across the meridian, taken over twelve months, at any part of the earth's surface.

Scanned with CamScanner

clock which measured the period of rotation of the earth to an exceptionally high order of accuracy and this showed clearly the irregularity in the rate of rotation of the earth. Caesium or atomic clocks are now used as standard clocks. The second is defined as the time for 9192631770 cycles of vibration of a particular radiation from the caesium-133 atom.

Volume

Measurements of volume were originally based on the *litre* (*l*); this is defined as the volume occupied by a mass of 1 kg of pure water at its temperature of maximum density and at standard atmospheric pressure. The *millilitre* (ml) is 1/1000 of a litre. In the SI system, however, volumes are based on the cube whose side is 1 m; that is the cubic metre. This volume is written 'm^3'. Volumes are also measured in cubic centimetres (cm^3), although this is not an SI unit. $1\,cm^3 = 1$ ml.

Water has a density (mass/volume) of 1000 kg/m^3 in SI units (equivalent to 1 g/cm^3); mercury has a density of 13600 kg/m^3 in SI units (equivalent to 13·6 g/cm^3).

Vernier scale

P. Vernier showed how lengths can be measured accurately. He used a subsidiary scale, now called a *vernier scale*. Fig. 1.2a shows the vernier scale Y below the main scale X, which is graduated in centimetres and millimetres. 10 divisions on Y make 9 mm. Thus the length between each division on Y is 0·9 mm, which is 0·1 mm or 0·01 cm less than the length of the divisions on X.

Fig. 1.2b shows the vernier scale Y moved to measure the length of a rod R.

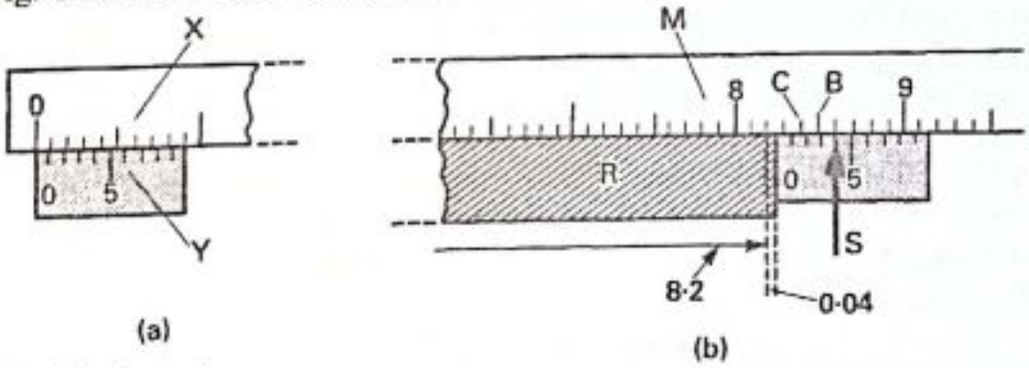


Fig. 1.2 Vernier scale

The length of R is greater than 8·2 but less than 8·3 cm. We now note the division S on Y which coincides with a division on M. This is the fourth division on Y. The third division on Y is thus 0·01 cm in front of the graduation B, the 2nd division on Y is 0·02 cm in front of the graduation C, and so on. Hence the zero of Y is 0·04 cm in front of the 8·2 cm reading on M. Hence the length of R is 8·24 cm.

Fig. 1.3 shows a form of *vernier calipers*. The object measured is placed between the jaws X and Y, and its length is read from the main and vernier scales as explained above.

Micrometer screw gauge

The micrometer screw gauge is an instrument for measuring accurately the diameters of wires or thin rods. It uses an accurate screw of known 'pitch' such as

Scanned with CamScanner

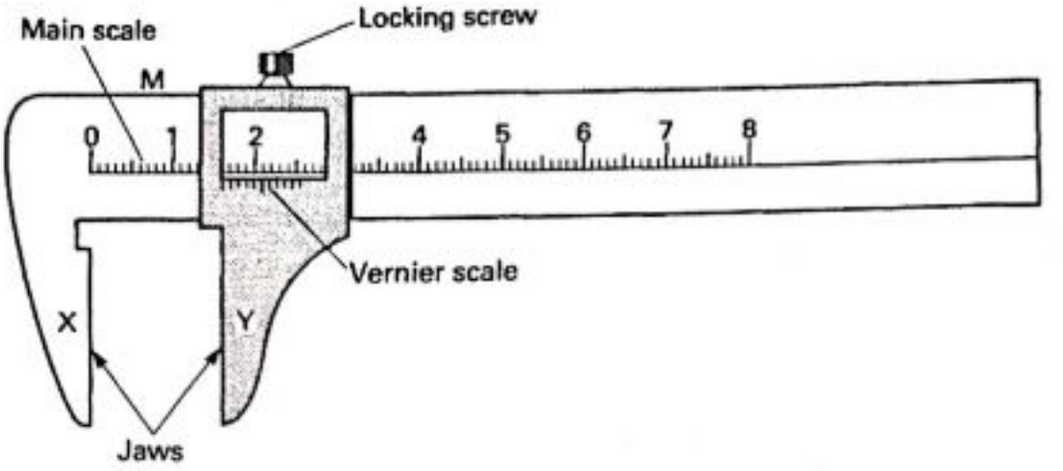


Fig. 1.3 Vernier calipers

0·5 mm, shown inset in Fig. 1.4. This means that for one revolution of the screw the spindle X moves forward or back 0·5 mm.

To measure the diameter of a wire or rod A, one side is rested on the end Y and the thimble is turned until the end of the spindle X just touches A. Further turning of the thimble does not make the spindle move forward because a ratchet stops excessive pressure on A. A locking device is sometimes included so that the diameter reading can be read after A is taken out.

An engraved linear millimetre scale L shows the forward movements of the spindle. A circular scale C on the thimble helps to measure fractions of millimetres. In Fig. 1.4, 50 circular divisions = 0·5 mm, or 1 circular division = 0·01 mm. So from Fig. 1.4,

$$\begin{aligned}\text{diameter of rod} &= 2{\cdot}0\text{ mm (scale L)} + 46 \times 0{\cdot}01\text{ mm (scale C)}\\ &= 2{\cdot}46\text{ mm}\end{aligned}$$

Some precautions are needed when using the screw-gauge to measure the diameter of a wire, for example (a) when the gap between X and Y is closed, check if the reading is zero; if not, add or subtract the error to your final reading of the diameter: (b) with the wire in the gap, make a firm but gentle contact with the screw, that is, do not overscrew: (c) you should measure the diameter at three different places along the wire to allow for lack of uniformity; at each place, measure two perpendicular diameters to allow for any circular defect and then take the average of the six readings to get the diameter.

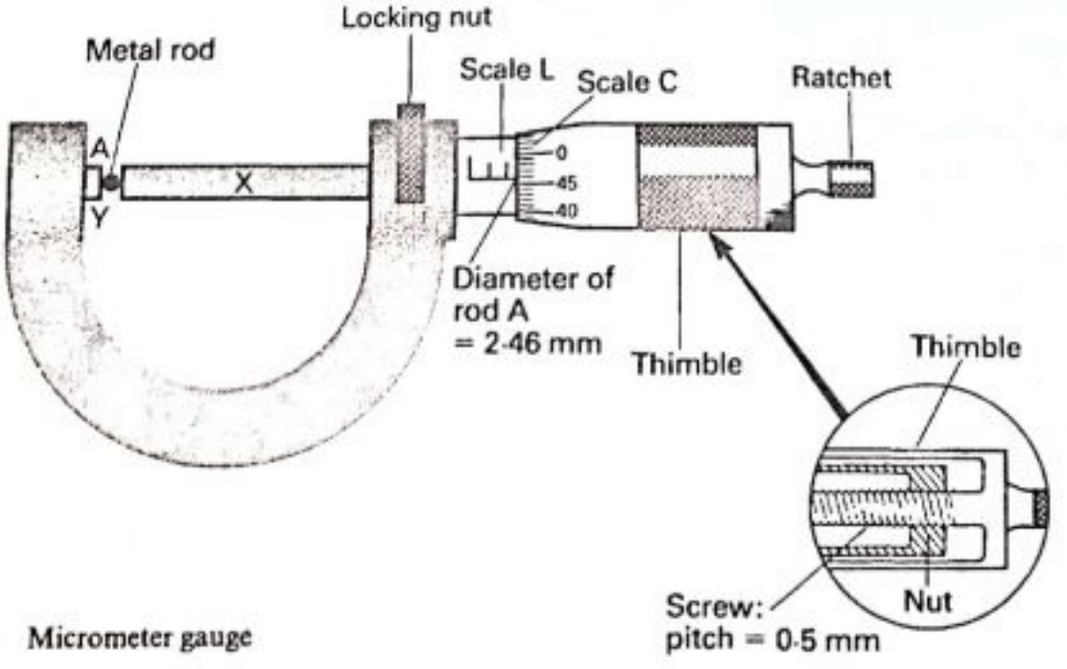


Fig. 1.4 Micrometer gauge

Scanned with CamScanner

Measurement of mass

The *mass* of an object can be measured by comparing it with standard masses. These are derived from copies of the International Kilogram Mass. The chemical balance is used to compare masses in this way. Extremely sensitive balances can be designed. The National Physical Laboratory at Teddington in England has a balance which can measure to one-millionth of a gram. At this laboratory very precise weighings are carried out to assist science and technology.

School balances do not require such high precision. The design of chemical balances has altered so much in recent years that the types used will generally differ from school to school. The top pan balance gives direct reading of mass. It does this by comparing the unknown mass with movable known masses inside it. As in the common balance, the unknown and known masses are balanced using a lever arrangement.

If an object is taken say from London, England, to Lagos, Nigeria, or anywhere else in the world, and weighed on any other chemical balance, its mass will be exactly the same (see page 40). So a chemical balance *compares masses.*

Measurement of weight

A spring balance provides a quick method of measuring *weight*. The object X (Fig. 1.5) is hung from the hook and the spring is then pulled out by a length proportional to the weight of X. The weight of X is the force on X due to the earth's gravitational pull. The spring balance has a 'uniform' scale, that is equal divisions represent equal changes in weight along the whole scale.

Weights are measured in newtons (N) in SI units, as explained on page 39.

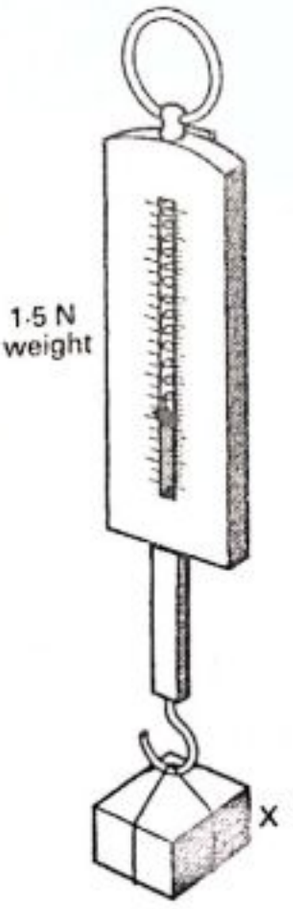


Fig. 1.5 Spring balance measuring weight

Scanned with CamScanner

Unlike the chemical balance, the reading on the spring balance will vary slightly if the same mass is taken to different parts of the world. At the north pole, for example, it will weigh slightly more than at the equator. This is because the gravitational pull is stronger at the poles and so the spring is stretched more. A spring balance measures weight, not mass. The difference between weight and mass is explained more fully on page 39 to 40.

Use of graphs

The results of experiments are often represented on a graph. As an example, suppose the electrical resistance R of six lengths l of a reel of wire are measured, where R is in ohms (Ω) and l is in metres (m). The results are:

l/m	5·0	6·0	7·0	8·0	9·0	10·0
R/Ω	2·0	2·4	2·8	3·2	3·6	4·0

Fig. 1.6 shows the graph obtained when R is plotted against l. It is a *straight line passing through the origin* 0. So we conclude that R is *directly proportional* to the length l, or $R \propto l$. If the straight line did not pass through the origin 0, we could not say that R is directly proportional to l, but we could say still that R increases in proportion with the increase in l.

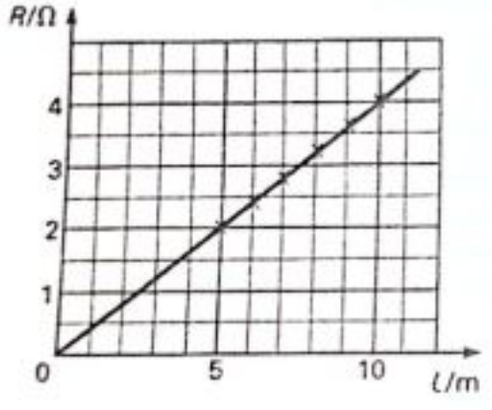


Fig. 1.6 R directly proportional to l

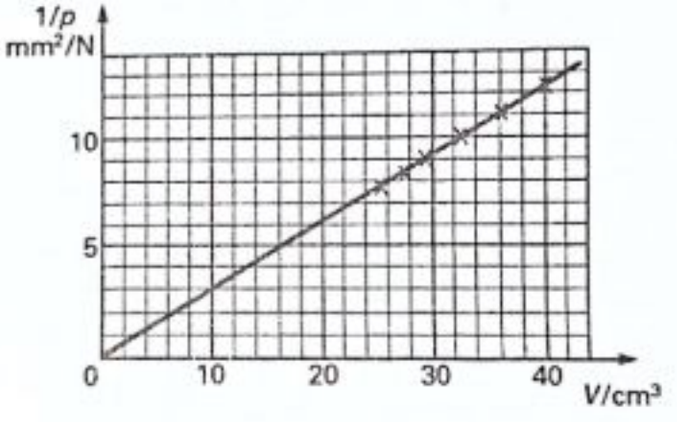


Fig. 1.7 $1/p$ directly proportional to V

In the experiment just described, the resistance R increases as l increases. Suppose, however, that the volume V of a gas such as air is measured at several pressures p, where V is in cm^3 and p is in N/mm^2. Typical results are shown in the table.

p(N/mm²)	0·08	0·09	0·10	0·11	0·12	0·13
V/cm³	40	36	32	29	27	25
$1/p$	12·5	11·1	10·0	9·1	8·5	7·7

This time the volume V decreases as the pressure p increases. We therefore calculate the reciprocal of p, or $1/p$, as shown in the table. When the graph of $1/p$ against V is plotted, a straight-line graph is obtained which passes through the origin (Fig. 1.7). So we deduce $V \propto 1/p$, that is the volume is *inversely proportional* to the pressure. Straight-line graphs are widely used in physics experiments to find the relationship between two quantities.

Scanned with CamScanner

Limitations of experimental measurement · Significant figures

If it is measured with a ruler graduated in millimetres, the length of a rod may be between 123 mm and 124 mm. It would be wrong to give the length as 123·6 mm, for example, because the graduations on the ruler used are only millimetres and not tenths of a millimetre. So the length of the rod is given as 124 mm or 12·4 cm, which is a result to *three significant figures.*

In an experiment to measure the resistance R by a voltmeter-ammeter method (page 396), the voltmeter may be read as 3·4 V and the ammeter may read as 1·2 A, each to two significant figures.

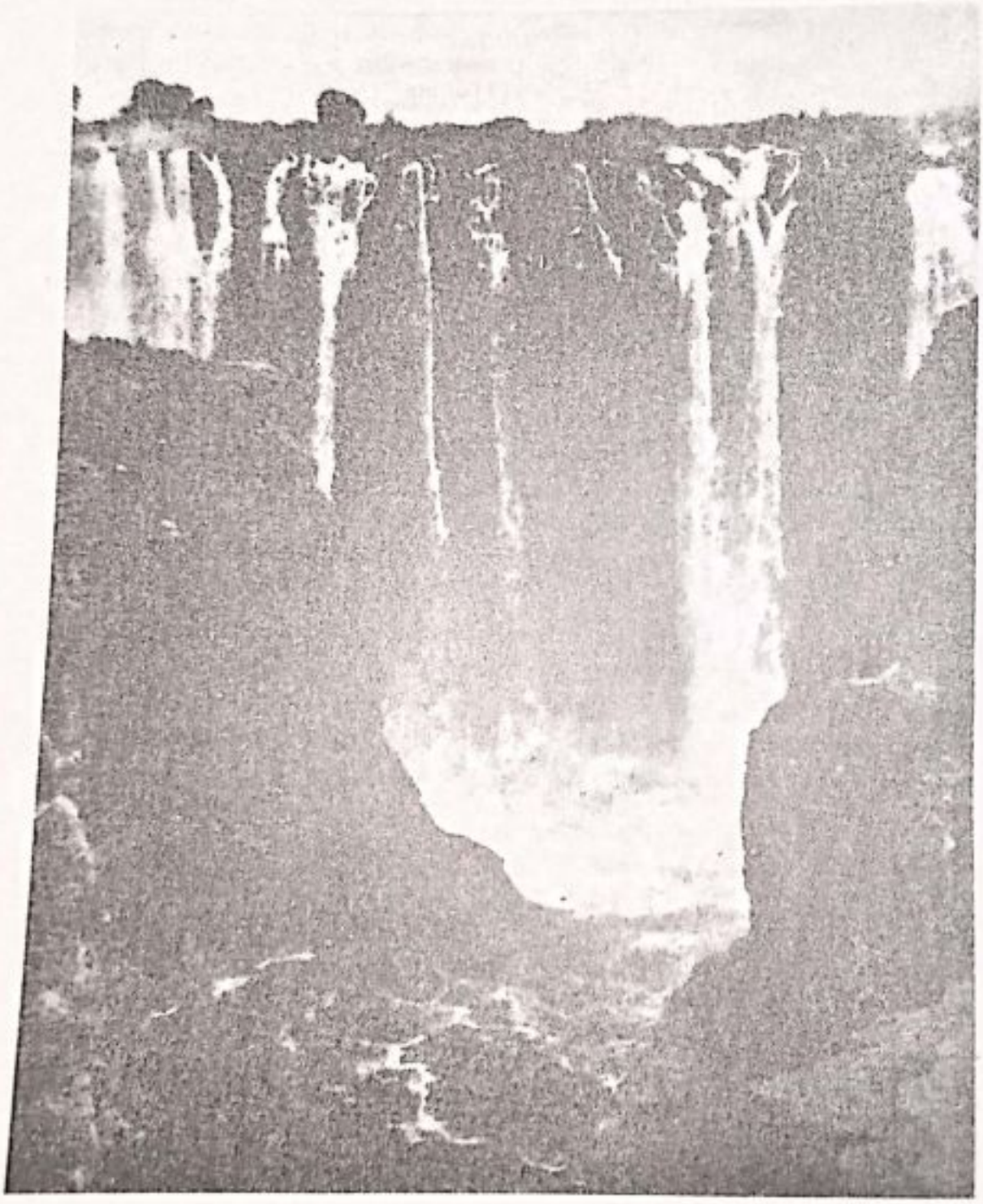
From $R = V/I$,

$$R = \frac{3{\cdot}4}{1{\cdot}2} = 2{\cdot}833\,\Omega\,(\text{ohms})$$

An answer for R of 2·833 Ω would be wrong because the measurements of 3·4 V and 1·2 A have an accurancy of only two significant figures. So the final result for R is given as 2·8 Ω to *two significant figures.*

Scanned with CamScanner

MECHANICS AND FLUIDS



Part of the Victoria Falls on the Zambesi River. The average height of this waterfall is just over 100 metres. The temperature of the water at the bottom is slightly higher than at the top because some of the gravitational potential energy of the water at the top is transformed to heat when it reaches the bottom. (*Paul Popper Ltd*)

Scanned with CamScanner

2. Dynamics · Speed, Velocity, Acceleration

Dynamics is about the motion of objects and the forces acting on them. For example, it deals with the forces that keep an aeroplane in flight through the air, or a speedboat in motion through water, or a car or train in motion along a road or track. All these machines were developed through the skills of scientists and engineers, who studied and understood the principles of dynamics.

The founder of mechanics is generally recognised to be Sir Isaac Newton. He published a book called *Principia* in 1687 in which 'Laws of Motion' were clearly stated for the first time.

Speed

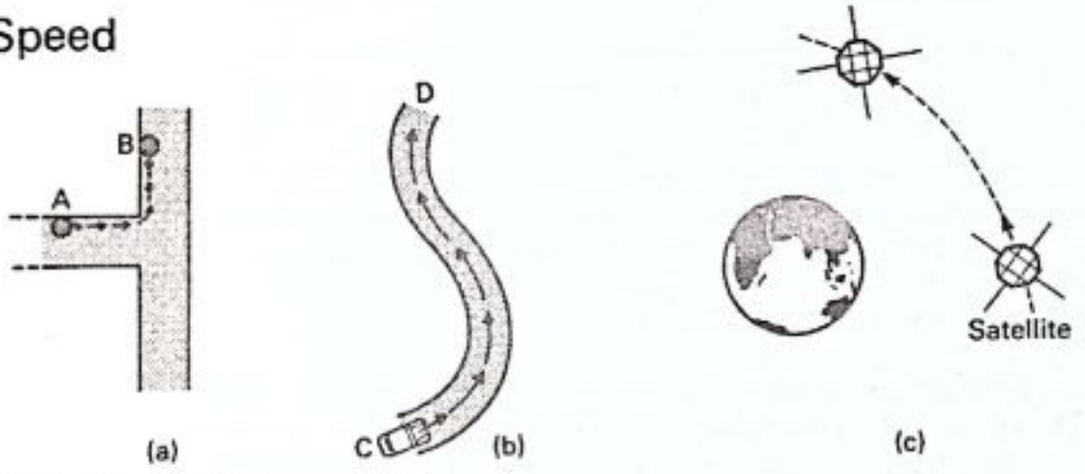


Fig. 2.1 Speed of objects

If you run round the corner of a street from A to B to see a friend (Fig. 2.1a) and travel a distance of 24 m in 20 s, then your average speed is

$$\frac{24\,\text{m}}{20\,\text{s}} \text{ or } 1{\cdot}2\,\text{m/s}$$

Similarly, if a car takes 2 h to travel along a winding road from a town C to another town D 96 km away (Fig. 2.1b) the average speed of the car is

$$\frac{\text{distance}}{\text{time}} = \frac{96\,\text{km}}{2\,\text{h}} = 48\,\text{km/h}$$

The term 'speed' is always used about a moving object *when it changes direction*. So if a satellite used for world television communication takes 24 h to move round the earth in a circular path 60000 km long (Fig. 2.1c)

$$\text{average speed} = \frac{60000\,\text{km}}{24\,\text{h}} = 2500\,\text{km/h}$$

Scanned with CamScanner

If the satellite moves through equal distances in equal times, no matter how small the times may be, we say it has a constant or *uniform* speed.

In calculations it is sometimes necessary to change 'kilometres/hour' to 'metres/second'. So remember that

$$36\ \text{km/h} = 10\ \text{m/s}$$

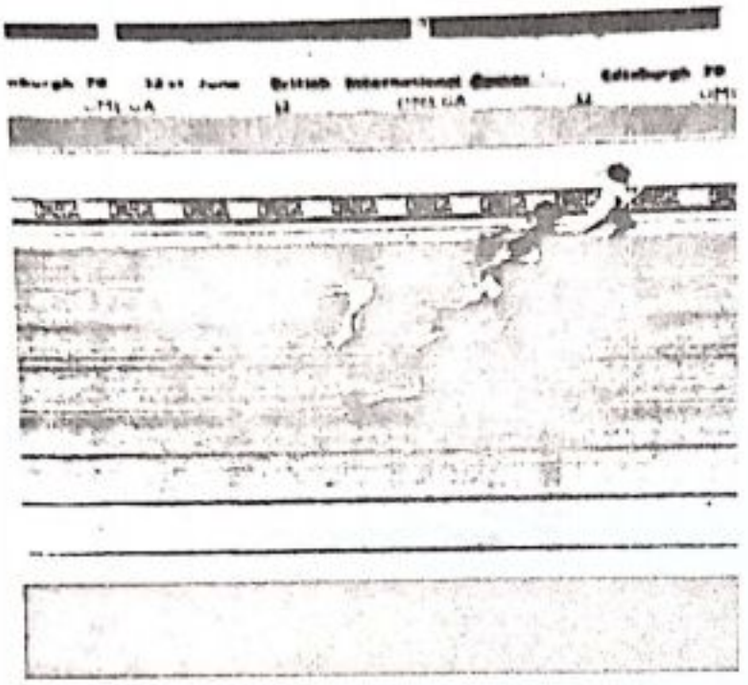


Plate 2A Finish of the men's 100 metres heat at the British International Games, Edinburgh. The time registered is just under 11·03 s. (*Omega Watch Company*)

Velocity

Consider a car moving round a circular track ABC with a uniform speed (Fig. 2.2a). At the instant it reaches A the car points along the direction of the arrow shown, which is the tangent to the circle. At other places, such as B and C, the car points in different directions. If the *direction* as well as the magnitude of the car's motion is considered, we speak of the *velocity* and not the speed. So the speed of the car may be the same at A, B or C as it goes round a circle. But the velocities v at the three points are different because the directions are different. Velocity, then, has direction and magnitude (see page 28).

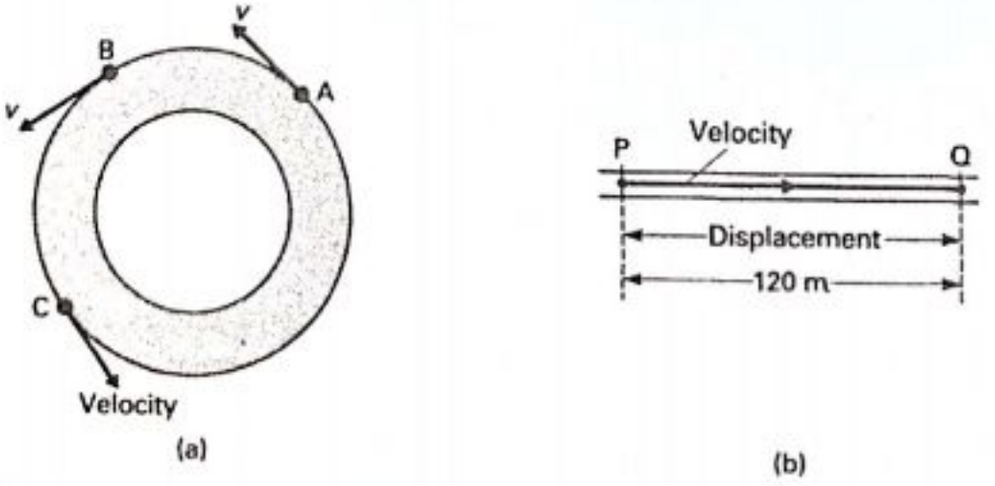


Fig. 2.2 Velocity has direction and magnitude; velocity as a vector

Scanned with CamScanner

If a car moves in a constant direction we use the word 'displacement' rather than 'distance'. Suppose a car takes 8 s to travel along a perfectly straight road PQ 120 m long (Fig. 2.2b). Then

$$\text{average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{120\,\text{m}}{8\,\text{s}}$$

$$= 15\,\text{m/s}.$$

If the displacement along the road in equal times is constant no matter how small the times may be, the velocity is said to be *uniform*.

A train with a uniform velocity of 96 km/h will travel 192 km in 2 h. So if the velocity is uniform

$$\text{velocity} = \frac{\text{displacement}}{\text{time}}$$

$$\text{displacement} = \text{velocity} \times \text{time}$$

$$\text{time} = \frac{\text{displacement}}{\text{velocity}}$$

Examples on velocity and speed

1. A car has a velocity of 72 km/h. How far does it travel in $\frac{1}{2}$ minute?

We must *always* use the same kinds of units in calculations.
Now 72 km/h = 20 m/s and $\frac{1}{2}$ min = 30 s
$\therefore$ displacement (distance) = velocity × time = 20 × 30 = 600 m

2. A car moving round a circular racing track takes 120 s to do a lap of 8 km. What is the speed in km/h?

$$\text{Time} = 120\,\text{s} = 2\,\text{min} = \tfrac{1}{30}\,\text{h}$$

So
$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{8\,\text{km}}{(1/30)\,\text{h}} = 8 \times \frac{30}{1} = 240\,\text{km/h}$$

Acceleration

When an object increases its velocity it is said to *accelerate*. A saloon car starting from rest may accelerate to a velocity of 48 km/h in 10 s; a sports car may accelerate from rest to 48 km/h in 3 s, a much greater acceleration. Acceleration is calculated by

$$\text{acceleration} = \frac{\text{velocity change}}{\text{time taken to make the change}}$$

In the case of the saloon car, therefore,

$$\text{average acceleration} = \frac{(48-0)\,\text{km/h}}{10\,\text{s}} = 4{\cdot}8\,\text{km/h per s}$$

For the sports car,

$$\text{average acceleration} = \frac{(48-0)\,\text{km/h}}{3\,\text{s}} = 16\,\text{km/h per s}$$

The 'zero' in the numerator of the acceleration value has been deliberately included. It emphasises that acceleration always refers to a velocity *change*, not to the actual velocity. So a car travelling with a uniform velocity of 48 km/h has no

Scanned with CamScanner

acceleration. A car accelerating from 48 to 72 km/h in 4 s has an average acceleration given by

$$\frac{\text{velocity change}}{\text{time}} = \frac{(72-48)\text{ km/h}}{4\text{ s}} = \frac{24\text{ km/h}}{4\text{ s}}$$
$$= 6\text{ km/h per s}$$

So if the acceleration remains constant after the car has reached 72 km/h, the velocity increases 2 s later by 2×6 or 12 km/h. The new velocity is then 12 km/h more than 72 km/h, or 84 km/h.

Uniform acceleration

Suppose the velocity of a car increases by 2 km/h in 3 s and then by 6 km/h in the next 3 s. The acceleration of the car is then irregular or *non-uniform*. If the acceleration is constant, we say it has uniform acceleration.

Uniform acceleration is defined as *the motion of an object whose velocity increases by equal amounts in equal time intervals, however small the time intervals may be*.

Suppose a train travels at 36 km/h and accelerates uniformly to 108 km/h in 10 s. Changing the velocities to metres/second, 36 km/h = 10 m/s and 108 km/h = 30 m/s. So

$$\text{acceleration} = \frac{(30-10)\text{ m/s}}{10\text{ s}} = 2\text{ m/s per s}$$

Since the 'second' is repeated twice, the unit of acceleration is usually given as 'm/s²'. Note that 'm/s' is a unit of velocity or speed.

Experiment shows that a body falling freely under gravity has a uniform acceleration of about 10 m/s², which means that the velocity increases by 10 m/s every second of its fall.

Deceleration

If the brakes of a bicycle or car are pressed the velocity of the vehicle decreases. The bicycle or car is then said to have a deceleration. *Uniform deceleration* means that the velocity decreases by equal amounts in equal times, no matter how small the times may be. Like acceleration,

$$\text{deceleration} = \frac{\text{velocity change}}{\text{time taken}}$$

but as the velocity decreases with time it can be considered as 'negative acceleration'.

Suppose a train slows uniformly from 98 to 48 km/h in 10 s. Then

$$\text{deceleration} = \frac{(98-48)\text{ km/h}}{10\text{ s}} = 5\text{ km/h per s}$$

So 2 s after the velocity of 48 km/h is reached, the velocity decreases by 5×2 or 10 km/h. The velocity is then 48 − 10 km/h or 38 km/h. It can be seen that 9·6 s after 48 km/h is reached the train comes to rest.

If an object is thrown up into the air, it undergoes a uniform deceleration of about 10 m/s² due to gravitational attraction; that is, its velocity decreases by 10 m/s for each second it rises.

Scanned with CamScanner

Formulae with uniform acceleration

Formulae are very useful tools of science because they express the general relationship between quantities.

Suppose an object is accelerating uniformly, and that u = initial velocity, v = final velocity at the end of a time t. Then the acceleration, which we shall denote by a, is given by

$$a = \frac{\text{velocity change}}{\text{time}}$$

or

$$a = \frac{v-u}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Suppose a train accelerates uniformly at the rate of 2 km/h per s for 10 s after it reaches a velocity of 30 km/h.

$$\text{Increase in velocity} = 10 \times 2\,\text{km/h} = 20\,\text{km/h}$$

So $$\text{final velocity after } 10\,\text{s} = 30\,\text{km/h} + 20\,\text{km/h} = 50\,\text{km/h}.$$

Using symbols, if a represents the magnitude of the uniform acceleration of an object, in a time t, the increase in velocity is $a \times t$ or at. If the initial velocity, at the beginning of the time, is u the final velocity v is given by

$$v = u + at \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

This formula, which should be memorised, can also be obtained directly from (1). From (1) we have

$$at = v - u, \qquad \text{so } v = u + at$$

Deceleration has the same units as acceleration, and so it can be regarded as 'negative acceleration' in formulae. This is illustrated by the following examples.

Examples on acceleration and deceleration

1. Acceleration

A car travelling at 36 km/h accelerates uniformly at 2 m/s². Find its velocity in km/h in 5 s time.

We must use the *same units* in problems. The velocity of 36 km/h is therefore changed to metre per second. Since 36 km/h = 10 m/s,

$$\text{initial velocity } u = 10\,\text{m/s}$$

Now $$a = 2\,\text{m/s}^2, t = 5\,\text{s}$$

$$\therefore \text{ Final velocity } v = u + at = 10 + (2 \times 5) = 10 + 10 = 20\,\text{m/s}$$

$$\therefore \ v = 72\,\text{km/h}$$

2. Deceleration

A train slows from 72 km/h with a uniform deceleration of 2 m/s². How long will it take to reach 18 km/h?

$$\text{Initial velocity } u = 72\,\text{km/h} = 20\,\text{m/s}$$
$$\text{final velocity } v = 18\,\text{km/h} = 5\,\text{m/s}$$
$$\text{acceleration } a = -2\,\text{m/s}^2$$
$$\text{From} \quad v = u + at$$
$$5 = 20 - 2t$$
$$\therefore \ 2t = 20 - 5 = 15$$
$$\therefore \quad t = \frac{15}{2} = 7{\cdot}5\,\text{s}$$

Scanned with CamScanner

Motion under gravity

Experiments show that the acceleration due to gravity, g, is about $10\,m/s^2$ (page 20). Thus if a ball is dropped from a height above the ground its velocity after 2 s is given by

$$\begin{aligned} v &= u + at \\ &= 0 + (10 \times 2) = 20\,m/s \end{aligned}$$

An object thrown upwards with an initial velocity of 30 m/s has a deceleration of $10\,m/s^2$. That is, $a = -10\,m/s^2$. So in 2 s time it will have a velocity given by

$$v = u + at = 30 - (10 \times 2) = 10\,m/s$$

The ball will come to rest, that is $v = 0$, at a time t given by

$$v = 0 = u + at = 30 - 10t$$

$$\therefore\ 10t = 30, \text{ or } t = \frac{30}{10} = 3\,s$$

Distance travelled with uniform acceleration

We now have to find the distance travelled by an object moving with uniform acceleration a for a time t from an initial velocity u.

First, consider an actual calculation. Suppose a train is moving with a velocity of 10 m/s and accelerates uniformly at $2\,m/s^2$ for 10 s. At the end of the 10 s

$$\begin{aligned} \text{velocity reached} &= \text{initial velocity} + \text{increase in velocity} \\ &= 10 + (10 \times 2) = 30\,m/s \end{aligned}$$

Now if the acceleration is constant, the velocity increases uniformly. In this case

$$\begin{aligned} \text{average velocity} &= \tfrac{1}{2}\,(\text{initial} + \text{final velocity}) \\ &= \tfrac{1}{2}\,(10 + 30) = 20\,m/s \end{aligned}$$

Note that the '$\frac{1}{2}$' is used only because the acceleration is uniform. If the acceleration is non-uniform the average velocity is a different fraction.

So
$$\begin{aligned} \text{distance travelled } s &= \text{average velocity} \times \text{time} \\ &= 20\,m/s \times 10\,s = 200\,m \end{aligned}$$

Formula for distance

We now obtain a general formula for the distance s. Suppose the initial velocity is u, and the acceleration is a for a time t. In this time the increase in velocity is $a \times t$. So the final velocity v is

$$\text{initial velocity} + \text{increase in velocity}$$

or
$$v = u + at$$

Since the acceleration is uniform,

$$\begin{aligned} \text{average velocity} &= \tfrac{1}{2}(u + v) = \tfrac{1}{2}(u + u + at) \\ &= \tfrac{1}{2}(2u + at) = u + \tfrac{1}{2}at \end{aligned}$$

$$\begin{aligned} \therefore\ \text{distance travelled, } s &= \text{average velocity} \times \text{time} \\ &= (u + \tfrac{1}{2}at) \times t \end{aligned}$$

$$\therefore\ s = ut + \tfrac{1}{2}at^2$$

Scanned with CamScanner

Note again that the numerical factor '$\frac{1}{2}$' is due to *uniform* acceleration. If the acceleration was *non-uniform*, the formula would not be used.

Examples on distance formula

1. A boy on a bicycle accelerated uniformly at 1 m/s² for 10 s from an initial velocity of 4 m/s. Calculate the distance travelled in this time.

We have $u = 4\,\text{m/s}, a = 1\,\text{m/s}^2, t = 10\,\text{s}$

So $s = ut + \frac{1}{2}at^2 = (4 \times 10) + (\frac{1}{2} \times 1 \times 10^2) = 40 + (\frac{1}{2} \times 1 \times 100)$

$= 40 + 50 = 90\,\text{m}$

2. A train travelling at 54 km/h accelerates uniformly at 2 m/s² for $\frac{1}{4}$ min. Find the distance travelled in this time.

We must change all the quantities to the same units before substituting in a formula.

Now $u = 54\,\text{km/h} = 15\,\text{m/s}, a = 2\,\text{m/s}^2, t = \frac{1}{4}\,\text{min} = 15\,\text{s}$

So $s = ut + \frac{1}{2}at^2 = (15 \times 15) + (\frac{1}{2} \times 2 \times 15^2) = 225 + (\frac{1}{2} \times 2 \times 225)$

$= 225 + 225 = 450\,\text{m}$

Equations of linear motion

So far we have obtained two useful formulae for motion in a straight line, which is called *rectilinear motion*. They are

$$v = u + at \quad \ldots \quad (1)$$

$$s = ut + \tfrac{1}{2}at^2 \quad \ldots \quad (2)$$

To eliminate the time t and obtain another useful relation, we use

$$\text{distance} = \text{average velocity} \times \text{time}$$

Now $\text{average velocity} = \dfrac{v+u}{2}$, and time $t = \dfrac{v-u}{a}$, from (1),

So $\text{distance } s = \dfrac{v+u}{2} \times \dfrac{v-u}{a} = \dfrac{v^2-u^2}{2a}$

Cross-multiplying, $2as = v^2 - u^2$

$$v^2 = u^2 + 2as \quad \ldots \quad (3)$$

The formulae (1), (2), (3) are the basic relations for motion under uniform acceleration.

Examples on linear motion

1. A ball is thrown vertically upwards from the ground with a velocity of 20 m/s. Calculate: (a) the maximum height reached; (b) the time to reach the maximum height; (c) the time to reach the ground again after the ball is thrown up; (d) the velocity reached half-way to the maximum height. (Assume acceleration due to gravity $g = 10\,\text{m/s}^2$.)

We have $u = 20\,\text{m/s}$, and $a = -10\,\text{m/s}^2$ since the ball has a deceleration as it rises.

(a) At the maximum height the ball comes momentarily to rest, so that the final velocity $v = 0$.

Using $v^2 = u^2 + 2as$

$\therefore \quad 0 = 20^2 - (2 \times 10 \times s) = 400 - 20s$

$\therefore \quad 20s = 400$

$\therefore \quad s = \dfrac{400}{20} = 20\,\text{m} = \text{maximum height}$

Scanned with CamScanner

(b) To find the time t to reach the maximum height, we use

$$v = u + at$$

Then $0 = 20 - 10t$, so $10t = 20$

$$\therefore\ t = \frac{20}{10} = 2\text{ s}$$

(c) The time to reach the ground again is twice the time to reach the maximum height, or 4 s. We can see this by finding the time taken to reach the ground from the maximum height. In this case

initial velocity $u = 0$, $a = 10\text{ m/s}^2$, $s = 20\text{ m}$

Using $s = ut + \frac{1}{2}at^2$

$$\therefore\ 20 = \tfrac{1}{2} \times 10 \times t^2 = 5t^2$$

$$\therefore\ t^2 = \frac{20}{5} = 4$$

$$\therefore\ t = 2\text{ s}$$

The time to reach the maximum height is also 2 s from (b), so giving a total time of 4 s.

(d) Half-way to the maximum height,

distance travelled $= \frac{1}{2} \times 20\text{ m} = 10\text{ m}$

Using $u = 20\text{ m/s}$, $a = -10\text{ m/s}^2$, $s = 10\text{ m}$,

then $v^2 = u^2 + 2as$

$= 20^2 - (2 \times 10 \times 10) = 400 - 200 = 200$

So $v = \sqrt{200} = 14\text{ m/s}$ (approx.)

2. A car travels with a velocity of 18 km/h. It then accelerates uniformly and travels a distance of 50 m. If the velocity reached is 54 km/h find the acceleration and the time to travel this distance.

Here $u = 18\text{ km/h} = 5\text{ m/s}$, $v = 54\text{ km/h} = 15\text{ m/s}$, $s = 50\text{ m}$

From $v^2 = u^2 + 2as$

$$\therefore\ 15^2 = 5^2 + (2 \times a \times 50)$$

$$\therefore\ 225 = 25 + 100a$$

$$\therefore\ 100a = 225 - 25 = 200$$

$$\therefore\ a = \frac{200}{100} = 2\text{ m/s}^2$$

Also $$\text{time} = \frac{\text{distance}}{\text{average velocity}} = \frac{50\text{ m}}{10\text{ m/s}} = 5\text{ s}$$

(or $v = u + at$

$\therefore\ 15 = 5 + 2t$, or $2t = 15 - 5 = 10$

$\therefore\ t = \frac{10}{2} = 5\text{ s}$)

Direct measurement of g by free fall

We can measure g, the acceleration due to gravity, by timing a free fall with a special type of electric clock which can measure to one-hundredth of a second, or a centisecond.

Experiment

The apparatus is shown in Fig. 2.3. When connected according to the manufacturer's instructions, the electromagnet M is switched off when the clock is switched on because of the way they are connected up together. A steel ball, with a piece of thin paper between itself and the electromagnet, is then held by M at a height h above a hinged plate X. The switch is pressed, the current to M is cut off and the ball begins to fall. At the same time the

Scanned with CamScanner

clock is switched on. When the ball strikes the plate X at B, contact with C is broken and the clock automatically stops.

The time t of fall is read to 1/100 s. The height h of fall is measured from the bottom of the ball when held at M to the plate X. The height h is varied by moving the electromagnet up or down in suitable steps and the readings of h and t are repeated.

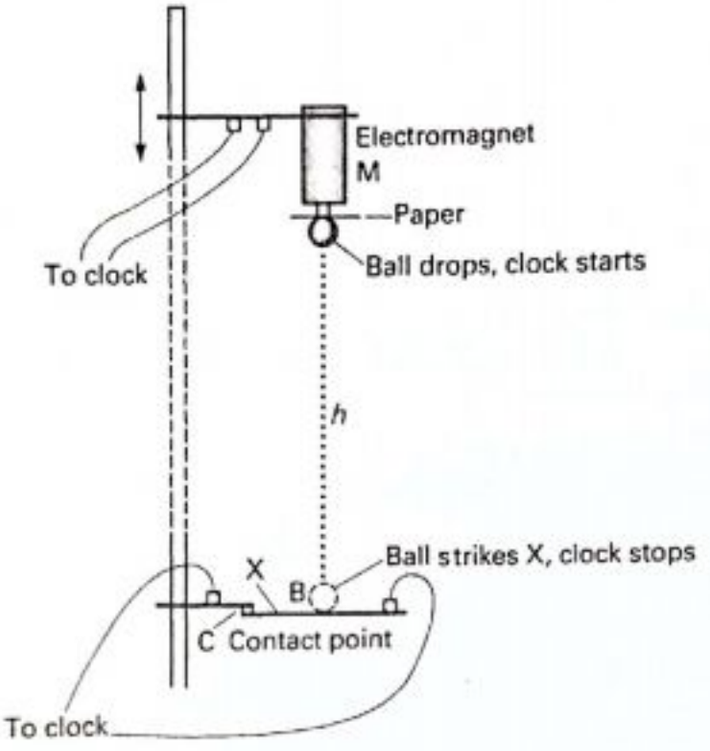
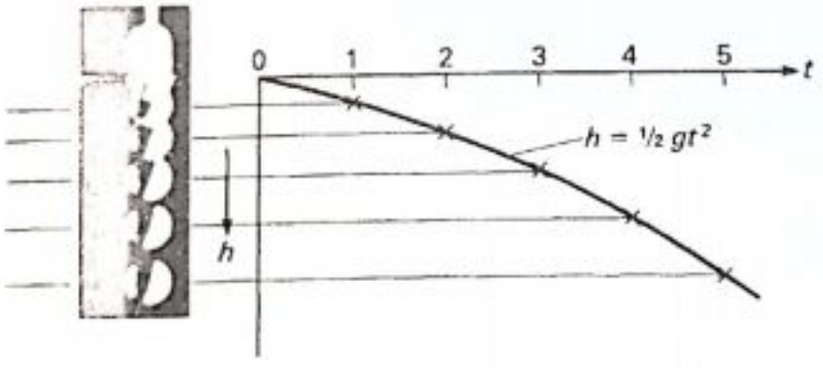


Fig. 2.3 Free fall method for finding g



Ball falling a distance h vertically under gravity, photographed at equal time intervals. Observe the increase in velocity and the graph of h against time t. (*Freefall multiflash photograph by Gordon Severn*)

Calculation: The ball drops from rest through a distance h. So $s = h$, $u = 0$ and $a = g$. From $s = ut + \frac{1}{2}at^2$ (see page 18)

$$h = 0 \times t + \tfrac{1}{2}gt^2$$
$$h = \tfrac{1}{2}gt^2$$
$$\therefore\ g = \frac{2h}{t^2}$$

Scanned with CamScanner

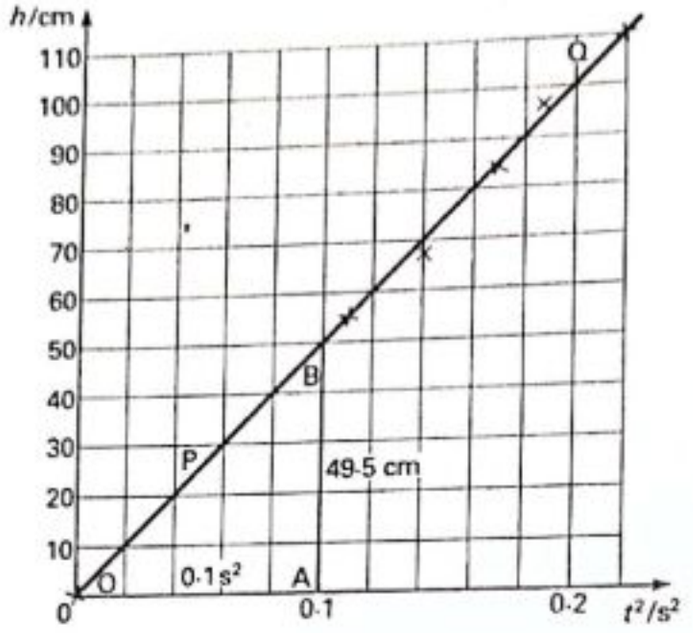


Fig. 2.4 Graph of h against t^2

A graph of h against t^2 (not t) is therefore plotted. A straight line OP, which goes through the origin, can be drawn through the points (Fig. 2.4). This means that h is directly proportional to t^2, or that *g is a constant*. From the graph,

$$\frac{h}{t^2} = \text{gradient (slope) of line OP} = \frac{AB}{OA}$$

$$\therefore\ g = \frac{2h}{t^2} = 2 \times \text{gradient}$$

Results: One set of results is shown below.

h/cm	0	54·7	67·5	84·1	96·0	109·5
t/s	0	0·333	0·369	0·412	0·441	0·469
t^2/s²	0	0·11	0·14	0·17	0·19	0·22

From the graph, $\text{gradient} = \frac{AB}{OA} = \frac{49{\cdot}5\,\text{cm}}{0{\cdot}1\,\text{s}^2}$

$$\therefore\ g = 2 \times \frac{49{\cdot}5}{0{\cdot}1} = 990\,\text{cm/s}^2 = 9{\cdot}9\,\text{m/s}^2$$

Notes. 1. The piece of thin paper is placed between the electromagnet and steel ball to stop the ball from 'sticking' to the iron core. Sources of error, however, are likely to be a short time delay (a) before the ball begins to fall, (b) for the contact at C to be broken.

2. Greater values of h give larger values of time of fall t. The time t can then be read to a greater degree of accuracy.

3. If the same experiment is repeated with magnetic materials of different mass in place of the ball, the same fall times for all are obtained for a given height h. So all objects, no matter what their mass may be, fall freely under gravity with the same acceleration.

4. A graph of h against t is a parabola and not a straight line (see graph, Fig. 2.3). A graph of h against t^2, however, is a straight line passing through the origin (Fig. 2.4). This is because h is directly proportional to t^2 in the equation $h = \frac{1}{2}gt^2$.

Scanned with CamScanner

Oscillations of a simple pendulum

As we see later, radio, light and sound waves, are all due to *oscillations*.

The *simple pendulum* was an early example of oscillation in mechanics. It consists of a small heavy object such as a lead bob or brass weight O suspended by a long thread PO from a fixed point P (Fig. 2.5a). When O is pushed slightly (displaced) and released it oscillates (vibrates) about its original undisturbed position at O, moving to-and-fro along the arc AOB of a circle.

The oscillations of the bob are due to the sideways pull, or component, of its weight (page 93). When the bob reaches the end of its swing at B, for example, this component acts along BL, the tangent to the arc. After moving through O the bob is slowed by the component as it rises to A, after which it is pulled back again to repeat the motion.

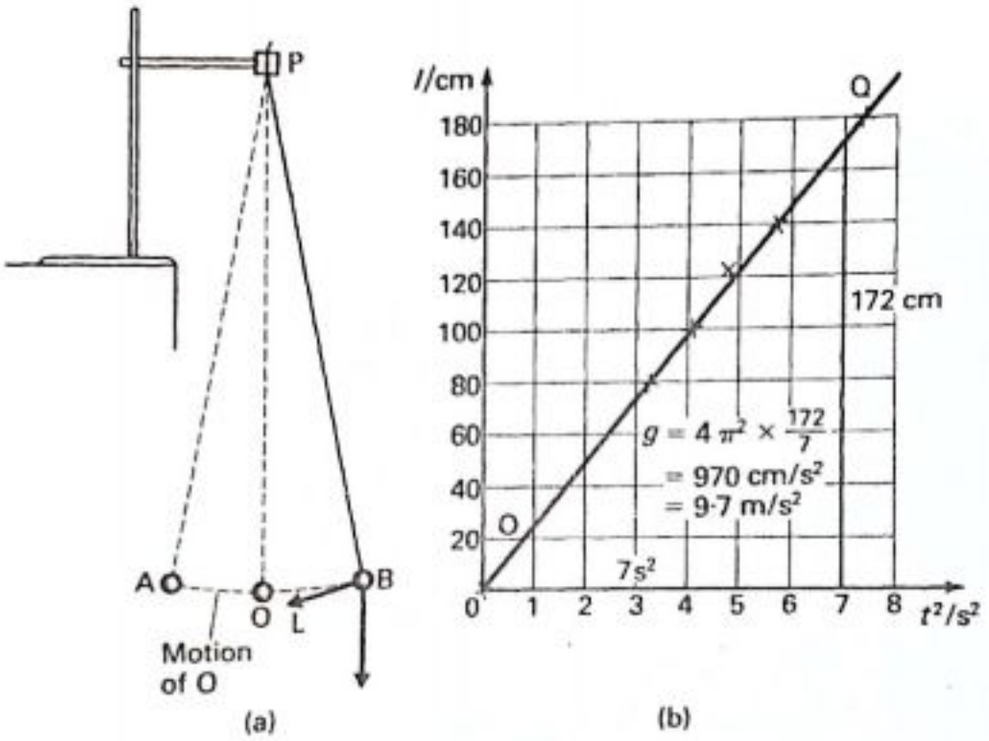


Fig. 2.5 g by the simple pendulum experiment

Pendulum period

The *period* T of an oscillation or vibration is defined as the time for a complete to and fro oscillation. In Fig. 2.5, for example, the period is the time taken for the bob to move from B to A *and* back to B. Or the time taken to move from O to B, then to A and back to O.

For the same pendulum length, experiment shows that T remains constant when the mass m of the bob is changed. This is because the weight W of the bob is proportional to its mass m, so W/m is the same for large or small masses.

Experiment also shows that T is constant as long as the swings are all small, even though the swings are different in size.

Measurement of g

The acceleration due to gravity, g, can be measured by a simple pendulum method.

Scanned with CamScanner

Experiment

Method: **As shown in Fig. 2.5a, clamp the end of a long thread (1 m to 2 m) between the two pieces of cork or coins, so that there is no sideways motion of the support as the attached bob swings. Use the longest length of thread. Give the bob a *small* displacement and time at least 20 complete swings or oscillations using the stop-clock.**

To help exact timing, set up a pointer or ruler behind the bob at the start, and start counting the swings as the bob passes the pointer in the *middle* of its oscillation. (One oscillation is when the bob again passes the pointer moving in the *same* direction.)

Repeat the measurement of time t for the same number n of oscillations and find the average value of t. Measure the length l of the pendulum from the point P where it leaves the clamp at the top to the centre of the bob. Repeat the whole experiment using four more different values of l.

Measurements:

l/cm	n	t/s	$T = n/t$

Calculation: Calculation beyond the scope of this book shows that the period T is related to l and g, the acceleration due to gravity at the place where the pendulum is used, by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

So, by squaring,

$$g = \frac{4\pi^2 l}{T^2}$$

g can be calculated from individual values of l and T but it is better to plot a graph of l against T^2 (Fig. 2.5b). Draw the best straight line OQ through the origin. The gradient of the line OQ is l/T^2 and g can be calculated from

$$g = 4\pi^2 \times \text{gradient}$$

The accuracy of the value of g obtained will depend on accurate timing of a large number of oscillations, on drawing a careful graph and on choosing large values for finding the gradient of the line.

Results: Some results are shown in the table below.

l/cm	100·0	120·0	140·0	160·0	180·0
T/s	2·02	2·21	2·39	2·55	2·71
T^2/s^2	4·08	4·88	5·71	6·50	7·34

From the graph, Fig. 2.5b,

$$g = 4\pi^2 \times \text{gradient} = 4\pi^2 \times \frac{172\,\text{cm}}{7\,\text{s}^2}$$

$$= 970\,\text{cm/s}^2 = 9{\cdot}7\,\text{m/s}^2$$

Displacement–time graphs

Displacement–time graphs provide useful information about the motion of an object. If the graph is a straight line such as OA the velocity is uniform, since equal distances are then travelled in equal times (Fig. 2.6a). Further, considering a time OP,

$$\text{velocity} = \frac{\text{displacement (distance)}}{\text{time}} = \frac{\text{QP}}{\text{OP}}$$

the ratio QP/OP is the *gradient* of the line OA. Hence

$$\text{velocity} = \text{gradient of OA.}$$

Scanned with CamScanner

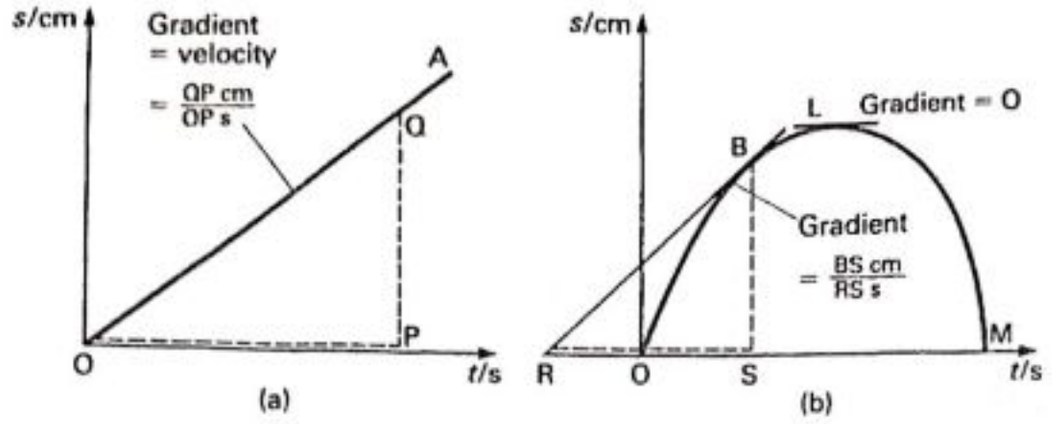


Fig. 2.6 (a) Uniform velocity graph (b) Variable velocity curve

Fig. 2.6b, however, shows the displacement–time curve OLM for the height of a ball thrown vertically upwards in the air. It reaches a maximum height at L, and then falls to the ground at a time M. This is an example of non-uniform velocity. The velocity at any height such as B is the gradient of the *tangent* to the curve at B, which is BS/RS. At the highest point L the gradient of the tangent is zero because the ball is at rest.

Velocity–time graphs

In practice, *velocity–time* graphs are more useful than displacement–time graphs for studying motion. The magnitude of the acceleration at any part of the motion, for example, can be found from the graph. Thus in Fig. 2.7

$$\text{uniform acceleration along OA} = \frac{\text{velocity change}}{\text{time}}$$
$$= \frac{\text{LA}}{\text{OL}}$$

The ratio LA/OL is the gradient of the line OA. So

acceleration = gradient of velocity–time graph.

When the acceleration is calculated we read LA from the velocity axis and OL from the time axis.

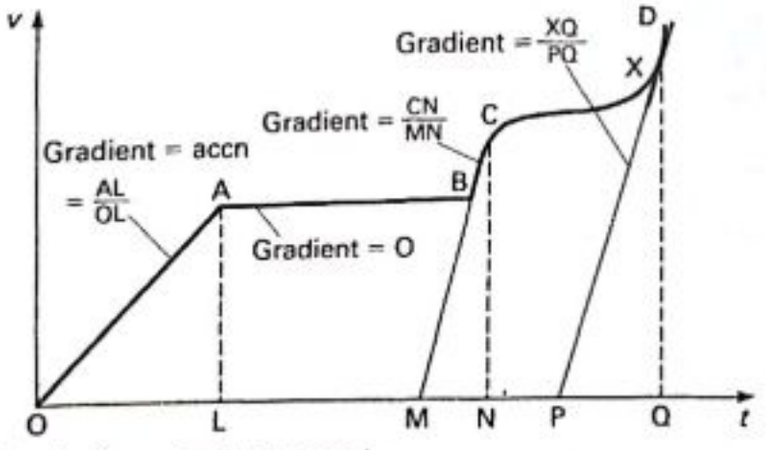


Fig. 2.7 Acceleration from velocity–time graph

In Fig. 2.7, AB represents uniform or constant velocity; the gradient or acceleration is zero. The velocity–time graph represented by BC is a line with a steeper gradient than OA, and so the acceleration here is greater. When the graph

Scanned with CamScanner

shows an irregular velocity change as along CD, the acceleration at the time X, for example, is the gradient of the tangent to the graph at this point, as shown.

Distance from velocity–time graph

The displacement or distance travelled can also be found from a velocity–time graph of the motion.

As an illustration, suppose that the speedometer of a racing car reads the following values of velocity v in the time t shown, starting from rest.

Time/s	0	2	4	6	8	10	12	18	24	30	32	34	36	38	40	42
Velocity/m/s	0	12	24	36	48	60	60	60	60	60	50	40	30	20	10	0

Fig. 2.8 shows the velocity–time graph of the motion. We can see from it that the velocity increased uniformly along OA, then became constant over AB and decreased uniformly to zero along BC.

In the time OH, the average velocity = 30 m/s or $\frac{1}{2}$AH. So

$$\text{distance travelled} = \text{average velocity} \times \text{time} = \tfrac{1}{2}\text{AH} \times \text{OH}$$

Now $\frac{1}{2}$AH × OH = area of triangle OAH. So the distance travelled can be calculated from the area between the graph OA and the time axis. This result is true for any motion, even an irregular one. So, generally, the distance travelled is given by

distance = area between velocity–time graph and time axis

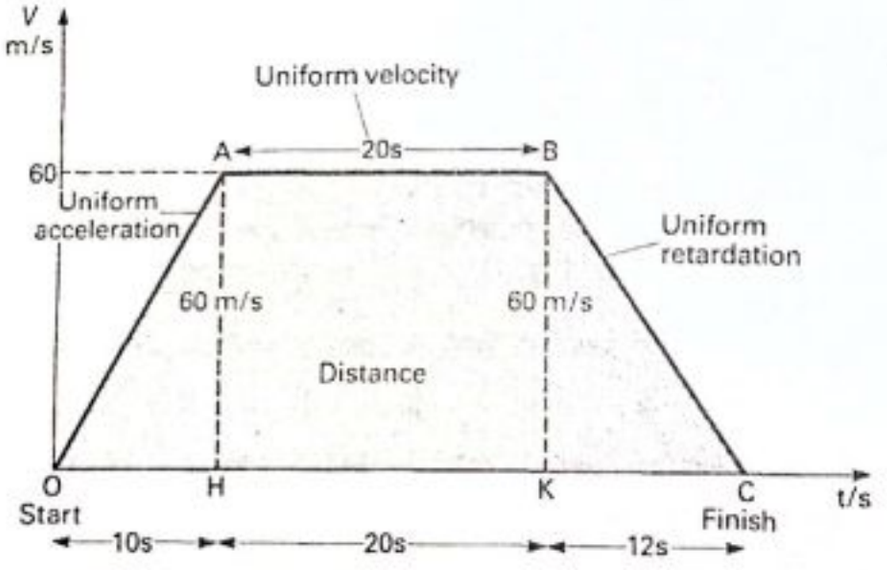


Fig. 2.8 Calculation from velocity–time graph

The respective distances travelled were therefore as follows.

$$\begin{aligned}\text{In time OH, distance} &= \text{area of triangle OAH} = \tfrac{1}{2}\text{OH} \times \text{AH}\\ &= \tfrac{1}{2} \times 10\,(\text{s}) \times 60\,(\text{m/s}) = 300\,\text{m}\\ \text{In time HK, distance} &= \text{area of rectangle HABK} = \text{HK} \times \text{AH}\\ &= 20\,(\text{s}) \times 60\,(\text{m/s}) = 1200\,\text{m}\\ \text{In time KC, distance} &= \text{area of triangle BKC} = \tfrac{1}{2}\text{KC} \times \text{BK}\\ &= \tfrac{1}{2} \times 12\,(\text{s}) \times 60\,(\text{m/s}) = 360\,\text{m}\end{aligned}$$

So total distance = 1860 m

Scanned with CamScanner

The total distance travelled would also be given by the area of OABC, which is a trapezium. In this case

$$\begin{aligned}\text{total distance} &= \tfrac{1}{2}\text{ sum of parallel sides} \times \text{height} \\ &= \tfrac{1}{2}(AB+OC) \times AH = \tfrac{1}{2}(20+42) \times 60 \\ &= 31 \times 60 = 1860\text{ m}\end{aligned}$$

From Fig. 2.8, the uniform acceleration along OA = gradient of OA = AH/OH = 60/10 = 6 m/s^2. The uniform deceleration along BC = gradient of BC = BK/KC = 60/12 = 5 m/s^2.

Other velocity–time graphs

Fig. 2.9a shows the velocity–time graph of a lift whose velocity increases uniformly along OA and then decreases uniformly in velocity to rest along AB. The distance travelled is the area of the triangle OAB.

Fig. 2.9b shows the velocity–time graph of a train between two stations. The train's velocity increased along OL as it left the station, then it travelled with fairly uniform velocity for a time along LM, and finally decreased in velocity and came to rest at N. The distance travelled is the area below the curve OLMN. Similar graphs are obtained in speed tests on railway engines.

Fig. 2.9c shows the velocity–time graph of a cricket ball thrown vertically upwards. The velocity decreases uniformly from OA to zero at the top of its flight in a time OP and then increases in velocity uniformly as it returns to the hand of the thrower at B. The gradient of the line APB = $-g$, where g is about 10 m/s^2 (page 19). The height reached (distance travelled) when the ball is thrown up is equal to the area of triangle AOP. The distance it falls from its maximum height is equal to the area of triangle PCB, which should be exactly the same as triangle AOP ignoring air resistance.

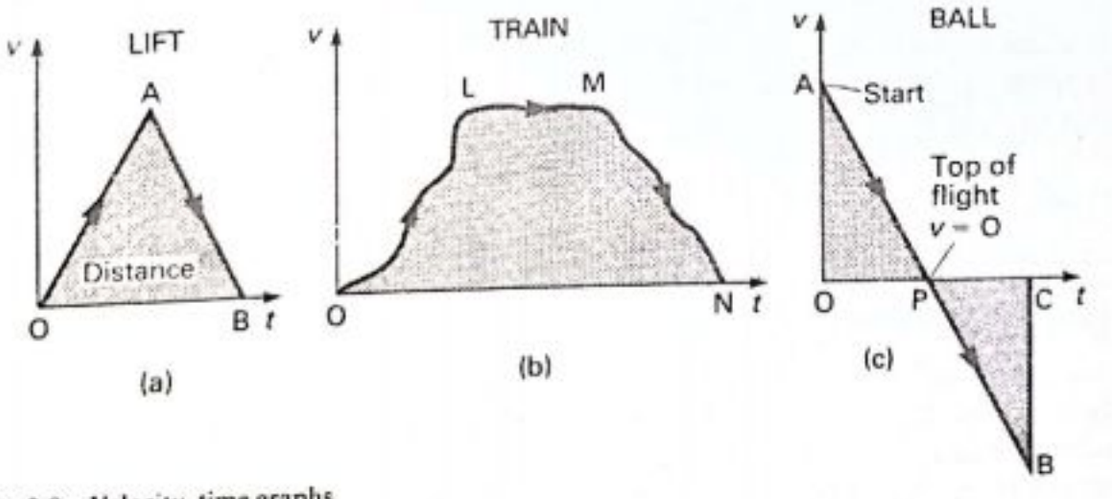


Fig. 2.9 Velocity–time graphs

We can summarise the information obtained from the graphs in this chapter as follows.

1. Displacement (s)–time (t). The *gradient* at a point represents velocity.
2. Velocity (v)–time (t).
   (a) The area between the graph and the time–axis represents displacement or distance.
   (b) The gradient at a point represents acceleration or deceleration.

Scanned with CamScanner

Scalars and vectors

In science there are many quantities which have only numerical values. For example, 'temperature' is a number on some scale; a temperature has no direction. These quantities are called *scalars*. Speed is a scalar quantity. A 'speed of 30 km/h' can have any direction; it can be measured along a winding road or a circular track, for example.

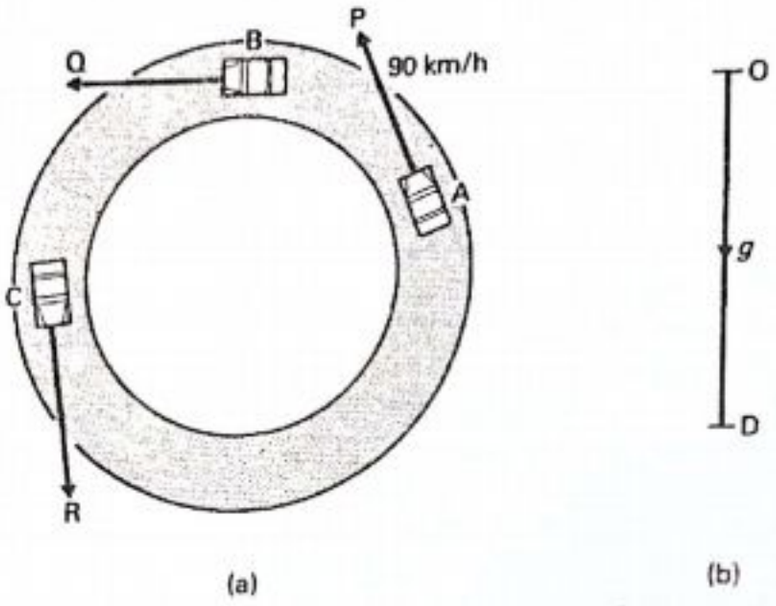


Fig. 2.10 Velocity and acceleration are vectors

On the other hand, many quantities in physics have direction as well as numerical value or magnitude. These quantities are called *vectors*. Velocity and acceleration are examples of vectors. A car travelling round a circular track may have a constant *speed* of 90 km/h (Fig. 2.10a). The *velocity* at the instant the car reaches a point A, however, is *90 km/h in the direction AP*, where AP is the tangent to the circle travelled by the car. When the car reaches B the velocity at this instant is *90 km/h in the direction BQ*. At C the velocity is *90 km/h in the direction CR*. The velocity of the car at A, B and C is therefore different; but the speed, which is a numerical value only, is the same at each point.

Representation of vectors

Acceleration is a vector quantity because it has direction as well as magnitude. A falling object, for example, has an acceleration towards the centre of the earth which is practically a vertical direction. So the acceleration due to gravity g, about 10 m/s^2, can be represented by a straight vertical line OD of length 10 cm, where 1 cm is 1 m/s^2 (Fig. 2.10b).

Fig. 2.11a illustrates a car moving with a uniform velocity v of 48 km/h along a road AO inclined at 50° to a road OB. The velocity v, a vector, would then be represented by a line parallel to AO and 8 cm long if you used a scale of 1 cm to 6 km/h.

Fig. 2.11b illustrates a bicycle accelerating at 3 m/s^2 down a road PQ inclined at 15° to the horizontal. The acceleration a, a vector, would be represented by a line parallel to PQ and 3 cm long, if you used a scale of 1 cm to 1 m/s^2.

Scanned with CamScanner

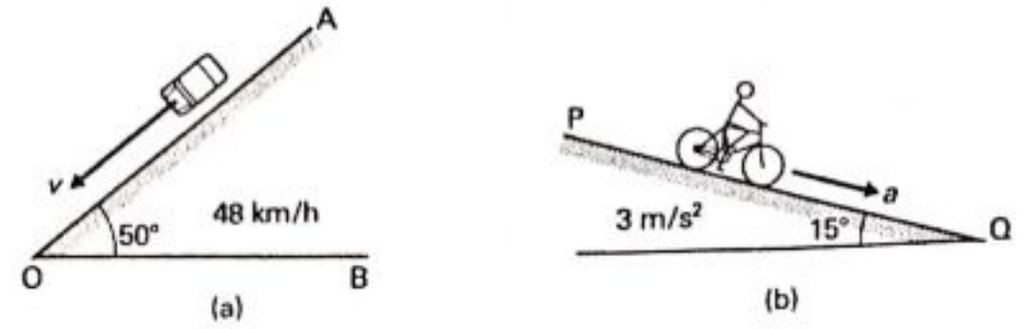


Fig. 2.11 Representation of velocity and acceleration

Addition of vectors : parallelogram and triangle of vectors

Suppose a boy or girl is swimming across a stream from one side to the other with a velocity of 5 km/h and there is a current downstream of 10 km/h (Fig. 2.12a). The total velocity of the swimmer is called the *resultant* of the two velocities. We cannot say that the resultant is 5 + 10 or 15 km/h because velocity is a vector quantity. We must take into account the direction of the velocity, as well as its magnitude when adding two velocities together.

One method of adding two vectors, a *parallelogram method*, is shown in Fig. 2.12a. A line OA is drawn to represent the current velocity of 10 km/h in magnitude and direction. On the same scale, a line OB is drawn to represent the swimmer's velocity of 5 km/h in magnitude and direction. For example, using 1 cm to represent 1 km/h, OA is 10 cm long and OB is 5 cm long at right angles to OA. A parallelogram OBCA is now completed as shown. It happens to be a rectangle here because the two velocities are at 90° to each other. *The resultant velocity is then represented in magnitude and direction by the diagonal OC through O.*

On measurement, OC is about 12 cm long and angle COA is about 26°. So the resultant velocity of the swimmer is about 12 km/h, and he or she actually swims in a direction about 26° to the bank.

Since AC = OB and AC is parallel to OB, the velocity of 5 km/h of the swimmer is also represented by the line AC. The resultant velocity can now be drawn by a different method. First, the line OA is drawn to represent the velocity of 10 km/h of the stream. Secondly, the line AC is drawn from A to represent the velocity of 5 km/h (Fig. 2.12b). The points A and C are then joined. OC, the third side of the triangle OCA, represents the sum or resultant of the two velocities. This may be called a triangle method of adding two vectors. It should be noted that the arrows on OA and AC follow each other round the triangle.

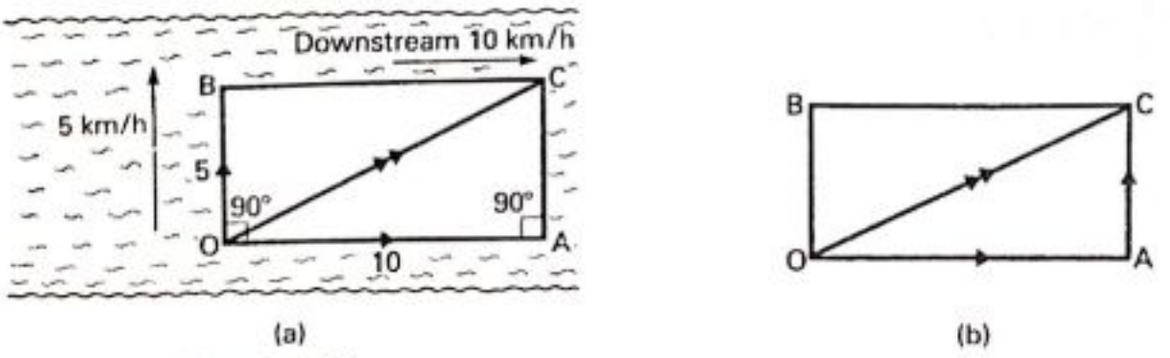


Fig. 2.12 Addition of velocities

Scanned with CamScanner

Components of g Cosine law

Galileo, one of the early great scientists, who lived from 1564 to 1642. showed how to lessen the effect of the acceleration due to gravity g. Instead of allowing a ball to fall freely to the ground, he placed it on a smooth inclined plane PAQ and let it roll down (Fig. 2.13a). The ball now takes longer to fall from the height P to AQ. Its acceleration is not g, which acts vertically downwards, but that part of g which acts down the plane. This is called the *component of* g down the plane.

To find the size of the component, we first draw a vertical line DE to represent g in magnitude and direction (Fig. 2.13b). From D, a line DL is drawn in the direction of the sloping side PA of the plane PAQ, and from E a line EL is drawn perpendicular to DL. Now the vector DE or g is the sum of the vectors DL and LE, as we explained using Fig. 2.12b. The acceleration g is therefore equivalent to an acceleration of magnitude DL down the sloping side PA of the plane PAQ plus an acceleration of magnitude LE in a perpendicular direction to DL.

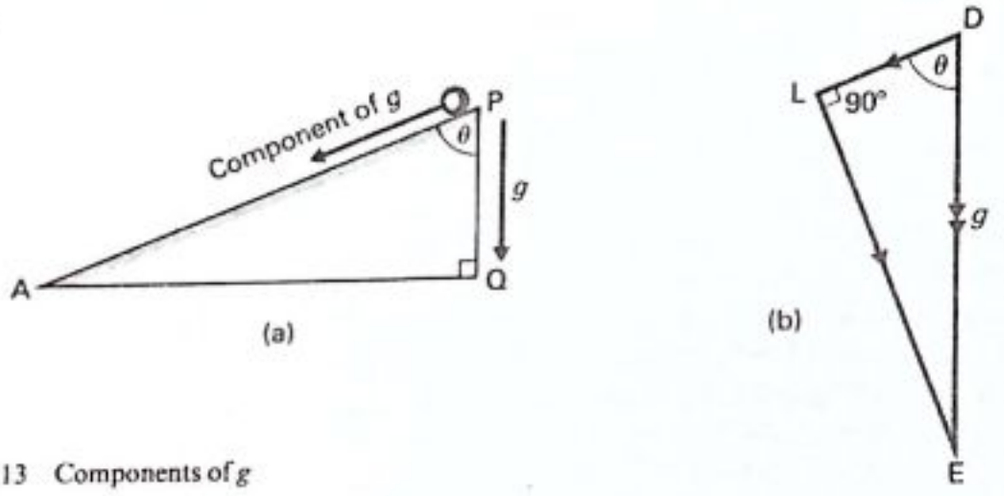


Fig. 2.13 Components of g

The vector LE has no effect in the direction DL perpendicular to itself. Consequently, the effective part, or component, of g or DE in the direction PA is the vector DL. So if DL makes an angle θ with DE, which is the same as the angle APQ of the plane

$$\text{component of } g = \text{DL} = \text{DE} \cos\theta = g \cos\theta$$

The other component of g, which is LE, acts perpendicular to DL, that is, at right angles to the sloping side PA of the plane.

Suppose the angle of inclination (θ) of the plane PAQ to the vertical is 60°. Then an object moving under gravity down the plane has an acceleration given by

$$g \cos 60°, \text{ which is about } 10 \times 0{\cdot}5 \text{ m/s}^2 \text{ or } 5 \text{ m/s}^2.$$

If the plane is made less steep, say 20° to the horizontal, then θ, the angle to the vertical, becomes 70°. The acceleration down the plane is then

$$g \cos 70°, \text{ which is about } 10 \times 0{\cdot}34 \text{ m/s}^2 \text{ or } 3{\cdot}4 \text{ m/s}^2.$$

This cosine rule for calculating components should be remembered for future use.

Horizontal and vertical motion under gravity

A simple experiment on horizontal and vertical motion under gravity can be carried out.

Scanned with CamScanner

Experiment

Two coins A and B are placed near the edge of a table, and then struck simultaneously with a rod pivoted at O near A (Fig. 2.14). Both coins will move at the same time sideways off the table. B, however, is shot forward with a much greater velocity than A, so that it lands much further away on the floor at D. A lands at C, close to the table, so that it falls almost vertically. On listening, *both coins are heard to reach the ground at the same time*. This is the result wherever the coins are placed. The time taken to reach the ground is therefore always the same. So the *vertical* motion of B, due to *g*, is not affected by its horizontal velocity *v*.

The horizontal and vertical motions of an object thrown into the air can be treated as completely independent as shown by the experiment above. This is illustrated in Fig. 2.14. The ball travels equal distances horizontally in equal times, as shown along OX in the photograph, because it moves forward horizontally with a uniform velocity. The vertical positions of the ball in equal times are shown along OY. The vertical motion is one of uniform acceleration g and the distance fallen in a time t is given by $s = \frac{1}{2}gt^2$, from $s = ut + \frac{1}{2}at^2$.

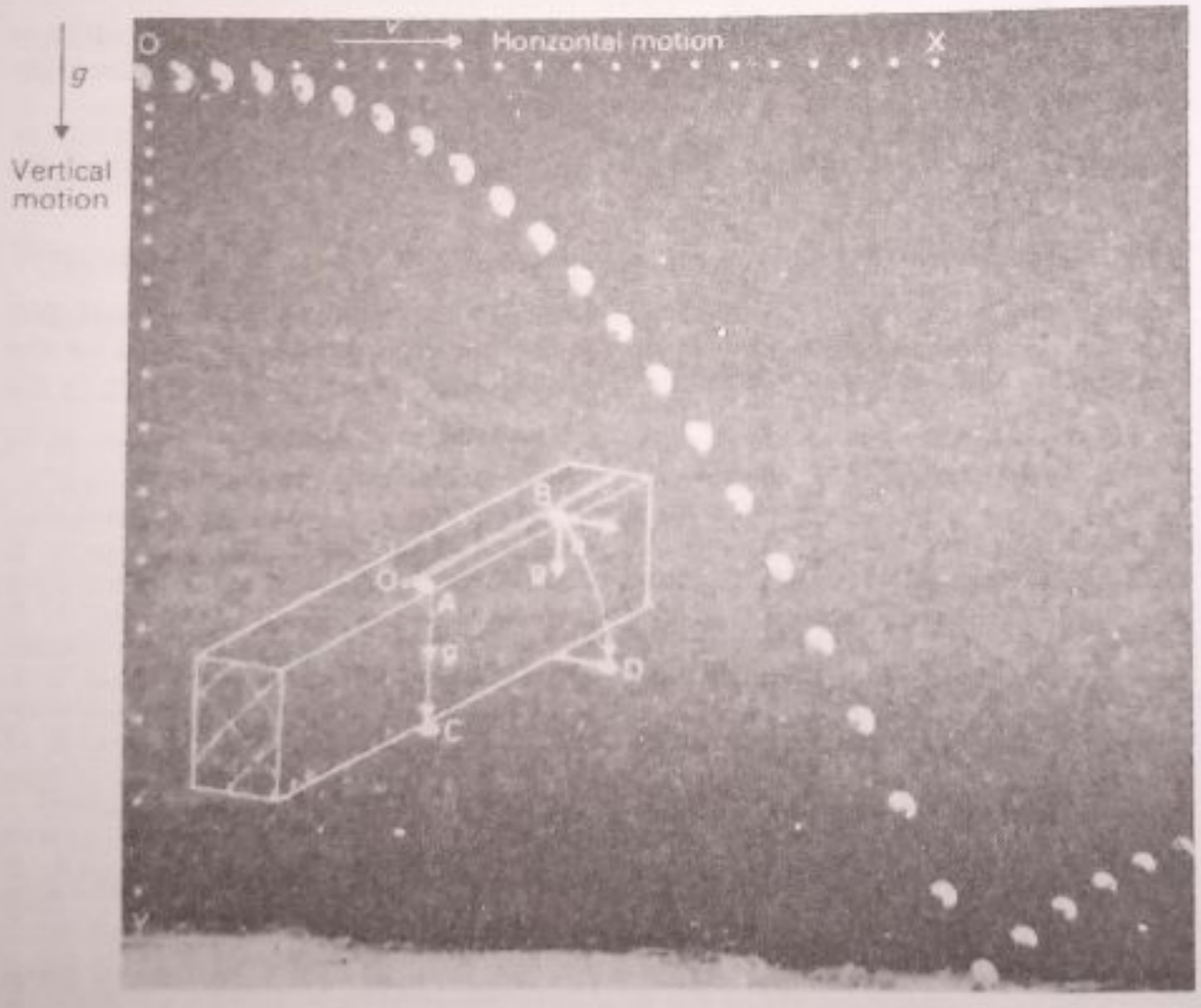


Fig. 2.14 Horizontal and vertical motion under gravity
The photograph shows the parabolic path of a ball thrown horizontally with uniform velocity and rebounding from the floor after falling. The ball travels equal horizontal distances in equal times. The ball falls vertically with uniform acceleration g. Note the acceleration and deceleration.
(*Freefall multiflash photograph by Gordon Severn*)

Scanned with CamScanner

Examples on motion and velocity–time graphs

1. Two stones are thrown vertically upwards from the same point with the same velocity of 20 m/s but at an interval of 2 s. When they meet, the second stone is rising at 10 m/s.

(a) For what time is the second stone in the air before they meet? (b) What is the velocity of the first stone when they meet? State any assumptions you make in obtaining your answers. (L)

(a) *Second stone* This stone moves upwards and its acceleration $= -g = -10\,\text{m/s}^2$

So from $v = u + at$

$10 = 20 - 10t$

So $10t = 20 - 10 = 10$, so $t = 1\,\text{s}$

The second stone has been in the air for 1 s before they meet.

(b) *First stone* This stone has been moving for $1 + 2$ or 3 s.

From $v = u + at$

$v = 20 - (10 \times 3) = 20 - 30 = -10\,\text{m/s}.$

The minus means that the first stone is moving *down* and its velocity is 10 m/s.

The assumptions are that the frictional force due to air resistance on the stones is negligible and that g is constant at $10\,\text{m/s}^2$.

2. An electric train moves from rest with a uniform acceleration of $1{\cdot}5\,\text{m/s}^2$ for the first 10 s and continues accelerating at $0{\cdot}5\,\text{m/s}^2$ for a further 20 s. It continues at constant speed for 90 s and finally takes 30 s to decelerate uniformly to rest.

(a) Draw a graph of speed against time for the journey. (b) From your graph, or otherwise, deduce the total distance travelled. (c) What is the average speed of the train for the whole journey? (L)

(a) Fig. 2.15 shows the speed (v)–time (t) graph OABCD. In 10 s, the speed $v = u + at = 1{\cdot}5 \times 10 = 15\,\text{m/s}$, and this enables the line OA to be drawn.

In a further 20 s at $0{\cdot}5\,\text{m/s}^2$, the speed increases by $20 \times 0{\cdot}5$ or 10 m/s. So the speed at 30 s is 25 m/s. The line AB can now be drawn. The line BC represents uniform speed for 90 s and CD the velocity while decelerating uniformly to rest in 30 s.

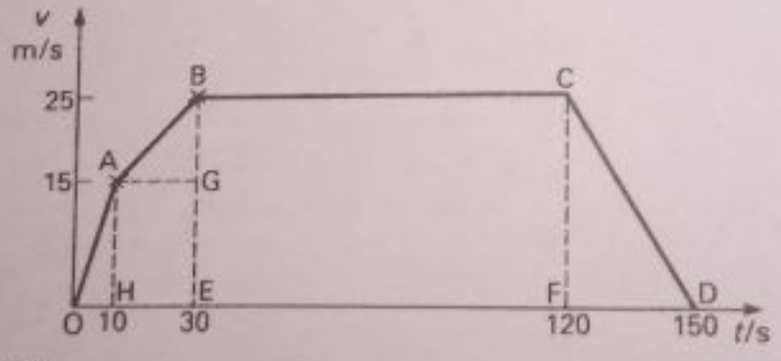


Fig. 2.15 Example

(b) The total distance travelled is represented by area between OABCD and time axis.

Area OAH $= \frac{1}{2} \times 10 \times 15 = 75\,\text{m}$

Area ABEH = area ABG + area AGEH $= (\frac{1}{2} \times 20 \times 10) + (20 \times 15) = 100 + 300 = 400\,\text{m}$

Area BCFE $= 90 \times 25 = 2250\,\text{m}$

Area CDF $= \frac{1}{2} \times 30 \times 25 = 375\,\text{m}$

$\therefore$ total distance $= 75 + 400 + 2250 + 375 = 3100\,\text{m}$

(c) Average speed for whole journey

$$= \frac{\text{distance}}{\text{time}} = \frac{3100}{150} = \frac{62}{3} = 20{\cdot}7\,\text{m/s}$$

Scanned with CamScanner

Summary

1. Speed = distance/time or distance per unit time. It is a scalar quantity–it has no direction.

$$36\ \text{km/h} = 10\ \text{m/s}.$$

2. Velocity = displacement/time or displacement per unit time. It is a vector quantity–it has direction as well as magnitude.

The velocity of a car moving round a circular track is tangential to the circle it is following at any instant.

3. Acceleration = velocity change/time. It is a vector and has units such as metre/second^2 (m/s^2).

4. Equations of motion for uniform acceleration are

$$v = u + at,\ s = ut + \tfrac{1}{2}at^2,\ v^2 = u^2 + 2as$$

5. The acceleration g due to gravity is about 9·8 m/s^2 or 10 m/s^2 (approx.). It can be measured by free-fall using a centisecond clock or by a pendulum method ($T = 2\pi\sqrt{l/g}$).

6. Velocity–time graph. With uniform acceleration the graph is a straight line whose gradient is equal to the acceleration. The distance travelled is represented by the area between the velocity–time curve and the time-axis.

7. Velocity addition. Either use the parallelogram method or the triangle method.

8. Components. (a) The acceleration due to gravity g acts vertically downwards; it has no component in a horizontal direction. So a ball thrown forward has a constant horizontal velocity and moves vertically downwards with acceleration g.

(b) The acceleration of an object due to gravity down a smooth inclined plane is $g\cos\theta$ if θ is the angle of the plane to the vertical.

Exercise 2 (Answers, page 640)

(Assume where necessary $g = 10\ \text{m/s}^2$).

1. A train moves with a uniform velocity of 36 km/h for 10 s. Calculate the distance travelled.

2. A car has a velocity of 10 m/s. It now accelerates at 1 m/s^2 for $\frac{1}{4}$ min. Find the distance travelled in this time and the final speed of the car.

3. A tennis ball is hit vertically upwards with a velocity of 20 m/s. What is its deceleration as it moves upwards? Calculate the maximum height reached by the ball.

4. In question 3, find the time taken for the ball to reach its highest point. What is the time taken by the ball to return to the thrower?

5. A car travels with uniform velocity of 30 m/s for 5 s and then comes to rest in the next 10 s with uniform deceleration.

Using a suitable scale, draw a velocity–time graph of the motion. From the graph, find (a) the deceleration, (b) the total distance travelled.

6. A stone is thrown horizontally with an initial velocity of 15 m/s from a tower 20 m high.

(a) How long does it take to reach the ground?

(b) At what distance from the base of the tower does the stone reach the ground?

7. What is a vector and a scalar quantity? Give *one* example of each.

8. A train, starting from rest, accelerates uniformly at 2 m/s^2 for 15 s. It then travels for 30 s at the velocity reached, and is brought to rest with uniform deceleration after 10 s more.

Draw a velocity–time graph of the motion. From the graph, find the total distance travelled by the train and its deceleration. Verify your results from formulae.

9. A car moves round a circular track with a uniform speed of 100 km/h. Show in a diagram vectors which represent the velocity at four consecutive positions A, B, C, D equally spaced round the track. Is (a) the velocity and (b) the speed the same at A, B, C and D?

Scanned with CamScanner

10. (a) If the bob of the pendulum shown in Fig. 2.16 takes $\frac{1}{4}$ second to swing between the extreme positions A and C, what is its frequency of oscillation? (b) State whether the frequency of the oscillations will increase, decrease or remain the same if (i) the length of the string is increased, (ii) the mass of the bob is increased (without altering its size), (iii) the distance between A and C decreases. (L)



Fig. 2.16

11. Sketch a graph showing the relation between time and distance travelled by a body falling freely from rest. How would you use this graph to determine (a) the velocity of the body at any instant, (b) the acceleration of the body? (N)

12. Explain the meaning of *uniform acceleration* and deduce expressions for (a) the velocity acquired in a time t, (b) the velocity acquired in a distance s, by a body which starts from rest and is subject to a uniform acceleration a.

A train moves from rest with an acceleration of 0·5 m/s^2. Find the speed, in km/h, which it reaches in moving through its own length, which is 100 m. (L)

13. What is meant by *uniform acceleration*? Sketch a velocity–time graph and a distance–(time)^2 graph for a body starting from rest and moving with uniform acceleration.

A train, starting from rest, accelerates uniformly so that it attains a speed of 48 km/h in 2 min. It travels at this speed for 5 min, and is then brought to rest with uniform deceleration in 3 min. Find the total distance travelled by the train.

14. Describe an experiment to determine the acceleration due to gravity.

Draw the velocity–time curve of a body which has an initial velocity u metre per second and moves for t second in a straight line with an acceleration a metre per second.

Use a graph to deduce an expression for the distance travelled by the body in this time. (N)

15. Derive, from first principles, an expression for the distance travelled by a uniformly accelerated body moving from rest, in terms of the acceleration and the time.

A ball dropped from rest on to horizontal ground 20 m below rebounds with $\frac{3}{4}$ of the velocity with which it hits the ground. Find the time that elapses between the first and second impacts of the ball with the ground. (L)

16. Define *uniform velocity, uniform acceleration*. Show how, from the velocity–time graph for a body, (a) the acceleration at any instant and (b) the average velocity can be calculated.

A car, starting from rest, is uniformly accelerated at 0·15 m/s^2 until it reaches a speed of 15 m/s. It travels at this speed for 3 min and is then uniformly retarded so as to come to rest 8 min after starting. Plot the velocity–time graph of the motion and use the graph to determine the average speed of the car. (O and C)

17. Explain the terms: *uniform acceleration, acceleration due to gravity*. Describe a method of determining the acceleration due to gravity at a given place. A tennis ball is hit vertically upwards and returns 6 s later. Calculate: (a) the greatest height reached by the ball; (b) the initial velocity of the ball. (O and C)

18. A man wishes to swim straight across a river 20 metres wide to point X directly opposite him. There is a uniform current of $\frac{1}{2}$ metre per second.

Find, by calculation or scale drawing, the direction in which the swimmer must head to do this, and the time he would take to reach X, when he swims at a steady speed of 1 metre per second. (C)

19. Describe an experiment to illustrate motion with uniform acceleration. Explain how you would establish that the acceleration is uniform.

A body moves from rest with uniform acceleration and travels 90 m in the fifth second. Calculate the velocity of the body 10 s after starting. (N)

20. What is meant by *uniform acceleration, gravitational acceleration*? Describe a method of measuring gravitational acceleration.

The speed of a train is reduced from 96 to 48 km/h in a distance of 800 m on

Scanned with CamScanner

applying the brakes. How much farther will the train travel before coming to rest, assuming the retardation remains constant, and how long will it take to bring the train to rest after the application of the brakes? (O and C)

21. Examination of a ciné-film of the motion of a heavy ball, thrown upwards inclined to the vertical, showed that the ball travelled sideways a distance of 10 metres in each successive second and that its height in successive seconds was 3, 25, 37, 39, 31, 13 metres. Draw, on squared paper, a scale diagram to show the path of the ball in its flight. Estimate how long it was in flight, how high it rose and how far it went.

Describe in general terms the vertical and horizontal motions of the ball and relate these to any forces acting on the ball. Assume air resistance to be negligible in this case, and then explain how the motion of the ball would have been affected by a strong wind blowing horizontally against the ball. Illustrate your answer by showing, on your scale diagram, a possible path for the ball. (C)

Multiple-Choice Questions

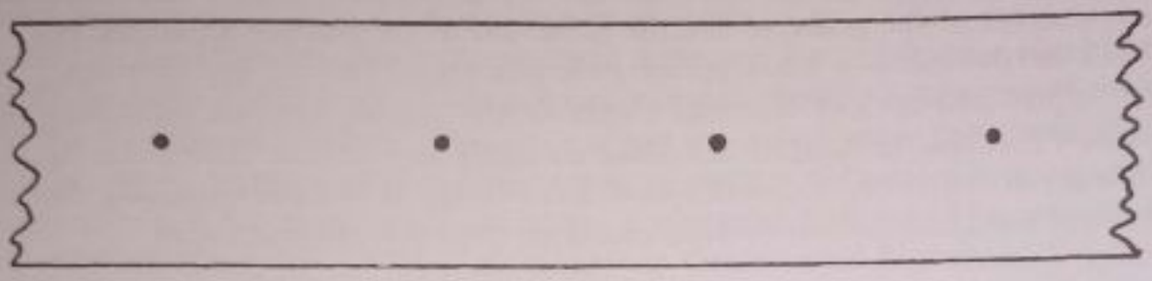


Fig. 2.17 (Magnified tape)

22. The tape shows dots equally spaced 1 cm when the vibrator of a ticker-timer made 50 dots per second. The speed of the tape in metres per second was

A 0·02 B 0·1 C 0·5 D 5·0 E 50

23. A car travels a distance of 200 m in 10 s at constant speed when moving round a complete circle. Which statement A to E is best true?

A The speed always changes in direction during 10 s
B the acceleration is 20 m/s^2
C the velocity is constant for 200 m
D the velocity changes during the 10 s
E the speed reverses after 10 s

24. In free fall, a ball of mass 1 kg drops 20 m from rest in 2 s. A ball of mass 2 kg drops 20 m from rest in

A 1 s B 2 s C 4 s D 5 s E 8 s

25. A train travels with uniform (constant) acceleration from rest. After a time t it has gone a distance of 50 m. In a time $3t$ from rest, the train travels a distance in metres of

A 150 B 200 C 300 D 400 E 450

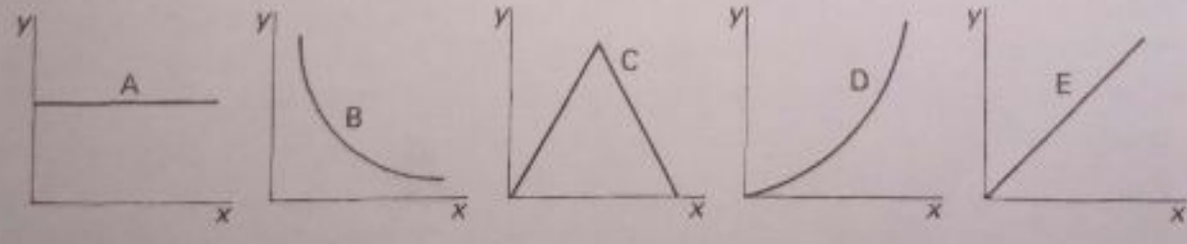


Fig. 2.18

Which of the graphs A to E in Fig. 2.18 shows the following motion:

26. The variation of velocity (y) with time (x) when a ball is dropped from a height above the ground.

27. The acceleration (y) of a falling ball with time (x).

28. The distance (y) with time (x) when a car accelerates uniformly from rest.

29. The velocity (y) of a lift (elevator) accelerating uniformly from rest and then decelerating uniformly to rest with time (x).

Scanned with CamScanner

3. Force, Momentum, Energy, Power

Sir Isaac Newton, 1642–1727, is recognised as one of the greatest scientists who ever lived. He made important discoveries in optics, such as the colours of white light. He also found a universal law of gravitation which explained the movements of the moon round the earth and of the planets round the sun. In 1687 he published a book called *Principia*, in which, for the first time, the principles and laws of mechanics were clearly stated. Another great scientist, Albert Einstein, developed new ideas on space and time in 1905 in his Special Theory of Relativity. Einstein's work has developed Newton's theories. But the laws of mechanics which Newton stated are basically still true today.

Force · Newton's first law

If you bump into someone while walking, you slow down. The *force* due to the collision has caused a change in your velocity. If a tennis ball is hit, or a football is kicked, the force will usually change its direction.

A metal disc used in ice hockey, called a puck, with dry ice (solid carbon dioxide) packed beneath its base, glides smoothly along a flat sheet of glass carefully levelled if it is given a gentle push. Carbon dioxide gas, released between the dry ice and the glass, forms a cushion of gas on which the puck moves. So friction is reduced to an extremely low value and we can use the puck to see what happens to the motion of an object when almost no force acts on it.

By using regular flashes of light in a dark room, produced by an electronic instrument called a stroboscope, a camera can take successive photographs of a moving puck in equal intervals of time. Fig. 3.1 shows the result when it moves on its own across a highly polished plate. We see that the puck, which has almost no frictional or other force on it, moves with *constant velocity* in a straight line.

Newton's first law summarises this. *Every object continues in its state of rest or uniform motion in a straight line unless unbalanced forces act on it.*

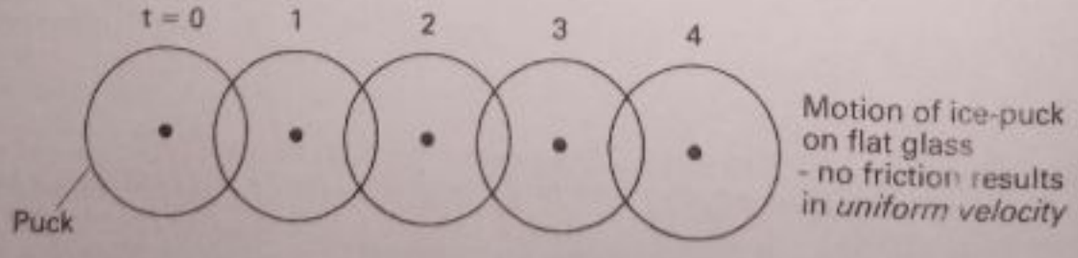


Fig. 3.1 Motion of a puck with no force on it

A rocket, fired from the earth to reach the moon, eventually reaches outer space. Here the effect of the earth's gravitational attraction is very small. So if no

Scanned with CamScanner

other force acts on it, the rocket continues to move in a straight line with uniform velocity. When it comes into the gravitational fields of the moon, however, its path is affected by the gravitational pull of the moon, and so its direction may change.

Passengers in a fast-moving car or bus which suddenly stops, continue to move forward because there is not enough force to stop them. Safety straps help to stop them moving forward.

Inertial mass

Newton's first law recognised that objects have a reluctance to move when they are at rest or to slow down or stop when they are moving. This is what we mean when we say objects have a certain amount of *inertia*.

A large block of stone can be pushed with difficulty. A small wooden block can easily be pushed. The *mass* of the stone or wooden block is a measure of their inertia. Newton said that the 'mass' of an object was a measure of the 'quantity of matter' in it, without stating the nature of matter. Masses are measured accurately against a standard mass by a chemical balance. The standard kilogram is a particular block of metal kept in the National Bureau of Weights and Measures in France.

Force, mass and acceleration

We have seen that a force produces a change of velocity when it acts on an object. So a force produces an *acceleration* or a *deceleration*, a negative acceleration.

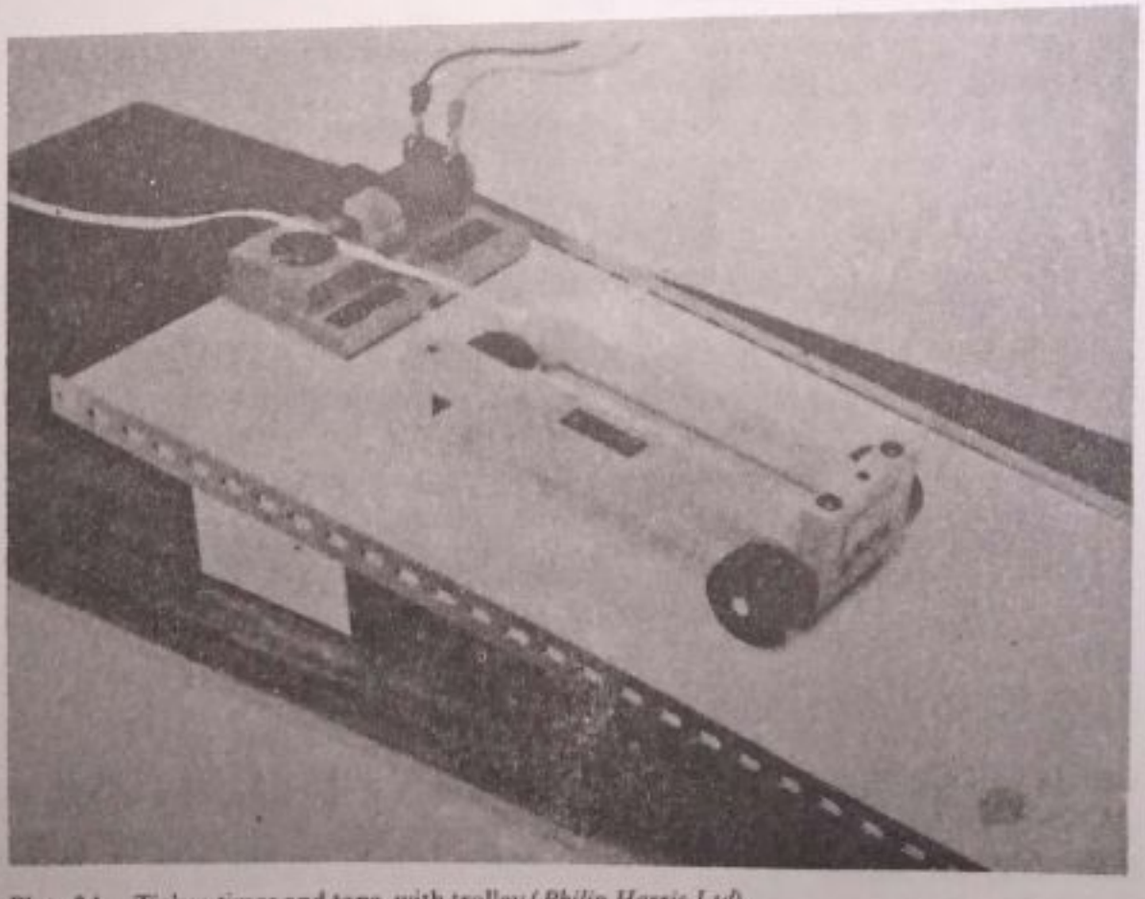


Plate 3A Ticker-timer and tape, with trolley (*Philip Harris Ltd*)

Scanned with CamScanner

Experiment

The acceleration produced by a force can be investigated by using a trolley and ticker-tape (Plate 3A). A long board B is set up as a runway and a preliminary run is made by giving the trolley a gentle push down the plane (Fig. 3.2). To compensate for friction, one end of B is raised until the trolley moves down the plane with uniform velocity when it is pushed. The ticker-tape then shows equally spaced dots.

A constant force to pull the trolley down the board is produced by stretching a piece of elastic L by a constant length. One end, with an eyelet E, is placed on a peg P on the trolley. The other end is pulled between the finger and thumb until it is exactly at the pegs R and S or at another fixed point like the axle of a wheel.

After trials the trolley is pulled along by the hand holding the stretched length of elastic, which is kept a constant length all the time. The trolley now moves down the plane under a constant force, which may be called 'one unit' of force. Two and then three units of force are obtained by attaching first one more and then two more pieces of elastic to P, each stretched exactly the same length as before. After each run the tape is taken off and a fresh tape is tied to the trolley.

Results: Fig. 3.2 shows the results when the three tapes are each cut into 10-tick units and pasted beside each other. The acceleration is the respective gradient of the line joining the tops of the strips. So

ratio of accelerations = 7·3 to 4·8 to 2·4 = 3:2:1

and ratio of forces = 3:2:1

Conclusions: 1. A constant force produces a constant acceleration.
2. The acceleration is directly proportional to the force for a constant mass.

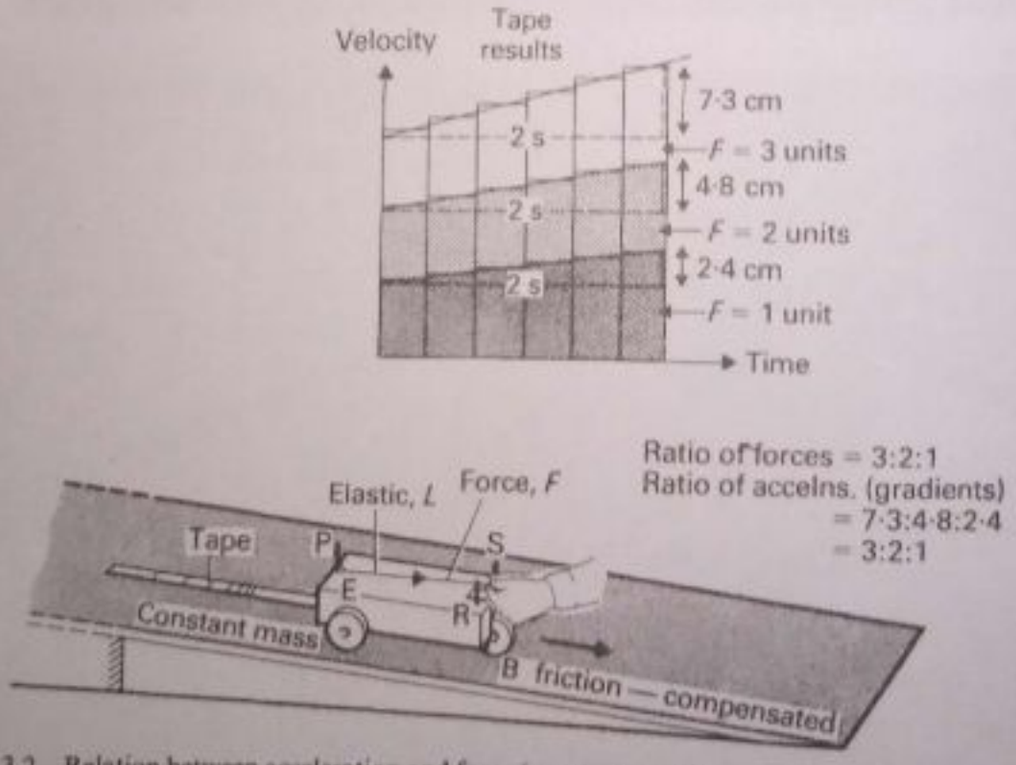


Fig. 3.2 Relation between acceleration and force (constant mass)

We can alter the mass m by using one trolley, then putting another on top so that there are two trolleys, and finally using three trolleys on top of each other. Each time the pulling force is kept constant by using one piece of stretched elastic. The results show that the acceleration a decreases in the ratio 3:2:1 as the

Scanned with CamScanner

mass m increases in the ratio 1:2:3. So a is proportional to $1/m$ when the force F is constant.

The results above can be summarised by stating that

$$F \propto ma$$

Force = mass × acceleration

The newton, symbol N, is the unit of force in the SI system. It is defined as *the force acting on a mass of* 1 *kilogram which gives it an acceleration of* 1 *metre per second*^{2}. From this definition, it follows by proportion that a force on a mass of 10 kg which gives it an acceleration of 2 m/s^2 has a magnitude of 10×2 or 20 newtons.

Generally, then, the relation between force F, mass m and the acceleration produced, a is

$$F = ma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where F is in newtons, m in kilograms and a in metres per second^2. So if a force of 50 newtons (50 N) produces an acceleration of 2 m/s^2, the mass m of the object is given by

$$50 = m \times 2, \text{ or } m = 25 \text{ kg}$$

If a car of mass 2000 kg is acted on by a force of 4000 N, the acceleration a of the car, from $F = ma$, is given by

$$4000 = 2000 \times a$$

So

$$a = \frac{4000}{2000} = 2 \text{ m/s}^2$$

Force due to gravity

If we slip and fall over we realise immediately that the earth attracts our mass. A ball thrown into the air comes down again for the same reason. All objects, tiny as well as large, are attracted by the earth. This is called *gravitational attraction*, or the force due to gravity. The molecules of air in the atmosphere are kept round the earth by the gravitational attraction; a thin 'blanket' of gas, the atmosphere, stretches more than 11 kilometres high round the earth.

The force due to gravity on an object is called its weight. All objects near the earth's surface fall freely with the same acceleration, g. Since g is about 9·8 m/s^2, or 10 m/s^2 in round figures, we can find the weight of a 1 kg mass by

$$F = ma = mg = 1 \times 10 = 10 \text{ N}$$

and the weight of a 50 kg mass by

$$F = mg = 50 \times 10 = 500 \text{ N}$$

So the larger the mass of an object, the greater is its weight. In fact,

$$weight \propto mass$$

Since weight $F = mg$, then $g = F/m$. Using a mass of 1 kg, we see from above that $F = 10$ N in this case. So in the earth's gravitational field, the 'force per unit mass' is 10 N/kg. We can therefore write

$$g = 10 \text{ m/s}^2 \text{ or } 10 \text{ N/kg}$$

Scanned with CamScanner

The weight of an average size apple of mass 100 g is about 1 N. For future use, note that, in round figures

$$\text{weight of 1 kg mass} = 10\ \text{N} \quad \ldots\ldots\ldots (1)$$

and

$$\text{weight of 1 g mass} = 0{\cdot}01\ \text{N} \quad \ldots\ldots\ldots (2)$$

Gravity above earth · The moon

When a satellite is sent up into space the force of attraction of the earth becomes less as it gets further away. So the weight of the satellite diminishes, although its mass remains constant. The acceleration due to gravity, from $F = ma$, thus also decreases. At a distance of about 6400 km from the earth, a distance from the earth's centre of twice its radius, the acceleration due to gravity diminishes to one-quarter of its magnitude at the earth's surface, or to about $2{\cdot}5\ \text{m/s}^2$.

In July 1969, America succeeded in landing a spacecraft, Apollo 11, on the moon. On this planet the magnitude of the gravitational attraction is about one-sixth that of the earth, since the mass of the moon is much less than that of the earth. The astronauts were seen to 'float' in walking over the moon's surface. Theoretically, high-jumpers could leap very high distances for the same reason. The moon does not have any atmosphere and scientists think this is because of the weak gravitational attraction of the moon on gaseous matter.

Mass and weight

The *mass* or amount of matter or 'stuff' in a 1 kg box of chocolates carefully sealed is 1 kg all over the world; it would be the same at the equator or at the poles, or even if it were landed on the moon. On the other hand, if the force due to gravity is measured all over the world by attaching the box to a spring balance (Fig. 3.3a), the extension of the spring will vary slightly from place to place. It will be greater at the north and south poles of the earth, for example, than at the equator. So the *weight* is greater at the poles. This is explained by the fact that the earth is not perfectly spherical–it is flattened slightly at the poles. Objects here are therefore nearer to the centre of the earth, and so the force of attraction is greater. As we have seen, if m is the mass of an object and g is the acceleration due to gravity at a particular place

$$\text{weight of object} = \text{mass} \times \text{acceleration} = mg$$

We can now see why a common balance compares masses and not weights. As an illustration, suppose an unknown mass m in one scale-pan is counter-balanced exactly by a mass of 4 kg on the other scale-pan (Fig. 3.3b). The balance is a lever with equal arms a. On weighing, therefore, the moments of the forces about the pivot are equal (page 67), or

$$\text{weight on one scale-pan} \times a = \text{weight on other scale-pan} \times a$$
$$\therefore\ \text{weight on one scale pan} = \text{weight on other scale-pan}$$

Assuming the scale-pans have equal weight, we then have

$$mg\ (\text{weight}) = 4g\ (\text{weight})$$

Since g is the same, it can be cancelled on both sides, so that $m = 4$ kg. The same result, $m = 4$ kg, would be obtained with a common balance at any part of the world since it is independent of g. So a common balance compares masses.

Scanned with CamScanner

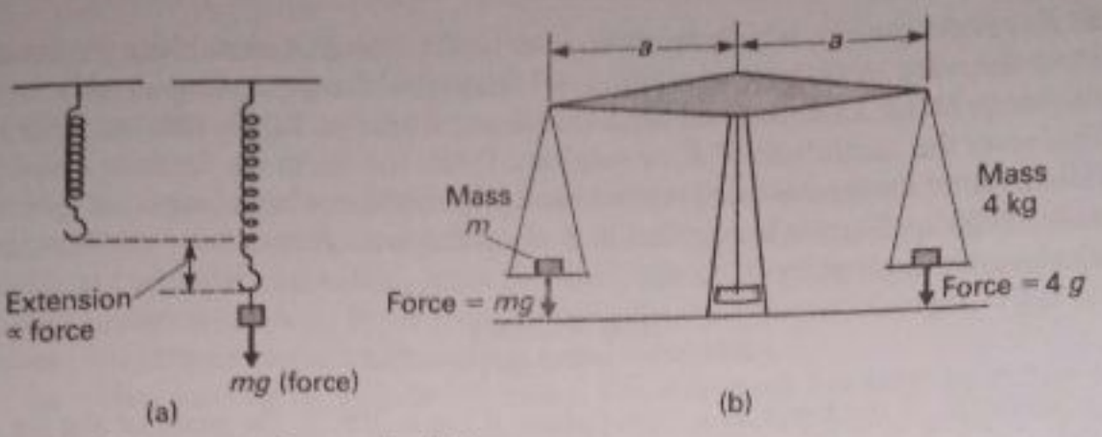


Fig. 3.3 Measurement of mass and weight

Example on $F = ma$

A box of mass 80 kg is raised vertically with a uniform acceleration a by a force of 1000 N in a rope. Assuming $g = 10\ \text{m/s}^2$, calculate a.

$$\text{Weight of box} = mg = 80 \times 10 = 800\ \text{N}$$

The weight acts downwards and opposite in direction to the force 1000 N.

$$\therefore \text{ net upwards force } F = 1000 - 800 = 200\ \text{N}$$

From
$$F = ma$$
$$200 = 80 \times a$$
$$\therefore\ a = \frac{200}{80} = \frac{5}{2} = 2{\cdot}5\ \text{m/s}^2$$

Application of $F = ma$ · Force on astronauts

When an astronaut is strapped into the seat in the spacecraft the upward force on him from the reaction of the seat is balanced by his weight mg, and no discomfort is experienced. At blast-off the spacecraft and astronaut are lifted with a tremendous acceleration from the firing pad. If R is then the reaction of the seat on him the *resultant upward* force F is $(R - mg)$. So if the upward acceleration is $15g$, from $F = ma$ we have

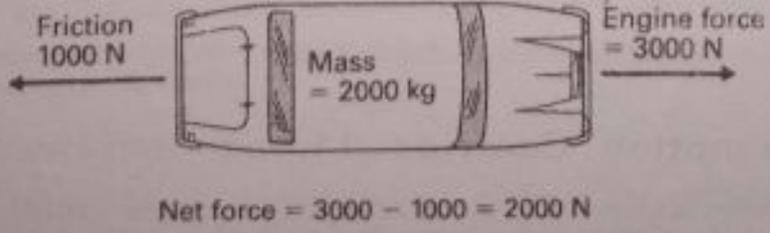
$$R - mg = m \times 15g = 15mg$$

So
$$R = 15mg + mg = 16mg$$

This is an exceptionally large force for the human body to withstand. Experiments which reach this high acceleration are carried out as part of the training of an astronaut by whirling him round at very fast speeds.

Frictional forces and motion

We now consider the effect of friction on the motion of an object. Suppose the force on a car of mass 2000 kg due to the engine is 3000 N and the frictional force at the ground and the air resistance together total 1000 N (Fig. 3.4). First, note *all*



Net force = 3000 − 1000 = 2000 N

Fig. 3.4 Forces on a car

Scanned with CamScanner

the forces on the car, which are 3000 and 1000 N. Second, note their direction, since they are vectors (page 28), and draw the corresponding arrows in a diagram. Third, calculate the *resultant force*, which is $3000 - 1000$ or 2000 N. This gives the magnitude of F, or resultant force, for use in the formula $F = ma$. Fourth, note the *mass moved*, m; this has no connection whatsoever with 'forces', and is 2000 kg. Remembering that in $F = ma$ the force F must be in newtons, the acceleration is given by

$$2000 = 2000a$$
$$\therefore\ a = 1\ \text{m/s}^2$$

Similarly, if the force in a rope pulling a log is 100 N, the mass of the log is 40 kg, and the frictional force is 20 N, the resultant force $F = 100 - 20 = 80$ N. From $F = ma$

$$80 = 40a$$
$$\therefore\ a = \frac{80}{40} = 2\ \text{m/s}^2$$

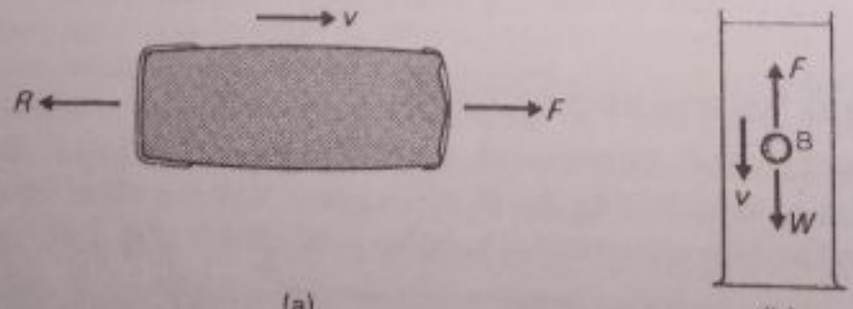
Uniform velocity and force · Terminal velocity

At first sight it may be puzzling how a car can travel along at a uniform or constant velocity if the engine provides a force on the car.

When the car first started off, the force due to the engine was greater than the frictional force. The resultant force produced an acceleration. As the speed of the car increased, the ground frictional force at the car wheels and the air resistance both increased. Eventually, when the total frictional force R is equal to the engine force F, the resultant force on the car is zero (Fig. 3.5a). So there is now no acceleration and the car moves with a constant velocity v. When an object is stationary or moving with uniform velocity, the resultant force on it is zero.

Suppose a small steel ball-bearing B is dropped into a thick liquid such as glycerine or syrup in a tall jar (Fig. 3.5b). At first, the effective weight W of B is greater than the opposing frictional (viscous) force F on B due to the liquid. So B accelerates downwards. As the velocity of B increases, however, F increases. When F becomes equal to the weight W the resultant force on B is zero.

B then continues to move with constant velocity. This maximum velocity is called the *terminal velocity* of B. For a similar reason, an object released from an aeroplane reaches a maximum or terminal velocity owing to the viscosity (friction) of the air.



(a) (b)

Fig. 3.5 Forces on objects moving with uniform velocity

Circular motion · Centripetal force

If a heavy stone is attached to the end of a long string and whirled steadily round in a horizontal circle by holding the other end of the string, we can feel that a

Scanned with CamScanner

force is needed to make the stone move in its circular path. If the stone is whirled faster the force increases. A far greater force can be seen exerted by the muscles of a hammer-thrower as he whirls the heavy hammer round at a fast speed in a circle.

Fig. 3.6 shows an object moving at constant speed round a circle centre O. The velocity v (a vector) at points such as A, B, C is directed along the *tangent* to the circle at the point concerned. Now the direction of the tangent changes as the object moves from A to B, or from B to C. So the velocity changes. Thus the object has an acceleration although its speed is constant.

An object moving in a circle requires a force directed towards the centre to keep it from moving along the tangent path. This is called the *centripetal force* on the object. A force produces an acceleration. So *the acceleration of the object must be directed towards the centre*, as shown in Fig. 3.6.

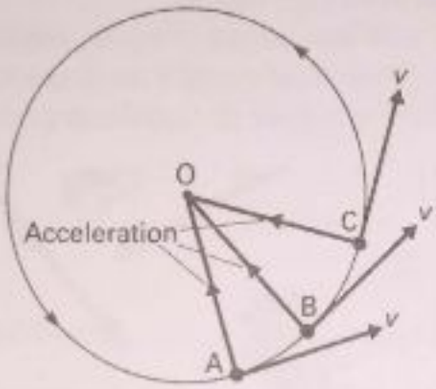


Fig. 3.6 Forces on an object moving in a circle

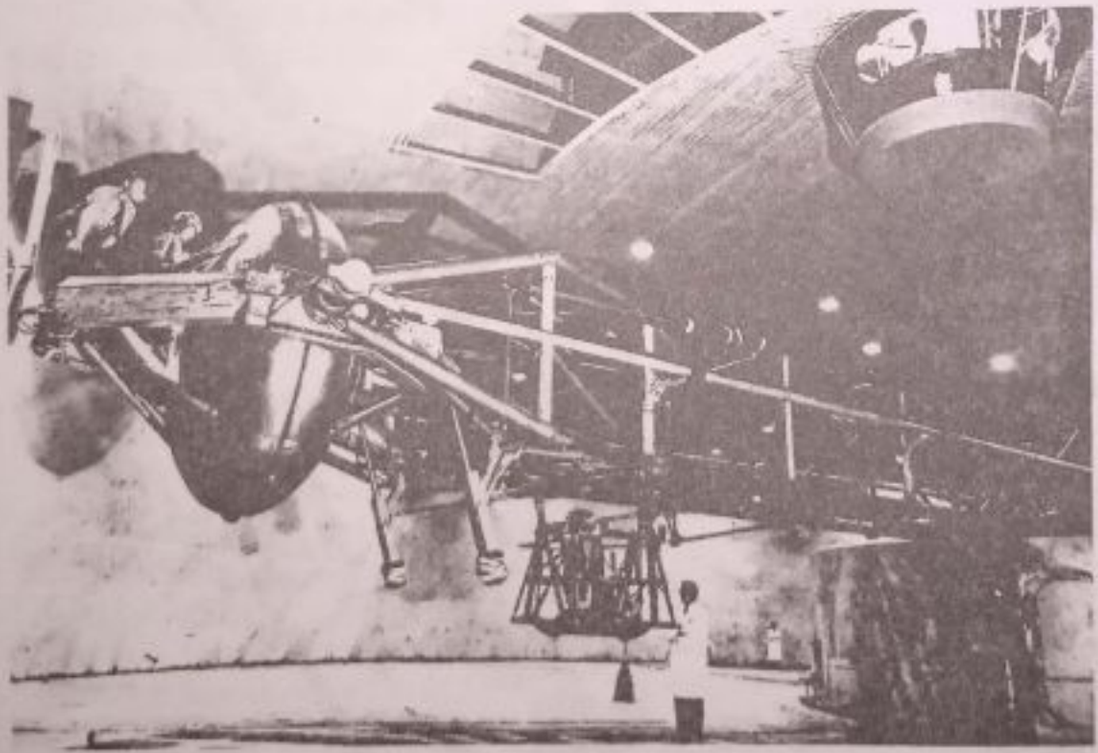


Plate 3B Astronauts are subjected on a centrifuge to about 16 *g* acceleration, corresponding to the highest force met in an actual mission.

Some centripetal forces

In their training, astronauts are strapped into chairs at the end of a long beam and whirled in a horizontal circle at high speeds. If the acceleration towards the

Scanned with CamScanner

centre is 16g their bodies are subjected to a force of 16*mg*. This is what happens at blast-off when they are strapped in a spacecraft.

When a bicycle or car moves round a circular path, the centripetal force is provided by the frictional force between the ground and the tyres (Fig. 3.7a). If the frictional force is not enough to provide the necessary centripetal force at high speeds the bicycle or car will move outwards or skid.

Orbits and weightlessness

The centripetal force required to make planets such as the earth move continuously in their orbits round the sun is provided by the gravitational attraction between the sun and the particular planet (Fig. 3.7b). The centripetal force on the moon as it rotates round the earth is provided by the gravitational attraction between the earth and the moon.

A spacecraft in orbit, and an astronaut inside, require a certain centripetal force to keep them moving round the earth. This is provided by the gravitational attraction of the earth. If the weight of the astronaut at this height is just equal to



Plate 3C Astronaut floating in skylab workshop (*Elisabeth Photo Library*)

Scanned with CamScanner

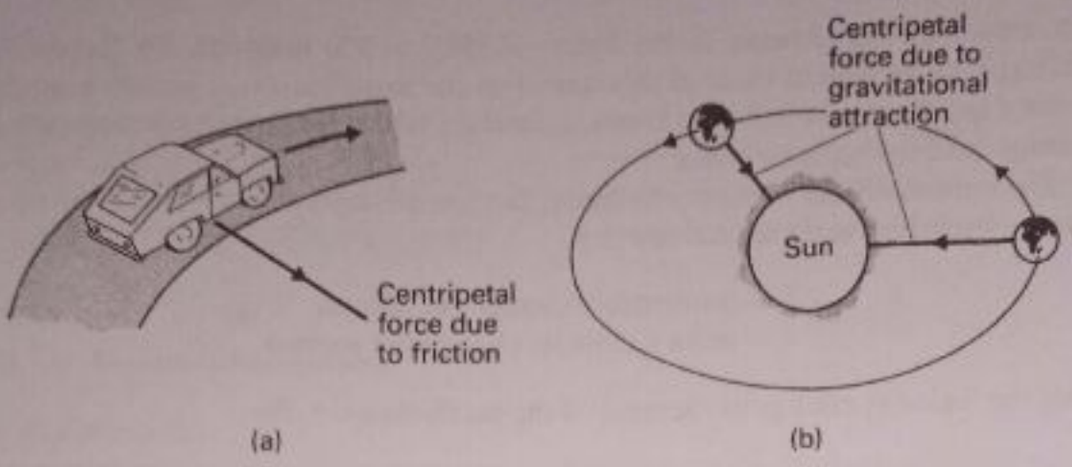


Fig. 3.7 Types of centripetal force

the required centripetal force, he will have no tendency to sink to the floor of the spacecraft or to rise to the ceiling. Other objects in the spacecraft also have this kind of 'weightlessness', and they will float about if not tied down in the spacecraft.

Momentum

Force and momentum

We now consider an important quantity a moving object has, called its linear *momentum*.

The momentum of a moving object was defined by Newton as the product of its mass m and its velocity v, or mv. So a boy of mass 50 kg moving with a velocity of 1 m/s has a momentum of 50×1 or 50 kg m/s. Now we have already seen that a force produces a velocity change when it acts on an object. So, a momentum change occurs. Newton defined force in his second law by stating *force is directly proportional to the rate of change of momentum produced.*

To show how we apply this law, suppose a boy or girl on a bicycle, total mass 50 kg, has a velocity of 1 m/s and that, when pedalling faster for 5 s, the velocity increases to 3 m/s. Then, since 'rate of change' means 'change per second'

$$\text{average force} = \frac{\text{momentum change}}{\text{time}} = \frac{(50 \times 3 - 50 \times 1)\ \text{kg m/s}}{5\ \text{s}}$$

$$= \frac{150 - 50}{5} = \frac{100}{5} = 20\ \text{kg m/s}^2$$

1 kg m/s^2 is the force acting on a mass of 1 kg which gives it an acceleration of 1 m/s^2. By definition, this force is 1 newton. So the force in our example above is 20 newtons. If the cyclist pedals harder, so that his or her velocity increases from 1 m/s to 3 m/s in 2 s instead of in 5 s, the momentum change per second is greater than before. The force is now given by

$$\frac{50 \times 3 - 50 \times 1}{2} = 50\ \text{kg m/s}^2 = 50\ \text{N}$$

Suppose the cyclist, moving with a momentum of 150 kg m/s, applies the brakes slowly and comes to rest steadily in 6 s. Then the momentum change per second = 150/6 = 25 newtons. If the cyclist pulls up sharply and comes to rest in

Scanned with CamScanner

$\frac{1}{2}$ s, however, the slowing down force $= 150/\frac{1}{2} = 300$ newtons. So the cyclist suffers a more violent force in this case. For the same reason, a person jumping from a height should bend the knees on landing. This makes a slower momentum change and so the force is less.

Since momentum = mass × velocity, the force F acting on a constant mass is given, from Newton's second law, by

$$F = \text{momentum change per second}$$
$$= \text{mass} \times \text{velocity change per second}$$

But the 'velocity change per second' is the acceleration a. So

$$F = ma$$

This result was first given on page 39. Note that 'momentum' is a *vector*; its direction is that of the velocity.

From Newton's law, we see that if a constant force F acts for a time t, then $F \times t$ = momentum change. The product $F \times t$, or *force* × *time*, is called the *impulse* of the force. Impulse produces momentum change.

Example on momentum

(a) State a unit of momentum in the SI system.
(b) Sand falls gently at a constant rate of 50 g/s onto a horizontal belt moving steadily at 40 cm/s. Find the force in newtons on the belt exerted by the sand.

(a) Since force F = momentum change/time, then momentum change = $F \times$ time. A momentum unit is thus *newton second*. Also, as momentum is mass × velocity, another unit is kg m/s.

(b) F = momentum change per second = mass of sand per second × velocity change

Neglecting the sand's initial velocity, and using 50 g = 0·05 kg and 40 cm = 0·4 m,

$$F = 0{\cdot}05\ (\text{kg/s}) \times 0{\cdot}4\ (\text{m/s})$$
$$= 0{\cdot}02\ \text{N}$$

Action and reaction

When you lean on a table with your elbow the downward force of your elbow on the table, called the 'action' force, produces an opposite upward force called the 'reaction'. This is the force on your elbow due to the table. When you press harder on the table, so increasing the 'action', the reaction also increases. Newton, in his third law of mechanics, stated that *action and reaction are equal and opposite*.

So if a person strikes a brick wall with the knuckles the force on the wall is equal and opposite to the force on the knuckles on impact. Although the forces are equal the effect on the wall is negligible but that on the knuckles is considerable! The earth attracts you with a force equal to your weight; from the law of action and reaction, you attract the earth with an equal and opposite force.

Momentum changes due to action and reaction

The momentum change produced by forces of action and reaction can be studied by using two trolleys A and B (Fig. 3.8a).

Scanned with CamScanner

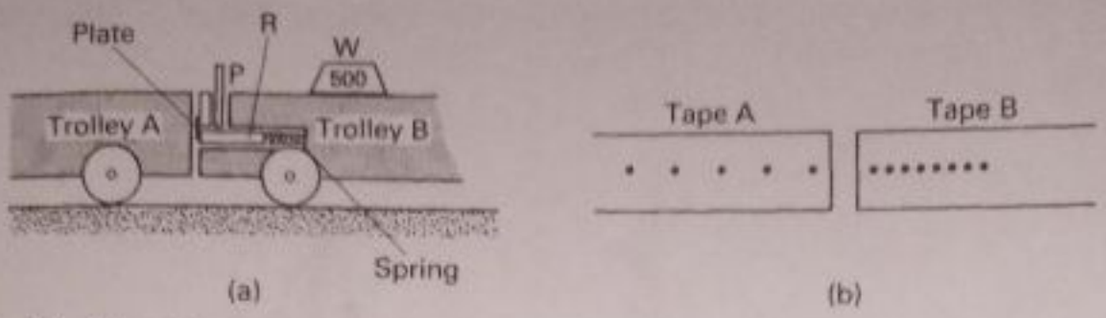


Fig. 3.8 Momentum measurement

Experiment

One trolley B, with a mass *W* of say 500 g on it, has a projecting wooden rod R. This is pushed back to compress a spring and then held in this position by a plate. Trolley A is then placed in contact with trolley B. Ticker-tapes are attached to A and B.

A projecting wooden peg P is tapped sharply to free the plate. The released spring gives a momentum to A and B, which move off in opposite directions. The tapes are then removed and the *initial* velocities are now calculated from the spacing of the dots (see Fig. 3.8b).

Results: In one experiment, results were as follows.

Trolley A: mass = 620 g, initial velocity = 20 units
Trolley B: mass = 1120 g, initial velocity = 11 units

Calculation: Momentum change of A = $620 \times 20 = 12400$ units
Momentum change of B = $1120 \times 11 = 12320$ units

Conclusion: Allowing for experimental error,
momentum change of A = momentum change of B

Similar results were obtained on varying the masses A and B.

The result above can easily be explained. From Newton's law, action and reaction are equal and opposite. So the force on A by the rod is equal to the force on B. The momentum change per second of A is hence equal to that of B. But the forces on A and B act for the same time. So

momentum change of A = momentum change of B

When a bullet is fired from a rifle, the rifle jerks backwards. Suppose a bullet of mass 15 g leaves a rifle of mass 3000 g with a velocity of 100 km/h. Since

momentum change of rifle = momentum change of bullet
$\therefore \quad 3000 \times v = 15 \times 100$

where v is the backward velocity of the rifle.

$$\therefore \quad v = \frac{1500}{3000} = 0{\cdot}5 \text{ km/h}$$

Applications of momentum change

There are many applications of momentum change caused by forces of action and reaction. Whittle used the reaction force when he invented the *jet aeroplane*. Here a jet of very hot gases is pushed out or expelled from behind the aeroplane; the speed and mass per second of the gas are so high that the gas has considerable momentum (Fig. 3.9a). An equal and opposite momentum is given to the

Scanned with CamScanner



Plate 3D Hot gases sent out from the rear of a rocket engine at very high speed produce an enormous reaction force (about 30 million newtons) which lifts the spacecraft into space

aeroplane, which then has a big forward thrust or force. The same principle is used in *rockets* for launching satellites. Jets of hot gas steam downwards; an equal and opposite momentum is given to the rocket, which then moves upwards. *Sprinklers* on lawns throw water out in one direction. The sprinkler itself spins round in the opposite direction (Fig. 3.9b).

Scanned with CamScanner

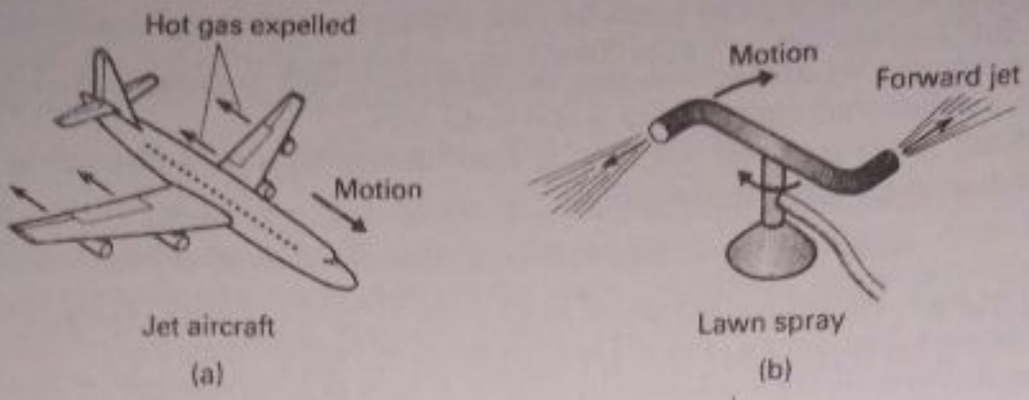


Fig. 3.9 Forces of reaction

Conservation of linear momentum

Action and reaction forces also occur when objects collide. The momentum changes can again be studied with two trolleys and ticker-tapes, as shown in Fig. 3.8. The trolley A is given a push so that it travels with uniform velocity and collides with trolley B, which is stationary. Both trolleys then stick together and move off with the same velocity.

The results show that, when objects collide, *their total momentum before collision = their total momentum after collision.*

Provided, then, that no forces other than those due to collisions act on them, the total momentum of colliding objects is constant. This is called the *Principle of the Conservation of Linear Momentum.*

We can see why the principle is true from the fact that action and reaction are equal and opposite. Suppose a moving ball X collides head-on with another moving ball Y. The 'action' force on Y produces a momentum change in Y of say +100 units. The 'reaction' force on X then produces a momentum change in X of −100 units, since the forces are equal and act for the same time. The *total* change of momentum due to the collision = +100 − 100 = 0. So the initial total momentum of X and Y = the final total momentum after collision.

Examples on momentum

1. A ball A of mass 100 g moving with a velocity of 5 m/s makes a 'head-on' collision with a ball B of mass 200 g moving with a velocity of 1 m/s in the opposite direction. If A and B stick together after the collision calculate their common velocity v in the direction of A.

Remembering that momentum is a vector quantity, that is, it has direction as well as magnitude,

initial total momentum of A and B $= (+100 \times 5) - (200 \times 1) = 500 - 200 = 300$

final total momentum after collision $= (100 + 200)v = 300v$

From the Principle of Conservation of Linear Momentum

$$300\,v = 300, \text{ or } v = 1 \text{ m/s}$$

This is the velocity in the direction of A; if v had worked out to be a minus quantity, the velocity would have been in the opposite direction.

2. A rocket expels gas at the rate of 0·5 kg/s. If the force produced by the rocket is 100 newtons, what is the velocity with which the gas is expelled?

$$\text{Force} = \text{rate of change of momentum}$$
$$= \text{mass of gas per second} \times \text{velocity } (v)$$
$$\therefore \quad 100 = 0{\cdot}5 \times v$$
$$\therefore \quad v = \frac{100}{0{\cdot}5} = \frac{1000}{5} = 200 \text{ m/s}$$

Scanned with CamScanner

Work

In mechanics we usually associate the term work with movement. An engine which pulls a train along a track is said to do *work*.

Work is always done whenever a force moves through a distance in the direction of the force. By definition

$$\textit{Work done } W = \textit{force} \times \textit{distance moved in direction of force} \quad \ldots \quad (1)$$

The *joule*, symbol J, is the unit of work in the SI system of units. This is defined as *the work done when a force of* 1 *newton moves through a distance of* 1 *metre in its own direction*. So the work done by a force of 50 N moving through 10 metres in its own direction is

$$50 \times 10 = 500\,\text{J}$$

Memorise that:

$$1\ \textit{newton} \times 1\ \textit{metre} = 1\ \textit{joule}$$

Approximately 1 J of work is done in raising an average-size apple (about 1 N weight) steadily from the floor to a table of height about 1 m. The joule is also the unit of energy of all kinds, for example mechanical energy, electrical energy and heat energy.

As we see later, there are many different examples of work. Muscles in the human body can do work by throwing, lifting or pulling objects. A world class shot putter can throw a 7 kg shot more than 21 m.

Calculations of work

From the relation (1) for work W, if an engine force of 5000 N pulls a train 100 m

$$W = 5000 \times 100 = 500\,000\,\text{J}$$

If a sledge or cart is pulled 40 m with a horizontal force of 200 N

$$W = 200 \times 40 = 8000\,\text{J}$$

It should be carefully noted that in calculating the work done from (1), the 'distance moved' must be that *in the direction of the force*. A horse pulling a barge C (Fig. 3.10a) along a canal by a rope at an angle 60° to the bank may produce a tension (force) of 1000 N in the string, but the distance moved along the canal from A to B, for example, is not in the same direction as the tension. The force pulling the barge in the direction A to B is actually the *component* in the direction AB of the tension. This is 1000 N × cos 60 or 500 N (see page 94). So if AB is 40 m

$$\begin{aligned}\text{work done } W &= \text{force in newtons} \times \text{distance in metres} \\ &= 500 \times 40 = 20\,000\,\text{J}\end{aligned}$$

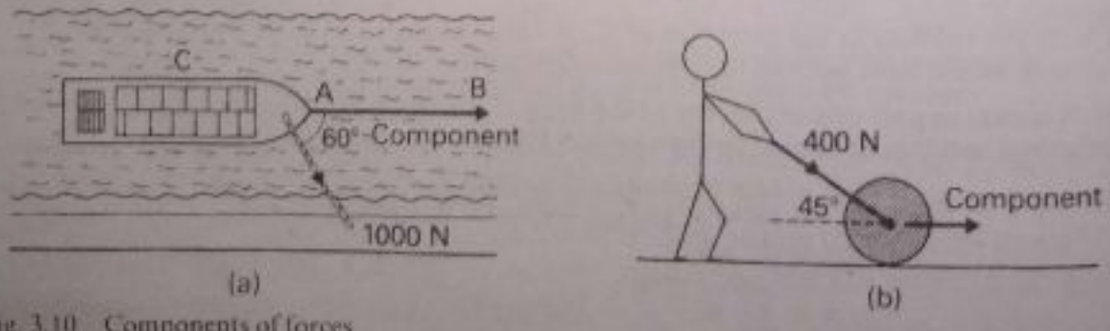


Fig. 3.10 Components of forces

Scanned with CamScanner

Similarly, if a lawn roller is pushed with a force of 400 N at an angle of 45° to the lawn, the force acting along the ground is 400 cos 45° or 280 N (Fig. 3.10b). So if the roller is pushed along a distance of 20 m,

$$\text{work done } W = 280 \times 20 = 5600 \text{ J}$$

Fig. 3.11a shows an object weight W on a smooth inclined plane, which is displaced steadily from X to Y higher up the plane by a force F. The reaction R of the plane on the object acts at 90° to the plane, and since it has no component along XY, *R does no work*. The work done by F is $F \times$ XY; or, since $F = W \cos\theta$, where θ is the angle between the vertical and the plane, the work done is

$$W \cos\theta \times \text{XY}$$

Fig. 3.11b shows an object whirled round in a horizontal circle centre O by a string attached to it, which has a force or tension F. The velocity or displacement of the object is along the *tangent* to the circle at points such as P and Q as it moves; and so the displacement at P, for example, acts at 90° to F. So no work is done by F as the object is whirled round the circle.

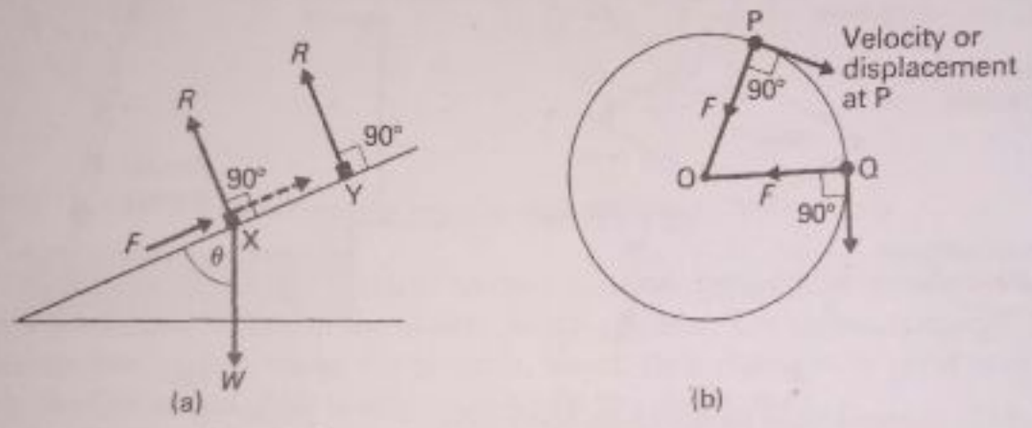


Fig. 3.11 Forces at 90° to displacements

Energy

A pile-driver is a heavy concrete weight used, for example, to drive a thick stake or pile of timber into the sea-bed when constructing foundations for a pier or bridge. The pile-driver is first raised high above the pile by an engine. Here, the pile-driver is stationary. But when it falls and crashes into the pile it drives it into the sea-bed. So the pile-driver does work against the resistance of the sea-bed.

We now see that the pile-driver had energy when it was held stationary above the pile, that is, *it was capable of doing work*. If it was raised higher it was capable of driving the pile further into the sea-bed when it was dropped. So in this new position it had more energy than before.

Energy is defined as the capacity for doing work.

Of course, the energy of the stationary pile-driver high above the pile comes from the energy spent or work done by the engine in raising it to this height. Suppose the pile-driver has a weight of 660 N and is raised 20 m steadily. The force lifting the pile-driver is then 660 N. So

$$\begin{aligned}\text{energy of stationary pile} &= \text{work done}\\ &= \text{force} \times \text{distance}\\ &= \text{weight of pile} \times \text{distance}\\ &= 660\,(\text{N}) \times 20\,(\text{m}) = 13\,200\,\text{J}\end{aligned}$$

Scanned with CamScanner

Potential energy

When it is stationary high above the pile, the pile-driver is said to have potential energy.

Potential energy is defined as energy possessed due to level or position.

There are many examples of potential energy. A stationary brick on a shelf above the ground has potential energy; if it is pushed off the shelf and crashes into a sheet of glass near the floor it will splinter the glass into many pieces (Fig. 3.12a). The moving splinters of glass, some of which may be thrown through big distances, show that the stationary brick on the shelf had potential or stored energy which it then transferred to the glass. The flying pieces of glass have kinetic energy, or energy of motion, discussed soon.

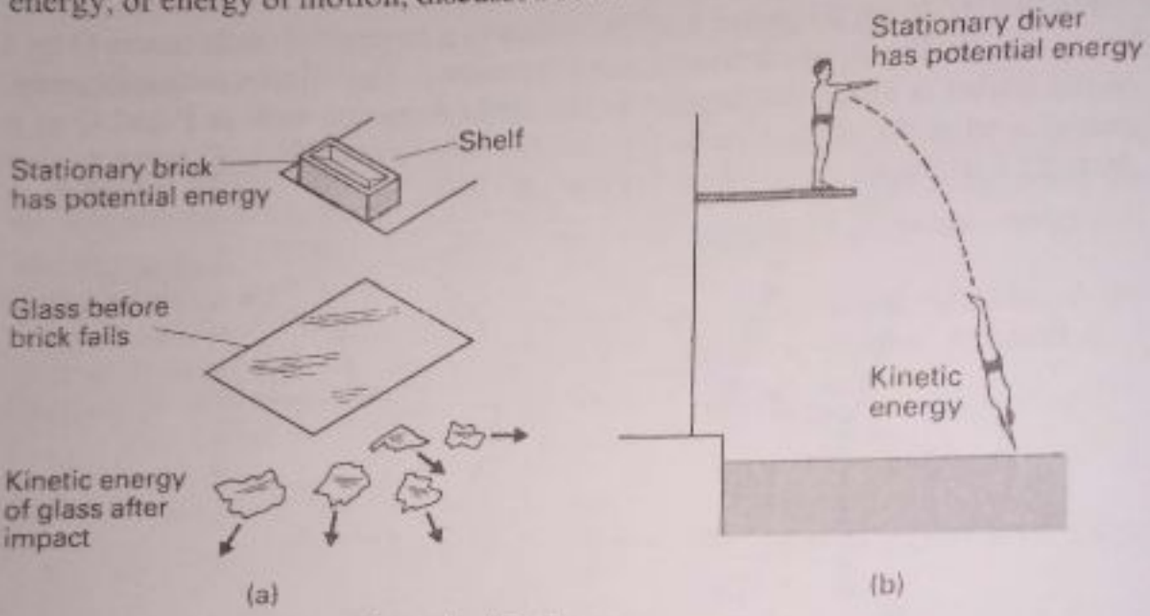


Fig. 3.12 Potential energy transformed to kinetic energy

A high-diver on a platform, ready to dive, has a potential energy equal to the work done in raising himself (Fig. 3.12b). So

$$\textit{potential energy} = \textit{weight} \times \textit{height}$$

Just before he dives into the water the potential energy is transformed to kinetic energy, which is then used up in doing work against the resistance of the water.

If the diver has a mass of 60 kg, his weight is $mg = 60 \times 10 = 600$ N, assuming $g = 10\ \text{m/s}^2$. So if the weight is 6 m above the water

$$\begin{aligned}\text{potential energy} &= \text{weight } (mg) \times \text{height } (h)\\ &= 600\ (\text{N}) \times 6\ (\text{m}) = 3600\ \text{J}\end{aligned}$$

Generally, potential energy, P.E., can always be calculated from

$$\text{P.E.} = mgh$$

Hydroelectric power stations use the potential energy of water as it falls to a lower level under gravity. It then moves with much kinetic energy and pushes against blades attached to a large wheel, which therefore turns round. The wheel drives an electrical generator and electrical energy is produced.

Other potential energies

The examples above are of potential energy due to work done in moving against the force of gravity. A different example of potential energy is that of a wound

Scanned with CamScanner

spring. Unwound, the spring has a certain amount of potential energy. When it is coiled it has a greater amount of potential energy (Fig. 3.13a). If it were inside a clockwork car, for example, as the spring gradually unwinds, the wheels of the car move. So work is done as the potential energy is used up. The potential energy of the spring comes from the work done in compressing the spring against the elastic forces in the metal, which are due to forces between their molecules (page 152).

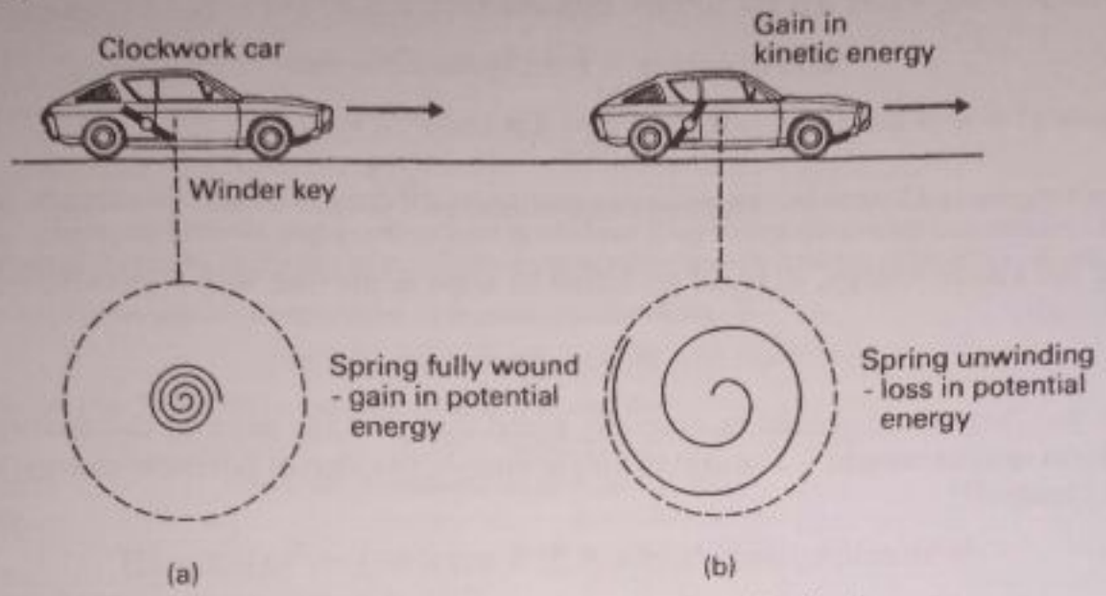


Fig. 3.13 Molecular potential energy of wound spring transformed to kinetic energy

The stretched elastic of a catapult has potential energy equal to the work done against molecular forces in the elastic. When released, the elastic transfers its molecular energy to a stone, for example, which then moves with great speed. The bow of an archer gives energy to an arrow in a similar way.

Kinetic energy

When a fast-moving ball crashes into a window pane splinters of glass are thrown through big distances. A moving train coming to rest at a station may compress powerful springs at the buffers. Since it can do work, these examples show that a moving object has energy. It is called kinetic energy.

Kinetic energy is energy due to motion. In contrast, potential energy is the energy of a stationary object due to its level or position.

In a general way we can see what factors affect the kinetic energy of a moving object. A massive rugby player running with the ball can charge his way through direct opposition more easily than a lighter man moving with the same speed. So the magnitude or size of the kinetic energy depends on the *mass* of the moving object; the greater the mass, the greater is the kinetic energy.

The kinetic energy also depends on the speed or *velocity* of the moving object. The faster a rugby player runs, the more easily can he push through opposition directly in his path. So the greater the velocity, the greater is the kinetic energy of a moving object.

A bullet is a very light object. But because it moves with very high speed it has a large amount of kinetic energy and penetrates deeply into the flesh. A cricket ball is much heavier. Moving with far less speed than a bullet, it can knock a stump out of the ground. Similarly, a heavy mallet moving with slow speed can make a peg penetrate deeply into the ground.

Scanned with CamScanner

Formula for kinetic energy

We can find the kinetic energy of an object of mass m moving with velocity v by calculating the work done by the force in giving the mass a velocity v from rest.

Suppose the force F acts for a distance s on the mass.

Then work done $= F \times s =$ kinetic energy gained

Now $F = ma$, where a is the acceleration due to F.

So kinetic energy $= F \times s = ma \times s = mas$

Since $v^2 = u^2 + 2as$ and $u = 0$, then $v^2 = 2as$ and $v^2/2 = as$.

So $$\text{kinetic energy} = mas = \frac{mv^2}{2}$$

So the kinetic energy, K.E., of an object of mass m moving with a velocity v is given by

$$\text{K.E.} = \tfrac{1}{2}mv^2$$

In this formula, it should be carefully noted that m is the *mass* of the moving object (not its 'weight'). The unit of kinetic energy, like that of potential energy, is the joule (J):

With m in kg and v in m/s, K.E. is equal to $\frac{1}{2}mv^2$ in joules (J)

Kinetic energy calculations

A boy or girl of mass 50 kg running with a velocity of 2 m/s would have kinetic energy given by

$$\text{K.E.} = \tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 50 \times 2^2 \text{ J}$$
$$= \tfrac{1}{2} \times 50 \times 4 \text{ J} = 100 \text{ J}$$

If the velocity was doubled, the kinetic energy would increase *four* times as much, since it is proportional to the *square* of the velocity.

A bullet weighing 20 g (0·02 kg) moving with a velocity of 108 km/hour, or 30 m/s, would have a kinetic energy given by

$$\text{K.E.} = \tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 0{\cdot}02 \text{ (kg)} \times 30^2 \text{ (m/s}^2\text{)}$$
$$= \tfrac{1}{2} \times 0{\cdot}02 \times 900 = 9 \text{ J}$$

Examples on momentum and energy

1. A stationary sledge of mass 4 kg is acted on by a force of 8 N for 5 s. Calculate the gain of (a) momentum, (b) energy of the sledge.

(a) Since force (F) = rate of change of momentum

$$= \frac{\text{momentum gain}}{\text{time } (t)}$$

$F \times t =$ momentum gain by sledge

$\therefore$ 8 N × 5 s = momentum gain = 40 N s

(b) Uniform acceleration of sledge, $a = F/m = 8 \text{ N}/4 \text{ kg} = 2 \text{ m/s}^2$

$\therefore$ velocity reached in 5 s, $v = u + at = 0 + (2 \times 5) = 10 \text{ m/s}$

$\therefore$ K.E. $= \tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 4 \times 10^2 = 200 \text{ J}$

Scanned with CamScanner

2. A ball of mass 0·2 kg is dropped from a height of 20 m. On impact with the ground it loses 30 J of energy. Calculate the height it reaches on the rebound. (Assume $g = 10\,\text{m/s}^2$)

Initial energy of ball = potential energy
$= mgh = 0{\cdot}2 \times 10 \times 20 = 40\,\text{J}$

So energy after impact $= 40\,\text{J} - 30\,\text{J} = 10\,\text{J}$

At top of rebound, energy of ball = potential energy = mgh, where h is the height of rebound. So $mgh = 10\,\text{J}$. Therefore

$$0{\cdot}2 \times 10 \times h = 10$$

$$h = \frac{10}{0{\cdot}2 \times 10} = 5\,\text{m}$$

3. A stationary bomb of mass 5 kg explodes into one part A of mass 2 kg flying off with a velocity of 60 m/s and another part B of mass 3 kg flying off with a velocity v in the opposite direction. Find (a) v, (b) the total kinetic energy produced by the explosion.

(a) From law of conservation of momentum for A and B

final total momentum = initial total momentum

From Fig. 3.14, final total momentum of A and B =
(mass × velocity of B) − (mass × velocity of A) $= 3v - (2 \times 60)$

Now initial total momentum of A and B $= 0$

So $3v - 120 = 0$

from which $v = 40\,\text{m/s}$

(b) Total kinetic energy of A and B

$= (\frac{1}{2} \times 2 \times 60^2) + (\frac{1}{2} \times 3 \times 40^2) = (\frac{1}{2} \times 2 \times 3600) + (\frac{1}{2} \times 3 \times 1600)$
$= 3600 + 2400 = 6000\,\text{J}$



Fig. 3.14 Example

Potential and kinetic energy under gravity

Potential energy can be transformed to kinetic energy, and kinetic to potential energy. As an illustration, consider a box of mass 2 kg allowed to drop freely from rest from a height 5 m above the ground. At this height its energy is only potential energy, P.E. Assuming the weight of 1 kg mass is 10 N,

P.E. = weight × height $= 20\,\text{N} \times 5\,\text{m} = 100\,\text{J}$ . . . (1)

Just before it hits the ground the energy of the box is only kinetic energy, K.E. The velocity v is given by

$$v^2 = u^2 + 2as$$

Since $u = 0$, and $a = g = 10\,\text{m/s}^2$ (approx.)

$$v^2 = 0 + 2 \times 10 \times 5 = 100$$

$$v = \sqrt{100} = 10\,\text{m/s}$$

$$\therefore \text{K.E.} = \tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 2 \times 10^2\,\text{J} = \tfrac{1}{2} \times 2 \times 100\,\text{J}$$
$$= 100\,\text{J} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2), it follows that the potential energy at the height of 5 m is equal to the kinetic energy near the ground. So all the potential energy has been transformed to kinetic energy.

Scanned with CamScanner

When the box has fallen through a distance of 3 m say, from rest, the energy at this instant is partly potential and partly kinetic. The potential energy is given by

$$\begin{aligned} \text{P.E.} &= \text{weight} \times \text{height above ground} \\ &= 20\,\text{N} \times 2\,\text{m} = 40\,\text{J} \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The velocity v at this instant is given by

$$\begin{aligned} v^2 &= u^2 + 2as = 0 + 2 \times 10 \times 3 = 60 \\ \text{K.E.} &= \tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 2 \times 60\,\text{J} \\ &= 60\,\text{J} \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

From (3) and (4)

$$\text{total energy} = \text{P.E.} + \text{K.E.} = 40 + 60 = 100\,\text{J}$$

It therefore follows that the total mechanical energy of the box is *constant* as it falls to the ground. After hitting the ground and coming to rest the mechanical energy is zero. Most of this energy is transformed to heat energy; the internal (vibrational) energy of the box molecules is increased (page 146).

Energy of pendulum · Galileo's experiment

The change of P.E. to K.E., and K.E. to P.E., takes place throughout the motion of the bob of a simple pendulum (Fig. 3.15).

At the end A of its swing, for example, the bob has maximum P.E. and zero K.E. When it moves back to D, some of the P.E. is transformed to K.E. At its lowest point B, all the P.E. is transformed to K.E. So the K.E. is a maximum at B. When the bob swings up to C, its highest point on the other side, its P.E. is maximum again and its K.E. is zero.

Galileo showed that the kinetic energy at B was practically equal to the potential energy at A by placing a rigid peg at X well above B. The bob was released from A, and on reaching B the string was stopped by X so that the bob swung upwards. The bob reached a height P, practically on the same horizontal level as A. So the kinetic energy at B was equal to the potential energy at A. If the peg was moved to a position Y and the experiment repeated, the bob again swung up to Q at the same level as P or A. So Galileo's experiment shows that potential

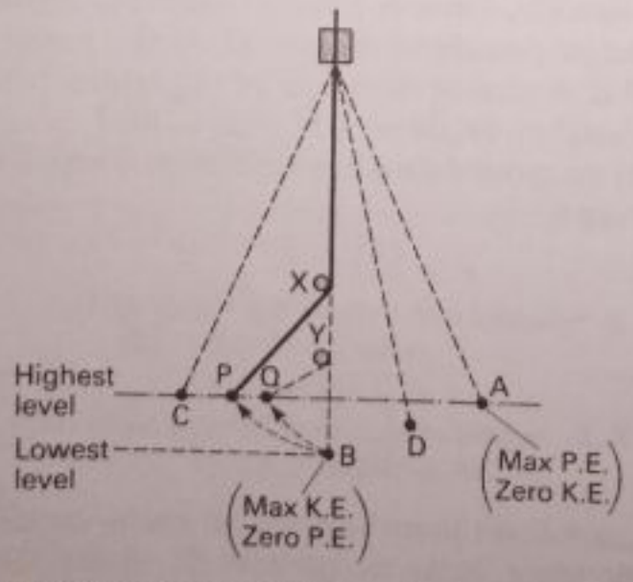


Fig. 3.15 Mechanical energy transfers in pendulum motion

Scanned with CamScanner

and kinetic energy in the earth's gravitational field can be transformed from one form to the other.

Effect of friction

Close examination of the bob of a swinging pendulum shows that the height reached on the other side of the swing is just below the original height or level. Some mechanical energy has been lost. This is due to *friction* between the bob and the air. Eventually the bob comes to rest; its energy is completely used up in doing work against the frictional forces.

We do not get back this mechanical energy–it is all transferred to heat energy. When a card is attached to the bob of a pendulum so that its plane is perpendicular to the motion of the bob, the bob comes to rest much more quickly. The frictional force is much greater in this case as the card sweeps through the air. So the mechanical energy is transformed to heat energy more quickly.

Principle of conservation of energy

The scientific meaning of energy grew slowly over the centuries. Among famous scientists who helped to make clear its meaning were Kelvin of Scotland, Joule of England, and Clausius and Helmholtz of Germany, all of whom lived in the 19th century.

'Energy' means 'the capacity for doing work'. Today, many different forms of energy are recognised. We have already discussed *mechanical energy*. A steam engine uses *heat energy* to do work. An electric motor uses *electrical energy* to drive an electric train. *Light energy*, falling on a light meter used in photography, makes a pointer move across a scale. *Sound energy* causes a microphone plate to vibrate. *Chemical energy* is the energy in our food which makes us grow and also gives us muscular energy to move objects. *Nuclear energy*, the energy in the nucleus of atoms, produces heat energy which can be used to generate electrical power.

By suitable machines or apparatus, energy can be changed from one form to another. This is illustrated on page 3. It is important to note that *in a given or closed system, the total amount of energy is always constant, although energy may be transformed from one form to another.*

This statement is known as the *Principle of the Conservation of Energy*. It is recognised today as one of the most important principles in science.

Power

Two boys of the same weight each set out from the bottom of a hill to reach the top walking at a steady speed. One, X, is strong and athletic. The other, Y, is the opposite type, weak and not athletic. X can reach the top in a much shorter time than Y. Walking steadily, each has overcome the same frictional forces at the ground, and each has raised the same weight through the same height. So, each has done the same amount of work. X, however, has done it in a *shorter time* than Y. We say that X has a greater *power* than Y. In the same way, a large engine can work faster than a small engine of the same kind and is said to have a greater power.

Scanned with CamScanner

Power is defined as *the rate of doing work or expending energy*, or as the work done or energy expended per second. Thus

$$\text{power} = \frac{\text{work done or energy expended}}{\text{time taken}} = \frac{\text{force} \times \text{distance moved}}{\text{time}}$$

Units of power

Suppose an engine raises 100 kg of water steadily through a height of 30 m in 10 s. The upward force used is equal to the weight of water raised = mass $\times g$ = 100×10 N, using $g = 10$ N/kg (approx.)

So the power (P) at which the engine works is calculated by

$$P = \frac{\text{force} \times \text{distance moved}}{\text{time}} = \frac{100 \times 10\,(\text{newton}) \times 30\,(\text{m})}{10\,\text{s}} = \frac{30000\,\text{J}}{10\,\text{s}}$$

$$= 3000\,\text{J/s}$$

The *watt*, W, is a practical unit of power. It is defined as 1 J/s rate of working.

So $P = 3000$ watts $= 3$ kilowatts (kW)
since 1 kW = 1000 watts.

As we see later, electrical energy is measured in joules and electrical power in watts (or kilowatts). A lamp of 150 watts uses energy at the rate of 150 J/s. It produces a brighter light than a lamp of 100 watts.

Suppose a car travels with a uniform velocity of 72 km/h or 20 m/s along a horizontal road and overcomes a constant frictional force of 500 N while moving. Then

engine power = work done per second = force $\times$ distance per second
$= 500 \times 20$ J/s
$= 10000$ W $= 10$ kW

1 *horse-power* (h.p.) is 746 watts, about $\frac{3}{4}$ kW

$$\therefore \text{ engine power} = \frac{10}{3/4} = \frac{10 \times 4}{3} = 13\tfrac{1}{3}\,\text{h.p.}$$

Since 1 h.p. is about 750 W, a small 1/6 h.p. motor in a vacuum cleaner uses about 125 W when working.

Estimation of power

A rough estimate of a pupil's power can be made by asking him or her to walk steadily up a flight of stairs.

Experiment

Measure the height of each step and so find the height of a flight of stairs about 5 metres high. Using a stopwatch, walk steadily up these steps to the top and find how long this takes.

Scanned with CamScanner

Result: Suppose the time taken is 6·2 s, the height is 5·0 m and the weight of the pupil is 500 N. Then

$$\text{average power} = \frac{\text{work done}}{\text{time}} = \frac{500 \times 5}{6{\cdot}2} = \frac{2500}{6{\cdot}2} = 400\,\text{W (approx.)}$$

Find the new value of the power when running steadily up the stairs.

In medical research, the power of an athlete is measured by cycling on rollers fitted with a friction brake. This introduces a known frictional force. The athlete does work against this force and the power is calculated from the speed with which he or she pedals.

Examples on energy, power and force

1. A car of mass 1000 kg is travelling along a straight road at a constant velocity of 15 metres per second. The engine then develops 3·0 kilowatts. Calculate the force resisting the motion. If the engine is switched off, how much energy will the car lose in coming to rest? Describe the energy changes in this process. (L)

Since the car travels at a constant velocity, its acceleration is zero. So the resultant force is zero.

$$\therefore \text{ force resisting motion} = \text{force } F \text{ due to engine}$$

But

$$F \times \text{distance per second of car} = \text{work done per second by engine}$$
$$= \text{power of engine} = 3000 \text{ watts}$$
$$\therefore F \times 15 = 3000$$
$$\therefore F = \frac{3000}{15} = 200\,\text{N} = \text{force resisting motion}$$

When the engine is switched off,

$$\text{kinetic energy of car} = \tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 1000 \times 15^2 = \tfrac{1}{2} \times 1000 \times 225$$
$$= 112\,500\,\text{J} = \text{energy lost coming to rest}$$

The kinetic energy of motion is transformed to heat energy in overcoming frictional forces and wind resistance when the car comes to rest.

2. A cable car is pulled up a slope by a constant force of 5000 N at a uniform speed of 6 m/s. It takes 4 min to complete the journey.
(a) How much work is done in getting the car to the top of the slope? (b) How much work would be done if the speed were 12 m/s (the force remaining the same)? (c) How does the power developed in (a) compare with (b)? (L)

(a) Work done = force × distance moved in 4 min (4 × 60 s)
$$= 5000\,(\text{N}) \times 6 \times 4 \times 60\,(\text{m}) = 5000 \times 1440\,\text{J}$$
$$= 7\,200\,000\,\text{J} = 7{\cdot}2 \times 10^6\,\text{J}$$

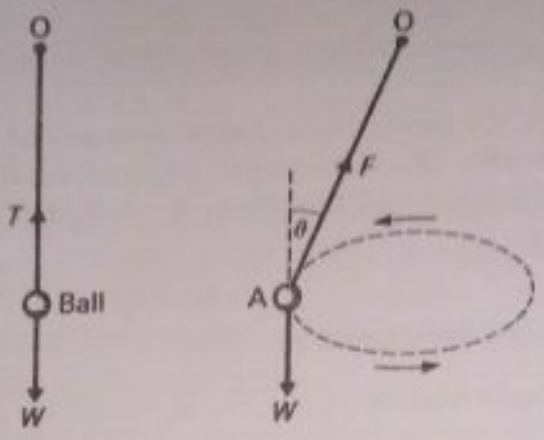
(b) Since the resistive force to the motion is again 5000 N, the work done at 12 m/s is the *same* as at 6 m/s. The time taken would be less.

(c) The work done in (a) is completed in 4 min; the work done in (b) is completed in 2 min since the speed is twice as great. So the *power* or rate of working in (a) is half that in (b).

3. A boy holds the end O of a rubber cord from which hangs a solid metal ball.
(i) When the ball is still, as in Fig. 3.16a, the rubber stretches. Draw a diagram showing the forces acting on the ball. What do you know about these forces?
(ii) In Fig. 3.16b, the boy whirls the ball in a horizontal circle, keeping his hand still. Draw a diagram showing the forces acting on the ball in the position A. Explain carefully why the rubber cord stretches more in (b) than in (a).
(iii) Explain why each of the forces in (b) does no work as the ball rotates.
(iv) If the rubber cord breaks when the ball is at A, state clearly the direction in which the ball will move, giving a reason for your answer.

Scanned with CamScanner

(v) At A the ball is 1·25 m above the ground. Calculate the tim it will take to reach the ground after the cord breaks. (The time is the same as if the ball were to fall vertically from rest.)
(vi) If at the moment the cord breaks the ball is moving at 1·5 m/s, what will be the horizontal distance from A when it reaches the ground? (I



(a) (b)

Fig. 3.16 Example

(i) The forces on the ball are its weight W and the tension (force) T in the rubber cord (Fig. 3.16a). These forces are equal.

(ii) When the ball is whirled in a horizontal circle, Fig. 3.16b, the force in the cord must provide a vertical force to balance the downward weight W plus a force to keep the ball moving in a circle. So the force F in the cord must be greater than the force T in Fig. 3.16a, which was only equal to W. Since the force in the cord is greater, the cord stretches more in (b) than in (a).

(iii) The weight W of the ball in (b) is *perpendicular* to the direction in which the ball moves. So no work is done by W. The force F has a vertical component and a horizontal component, each of which is perpendicular to the direction of motion of the ball. So F does no work.

(iv) If the cord breaks at A, the ball will move in the direction of the *tangent* to the circle at the point A, because the velocity of the ball is along the tangent of the circle at A.

(v) From $s = \frac{1}{2}gt^2$, then with $g = 10\ \text{m/s}^2$,

$$1{\cdot}25 = \tfrac{1}{2} \times 10 \times t^2 = 5t^2$$

So

$$t^2 = \frac{1{\cdot}25}{5} = 0{\cdot}25$$

$$\therefore\ t = \sqrt{0{\cdot}25} = 0{\cdot}5\ \text{s}$$

(vi) Horizontal distance = velocity × time

$$= 1{\cdot}5 \times 0{\cdot}5 = 0{\cdot}75\ \text{m}$$

Summary

1. *Mass* is a measure of inertia. Unit: kilogram (kg), gram (g)
   Force produces acceleration. Unit: newton (N)
   Approximately, 1 kg mass has a weight of 10 N.
2. $F = ma$, where m is mass, a is acceleration, F is force. With friction or other opposing forces, F always represents the *resultant* force. Force is a vector. $g = 10\ \text{m/s}^2$ or 10 newtons per kilogram mass.
3. *Momentum* = mass × velocity. Force is equal to the rate of change of momentum. Momentum is a vector.
4. *Conservation of linear momentum.* When bodies collide or interact and no

Scanned with CamScanner

external forces act, the total momentum in any direction before collision = the total momentum in the same direction after collision.

5. *Work* = force × distance moved in direction of force. Unit: joule (J); 1 joule = 1 newton × 1 metre. Work is a scalar quantity.

6. *Energy* is the capacity for doing work, and has the same units as work.

Kinetic energy is energy due to motion and is $\frac{1}{2}mv^2$, where m is the mass and v is the velocity. *Potential energy* is energy due to level or position and is weight × height above a particular level. Energy is a scalar quantity.

7. *Conservation of energy.* In a given or closed system, energy can be changed from one form to another, but the total amount of energy is constant.

8. *Power* is the rate of doing work or expending energy. Unit: joule per second or watt (1 watt = 1 joule per second). Approx. 1 h.p. = 750 watts.

Exercise 3 (Answers, page 640)

(Assume where necessary, and unless stated otherwise in the question, $g = 10\,m/s^2 = 10\,N/kg$)

1. (a) Complete the following: (i) a newton is a unit of ..., (ii) a joule is a unit of ..., (iii) a watt is a unit of ....
(b) Which of these is a vector quantity? (N)

2. Name a unit of force. Define the unit you have named. (N)

3. A car of mass 500 kg accelerates at 2 m/s². What is the force acting on the car?

4. Find the acceleration of an object of mass 0·2 kg when a force of 0·12 N acts on it. If the object is accelerated from rest, find the distance travelled in $\frac{1}{4}$ min.

5. A sledge is pulled 6 m by a force of 80 N. What is the work done?

6. Calculate the kinetic energy of a trolley of mass 80 kg moving with a velocity of 6 m/s.

7. Niagara Falls are 50 m high. Calculate the potential energy of 5 kg of water at the top, relative to the bottom. What is the kinetic energy of this water just before it reaches the bottom, and what happens to the energy after the water reaches the bottom?

8. A 10 h.p. (7·5 kW) car travels at a uniform velocity of 15 m/s. State the work per second done by the engine. Hence calculate the frictional force overcome.

9. Calculate the kinetic energy of a bullet of mass 0·015 kg moving with a velocity of 400 m/s. If the bullet is brought to rest in 2 cm by a wooden block, find the average force of resistance due to the wood. Into what forms of energy is the mechanical energy of the bullet transformed?

10. Fill in the following table

Quantity	*Unit*
acceleration	...
force	...
...	joule
...	watt
...	kilogram
...	kilogram metre/second

(L)

11. A body of mass 0·5 kg, and initially at rest, is subjected to a force of 2 newtons for 1 second. Calculate (a) the change in momentum of the body during the second; (b) the change in kinetic energy of the body during the second. (N)

12. Explain the terms vector and scalar. Which of the following are vectors: speed, velocity, momentum, force, energy?

A ball of mass 100 g falls from a height of 3 m on to a horizontal surface and rebounds to a height of 2 m. Calculate the change (a) in momentum, (b) in kinetic energy, of the ball when it strikes the surface. (O and C)

13. A pump is used to spray water from a pool to form a fountain. Determine the minimum power of the pump if it ejects 45 litres per minute and the spray reaches an average vertical height of 4 metres. You may assume that 1 litre of water has a mass of 1 kg and that $g = 10$ metres per second².

Give one reason why the water, after returning to the pool, might have a different temperature from that at which it left the pump. (L)

Scanned with CamScanner

14. 'Energy is lost in the form of friction.' Criticise the truth of this statement from your knowledge of physics and explain the true connection between friction and the loss of energy. (L)

15. A body of mass 4 kg is acted on by a constant force of 12 N for 2 seconds. Calculate the kinetic energy gained by the body at the end of the time.

Two identical free-running trolleys are on a smooth horizontal runway. One trolley is at rest and the other approaches it at a constant speed of 20 cm/s. After impact the two trolleys together move at 10 cm/s. Explain this result, stating the physical principle involved. (N)

16. The graph in Fig. 3.17 shows the motion of a car which accelerates uniformly from rest to a velocity of 20 metres per second in 25 seconds and is brought to rest from that velocity with a uniform retardation of 4 metres per second^2 on a level, unimpeded, straight road. If the total distance travelled is 1 km, determine (a) the time during which the car is uniformly retarded, (b) the distance travelled at uniform velocity, (c) the total time taken to travel the 1 km.

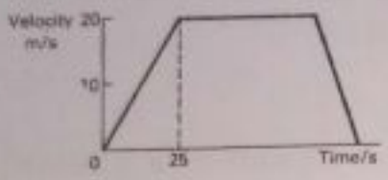


Fig. 3.17

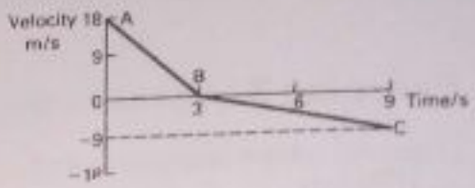


Fig. 3.18

If the engine exerts a constant force of 600 newtons during the initial acceleration, determine the constant force required to produce the retardation of the car. (L)

17. Explain the fact that a body on the moon has the same mass as, but would register less on a spring balance than, the same body on the earth. Explain why the spring balance shows this difference and why the beam balance or lever arm balance would not. (N)

18. A rocket is held fixed so that it ejects material horizontally with a speed of 160 m/s. If it ejects a mass of 1 kg each second, calculate the force needed to hold it fixed.

If the rocket were to be used to propel a sledge on a frozen lake, find, neglecting friction, the acceleration of the sledge at a time when the total mass was 150 kg. (O and C)

19. The lines AB, BC shown in Fig. 3.18 represent graphically the motion of an object which was projected up a rough incline with an initial velocity of 18 metres per second and returned to its starting point 9 seconds later with a velocity of 9 metres per second.

(i) Describe as fully as possible, stating any values you can determine, the motion, if any, of the object (a) between A and B, (b) at B, (c) between B and C.

(ii) Determine the distance the object travelled up the incline.

(iii) Name and explain the differences between the types of energy the object possessed due to its condition at A, B and C respectively.

(iv) Explain how you would conclude that the object had less energy at C than at A. Account for this loss of energy. (L)

20. Define momentum and kinetic energy. A trolley A of mass 2 kg travelling at 5 m/s collides with a stationary trolley B of mass 3 kg. After the collision the two travel on together at 2 m/s.

(a) What is the momentum of A before the collision?
(b) What is the momentum of A after the collision?
(c) Account for the change in the momentum of A.
(d) What is the kinetic energy of A before the collision?
(e) What is the kinetic energy of *each* trolley after the collision?
(f) During the collision, the kinetic energy gained by B is less than the kinetic energy lost by A. How much kinetic energy is unaccounted for, and what has become of it? (O)

21. Explain the meaning of the terms momentum, kinetic energy, force.

A motor car weighing 1000 kg travels at 48 km/h. Calculate (a) the momentum of the car, (b) the kinetic energy of the car, and find the braking force that can bring it to rest in 40 s. (O and C)

22. What is meant by (a) *force*, (b) *uniform acceleration*?

Sketch a graph showing the relation between time and distance fallen by a body falling freely from rest. How would you use this graph to determine the velocity of the body at any instant?

Scanned with CamScanner

A body of mass 1 kg falls freely from rest through a height of 75 m and comes to rest having penetrated 7·5 cm of sand. Calculate (a) the time taken in falling, (b) the velocity when the body strikes the sand, (c) the average force exerted by the sand in bringing the body to rest. (N)

23. Describe briefly the changes in the potential and kinetic energies of the bob of a simple pendulum as it goes from one side of its swing to the other. (C)

24. Explain the terms *mass, momentum, force* and describe an experiment which illustrates the conservation of momentum.

A firework rocket is burning fuel at the rate of 14 g/s. All the products of combustion are ejected in one direction at a speed of 210 m/s. What is the greatest weight the rocket can have if it is going to move vertically upwards? Give your answer in newtons. (O and C)

25. State the principle of the conservation of momentum.

A mechanical toy is made in the form of a bomb which can be set to explode into three parts. One part flies off in the direction due N and one in the direction due S. Why must the third part either go N or S or remain at rest?

A bucket of mass 5·0 kg hangs from a spring balance. It is being filled from a tap above it which delivers 64·0 kg of water in 240 s. The water is in a vertical jet moving at 9·0 m/s just before it hits the water in the bucket and after impact its vertical velocity can be taken as zero. How much vertical momentum of the jet is being lost per second? How much force is necessary to cause this loss? What will the spring balance read when there is 2·0 kg of water in the bucket? ($g = 9{\cdot}8\ m/s^2$) (O and C)

Multiple-Choice Questions

26. A force F newtons gives a mass of 2 kg an acceleration of $4\ m/s^2$. A force $3F$ newtons will give a mass of 8 kg an acceleration in m/s^2 of

A 1 B 3 C 6 D 12 E 24

27. Which of the units in A to E below could be used for kinetic energy:

A joules per second B kilograms C newtons
D kilogram metres per second E newton × metre

28. A ball is thrown vertically upwards. Which graph A to E shows how the force (y) due to gravity on the ball varies with velocity (x)?

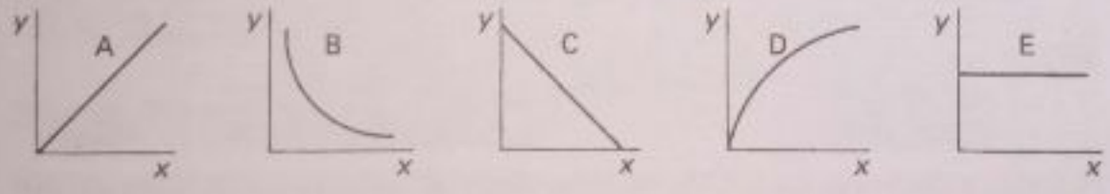


Fig. 3.19

29. A heavy ball is swung round in a horizontal circle of centre O with a constant speed. The acceleration of the ball in the circle is

A zero B along a tangent to the circle C away from O
D towards O E vertical

30. When a car moves round a circular path at constant speed, throughout its motion it has

A a displacement B velocity C acceleration D momentum
E kinetic energy

Which of these quantities is constant throughout?

Scanned with CamScanner

4. Forces and Moments

In dynamics we considered forces which keep an object in motion. In this chapter we consider how forces are used in machines such as levers and how they keep objects such as bridges stationary. This study is called statics.



Plate 4A Tower Bridge, London. The two halves of the roadway, each about 50 m long and 1000 tonnes mass, are raised by the turning effect or moment of forces produced by hydraulic machinery. Tall ships can then pass up the river. (*Fox Photos Ltd*)

Types of forces

A push and a pull are examples of forces. If someone pushes a pram they feel the force exerted by their muscles, which become taut (Fig. 4.1a). Ropes or strings

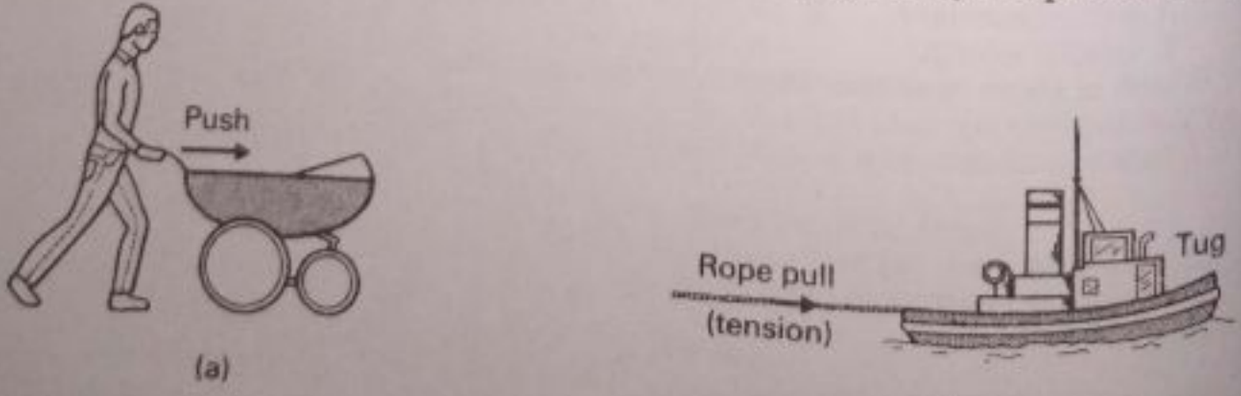


Fig. 4.1 Force may be push or pull

Scanned with CamScanner

are used to pull or support objects. When a tug uses a strong rope to pull a boat from a sandbank the rope becomes taut, and the force in the rope is called its tension (Fig. 4.1b). Fig. 4.2a shows the tensions T and S in a string which supports the weight W of a picture. The weight acts vertically downwards; the tensions together act upwards to support the weight.

As we saw on page 28, forces are vector quantities; that is, they have magnitude and direction. This is discussed further in a later chapter (page 91). *In all diagrams in statics, always draw the directions of the forces acting on the object, showing these with an arrow.* Fig. 4.2b for example, shows the forces on a ladder AB, whose weight W acts at a point C. If it rests against a smooth wall, a force of reaction P acts at right angles to the wall on the ladder. At B on the rough ground a force of reaction Q acts vertically upwards and a force of friction F acts along the ground and prevents the ladder slipping.

Fig. 4.2c shows forces R, S, T in the girders of part of a bridge structure, with joints at A, B and C.

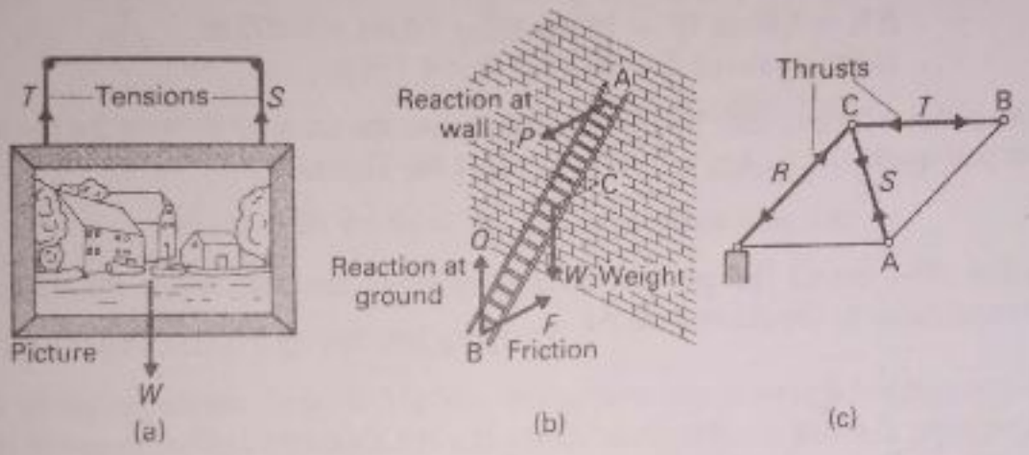


Fig. 4.2 Types and directions of forces

Moment of a force

Forces can produce turning effects, and this is widely used in everyday life and in machines. When a door is opened the force on the handle exerts a turning effect about the hinges (Fig. 4.3a). In the case of a bicycle, the force on the pedal produces a turning effect which keeps the wheels moving (Fig. 4.3b). The metal cap of a bottle of drink is levered off by a turning effect (Fig. 4.3c). Many

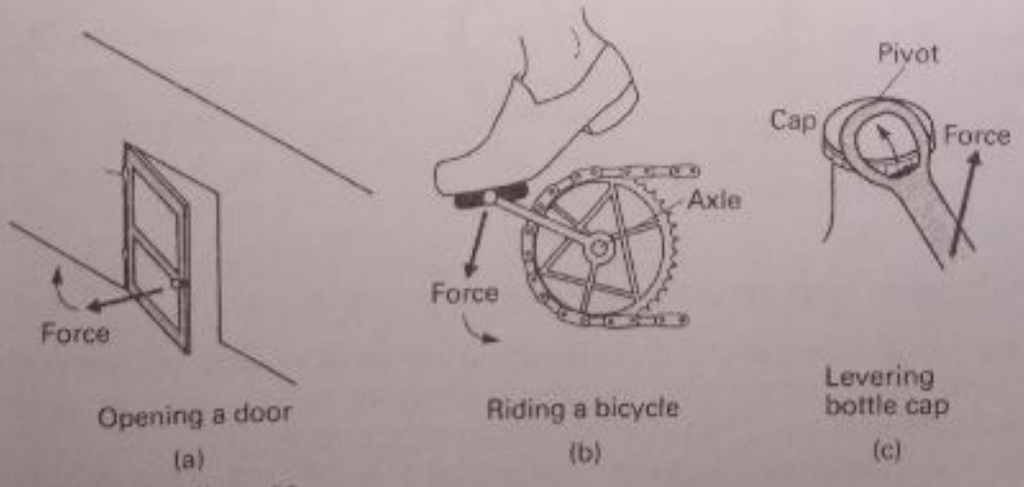


Fig. 4.3 Turning effects of forces

Scanned with CamScanner

machines in industry, such as printing presses and lathes, are kept in motion by the turning effects of forces.

The turning effect of a force is called its *moment*.

The moment of a force about a point or axis O = force × perpendicular distance from O to line of action of force, by definition. In the SI system of units, the force is measured in newtons (N) and the perpendicular distance is measured in metres (m). The moment is therefore expressed in *newton metre* (N m).

How perpendicular affects moment

Suppose the pedal P of a bicycle slips on its bearing and becomes horizontal to the ground (Fig. 4.4a). If the downward vertical force on the pedal is 20 N, the moment about the hinge at A = 20 × AX, where AX is the perpendicular length from A to the force. If the crank AP is 15 cm long and inclined at 30° to the vertical, then

$$AX = AP \sin 30° = 15 \sin 30° = 7{\cdot}5\text{ cm} = 0{\cdot}075\text{ m}$$
$$\therefore \text{ moment about A} = 20 \times 0{\cdot}075 = 1{\cdot}5\text{ N m}$$

If the pedal is turned to be in the correct position, the force of 20 N on the pedal is now perpendicular to AP, as shown in Fig. 4.4b. This time

$$\text{moment about A} = 20 \times AP = 20 \times 0{\cdot}15 = 3\text{ N m}$$

So the rider exerts the greatest turning effect or moment when the force is perpendicular to the crankshaft AP.

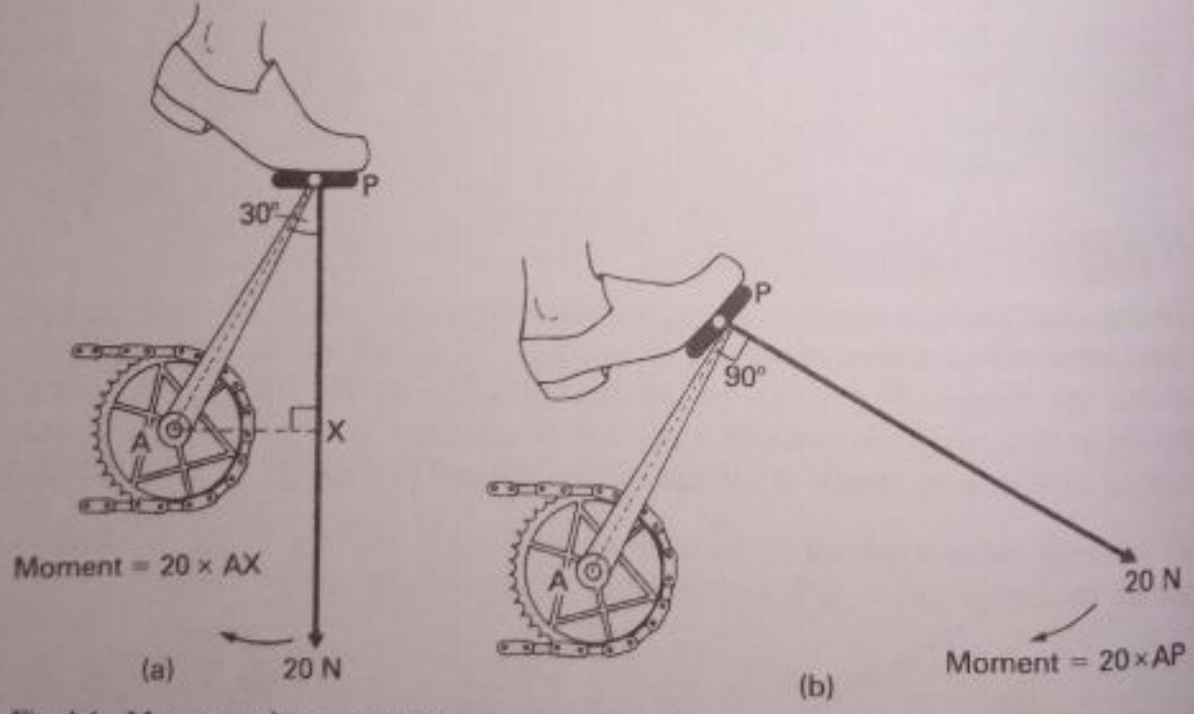


Fig. 4.4 Moment and perpendicular

Clockwise and anticlockwise moments

The moment (turning effect) of a force about a point or axis can be clockwise or anticlockwise. If we need to add or subtract moments, an anticlockwise moment can be called positive (+) and a clockwise moment negative (−).

Fig. 4.5a shows a light horizontal rod pivoted at B, with vertical forces of 40 N and 100 N at the ends. The 40 N force has an anticlockwise moment about B

Scanned with CamScanner

which is $40 \times 3 = 120$ N m. The 100 N force has a clockwise moment about B which is $100 \times x = 100x$ N m.

If the rod balances horizontally, the clockwise moment = the anticlockwise moment. So

$$100x = 120, \text{ or } x = 1{\cdot}2 \text{ m}$$

Fig. 4.5b shows a heavy box on a flat surface, with horizontal force P of 15 N pulling at the top edge. Its clockwise moment about C on the surface = $15 \times 2 = 30$ N m. The weight of the box is 20 N and this vertical force is 1 m from C as shown. Its anticlockwise moment about C = $20 \times 1 = 20$ N m. So the moment of the force P about C is greater than that of the weight, and the box overturns at C.

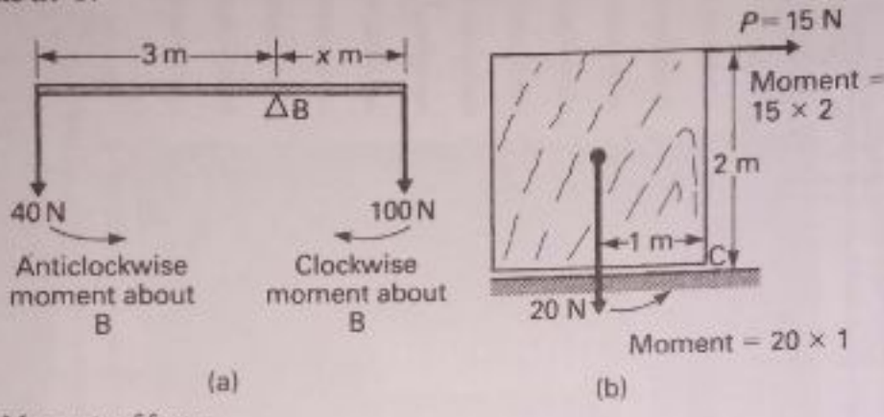


Fig. 4.5 Moments of forces

Equilibrium of pivoted object

With the apparatus shown in Fig. 4.6, we can investigate the relationship between the moments of force which keep a pivoted object such as a beam in equilibrium.

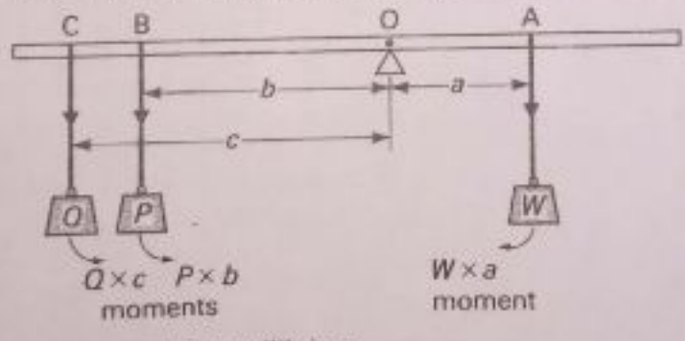


Fig. 4.6 Experiment on moments in equilibrium

Experiment

Suspend a light rod or ruler from its mid-point O. Attach a weight W such as 0·5 N at A and two smaller weights P and Q at B and C. Move W along the rod until it is balanced horizontally. Record the values of W, P and Q, and the respective horizontal distances a, b and c from the pivot O to the line of action of W, P and Q.

Measurements:

W/N	P/N	Q/N	a/m	b/m	c/m

Calculation:

Total clockwise moment about O = $W \times a$ N m

Total anticlockwise moment about O = $P \times b + Q \times c$ N m

Conclusion: The calculation shows that when a pivoted rod or object is in equilibrium, the total clockwise moments about the pivot of all the forces = the total anticlockwise moments about the pivot.

Scanned with CamScanner

Example

We can use moments to calculate the distance OR or a in Fig. 4.7 when the rod QR is in equilibrium.

Taking moments for all the forces about the pivot O

$$\text{total clockwise moments} = (10 \times a) + (3 \times 4) = 10a + 12$$

and $$\text{total anticlockwise moment} = 5 \times 8 = 40$$

For equilibrium of the rod QR

$$10a + 12 = 40$$

So $$10a = 40 - 12 = 28$$

$$\therefore\ a = 2{\cdot}8\ \text{m}$$

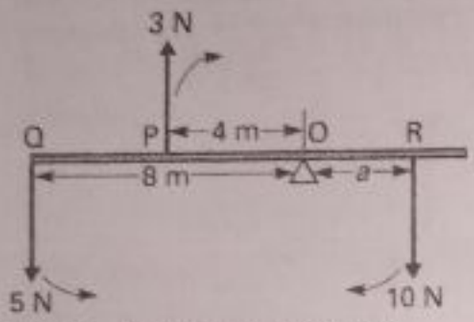


Fig. 4.7 Calculation of distance

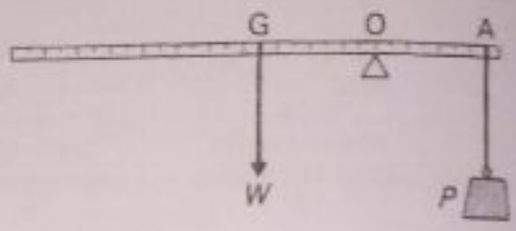


Fig. 4.8 Weight of ruler by moments

Measurement of weight of object by moments

The principle of moments can be used to find the weight W of a ruler or other object such as an umbrella or bat. As an illustration, consider the case of a metre ruler.

Experiment

Balance the ruler lengthwise on a pencil and so find roughly the position of its centre of gravity G (Fig. 4.8). Now suspend the ruler at a whole number O, such as 75 cm, near one end, attach a suitable weight P on the opposite side of O at A and move it until the ruler balances. Measure and record P, OA and OG.

Calculation: By moments about O, we have

$$W \times \text{OG} = P \times \text{OA}$$

So $$W = \frac{P \times \text{OA}}{\text{OG}}$$

Parallel forces

There are a number of cases in mechanics where an object is kept in equilibrium by parallel forces. One example is a bridge when traffic is stationary on it at one instant (Fig. 4.9). Suppose the weights of the individual vehicles are 20000 N, 5000 N and 10000 N. These forces act vertically downwards on the bridge, and so are parallel. The reaction forces, P and Q, at the supports X and Y act vertically upwards. Here five parallel forces keep the bridge in equilibrium.

Since the bridge does not move up or down, the total upward forces,

$$P + Q = \text{total downward forces, } (20000 + 5000 + 10000\ \text{N}),\ 35000\ \text{N}$$

Scanned with CamScanner

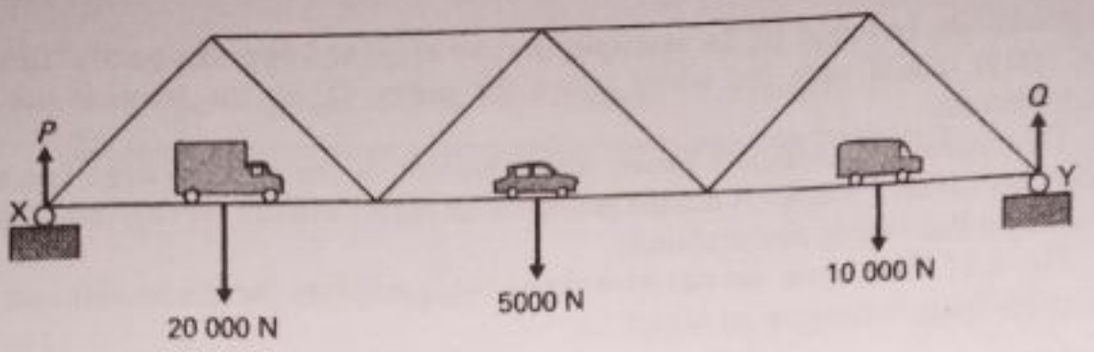


Fig. 4.9 Parallel forces in equilibrium

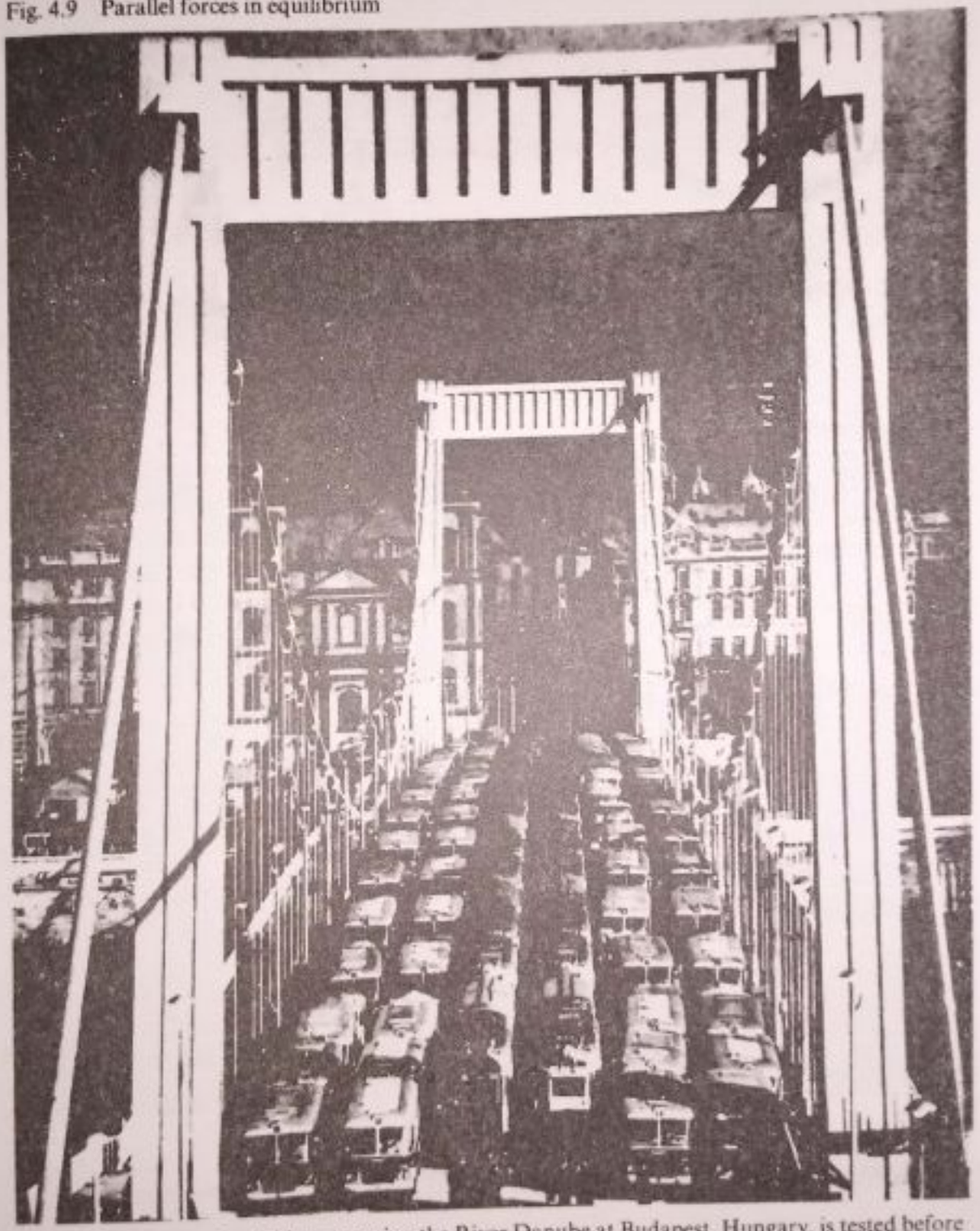


Plate 4B A suspension bridge, spanning the River Danube at Budapest, Hungary, is tested before opening by loading with vehicles (*Associated Press Ltd*)

Conditions for equilibrium of parallel forces

Although the total forces acting on an object in one direction may equal the total forces in the opposite direction, it does not necessarily follow that the object is in

Scanned with CamScanner

equilibrium. In Fig. 4.10, for example, the two equal and opposite parallel forces of 200 N would turn the wheel about the centre O. So the wheel is *not* in equilibrium.

Two equal and opposite forces which are not in the same straight line are together called a *couple*. A couple acting on an object will not move it in any one direction but simply *turn* it round.

Fig. 4.11 shows how we can investigate the conditions for the equilibrium of parallel forces acting on an object.

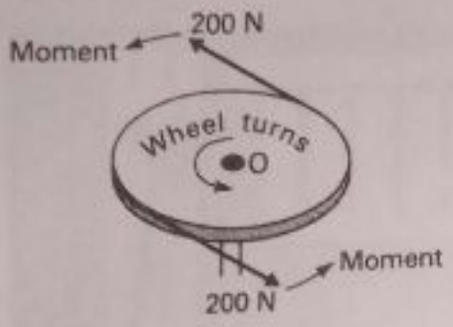


Fig. 4.10 Parallel forces not in equilibrium

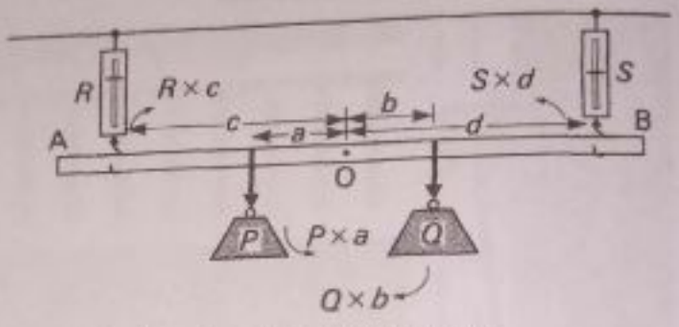


Fig. 4.11 Experiment on equilibrium of parallel forces

Experiment

Suspend two spring balances from a rod supported in two clamps (not shown) or by any other way. Place a light beam or ruler AB between the hooks of the balances, and attach suitable weights P and Q to AB. Adjust the positions of P and Q so that readings R and S which are not widely different are obtained on both balances, and AB is horizontal. Measure and record the values of P, Q, R and S, and the perpendicular distance to the forces from *any* point O on the rod; also measure distances a, b, c and d.

Measurements: In one experiment, results were

$$P = 5{\cdot}0\,\text{N},\ Q = 5{\cdot}0\,\text{N},\ R = 6{\cdot}5\,\text{N},\ S = 4{\cdot}2\,\text{N}$$
$$a = 0{\cdot}10\,\text{m},\ b = 0{\cdot}15\,\text{m},\ c = 0{\cdot}34\,\text{m},\ d = 0{\cdot}62\,\text{m}$$

Calculation:

Total clockwise moment about O $= Q \times b + R \times c = 5 \times 0{\cdot}15 + 6{\cdot}5 \times 0{\cdot}34 = 3{\cdot}05\,\text{N m}$

Total anticlockwise moment about O $= P \times a + S \times d = 5 \times 0{\cdot}10 + 4{\cdot}2 \times 0{\cdot}62 = 3{\cdot}10\,\text{N m}$

Conclusion: Allowing for experimental error:

1. Total forces in one direction equal total forces in opposite direction.
2. Total clockwise moments of the forces about any point are equal to the total anticlockwise moments of the forces about the same point.

Examples of parallel forces on beams or rods

1. A light beam AD rests on supports at B and C (Fig. 4.12). A load of 4 N is placed at O, where BO is 40 cm and CO is 60 cm. Find the reactions P and Q at the supports.

Since
$$\text{upward forces} = \text{downward forces}$$
$$P + Q = 4 \quad \ldots\ldots\ldots\

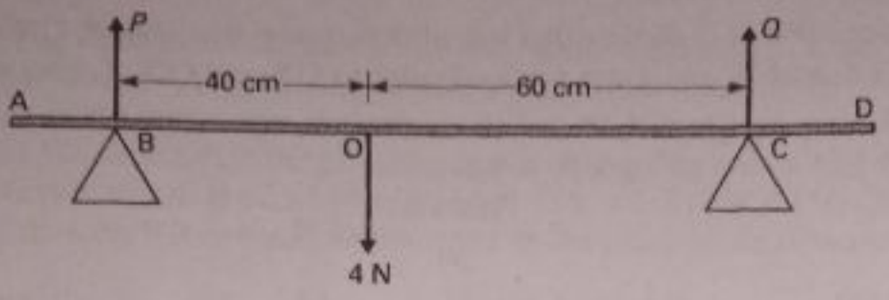


Fig. 4.12

Then
moment of P about C $= P \times BC = P \times 100$, clockwise
moment of 4 N about C $= 4 \times OC = 4 \times 60$, anticlockwise
moment of Q about C $= 0$

So
$$P \times 100 = 4 \times 60, \text{ or } P = \frac{240}{100} = 2{\cdot}4 \text{ N}$$

So, from (1),
$$Q = 4 - P = 4 - 2{\cdot}4 = 1{\cdot}6 \text{ N}$$

2. A light beam, with supports at B and C, has loads of 40 N, 20 N and 30 N placed on it as shown in Fig. 4.13.
(a) Calculate the reactions X and Y at the supports, (b) What additional weight W at A would make the beam *just* tilt about B?

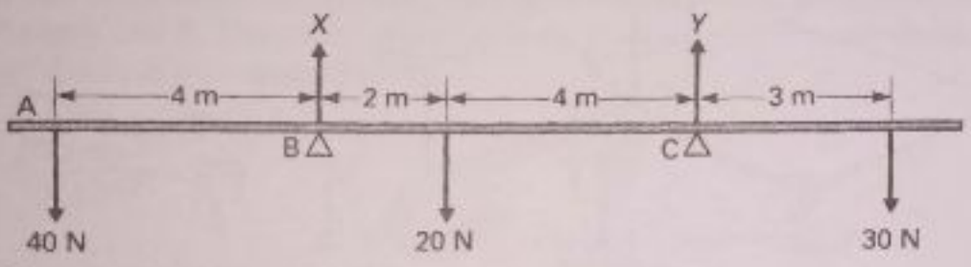


Fig. 4.13

(a)
Total upward force $= X + Y =$ total downward force
$= 40 + 20 + 30 = 90$ N . . . . . (1)

Taking moments about support at X

total anticlockwise moment $= (40 \times 4) + [Y \times (2+4)] = 160 + 6Y$,
total clockwise moment $= (20 \times 2) + [30 \times (2+4+3)] = 40 + 270 = 310$

So
$$160 + 6Y = 310$$
$$6Y = 310 - 160 = 150$$
$$Y = \frac{150}{6} = 25 \text{ N},$$

From (1),
$$X = 90 - Y = 65 \text{ N}$$

(b) When the beam just tilts about B, the reaction Y becomes zero. Taking moments about the support B

$$W \times 4 = (20 \times 2) + (30 \times 9) = 40 + 270 = 310$$
$$\therefore \; W = \frac{310}{4} = 77{\cdot}5 \text{ N}$$

Additional weight at A $= 77{\cdot}5 - 40 = 37{\cdot}5$ N

Levers

The turning effect or moment of a force is used in the lever. Fig. 4.14 shows three different types of levers.

(a) *Crowbar* (Fig. 4.14a) Here the force applied or effort P is exerted at one end N of the long arm ON of the bar. O is the pivot or 'fulcrum'. The opposing force

Scanned with CamScanner

or resistance W is at D on the other side of O. Suppose W = 200 N, ON = 50 cm, OD = 4 cm, and W and P are perpendicular to ON and OD. Taking moments about O,

$$P \times 50 = 200 \times 4$$

So
$$P = \frac{200 \times 4}{50} = 16\,\text{N}$$

So, using the lever, a small force of 16 N can overcome a large force of 200 N.

(b) *Bottle-opener* (Fig. 4.14b) This time the effort P and resistance W are both on the *same* side of the pivot or fulcrum O. Suppose W is 300 N, its perpendicular distance from O is 15 cm and the distance of the effort P from O is 1 cm. Taking moments about O

$$P \times 15 = 30 \times 1, \text{ so } P = 2\,\text{N}$$

A small force of 2 N can therefore overcome a resistance of 30 N.

(c) If *sugar tongs* or *tweezers* are used the effort P is *nearer* the fulcrum O than the resistance or weight W (Fig. 4.14c).

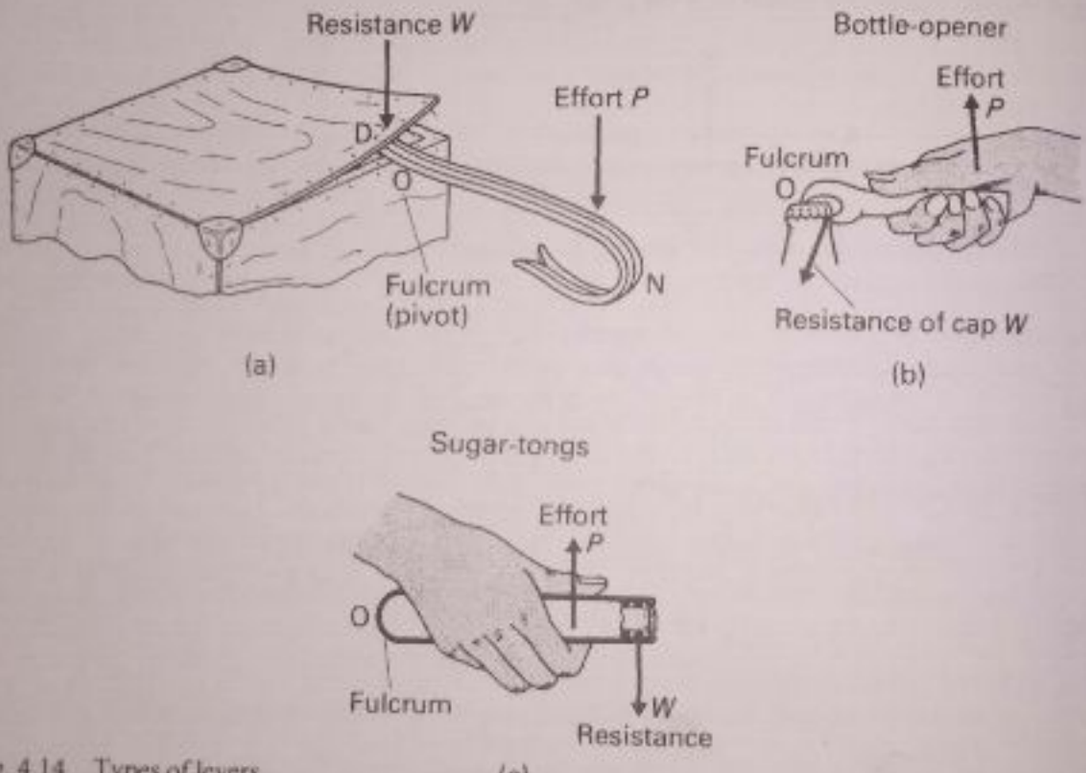


Fig. 4.14 Types of levers

To find P when a piece of sugar has a weight W of 0·1 N, its distance from O is 8 cm and the distance of the effort P from O is 5 cm, take moments about O. Then

$$P \times 5 = 0{\cdot}1 \times 8, \text{ or } P = 0{\cdot}16\,\text{N}$$

So an effort *larger* than the weight is now required.

Centre of mass

Consider two small masses A and B of 1 kg each connected by a rod of negligible mass (Fig. 4.15a). The *centre of mass* of A and B is the point O midway between them, that is, if the two masses connected together move we can consider their total mass of 2 kg concentrated at O for purposes of calculation.

Scanned with CamScanner

If the 1 kg mass at B is replaced by a 2 kg mass, the centre of mass O is now nearer to B than to A (Fig. 4.15b). Intuition tells us that the ratio OB:OA = 1:2; that is, we could replace the two masses by one of 3 kg at O in considering the motion of AB. Similarly if the 2 kg mass at B is replaced by one of 3 kg, the centre of mass O is nearer to B than before and OB:OA = 1:3 (Fig. 4.15c). In general, the more massive B is compared with A the nearer is the centre of mass to B.

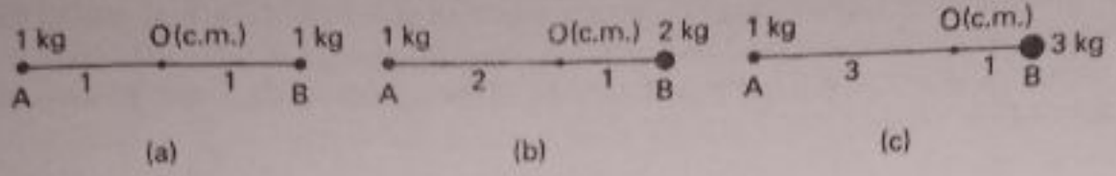


Fig. 4.15 Centre of mass (c.m.)

Centre of gravity

Suppose the rod AB connecting the masses of 1 kg and 2 kg is placed on a knife-edge at O (Fig. 4.16). The moment about O of the *weight* $2g$ of the 2 kg mass is exactly counterbalanced by the moment about O of the *weight* $1g$ of the 1 kg mass. So the rod is balanced horizontally. We call O the centre of gravity of the two weights at A and B. *The centre of gravity is the point where the resultant weight of the two masses acts or appears to act.*

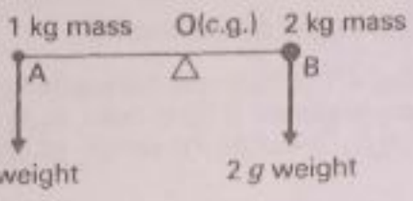


Fig. 4.16 Centre of gravity (c.g.)

In the three cases shown in Fig. 4.15, the centre of gravity (c.g.) coincides with the centre of mass (c.m.) at O. This may not always happen. For example, if the two masses are separated by very large distances, the Earth's gravitational intensity or acceleration of free fall, g, may be slightly greater at A than B. The c.g. would then be a little nearer A than the c.m., O, whose position does not depend on g. In the cases discussed in this book, the c.m. and c.g. will coincide in position.

Centre of mass and centre of gravity positions

A uniform rod AB is one whose mass per unit length is the same along its whole length. Its centre of mass is at its mid-point G (Fig. 4.17).

When the rod is placed on a knife-edge at G, the weights of all small lengths such as c on one side of G are exactly counterbalanced by the equal weights of

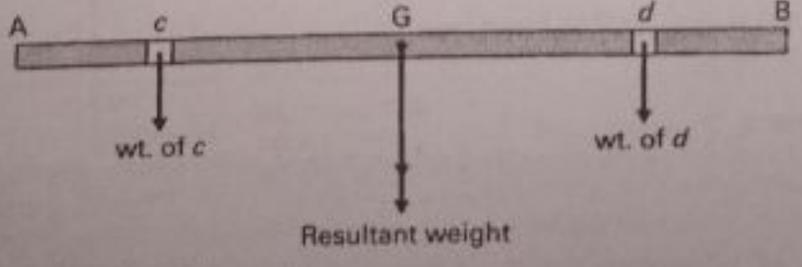


Fig. 4.17 Centre of gravity–uniform rod

Scanned with CamScanner

similar lengths such as d on the other side at the same distance from G. So the centre of gravity of the uniform rod is at its mid-point G.

Fig. 4.18 shows the position G of the centre of mass and centre of gravity G of (a) a uniform disc and (b) a uniform ring; in both cases G is at the centre. For a uniform thin sheet or lamina such as a square, rectangle or parallelogram, G is at the intersection of the diagonals (Fig. 4.18c).

A triangular uniform lamina has its centre of mass and centre of gravity G where the medians intersect. A median is the line joining a vertex of the triangle such as A in Fig. 4.18d to the midpoint M of the opposite side. Geometry shows that $AG = \frac{2}{3}AM$.

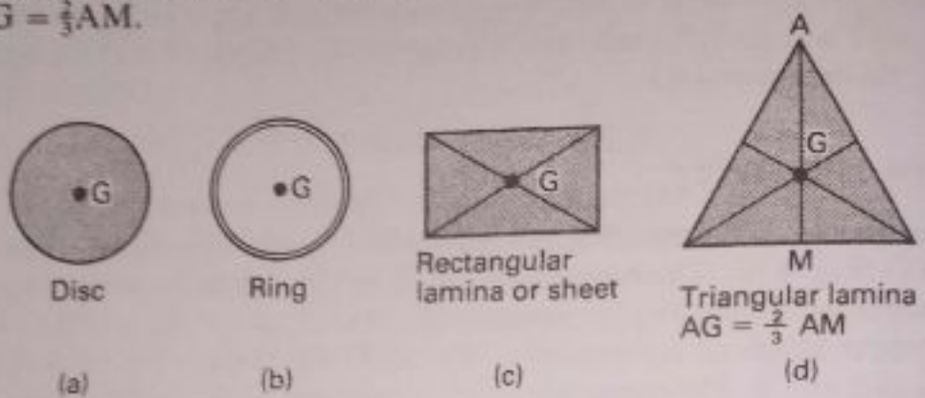


Fig. 4.18 Positions of centre of mass and centre of gravity

Example on calculating weight by balancing

The golf club shown in Fig. 4.19a balances horizontally on a fulcrum 0·75 m from the end A. When the fulcrum is placed 0·30 m from A, it balances horizontally with a weight of 6·0 N hung from A. Calculate the weight of the golf club. (O and C)

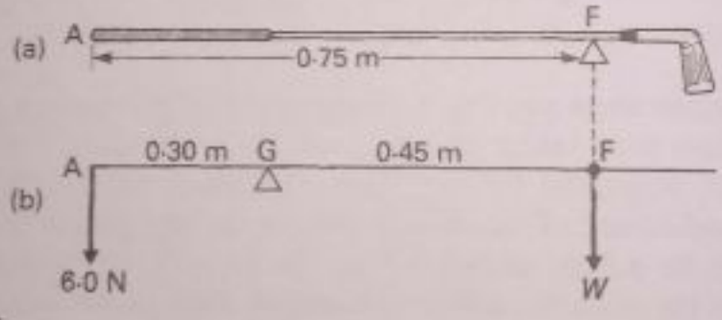


Fig. 4.19 Example

Since the golf club balances horizontally at the fulcrum F (Fig. 4.19a) the weight W of the club acts at F.

In Fig. 4.19b, the fulcrum is now at G. Taking moments about G

$$W \times 0{\cdot}45 = 6 \times 0{\cdot}30$$

$$W = \frac{6 \times 0{\cdot}30}{0{\cdot}45} = \frac{1{\cdot}80}{0{\cdot}45} = 4\,\text{N}$$

To find the c.g. of a lamina

Experiment

1. Approximate method

First, try to balance a lamina such as a thin wooden board on a point. Since the whole weight acts at the centre of gravity G, the position is roughly found in this way.

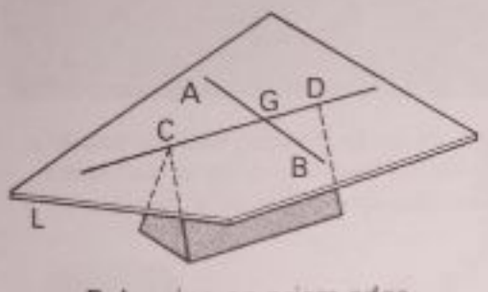
Scanned with CamScanner

To find G more accurately, balance the lamina L on a sharp edge, such as the edge of a triangular prism, and draw the line AB of the edge. The c.g. lies somewhere on AB. Now turn the lamina round through 90° and balance it again (Fig. 4.20). The new line CD of the edge is drawn on the lamina. Then the centre of gravity G is the point of intersection of AB and CD.

2. Accurate method

To find the centre of gravity more accurately than by the balancing method, suspend the lamina from a smooth horizontal support at A, near one edge, making sure it can swing freely. Now suspend a plumbline, which is a thread with a small weight attached at one end, from A. The thread is vertical. Draw this line on the lamina.

Now turn the lamina round and repeat the experiment by suspending it from another point Q (Fig. 4.21). Draw the new vertical line QR on the lamina. The point of intersection G of QR and AB is the centre of gravity of the lamina. This is because the lamina will always rest with its centre of gravity vertically below the point of suspension when it is free to move.



Balancing on prism edge

Fig. 4.20 Centre of gravity by balancing

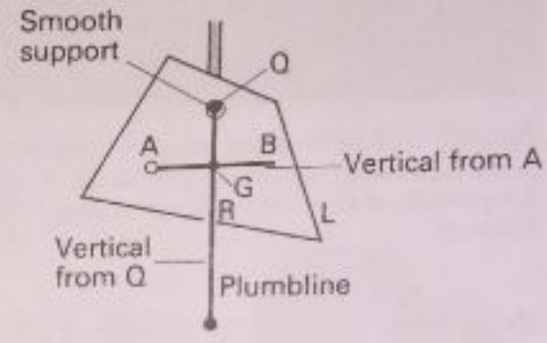


Fig. 4.21 Centre of gravity by plumbline

The c.g. of an object such as a stool can be found in a similar way. Suspend the stool freely by its legs in turn, and find the point of intersection of vertical lines from the suspension points by using threads tied to the legs to show these lines.

Stability and centre of gravity

The stability of an object is connected with the position of its centre of gravity and the moment its weight exerts about an axis. Suppose, for example, that a heavy cylinder with a high centre of gravity G, is held on an inclined plane (Fig. 4.22a). The vertical through G passes through the edge O of the base OA. When

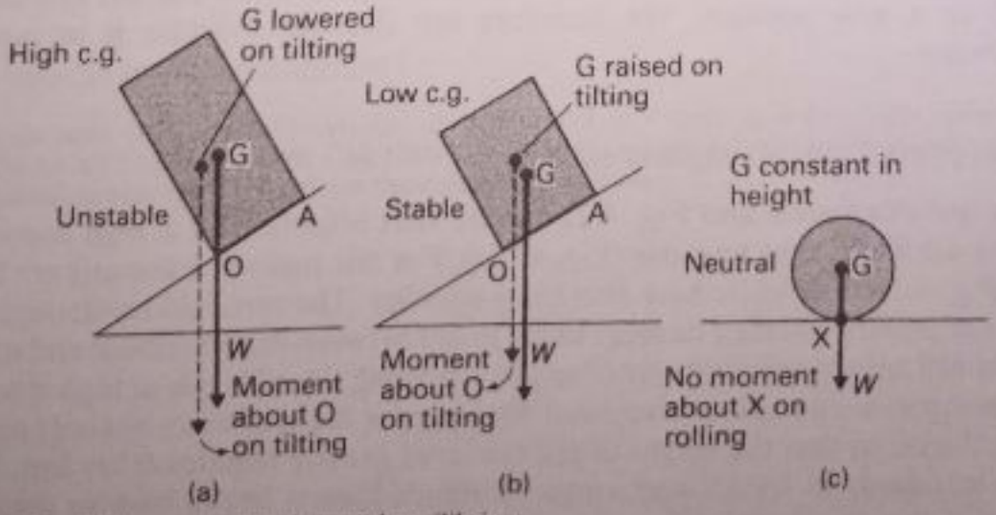


Fig. 4.22 Stable, unstable and neutral equilibrium

Scanned with CamScanner



Plate 4C A double-decker bus on a platform, undergoing a tilt test for stability with the help of a plumb-line and a chart showing the angle of tilt. All double-decker buses operating in the United Kingdom have to be tilted safely to an angle of 28° before entering service. (*Leyland Vehicles Limited*)

the cylinder is slightly displaced about O as shown, the weight now has an anticlockwise moment about O. This moves the cylinder further away from its initial position, and so it topples over. The cylinder in Fig. 4.22a is therefore said to be in *unstable equilibrium*.

Now consider a cylinder of the same mass but a lower centre of gravity G on the inclined plane (Fig. 4.22b). The vertical through G now passes through the base OA and to the right of O. So if the cylinder is slightly tilted about O and then released, the weight W now exerts a clockwise moment about O. This makes the cylinder return to its original position. So the cylinder is said to be in *stable equilibrium* in Fig. 4.22b.

Finally, suppose the cylinder is placed with its curved surface on a horizontal plane (Fig. 4.22c). The weight W acting through G now has *no* moment about the axis where the cylinder touches the plane. This is also true when the cylinder is rolled to a new position. We therefore say that the cylinder is in *neutral equilibrium*.

Height of centre of gravity and stability

From our discussion and Fig. 4.22 we see that objects with a high centre of gravity are likely to be unstable (Fig. 4.23a). For this reason racing cars are built low (Fig. 4.23b). A wider base also helps stability. The vertical line through the centre of gravity will then be more likely to fall between the wheelbase and so the weight will help to counterbalance any overturning round bends at high speeds.

Passengers are not allowed to stand on the upper decks of buses but only on the lower decks, so that the height of the centre of gravity remains fairly low. Tall lamp standards in homes and umbrella stands have a heavy base so that the centre of gravity is low.

Scanned with CamScanner

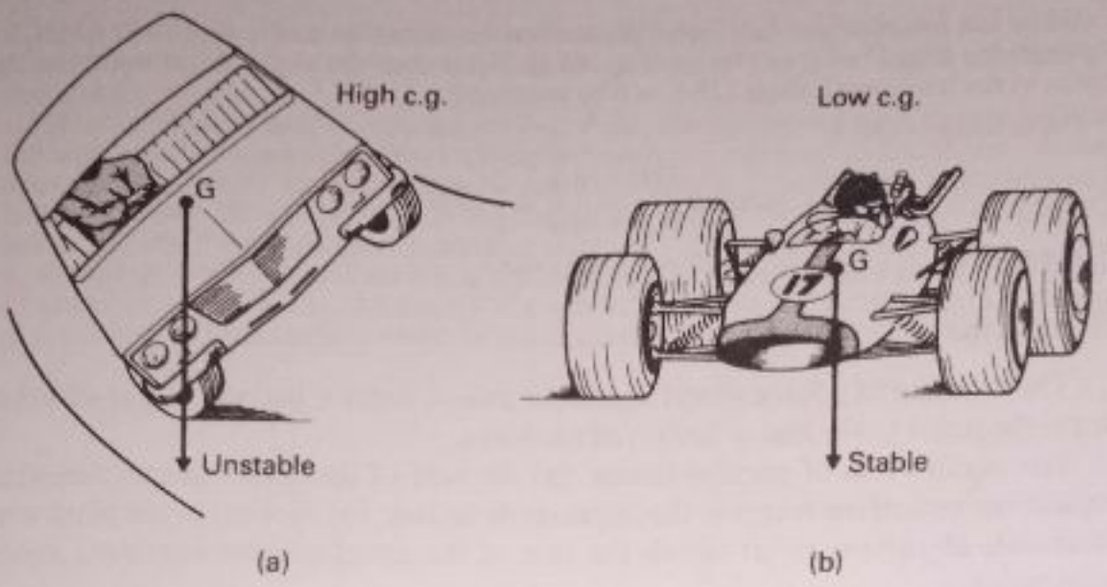


Fig. 4.23 Unstable and stable cars

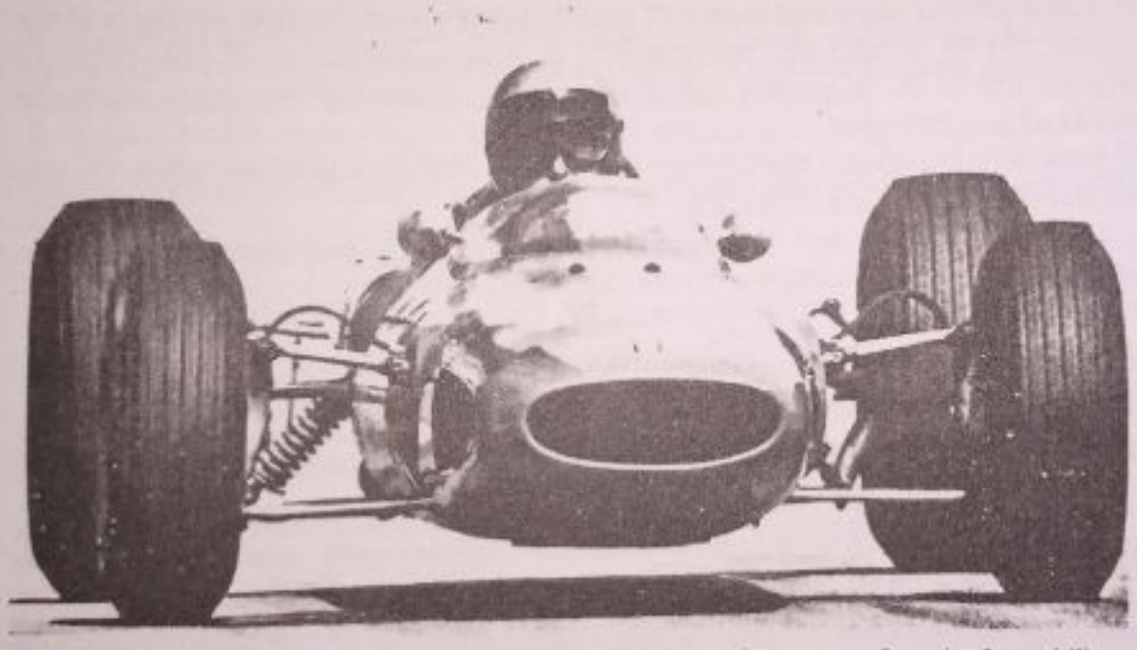


Plate 4D Racing cars such as the 2·4 litre Ferrari shown have a low centre of gravity for stability in going round bends (*Nigel Snowdon/Barnaby's Picture Library*)

Example on stable equilibrium

The diameter of a uniform cylinder is 0·2 m and its height is 0·4 m. The cylinder is placed on an inclined plane. Calculate the maximum angle to the horizontal to which the plane can be inclined before the cylinder falls over.

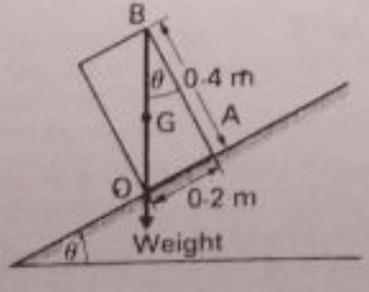


Fig. 4.24 Example

Scanned with CamScanner

When the cylinder just falls over, the vertical line from its centre of gravity G passes through the edge O of the cylinder (Fig. 4.24). If θ is then the angle of inclination of the plane to the horizontal, angle OBA $= \theta$ by geometry.

From triangle OBA,

$$\tan\theta = \frac{OA}{AB} = \frac{0{\cdot}2}{0{\cdot}4} = \frac{2}{4} = 0{\cdot}5$$

From tables, $\theta = 27°$ (approx.)

Summary

1. The moment of a force about a point or axis = force × perpendicular distance from the point to the line of action of the force.
2. For equilibrium of parallel forces: (a) the sum of the forces in one direction equals the sum of the forces in the opposite direction; (b) the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about that point.
3. In all classes of levers, the effort has a moment about the fulcrum which just overcomes the moment of the resistance about the fulcrum.
4. The centre of mass and centre of gravity (a) of a uniform disc or ring is at the centre, (b) of a uniform rectangular or square lamina is at the point of intersection of the diagonals, (c) of a triangular lamina is two-thirds along the median from any apex.
5. Stable equilibrium: the object returns to its original position when slightly displaced and released (low c.g.).

Unstable equilibrium: the object moves further away from its original position when slightly displaced and released (high c.g.).

Exercise 4 (Answers, page 640)

(Assume weight of 1 kg mass = 10 N and of 100 g mass = 1 N)

1. (a) In Fig. 4.25, a weight of 10 N is placed at A. What is its moment about C and about D?

(b) In pushing open a door, the moment exerted about the hinges was 15 N m. If the pushing force was 25 N and perpendicular to the door, what is the distance of the force from the hinges?

(c) In Fig. 4.26 a uniform ladder of weight 100 N rests against a smooth wall at Y and is prevented from slipping by the rough ground at X. The reaction R at Y is perpendicular to the wall. Find (i) the moment of the 100 N weight about X, (ii) the moment of R about X, (iii) the value of R if the moments in (i) and (ii) are equal.

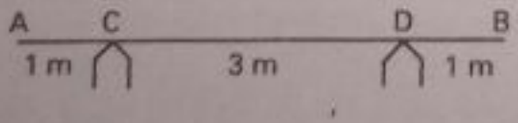


Fig. 4 25

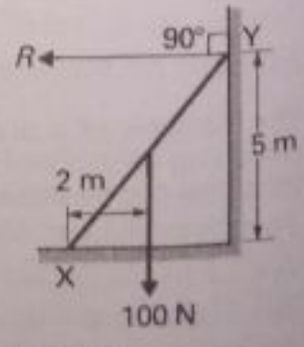


Fig. 4.26

2. AB is a uniform plank of length 5 metres and mass 80 kg. (Fig. 4.25). It is supported horizontally on two trestles, C and D, placed 1 metre from A and B respectively. What is the magnitude and direction of the minimum force which must be applied at A (a) to lift the plank just clear of the trestle at C, (b) to lift the plank just clear of the trestle at D? (L)

Scanned with CamScanner

3. Describe how you would determine experimentally the position of the centre of mass (centre of gravity) of a thin sheet of material of irregular shape and uniform thickness.

When a bunsen burner, shown in Fig. 4.27, stands on its base it can be tilted through a fairly large angle before it topples over but if stood on its barrel, with its base uppermost, a small displacement will cause it to topple. Explain these observations as fully as you can. Why, with the bunsen burner standing on its barrel, will it topple more easily in some directions than in others? (L)

4. A uniform half-metre scale AB is balanced horizontally across a knife edge placed 15 cm from A (Fig. 4.28). A mass of 30 g is hung from the end A. What is (a) the mass of the scale; (b) the force exerted on the scale by the knife edge? (N)



Fig. 4.27

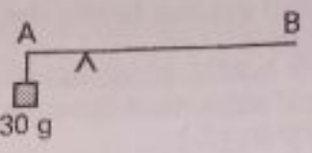


Fig. 4.28

5. A uniform rod 1 m long of mass 50 g is supported horizontally on two knife edges placed 10 cm from its ends. What will be the reactions at these supports when a 100 g mass is suspended 10 cm from the mid-point of the rod? (N)

6. Explain *stable*, *neutral* and *unstable* equilibrium and give *one* example of each. (L)

7. Define the centre of gravity of a body, and explain how you would determine the weight of a closed umbrella if you were given a metre rule, a straight edge and a 50 g mass. (L)

8. A simple weighing machine is made of a uniform bar 125 cm long and mass 5 kg and pivoted 2·5 cm from one end. Find the mass that must be suspended at the end of the long arm so as to balance a mass of 320 kg suspended at the end of the short arm. (N)

9. A uniform metre scale is balanced horizontally across a knife edge at the 20 cm mark with a 300 g mass hung by cotton from the 11 cm mark. Show in a diagram the three forces acting on the scale and calculate its mass. (N)

10. Explain how (a) the moment of a force, (b) the work done by a force, are each measured.

A trap door of mass 10 kg in a horizontal floor is made of a uniform piece of wood 1 m square, smoothly hinged along one side. What is the least vertical force required to raise the door?

What is the work done when the door is raised through an angle of 45°? (L)

11. Define *moment of a force about an axis*, *centre of gravity*. State the *principle of moments* and describe an experiment to test its validity.

A steel rod 7 cm long consists of two cylindrical parts, 3 cm and 4 cm long respectively, having a common axis. If the area of cross-section of the shorter part is twice that of the longer, find the position of the centre of gravity of the rod. (L)

12. Explain what is meant by *the moment of a force about an axis*. State the conditions of equilibrium for a number of coplanar parallel forces. Describe an experiment to verify them.

A uniform rod AB is 180 cm long and mass 2 kg. It balances horizontally on a single support when carrying loads of 1 kg and 3 kg at distances of 60 cm and 120 cm respectively from A. Find the position of the support and the magnitude of the downward thrust on it. (L)

13. Explain what is meant by *centre of gravity*, *stable equilibrium*.

State and explain, with the aid of diagrams, why the base of a bunsen burner is *heavily* weighted and of *large* area. (L)

14. What do you understand by the *principle of moments*? Describe how you would verify the principle experimentally for parallel forces.

A uniform rod of length 50 cm and mass 100 g is pivoted 8 cm from one end A. Loads are weighed by attaching them to A, and moving a mass of 500 g along the rod, on the opposite side to the pivot, until the rod balances horizontally. Find the position of the mass when a load of 2·4 kg is attached to A and calculate the total upthrust at the pivot. (N)

Scanned with CamScanner

15. What do you understand by *stable*, *unstable*, and *neutral equilibrium*? Give one example of each.

Describe how you would find the position of the centre of mass of a thin plane sheet of material of irregular shape.

Taking the mass of the earth to be 81 times that of the moon, the radius of the earth to be 6560 km and the distance between the centres of the moon and earth to be 60 times the earth's radius, find the distance of the centre of mass of the moon–earth combination from the centre of the earth. (O)

16. A double-decker bus standing on a sloping surface is tilted sideways. Draw a diagram to show the forces acting on it and explain how the design and use of the bus enable this tilt to be as large as possible without toppling.

An inclined plane has a surface rough enough to prevent slipping. Calculate, in each case, the maximum angle of inclination of this plane to the horizontal when (a) a uniform block 3 m by 2 m by 1 m, (b) a cylinder of length 1 m and radius 1 m are placed in turn on the surface in the position of their greatest stability.

Name and explain briefly the type of equilibrium portrayed by a tightrope walker without a pole. (L)

17. A milk bottle, suspended by a string which has been tied round its neck, is in equilibrium. Why must the string be vertical? State another condition which must be fulfilled. (O and C)

Multiple-Choice Questions

18.

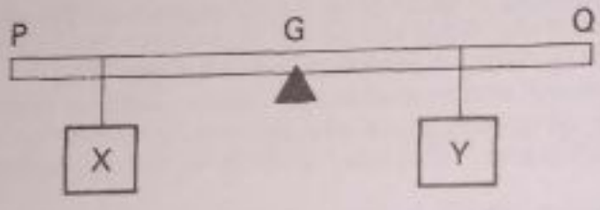


Fig. 4.29

In Fig. 4.29, a rod PQ is pivoted at its centre of gravity G and is in equilibrium with weights X and Y as shown. If a weight 2X is now added to X, the rod will be in equilibrium again by

A moving Y nearer to G,
B adding a weight 2X to Y,
C moving the weights 2X and X further away from G,
D adding a weight of 2Y to Y
E adding a weight of Y to Y

19.

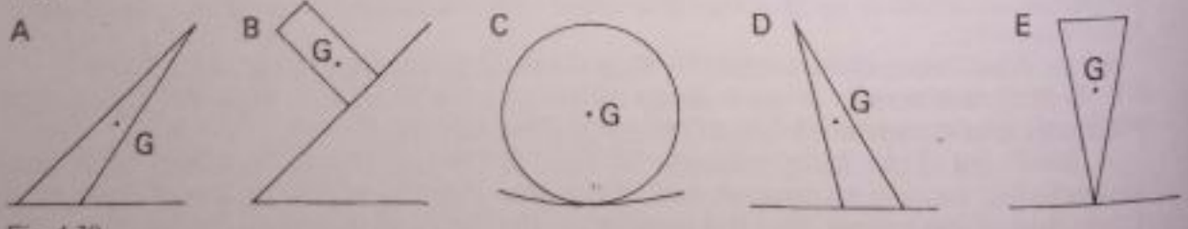
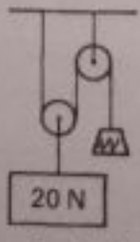


Fig. 4.30

Fig. 4.30 shows five objects A to E, where G is their centre of gravity. Which object is in stable equilibrium?

20. In Fig. 4.31, a weight W just supports the 20 N weight.

The value of W is



A 2 N B 5 N C 6·7 N D 10 N E 20 N

Fig. 4.31

Scanned with CamScanner

5. Machines · Resultant and Component of Forces

Machines

A heavy roller or barrel can best be raised up a step by rolling it up an inclined plank of wood with one end resting on the step. The inclined plank, or inclined plane, is being used as a *machine*; it enables a large weight or resistance to be overcome by a small effort. A lever, such as a bottle or can opener, is also an example of a machine. A screw is a machine; a small force applied at the end of the screw head is able to move the screw forward against the relatively large resistance of the wood. Pulleys are machines which enable heavy loads to be raised by much smaller forces.

Mechanical advantage

If a load W is raised steadily by a machine when an effort or force P is applied, the *mechanical advantage* of the machine is defined as the ratio W/P, or

$$\text{mechanical advantage (M.A.)} = \frac{\text{load } (W)}{\text{effort } (P)} \quad \ldots\ldots\ldots \quad (1)$$

Suppose an effort P of 20 newtons (20 N) is applied at one end of a crowbar and just overcomes the resistance W of 100 N at the lid of a case. Then

$$\text{mechanical advantage, M.A., of crowbar} = \frac{W}{P} = \frac{100}{20} = 5$$

If a pulley system raises a load of bricks of weight 400 N steadily when an effort of 40 N is applied

$$\text{mechanical advantage of the system} = \frac{400}{40} = 10.$$

In practice, part of the effort in a machine has to overcome the frictional forces present. The remainder of the effort has to raise the load steadily. It should therefore be remembered that *the mechanical advantage of a machine depends on the friction present.*

Velocity ratio

When machines are used, the small effort P moves through a large distance x, and the large resistance or load W moves through a small distance y in the same time

Scanned with CamScanner

(Fig. 5.1). The ratio of the distances per second moved steadily by the effort and by the load is called the *velocity ratio* of the machine.

$$\text{Velocity ratio (V.R)} = \frac{\text{distance moved by effort}}{\text{distance moved by load in same time}} = \frac{x}{y} \quad \ldots \quad (2)$$

Suppose, using a pulley, that the effort moves down a distance of 200 cm and the load is raised a distance of 40 cm in the same time. Then

$$\text{velocity ratio of the machine} = \frac{200\,\text{cm}}{40\,\text{cm}} = 5.$$

Usually the velocity ratio is much greater than 1.

As we have seen, the mechanical advantage W/P of a machine depends on the frictional forces that may be present, since part of the effort has to overcome them. On the other hand, the relative displacements of the load and effort depend only on the shapes and positions (or geometry) of the moving parts of the machine. The relative displacements do not depend on friction. So the velocity ratio of a machine is *independent* of friction.

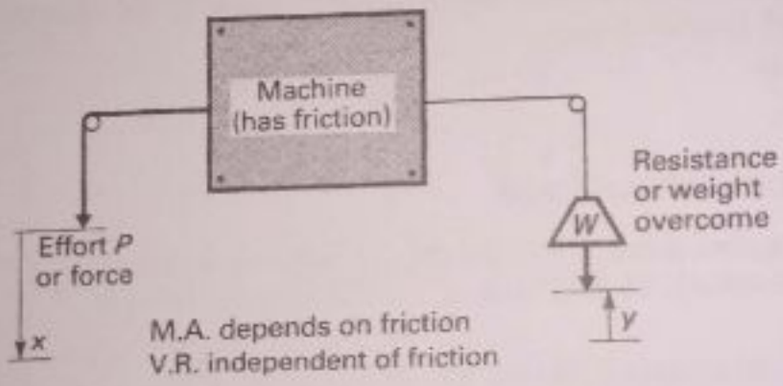


Fig. 5.1 Mechanical advantage and velocity ratio

Relation between efficiency, M.A. and V.R.

We can now derive a relation between mechanical advantage, velocity ratio and efficiency of a machine.

For any machine

$$\text{efficiency} = \frac{\text{work obtained}}{\text{work supplied}} \times 100\,\% = \frac{W \times y}{P \times x} \times 100\,\%$$

where y and x are the respective distances moved by W and P in the same time.

$$\therefore \text{ efficiency} = \frac{W}{P} \div \frac{x}{y} \times 100\,\%$$

But mechanical advantage $= W/P$

and velocity ratio $= x/y$

$$\therefore \text{ efficiency} = \frac{\text{M.A.}}{\text{V.R.}} \times 100\,\% \quad \ldots \quad (3)$$

Scanned with CamScanner

We have seen that velocity ratio is independent of friction. As the friction increases, however, the mechanical advantage decreases. So from (3), the efficiency of the machine becomes less with greater friction.

Example on efficiency

A machine with a velocity ratio of 6 requires 800 J of work to raise a load of 600 N through a vertical distance of 1 m. Find the efficiency and mechanical advantage of the machine.

$$\text{Efficiency} = \frac{\text{work obtained}}{\text{work supplied}} \times 100\%$$

$$= \frac{600 \times 1\,\text{J}}{800\,\text{J}} \times 100\% = \frac{3}{4} \times 100\%$$

$$= 75\%$$

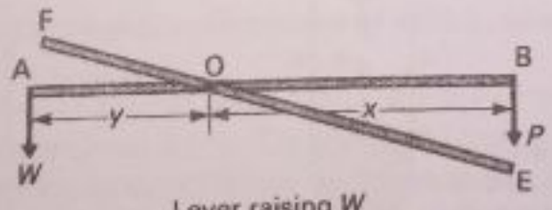
Since $$\text{efficiency} = \frac{\text{M.A.}}{\text{V.R.}} \times 100\%$$

$$75 = \frac{\text{M.A.}}{6} \times 100$$

$$\therefore \text{M.A.} = \frac{75 \times 6}{100} = 4{\cdot}5$$

Levers

Consider a lever AOB with a pivot at O, and suppose a small force or effort P is applied at the end B of the long arm (Fig. 5.2). If the load W is moved up through a *small* distance (AF) to F, the effort P moves down a distance BE in the same time. So the velocity ratio, V.R., = BE/AF = x/y = ratio of arms of lever. Hence if $x = 10y$, the velocity ratio = 10. In the absence of friction, the mechanical advantage $W/P = 10$; in practice, because of friction, the mechanical advantage is much less than 10.



Lever raising W

Mechanical advantage $= \frac{W}{P}$

Velocity ratio $\frac{BE}{AF} = \frac{x}{y}$

Fig. 5.2 The lever as a machine

Pulleys

Pulleys are machines used by builders for lifting heavy loads to higher floors. Cranes used at docks for lifting heavy cargoes in and out of ships are forms of pulleys.

A simple pulley is a fixed wheel with a rope passing round a groove in the wheel's circumference. A load W is attached to one end of the rope. It is easier to raise the load by pulling down at the other end, rather than lifting it up directly,

Scanned with CamScanner



Plate 5A A giant crane, capable of lifting 800 tonnes. It is shown raising a steel structure to be used as part of an offshore oil platform over 100 m tall. (*John Laing and Son Ltd*)

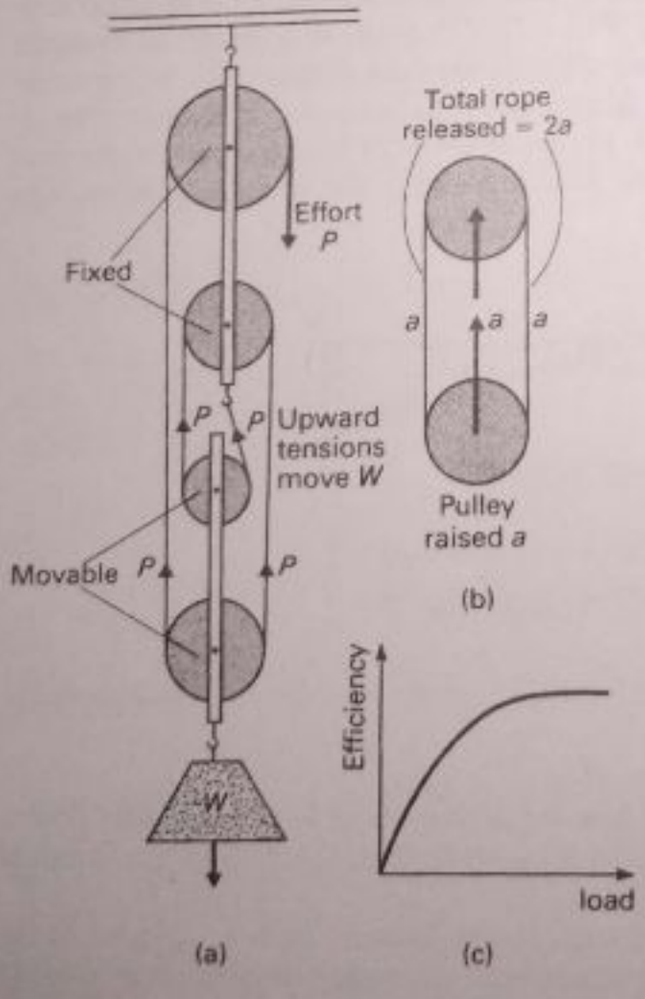


Fig. 5.3 Action of pulleys

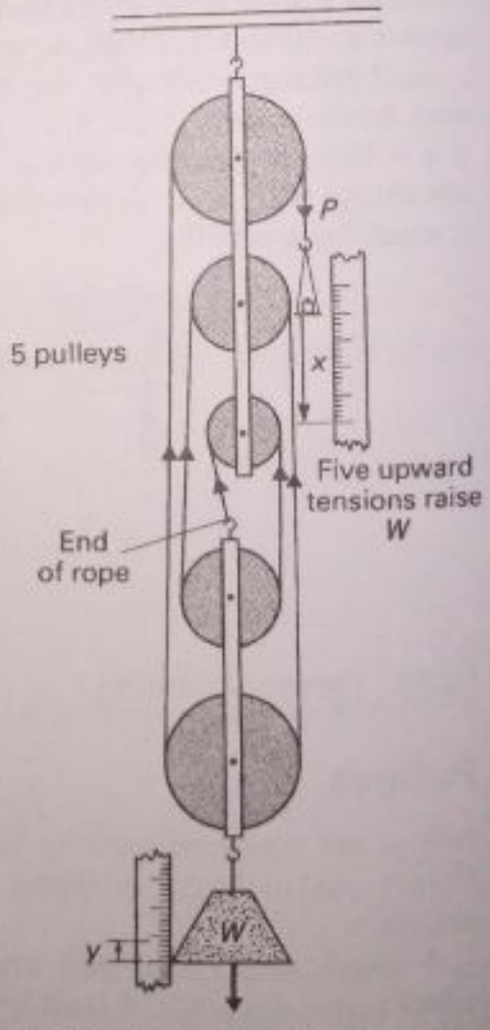


Fig. 5.4 Five-pulley system

Scanned with CamScanner

because the hauler can use part of his weight as a pulling force in addition to his muscles.

A more practical form of pulley system is to use a number of pulleys and to pass a continuous rope round them. Fig. 5.3a shows a system of four pulleys. The upper two are fixed, and the lower two are movable. The load W to be raised is attached to the lower pulleys, and the effort P is applied to the end of the rope passing round the top pulley. This is a *block and tackle* system.

Suppose the rope is pulled down with a force P equal to 100 N. The tension everywhere in the rope is then 100 N if frictional forces are neglected. Now there are four pieces of rope pulling upward on the load, two round each of the lower two pulleys. So the total upward pull is 4×100 or 400 N. Thus the load W raised is 400 N when the effort P is 100 N.

In practice, owing to the weight of the lower block and the friction at the pulley wheels, the force P needed to raise a load of 400 N is more than 100 N, say 120 N. In this case,

$$\text{mechanical advantage} = \frac{\text{load}}{\text{effort}} = \frac{400}{120} = 3{\cdot}3$$

As the load increases, the weight of the lower block becomes a smaller proportion of the load. So the mechanical advantage and the efficiency rise to a maximum as the load increases.

Velocity ratio

When the lower two pulleys move up a vertical distance a, corresponding to a movement a of the load W, each pulley releases a length a of rope on each side or a total length $2a$ (Fig. 5.3b). The effort P then moves down a distance of $2 \times 2a$ or $4a$. So

$$\text{velocity ratio} = \frac{\text{distance moved by } P}{\text{distance moved by } W \text{ in same time}} = \frac{4a}{a} = 4$$

Thus *the velocity ratio is equal to the number of pulleys*, which is a general result for this system of pulleys.

With an odd number of pulleys such as five, the upper set of fixed pulleys is three and the lower movable set is two. The end of the rope is now attached to the lower frame, instead of to the upper frame, as shown in Fig. 5.4, giving a velocity ratio of 5.

Measurement of mechanical advantage, velocity ratio, efficiency

The mechanical advantage, velocity ratio and efficiency of a pulley system can be measured as follows.

Experiment

M.A. A pulley system is supported and a suitable heavy weight W is added to the lower block (Fig. 5.4). A scale-pan, previously weighed, is attached to the end of the string. Increasing weights are now added to the scale-pan until the weight W *just* rises steadily.

Scanned with CamScanner

The effort P to just raise W is the sum of all the weights plus that of the scale-pan, and this is recorded. Then calculate the mechanical advantage, M.A., from

$$\text{M.A.} = \frac{W}{P}$$

V.R. To measure the velocity ratio two vertical rulers are set up in clamps near W and the scale-pan. (Fig. 5.4). Move the scale-pan down until W rises through a suitable distance y. Record the larger distance x moved by the scale-pan. Then calculate the velocity ratio from

$$\text{V.R.} = \frac{x}{y}$$

Efficiency Work done on load $= W \times y$ and work done by effort $= P \times x$. Work out the efficiency from

$$\text{efficiency} = \frac{W \times y}{P \times x} \times 100\,\%$$

Repeat your experiment with a heavier load for W and compare your results with the first experiment.

Example on pulleys

A block and tackle system of 5 pulleys is used to raise a load of 500 N steadily through a height of 20 m. The work done against friction is then 2000 J. Calculate (a) the work done by the effort, (b) the efficiency of the system, (c) the effort applied.

(a) Work done by effort = work done raising load + work done against friction
$= (500 \times 20) + 2000$
$= 10000 + 2000 = 12000\ \text{J}$

(b) $$\text{Efficiency} = \frac{\text{work obtained}}{\text{work supplied}} \times 100\,\%$$
$$= \frac{500 \times 20}{12000} \times 100\,\% = \frac{5}{6} \times 100\,\% = 83\,\%\ \text{(approx.)}$$

(c) Since $\dfrac{\text{M.A.}}{\text{V.R.}} = \text{efficiency}$

$\text{M.A.} = \text{efficiency} \times \text{V.R. (the number of pulleys)}$
$= \dfrac{5}{6} \times 5 = \dfrac{25}{6}$

If P is the effort, $\dfrac{25}{6} = \dfrac{500}{P}$, so $25P = 6 \times 500 = 3000$

$$P = \frac{3000}{25} = \frac{600}{5} = 120\ \text{N}$$

Inclined plane

Loads such as heavy lawn mowers or barrels can be raised with the aid of an inclined plane. The pyramids in Egypt are thought to have been built by slaves hauling loads to heights by inclined planes. The stone pillars at Stonehenge, England, are considered to have been stood on their ends by the same method.

Suppose that a drum Y of weight W is rolled steadily from the bottom O of an

Scanned with CamScanner

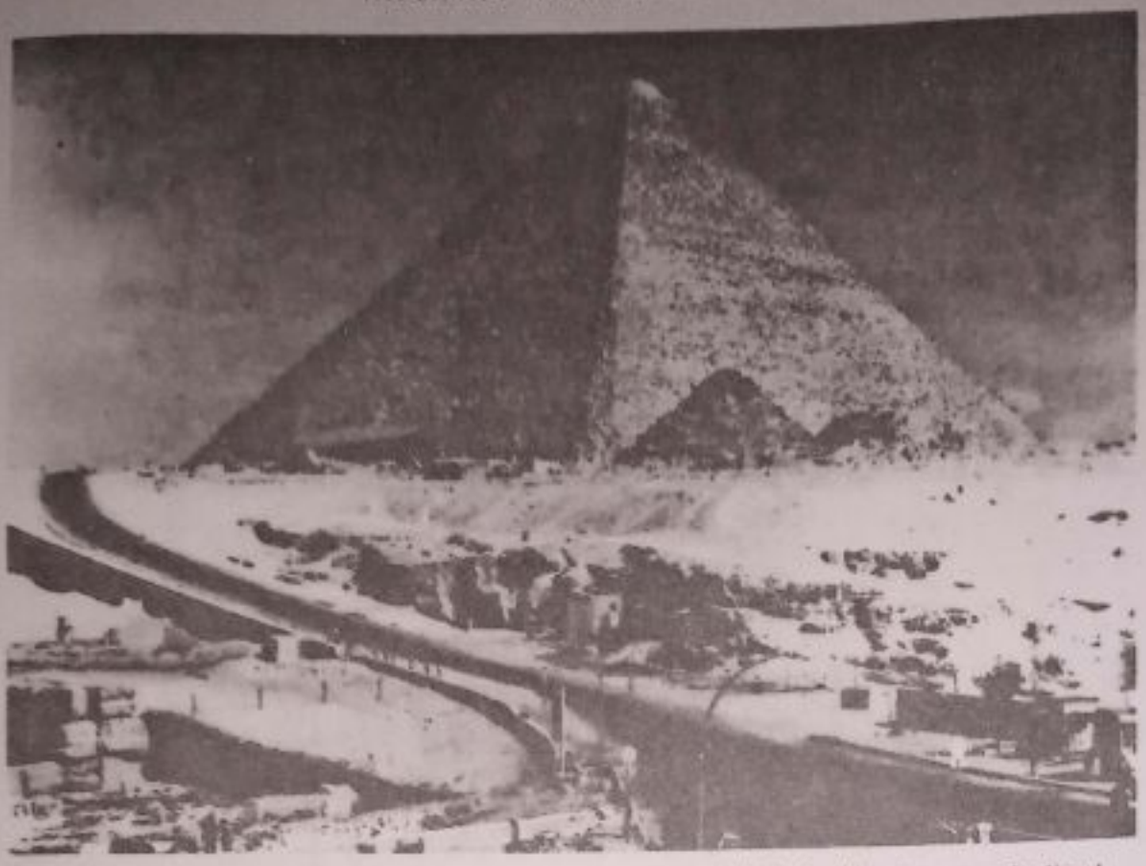


Plate 5B A pyramid at Giza, in Egypt, believed to have been built by hauling stones up inclined planes (*Barnaby's Picture Library*)

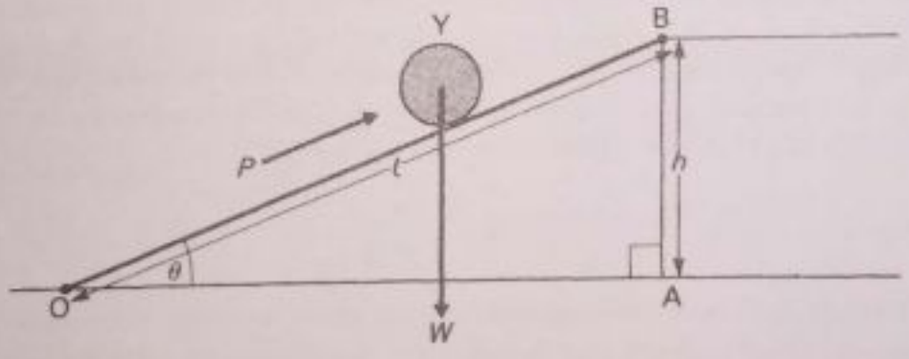


Fig. 5.5 Inclined plane

incline to the top B by using a constant effort P (Fig. 5.5). The distance l moved by the effort is then OB. The load is raised through a *vertical* distance h or AB. So

$$\text{velocity ratio, V.R.} = \frac{\text{OB}}{\text{AB}} = \frac{l}{h} = \frac{1}{\sin\theta}$$

where θ is the angle of inclination of the plane to the horizontal. So

if $\theta = 30°$, the velocity ratio $= 1/\sin 30° = 2$,
if $\theta = 20°$, the velocity ratio $= 1/\sin 20° = 3$ (approx.)

The smaller the angle of the plane, the greater is the velocity ratio.

If the friction is absent, an ideal case, then the work done by the effort is equal to the work done on the load.

Then $$P \times \text{OB} = W \times \text{AB}$$

So $$\frac{W}{P} = \frac{\text{OB}}{\text{AB}} = \frac{l}{h} = \frac{1}{\sin\theta} = \text{M.A.}$$

Scanned with CamScanner

In practice, owing to friction, the mechanical advantage is less than the velocity ratio (see Example below).

The following experiment can be carried out to measure the mechanical advantage, velocity ratio and efficiency of an inclined plane.

Experiment

Weigh a rough wooden or other suitable block, D, and record its weight *W*. Then place it on the surface AC of an inclined plane CAB–if the block slips, decrease the angle θ of the inclined plane (Fig. 5.6). Attach a length of string to the block, pass the string round a grooved wheel S at the top of the plane, and attach a previously weighed scale-pan T to the string so that T hangs freely as shown.

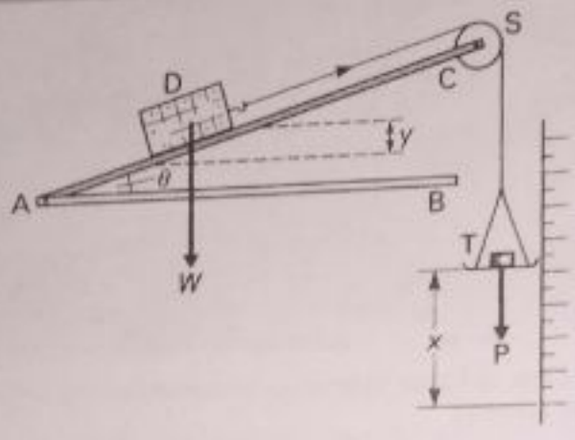


Fig. 5.6 To find M.A., V.R. and efficiency of an inclined plane system

(a) *Mechanical advantage* Add weights to the scale-pan until the block D rises steadily; give the scale-pan a slight downward push each time weights are added. The effort *P* is the sum of all the weights and that of the scale-pan.

Calculate $$\text{M.A.} = \frac{W}{P}$$

(b) *Velocity ratio* Using a ruler, move the scale-pan down a suitable distance *x* (this is the distance moved by the effort *P* in pulling the block up the plane) and measure the corresponding *vertical* distance *y* moved upwards by the block, load *W*. Repeat for another pair of values *x* and *y*. Then calculate

$$\text{V.R.} = \frac{x}{y}$$

Does the velocity ratio, V.R., equal the ratio 'length of inclined plane/height of inclined plane', l/h in Fig. 5.6?

(c) *Efficiency* Calculate this from

$$\text{efficiency} = \frac{W \times y}{P \times x} \times 100\%$$

Example on inclined plane

Fig. 5.7 shows a packing case of weight 500 N on the inclined plane AB. Calculate how much work would be done in sliding the packing case up the inclined plane from A to B. The frictional force is 50 N.

State, giving your reason, whether the packing case would remain stationary if it were placed half-way up the inclined plane.

Scanned with CamScanner

What effect would increasing the length of the inclined plane from B to the ground (keeping B at a height of 3 m above the ground) have on (a) the velocity ratio, and (b) the efficiency of the inclined plane considered as a machine? Give a reason for your answers. (You may assume that the frictional force does not change.) (L)

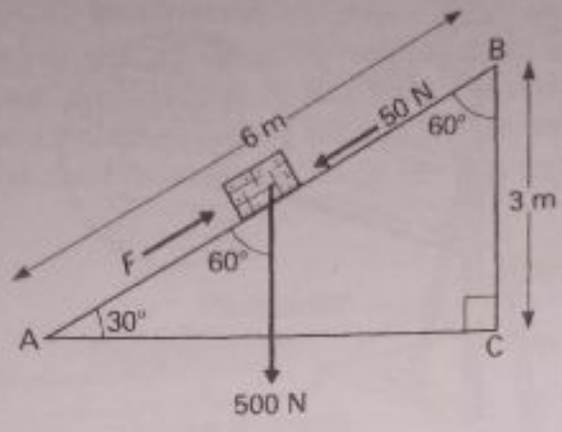


Fig. 5.7 Example

First part: The force F needed to push the case up the plane from A to B has to overcome the frictional force of 50 N and the component of the weight down the plane, which is $500 \cos 60° = 500 \times 0{\cdot}5 = 250$ N

So $$F = 50 + 250 = 300 \text{ N}$$

Hence $$\text{work done} = \text{force} \times \text{distance}$$
$$= 300 \text{ N} \times 6 \text{ m} = 1800 \text{ J}$$

If the case is on its own on the plane, the component of its weight down the plane is 250 N from above. This is greater than the frictional force of 50 N. So the frictional force would not stop the case sliding down the plane; that is, the case would not remain stationary.

Second part: (a) If the inclined plane is used as a machine and if the case is pushed from A to B by the force F or effort,

$$\text{velocity ratio} = \frac{\text{distance moved by effort}}{\text{distance moved by weight}} = \frac{\text{AB}}{\text{BC}}$$

So if AB is increased and BC is kept the same at 3 m, the velocity ratio *increases*.

(b) The efficiency is given by

$$\frac{\text{work obtained in lifting weight}}{\text{work done by effort}}$$

Now the work done in lifting the weight is always weight × height = weight × 3 m, and is therefore constant. The work done along the inclined plane by the force or effort would increase with a longer plane AB, since more work would be needed to overcome the frictional force along the plane. So, from the efficiency formula, the efficiency decreases when the plane is longer.

The screw and screw-jack

The thread of a screw can be regarded as a continuous inclined plane wrapped round a cylinder. When the screw is turned through one complete turn, the point moves through a distance p equal to the distance between consecutive threads. p is called the *pitch* of the screw.

A *screw-jack* can be used to raise heavy loads such as cars. In Fig. 5.8a, the car is raised through a distance p, the pitch of the screw R, when the brace B is turned by the force or effort through one revolution, a distance $2\pi r$, where r is the radius of the circle.

Scanned with CamScanner

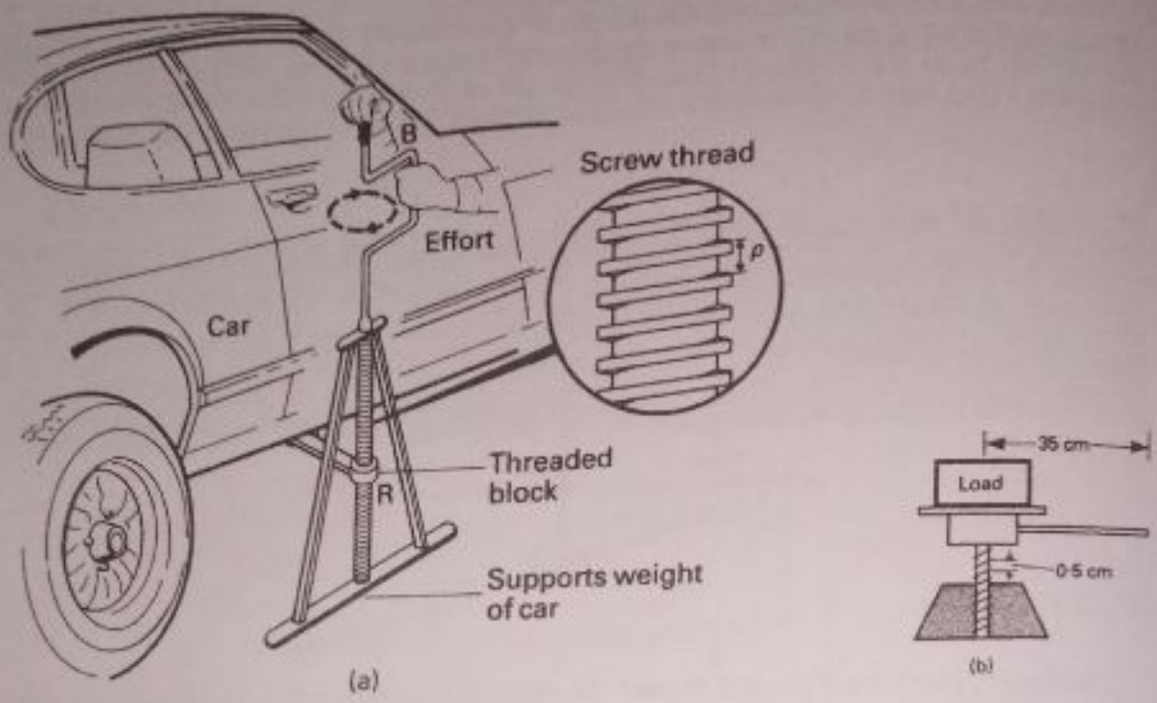


Fig. 5.8 Screw-jack

Example

The handle of the screw-jack in Fig. 5.8b is 35 cm long and the pitch of the screw is 0·5 cm. What force must be applied at the end of the handle when lifting a load of 2200 N if the efficiency of the jack is 40 %? (L)

When the handle is turned through one complete revolution, the effort P has moved through a distance $2\pi \times 35$ cm. The load W has then moved up vertically through a distance 0·5 cm, the pitch of the screw. So

$$\text{velocity ratio, V.R.} = \frac{\text{distance moved by effort } P}{\text{distance moved by load } W \text{ in same time}}$$

$$= \frac{2\pi \times 35}{0{\cdot}5} = 440$$

Now $$\text{efficiency} = \frac{\text{M.A.}}{\text{V.R.}} \times 100\,\%$$

So $$40 = \frac{\text{M.A.}}{440} \times 100$$

$$\therefore \text{M.A.} = \frac{40 \times 440}{100} = 176$$

$$\therefore 176 = \frac{\text{load } W}{\text{effort } P} = \frac{2200}{P}$$

$$\therefore P = \frac{2200}{176} = 12{\cdot}5 \text{ N}$$

Scanned with CamScanner

Resultant and component of forces

If two boys pull a sledge in the same direction by ropes with forces of 150 and 100 N, the total or *resultant* force is (150 + 100) or 250 N (page 68). If the forces act in opposite directions the resultant force is (150 – 100) or 50 N in the direction of the 150 N force.

Suppose that the ropes are inclined at 60° to each other and that the forces in each are again 150 and 100 N (Fig. 5.9). Experience shows that the sledge now moves forward more in the direction of the 150 N than in the direction of the 100 N force. Also the total or resultant force on the sledge is now less than (150 + 100) or 250 N.

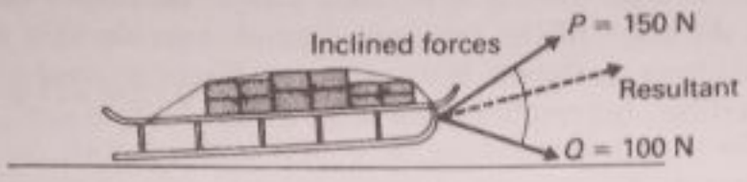


Fig. 5.9 Resultant (addition) of forces

Experiment

Support two pulleys X and Y from a horizontal rod held in two clamps, for example (Fig. 5.10a). Tie three pieces of string into a knot O at one end; pass one of the strings round X and one round Y and attach weights such as 2 N (200 g mass) and 1·5 N (150 g mass) to the ends; and suspend a weight *W* such as 3 N (300 g mass) from the third string.

When the weights have settled down so that O is in equilibrium under the three forces *P*, *Q* and *W*, place a sheet of paper behind the strings and mark their directions on the paper in pencil. On the paper, draw to a suitable scale lines to represent the two forces *P* and *Q*, 2 N and 1·5 N. These are represented by OA and OD in Fig. 5.10b. Complete the parallelogram OACD and measure the force represented by the diagonal OC. Compare this force with the value of *W*.

Now take the paper back to the strings, align OA and OD along the strings OX and OY in Fig. 5.10a, and then see if the diagonal OC is exactly opposite to the direction of W, which is a vertical force.

Result: allowing for experimental error, the result shows that the diagonal OC is (a) equal to *W* (b) in the same straight line as *W*. But *W* counterbalances exactly the *resultant* of *P* and *Q* since O is in equilibrium. The diagonal OC therefore represents the resultant of *P* and *Q*.

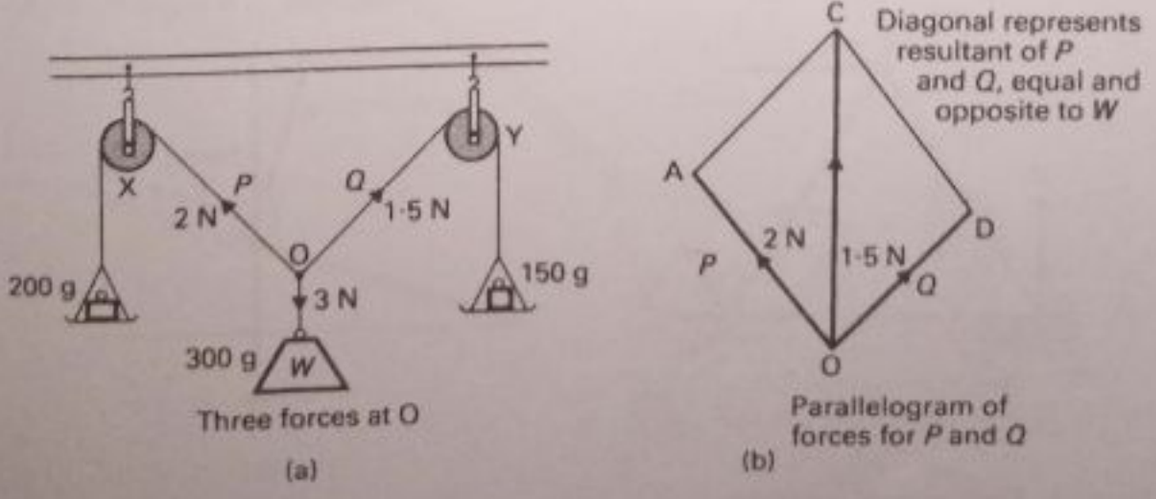


Fig. 5.10

Scanned with CamScanner

Parallelogram of forces

The conclusion of the experiment above is stated as a law called the *Principle of the Parallelogram of Forces. If two inclined forces are represented in magnitude and direction by the adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through the point of intersection of the two sides.*

Fig. 5.11 shows the resultant R of two forces of 40 N and 60 N when they act at angles to each other of: (a) 60°; (b) 90°; (c) 120°. In each diagram the side OA represents 40 N and the side OB represents 60 N on the same scale. The resultant OC in Fig. 5.11a is 87 N by drawing, the resultant OD in Fig. 5.11b is 72 N and the resultant OE in Fig. 5.11c is 52 N; each acts at the angles shown. So the magnitude and direction of the resultant depends considerably on the angle between the two forces, as shown. With two *equal* forces inclined at an angle of, say, 70° to each other, the resultant acts at an angle of 35° to each, or half-way between.

A force is a vector, that is, it is a quantity which has direction and magnitude (see page 28). Any two vectors, for example two velocities or two accelerations, can be added by the parallelogram method, as we have just done for two forces.

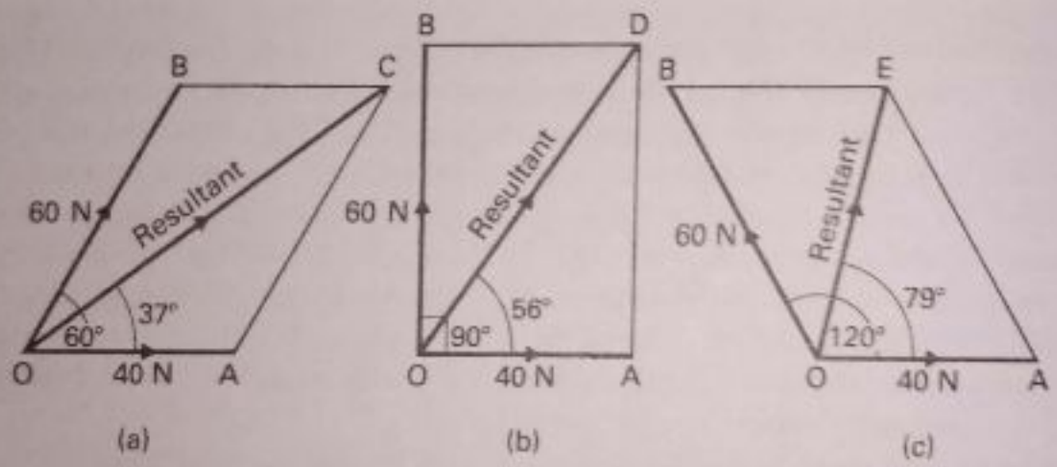


Fig. 5.11 Resultant of two inclined forces

Equilibrium of three forces acting at a point

Bridges and other structures must be kept in equilibrium by the forces acting on them. Fig. 5.12a shows a section of a bridge with a support at O. The joint at O is

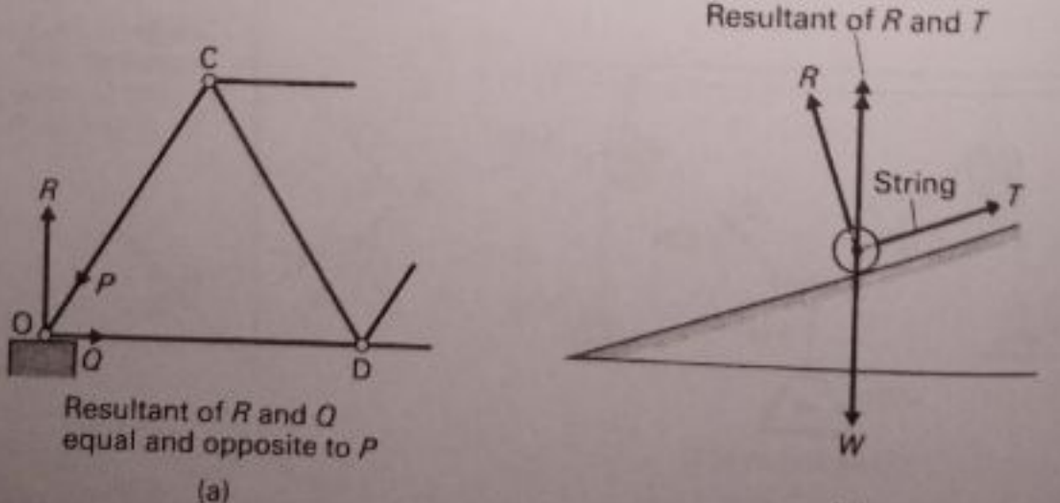


Fig. 5.12 Equilibrium of three forces

Scanned with CamScanner

kept in equilibrium by the forces P, Q in the girders OC, OD and the reaction R at O. For equilibrium, the resultant of R and Q must be equal and opposite to P.

Fig. 5.12b shows an object of weight W kept in equilibrium on a smooth plane by a string along the plane. The forces acting on the object are its weight W, the tension T in the string and the reaction R of the plane. When a plane is smooth, the reaction R always acts at right angles to the surface.

For equilibrium, the resultant of R and T must be equal and opposite to W. So the resultant acts vertically upwards in line with W, as shown.

Note that the directions of three forces keeping an object in equilibrium must all pass through one point. Otherwise the resultant of two of the forces cannot be balanced by the third force.

Components of forces

Sometimes only part of a force is used. A typical case is the use of a lawn-mower. Suppose it is pushed with a force of 200 N at a direction of 30° to the ground (Fig. 5.13a). The part of the force which pushes the mower horizontally is called its *resolved horizontal component*. The other part, which presses the mower vertically into the ground, is called the *resolved vertical component* of the force. The part of the force pushing a lawn-mower along the ground is its horizontal component.

The resolved components of a force can be found by using the parallelogram of forces. As an illustration, in Fig. 5.13b let CO represent in magnitude and direction the force of 200 N pushing a lawn-mower at an angle of 30° to the ground. From C draw a line CB parallel to the horizontal and a perpendicular vertical line CA. Complete the parallelogram CBOA, which is a rectangle in this case. Then, since CO is the diagonal of the parallelogram, CB and CA together have a resultant CO.

Now the force represented by CB is perpendicular to that represented by CA since CBOA is a rectangle. So CB has no effect in the direction of the force CA,

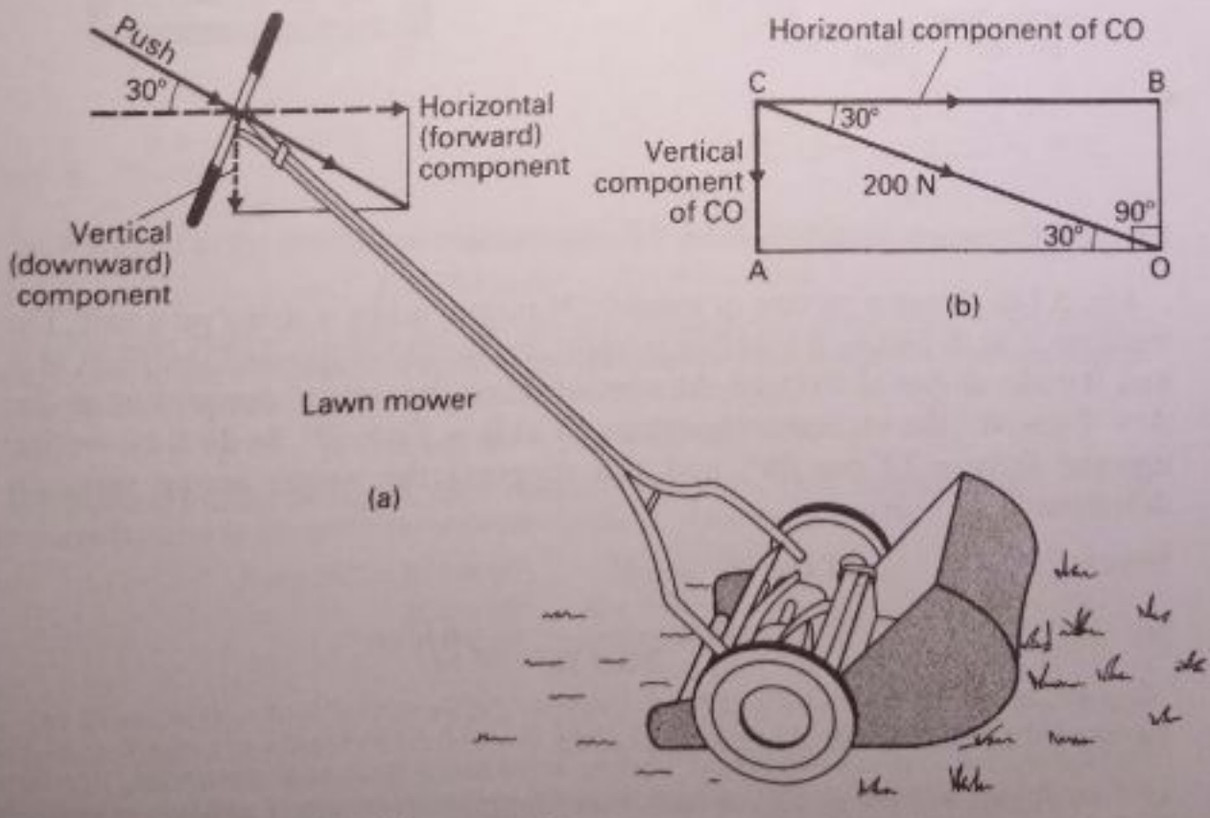


Fig. 5.13 Horizontal and vertical components

Scanned with CamScanner

and CA has no effect in the direction of CB. Therefore the resolved component of CO in a horizontal direction is CB; the resolved component in a vertical direction is CA.

From Fig. 5.13b, CBO is a right-angled triangle.

So $$CB/CO = \cos 30°$$
$$CB = CO \cos 30° = 200 \cos 30° = 173 \text{ N}.$$

Generally, then, the component of a force F in a direction inclined at an angle θ to it is always given by

$$component = F \cos \theta$$

The component in the vertical direction CA is $200 \cos 60° = 100$ N (Fig. 5.13b), since angle OCA is 60°. This downward force is balanced by the upward reaction of the ground. See also page 30.

Illustrations of components

Fig. 5.14a shows a hammer striking a nail in wood with a glancing (sideways) force. If the force is 60 N at 60° to the vertical, then the vertical component of the force is $60 \cos 60° = 60 \times 0{\cdot}5 = 30$ N. This is the magnitude of the force driving the nail down into the wood. If the hammer strikes the nail horizontally with a force of 60 N, the nail is not driven into the wood. This time the vertical component $= 60 \cos 90° = 0$.

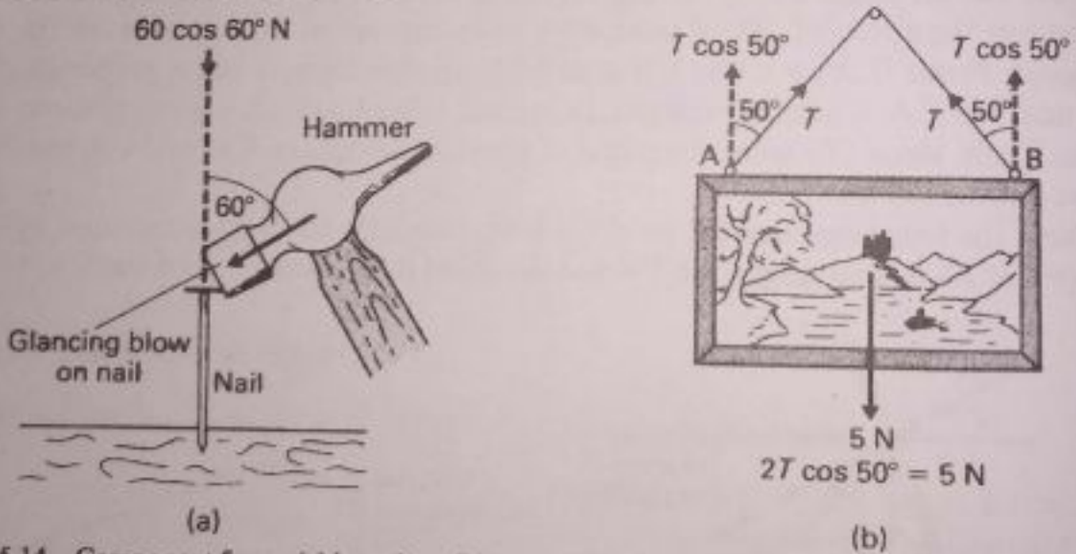


Fig. 5.14 Component forces (a) in action; (b) in equilibrium

Fig. 5.14b shows a picture of weight 5 N supported by a string on a nail. The tensions T at A and at B together support the weight. Suppose the strings at A and B make angles of 50° with the vertical. Then the vertical component of T at A $= T \cos 50°$; the vertical component of T at B $= T \cos 50°$ So the total vertical upward force $= 2T \cos 50°$, and this supports the weight acting vertically downward.

Hence $$2T \cos 50° = 5 \text{ N}$$

So $$T = \frac{5}{2 \cos 50°} = 3{\cdot}9 \text{ N}$$

Examples on resultants and components

1. Two strings at right angles to each other support an object O of weight W (Fig. 5.15a). If the forces in the strings are 12 N and 5 N, calculate W.

Scanned with CamScanner

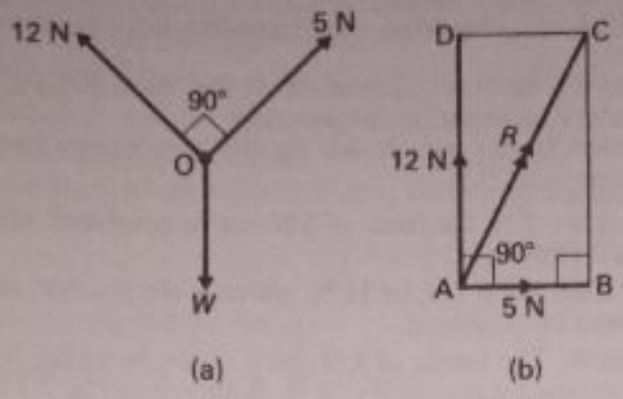


Fig. 5.15 Example

We first find the resultant R of the two forces 12 N and 5 N by the parallelogram of forces method. In Fig. 5.15b, AB represents 5 N, AD represents 12 N at 90° to AB, and ABCD is the completed parallelogram. The resultant R is represented by the diagonal AC. Applying Pythagoras' theorem to triangle ABC,

$$R^2 = 5^2 + 12^2 = 25 + 144 = 169$$

So $$R = \sqrt{169} = 13\,\text{N}$$

But W is equal and opposite to the resultant of 5 N and 12 N, since O is in equilibrium. So $W = R = 13\,\text{N}$

2. In Fig. 5.16a, an object O of weight 20 N is prevented from sliding down a smooth inclined plane by a force F in a string acting up the plane. The angle of the plane to the horizontal is 30°. Calculate, by using components, (a) F and (b) the reaction R of the plane. (c) What *horizontal* force is needed to keep O from sliding down the plane?

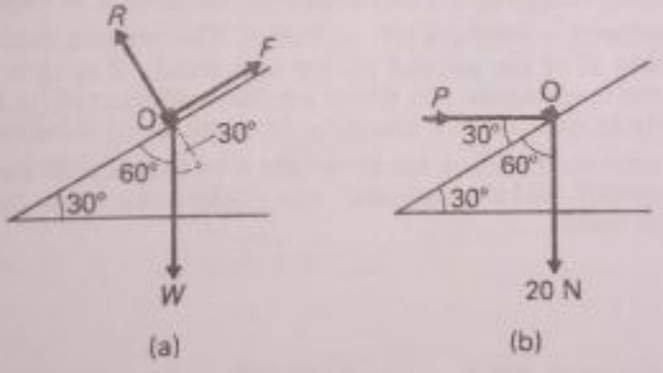


Fig. 5.16 Example

(a) F is equal to the component of the weight 20 N down the plane.

$$\therefore\ F = 20\cos 60^\circ = 20 \times 0{\cdot}5 = 10\,\text{N}$$

(b) Since the plane is smooth, the reaction R is *perpendicular* to the plane. Since there is equilibrium in this direction, R is equal to the component of the weight 20 N perpendicular to the plane. (F has no component perpendicular to the plane.)

So $$R = 20\cos 30^\circ = 20 \times 0{\cdot}867 = 17{\cdot}3\,\text{N}$$

(c) Suppose P is the horizontal force needed (Fig. 5.16b). Then the component of P up the plane is equal to the component down the plane of the weight 20 N.

So $$P\cos 30^\circ = 20\cos 60^\circ$$

$$P = \frac{20\cos 60^\circ}{\cos 30^\circ} = \frac{20 \times 0{\cdot}5}{0{\cdot}867} = 11{\cdot}5\,\text{N}$$

3. (a) State, with a brief explanation, whether it is possible, given forces of 3 N and 8 N, to produce a resultant of (i) 5 N, (ii) 15 N, (iii) 8 N.

(b) An elephant is dragging a tree trunk along a horizontal surface by means of an attached rope which makes an angle of 30° with the horizontal. The tension in the rope is 4000 N.

Scanned with CamScanner

(i) By scale drawing, or otherwise, determine the horizontal force exerted by the rope on the tree trunk.

(ii) Determine also the vertical component of the force exerted by the rope on the tree trunk. Explain why it serves a useful purpose.

(iii) Name the other forces which act on the tree trunk, and show clearly the directions in which they act. (L)

(a) (i) Since 5 N = 8 N − 3 N, the force of 5 N can be produced when the forces of 8 N and 3 N act in opposite directions.

(ii) 15 N is greater than 8 N + 3 N, or 11 N, which is the greatest resultant force of 8 N and 3 N. So 15 N cannot be produced.

(iii) Forces are vectors. The forces of 3 N and 8 N can be added by the triangle vector method (page 29) to produce a resultant of 8 N, as shown in Fig. 5.17a.

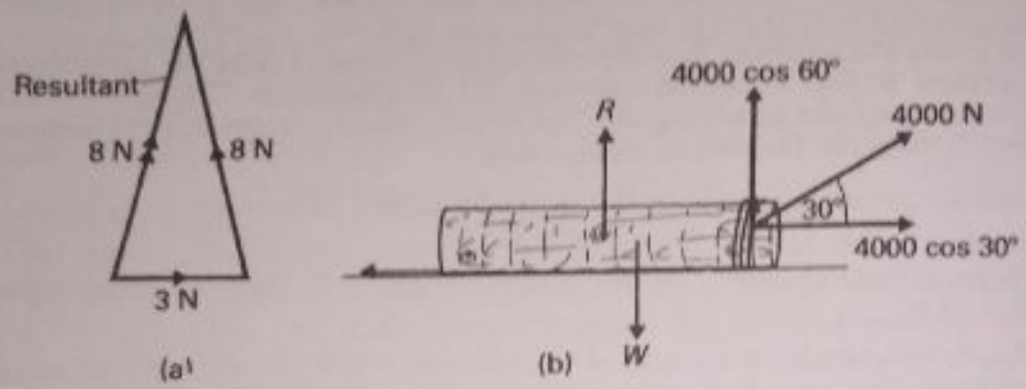


Fig. 5.17 Example

(b) (i) By calculation, using Fig. 5.17b, horizontal component = 4000 cos 30° = 3464 N

(ii) Vertical component = 4000 cos 60° = 2000 N. This serves a useful purpose because it lessens the reaction R of the ground on the tree trunk. R is then 4000 − 2000 N, or 2000 N. The frictional force equals μR, where μ is the coefficient of friction (page 155). So the frictional force to be overcome in dragging the trunk along is reduced.

(iii) The other forces on the trunk are its weight W which acts downwards, the normal reaction R acting upwards and the frictional force F which acts in the opposite direction to the movement of the trunk.

Summary

1. Mechanical advantage M.A. = load (W)/effort (P)

Velocity ratio V.R. = (distance moved by effort)/(distance moved by load in same time).

$$\text{Efficiency} = \frac{\text{work obtained (raising } W)}{\text{work supplied (by } P)} \times 100\,\%$$

2. $$\text{Efficiency} = \frac{\text{M.A.}}{\text{V.R.}} \times 100\,\%$$

M.A. depends on friction; V.R. is independent of friction.

3. In the block and tackle pulley system, one string is wound all round the pulleys. The velocity ratio $= n$, where n is the number of pulleys. The mechanical advantage increases when n increases.

4. In the inclined plane, the velocity ratio is $1/\sin\theta$, where θ is the inclination to the horizontal. The velocity ratio increases when the angle of inclination decreases.

5. In the lever, the velocity ratio is the ratio of the lengths of the two arms of the lever. The mechanical advantage increases when the ratio increases.

Scanned with CamScanner

6. In the *screw-jack*, the velocity ratio is $2\pi a/p$, where a is the radius of the moving arm and p is the pitch of the screw.
7. The resultant of two forces is a single force which has the same effect in magnitude and direction as the two forces acting together. It is given by the *diagonal* of the parallelogram of forces.
8. The resolved component of a force F in a direction inclined at an angle θ is $F\cos\theta$.

Exercise 5 (Answers, page 640)

(Where necessary, assume $g = 10\ \text{m/s}^2$ or 10 N/kg)

1. (a) In using a pulley system, an effort of 100 N is needed to raise steadily a load of 800 N. What is the mechanical advantage?
(b) If this effort moves through a distance of 20 m and the load is then moved up a distance of 2 m, calculate the velocity ratio.
(c) In this pulley system, find the work done by the effort and the work obtained by raising the load. From the results, calculate the efficiency.
(d) Two cables, each at an angle of 30° to the horizontal, support a lamp of weight 50 N. Find (i) the magnitude of the vertical component of the tension T in each cable, (ii) the tension T.
2. Fig. 5.18 represents a block and tackle with the load rising at a uniform speed. From the information given, calculate (a) the velocity ratio, (b) the mechanical advantage and (c) the efficiency of the machine. (N)

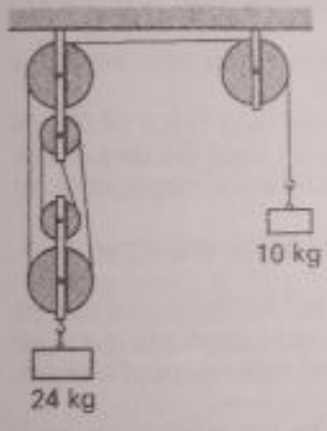
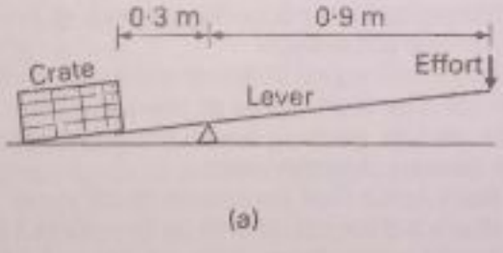
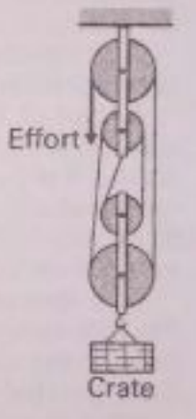


Fig. 5.18



(a)



(b)

Fig. 5.19

3. (a) What is meant by the terms *force* and *work*?
(b) How much work is needed to raise a crate of mass 50 kg through a height of 2 m?
(c) A lever is used to raise one edge of the crate so that a sling may be passed underneath it. Using the figures shown in Fig. 5.19a, calculate what fraction of the weight of the crate is supported by the lever when an effort of 80 N is applied at the end of the lever.
(d) With the sling round the crate, the pulley system shown in Fig. 5.19b is used to lift the crate through a height of 2 m.
(i) What is the velocity ratio of the pulley system? (ii) What effort would have to be applied to the pulley system, if it were 100% efficient, in order to raise the crate? (iii) If, in fact, the pulley system is only 80% efficient, how much work is needed to raise the crate through the 2 m, using the pulley system? (iv) Give *two* reasons why the pulley system is not 100% efficient. (O and C)
4. A block and tackle with a velocity ratio of 5 is used to raise a mass of 20 kg through a vertical distance of 50 cm at a steady rate. If the effort is equal to 50 newtons, determine (a) the distance moved by the effort, (b) the work done by the effort in lifting the load, (c) the loss in energy involved in operating the machine. (L)

Scanned with CamScanner

5. With the aid of labelled diagrams, describe and explain *one* application in each case of (a) a simple lever, (b) an inclined plane. Suggest suitable dimensions for each machine, and state the velocity ratio these dimensions would produce.

A lift carrying 120 kg weight of bricks travels to the top of a building 10 m high in 15 seconds. If the useful output of the engine driving the lift mechanism is 1 kilowatt calculate the efficiency of operation of the lift mechanism. (N)

6. (a) Describe how you would determine by experiment the efficiency of an inclined plane used to raise a load.

(b) A horizontal uniform metal bar PQ of mass 0·5 kg and 50 cm long is pivoted at P (Fig. 5.20). The bar is held stationary by the light string QR so that it makes an angle of 30° with the bar. Calculate the moment about P due to the weight of the bar and hence or otherwise calculate the tension in the string. (N)

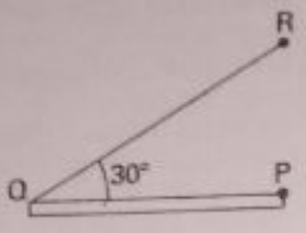


Fig. 5.20

7. Draw a labelled diagram of a single rope pulley block and tackle having a velocity ratio of 5. If, using the machine, a load of 500 N is raised a distance of 2 m by an effort of 160 N, find (a) its efficiency; (b) the work wasted in the machine. (L)

8. Define *mechanical advantage* and *velocity ratio*.

Describe how you would determine, by experiment, the efficiency of a machine used for lifting a heavy load.

A load of 21 000 N placed on the top of a screw-jack is lifted by a force of 300 N applied at right angles to the operating arm at a distance of 35 cm from the axis of the screw. If the screw has two threads to 2·5 cm, find its mechanical advantage, velocity ratio and efficiency. (L)

9. Illustrate, by referring to a simple pulley system, the meaning of the terms *machine, velocity ratio, mechanical advantage*.

Two men are rolling a 150 kg oil drum up a plane inclined at 30° to the horizontal, by the following method. The men stand at the top of the plane and each pulls a rope fixed to the plane at the top, passing down the plane, under and half-round the drum, then, parallel to the plane, up to the man.

What is (a) the least force that each man must exert, and (b) the least work each does in rolling the drum a distance of 3 m up the plane? (O and C)

10. Define *mechanical advantage, velocity ratio* and *efficiency* as applied to a machine.

Describe a simple screw-jack, and explain its mode of action.

In such a machine the pitch of the screw is 5 mm and the length of the effort arm is 30 cm. If the efficiency is 10 %, find the load that can be raised by an effort of 20 N applied at right angles to the effort arm. (O)

11. A force of 5 N and force of 8 N act at the same point and are inclined at 45° to each oth Find the magnitude of the resultant force. (N)

12. What is meant by the *parallelogram of forces*? Describe an experiment to illustrate it.

Horizontal forces of 100, 70 and 40 N act on a body on a horizontal plane in directions N, S and E respectively, their lines of action passing through its centre of gravity. Find graphically or by calculation the resultant force in magnitude and direction.

If the mass of the body is 10 kg and there is a frictional force of 20 N, with what acceleration does the body start to move? (L)

13. What is meant by the *components of a force*? Draw a diagram to illustrate your answer.

A small trolley is set in motion on a horizontal surface by a force of 250 newtons applied by means of a rope inclined at an angle of 20° to the horizontal. What *horizontal* force may be used instead? (C)

14. What is meant by (a) the resultant of a number of forces acting at a point; (b) the

Scanned with CamScanner

component of a force in a given direction? How would you find, experimentally, the resultant of two given forces acting in different directions at a given point?

An electric lamp, of mass 900 g, hangs on a flex, which is pulled aside by a horizontal string until the flex makes an angle of 30° with the vertical. Find the tension in the string and in the flex. (L)

Multiple-Choice Questions

15. Using an inclined plane, an object of weight 50 N is pushed a distance a up the plane. The object then moves up vertically a distance b. Which of the statements A to E is/are NOT true?

A the work output is $50 \times a$
B the mechanical advantage is b/a
C the velocity ratio is a/b
D the velocity ratio is b/a
E the efficiency is $50a/b$

16. In a single rope 4-pulley system, the mechanical advantage is less than 4 because

A the effort may vary
B the load is always raised
C the upper pulleys do not move
D friction acts on the pulleys
E the weight of the lower pulleys can be neglected

17. Two forces of 8 N and 4 N respectively can NOT have a resultant of

A 4 N B 5 N C 8 N D 12 N E 13 N

18.



Fig. 5.21

Fig. 5.21 shows a weight W moving down a rough inclined plane XY. The forces acting along XY on the weight is/are

A only W
B W and the reaction of the plane
C W and the frictional force
D the component of W along XY and the frictional force
E frictional force only

Scanned with CamScanner

6. Density · Forces due to Fluids

Density

If cubes of equal volumes of different solids, such as wood, aluminium, lead, glass and copper, are lifted, you notice immediately that lead is heavy but aluminium and wood are light. So the weight or *mass* of each material in equal volumes is different (Fig. 6.1).

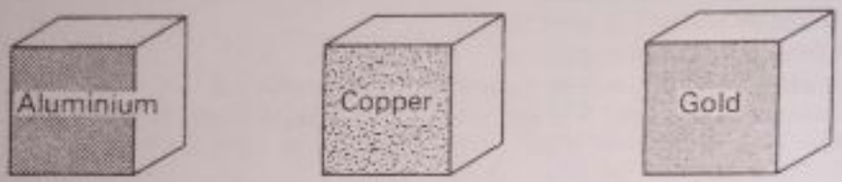


Fig. 6.1 Masses of 1 cm^3 of some metals

The *mass per unit volume* of a substance is called its *density*. So a block of lead of mass 69 g and a volume of 6 cm^3 has a density of 69 g/6 cm^3 or 11·5 g/cm^3. Generally

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

The unit of density is g/cm^3 or kg/m^3.

The mass of 1 litre ($1/1000\ m^3$) of water at a temperature of 4°C is 1 kilogram.

So density of water = 1 g/cm^3 = 1000 kg/m^3

A mass of water of 50 g thus has a volume of 50 cm^3. A mass of water of 2000 kg has a volume of 2 m^3.

Measurement of density

If a solid has a rectangular shape, its density can be found by first measuring its mass on a chemical balance, and then measuring the dimensions and calculating the volume. By calculation

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

The density of an irregularly shaped solid can be found by (a) measuring its mass on a balance, (b) then measuring its volume by *completely* immersing it in water inside a measuring cylinder and noting the change in the water level reading. If the sample is too big for a measuring cylinder, its volume can be

Scanned with CamScanner

found by measuring the volume of water overflowing from a so-called Eureka can, as shown in Fig. 6.2.

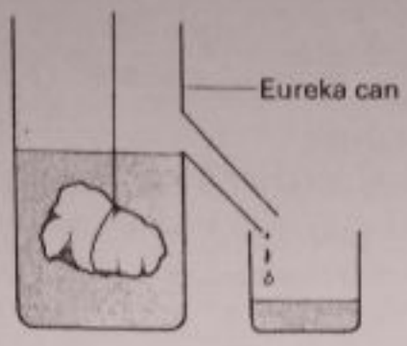


Fig. 6.2 Measuring volume with a Eureka can

Relative density

Water is the most common liquid, and so it is often useful to compare the mass of a substance with the mass of an equal volume of water. This is called its *relative density* (formerly, specific gravity).

$$\text{So} \qquad \text{relative density} = \frac{\text{mass of substance}}{\text{mass of equal volume of water}}$$

If unit volume is chosen as the equal volume of the substance and water, the masses are then numerically equal to the densities.

$$\text{So also} \qquad \text{relative density} = \frac{\text{density of substance}}{\text{density of water}}$$

Note that relative density has no unit, whereas density has a unit.

The relative density of lead is 11·4. So 1 cm^3 of lead is 11·4 times as heavy as 1 cm^3 of water, which has a mass of 1 g. The density of lead is thus 11·4 g/cm^3. The density of a material in g/cm^3 is therefore numerically equal to its relative density. This is not the case when the density is in kg/m^3, because the density of water is 1000 kg/m^3.

Liquids

Liquids such as water can be poured to fill the volume of a glass or bottle of any shape. Therefore, unlike solids, liquids have a fixed volume but no geometrical form or shape. In addition, an object such as a pencil can be moved about freely inside a liquid, but this cannot be done in a solid.

If a can of oil and a lump of metal of about the same volume are lifted we can tell that the density of the liquid is much less than that of the solid. The liquid metal mercury, however, is much denser than most metals, as one realises on lifting a small jar of mercury.

From the definition of density

$$\text{density of a liquid} = \frac{\text{mass of liquid}}{\text{volume of liquid}}$$

A measuring cylinder or burette can be used to measure the volume of a liquid. We can fill a burette with alcohol or other suitable liquid in an experiment, weigh

Scanned with CamScanner

a beaker and then run 50 cm^3 of liquid into it. On reweighing the beaker the mass of liquid can be found. Then

$$\text{density} = \frac{\text{mass of liquid}}{50\ cm^3}$$

and the density can be calculated.

Some liquid density values are given below.

Mercury	13·6 g/cm^3 or	13600 kg/m^3
Turpentine	0·87 g/cm^3 or	870 kg/m^3
Alcohol	0·79 g/cm^3 or	790 kg/m^3
Paraffin	0·79 g/cm^3 or	790 kg/m^3

Mercury has a very high density. Light oils have densities less than 1 g/cm^3, the density of water. The densities given are all measured at room temperature. When a liquid is heated its mass does not alter, but its volume usually increases. So the density decreases. Water has an unusual density change with temperature, as we discuss later (page 183).

Measurement by density bottle

The density bottle is one specially designed to measure the density of liquids. Its main feature is a tight-fitting stopper with a fine hole through it (Fig. 6.3). The bottle is first completely filled with liquid and the stopper is firmly fitted. The excess liquid passes through the fine hole and is then carefully wiped off with a cloth. In this way the volume of liquid filling the bottle is always exactly the same.

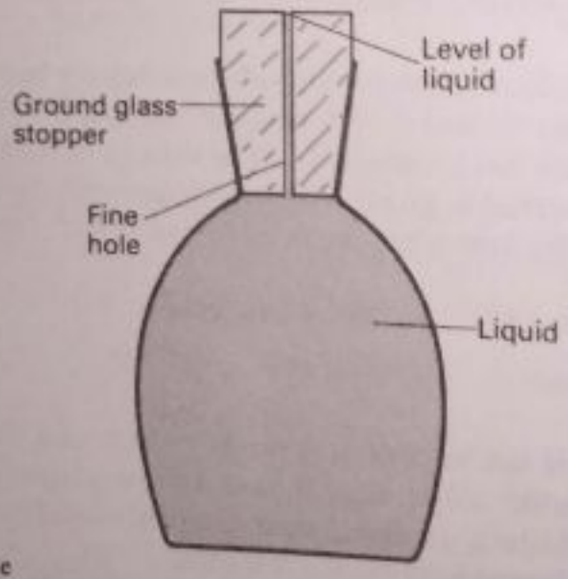


Fig. 6.3 Density bottle

Experiment

Weigh a clean and dry density bottle on a chemical balance with its stopper (call this mass m_0). Fill it completely with water, wiping the excess with a cloth. Reweigh the bottle (m_1). Empty the bottle and dry the inside. Then fill it with a suitable liquid such as paraffin or oil and reweigh (m_2). Record the values m_0, m_1, m_2.

Calculation:

mass of liquid filling bottle $= m_2 - m_0$

mass of water filling bottle $= m_1 - m_0$

Scanned with CamScanner

The mass of water in grams filling the bottle is numerically equal to the *volume* of water in cm^3, since 1 g of water has a volume of 1 cm^3. So

$$\text{density of liquid} = \frac{\text{mass}}{\text{volume}} = \frac{m_2 - m_0}{m_1 - m_0}$$

Result: Suppose the mass of water filling the bottle is 25·0 g; so the volume of the bottle is 25·0 cm^3. If the mass of the other liquid filling the bottle is 20·0 g, then

$$\text{density of liquid} = \frac{20{\cdot}0\,\text{g}}{25{\cdot}0\,\text{cm}^3} = 0{\cdot}80\,\text{g/cm}^3$$

Density of air

Gases are much lighter than liquids and solids of the same volume. So they have a much smaller density.

Air is the most common gas. The density of air can be measured roughly by pumping air into a suitable container, as we now describe.

Experiment

A 50 litre plastic container A with a tap T is weighed containing air at atmospheric pressure. The tap is left open to make sure the air is at atmospheric pressure (Fig. 6.4). Air is now pumped into the container until it is very hard, using a bicycle or foot pump and a one-way valve in the connecting tube. The tap is then closed and A is reweighed (call this mass m_1). The increase in mass should be about 7 g; if not, more air is pumped in.

By means of tubing attached to the tap T, the air is now released from the container and collected over water (Fig. 6.5). The collecting vessel is a transparent plastic box B with internal dimensions 10 cm by 10 cm or a cross-sectional area of 100 cm^2. The open end is placed under the water surface as shown. A line L is marked on the box at a distance of 10 cm from the closed or upper end F, so that a volume of 1000 cm^3 or 1 litre is defined.

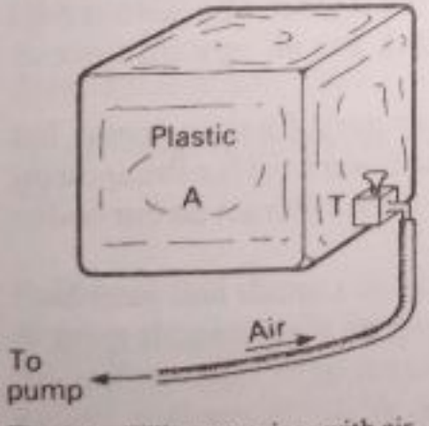


Fig. 6.4 Filling container with air

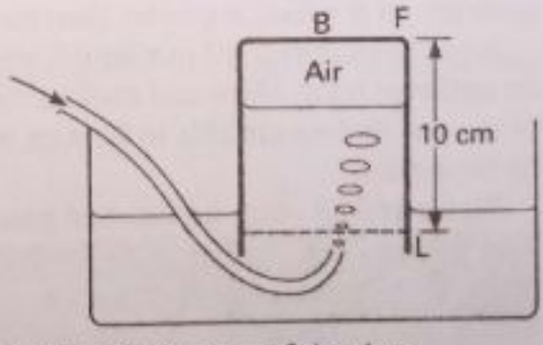


Fig. 6.5 Measurement of air volume

When the box is adjusted so that the water levels inside and outside are in line with the mark L, one litre of air at atmospheric pressure has been collected. The tap T on the plastic container is now closed, the collecting vessel B is refilled with water and the process is repeated. A whole number of litres of air together with an estimated fraction of a litre is then collected. The plastic container A (which will still have some air in it) is finally reweighed (m_2), and so the mass of air collected and its volume are known.

Scanned with CamScanner

Measurements:	Mass of container A + pumped air/g (m_1)	
	Mass of A after air removed/g (m_2)	
	Volume of air collected/litre (l)	

Calculation: Density of air at atmospheric pressure = mass/volume = $\frac{m_1 - m_2}{\text{volume}}$ g/l

Result: Accurate experiments show that at standard temperature (0°C) and standard pressure (760 mm mercury or 760 mmHg), the density of air is about 1·3 g/l, which is 1·3 kg/m³ or 0·0013 g/cm³.

Density of gases

The volume, and hence the density, of a gas is very sensitive to changes of temperature and pressure (page 188). The densities of some gases at standard temperature (0°C) and pressure (760 mmHg) are

Hydrogen	0·00009 g/cm³	or	0·09 kg/m³
Nitrogen	0·00125 g/cm³	or	1·25 kg/m³
Helium	0·00018 g/cm³	or	0·18 kg/m³

It is interesting that accurate measurements of the density of air led to the discovery of the rare gases in the atmosphere. One is argon, used for filling electric lamps; two others are helium and neon. Lord Rayleigh and Sir William Ramsay found that nitrogen gas obtained from the atmosphere was very slightly denser than nitrogen gas obtained from a chemical reaction. They came to the conclusion that there was an impurity in the 'atmospheric' nitrogen. After further investigations they discovered argon and other rare gases.

Forces due to fluids

Upthrust

If a cork is held below the surface of a liquid and then released it immediately rises to the surface. So when it is inside the liquid the cork has an upward force or *upthrust* on it which is greater than its weight.

Any object immersed in a liquid, whether all or only part of it is immersed, has an upthrust on it. Ships and submarines would sink if it were not for the upthrust of the sea, and we are able to float on water because of the upthrust on our bodies by the liquid.

Water and all other liquids and gases, are called *fluids*. Liquids and gases both transmit and exert pressure. So both exert an upthrust on objects inside them. A floating balloon has an upthrust on it due to the pressure of the surrounding air, which counterbalances the weight of the balloon. A liquid exerts a greater pressure than a gas because it is more dense, and so it exerts a much greater upthrust.

Upthrust measurement in liquids

The upthrust on a small object in a liquid can be measured in the laboratory by using a spring-balance. Fig. 6.6a shows a solid object, A, such as a metal, attached to a spring-balance by a string B. The reading 1·2 N is the weight W of

Scanned with CamScanner

A. Fig. 6.6b shows A now totally immersed in a liquid. The spring-balance reading decreases to 0·9 N. This is the size of the tension (force) T in the string tied to A. It is less than 1·2 N because the liquid pushes upwards or exerts an upthrust on A. From the two readings,

$$\text{upthrust} = 1{\cdot}2\,\text{N} - 0{\cdot}9\,\text{N} = 0{\cdot}3\,\text{N}$$

The reading of 0·9 N in Fig. 6.6b has been called the 'apparent weight' of A in the liquid. Actually it is the tension T in the string supporting A. The reading of 1·2 N in Fig. 6.6a is the true weight W of A. So the 'apparent loss in weight' is equal to the upthrust.

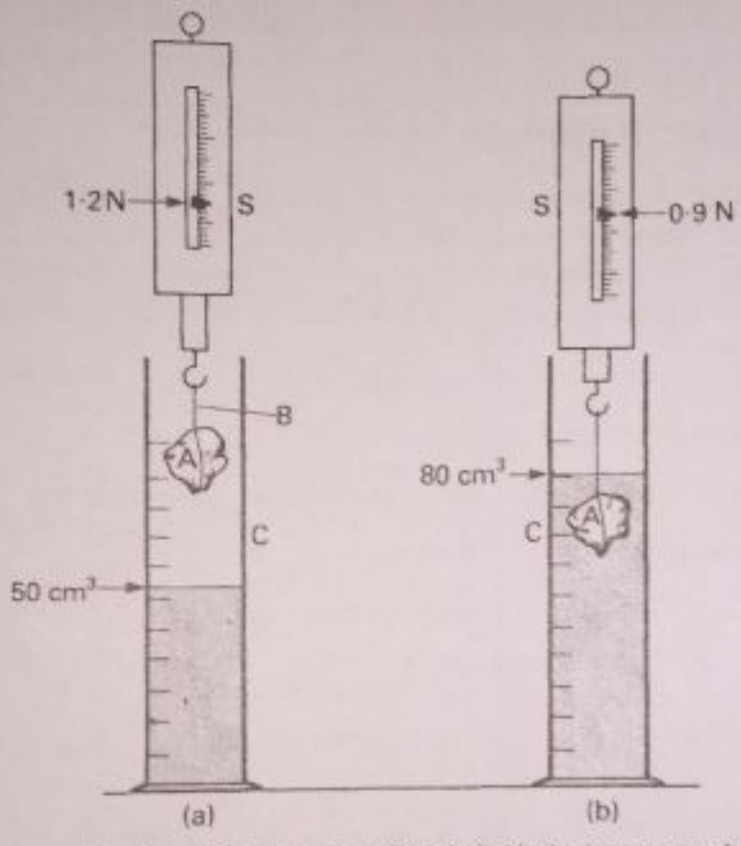


Fig. 6.6 Investigation of Archimedes' Principle. (a) Weight in air (b) Apparent weight in water

Magnitude of upthrust

We can investigate the magnitude of the upthrust due to a liquid in the following way.

Experiment

Suspend a large piece of metal or other solid object A by a thread from a spring-balance S and note the weight of A from the reading (Fig. 6.6a).

Fill a suitable measuring cylinder C with water to a convenient graduation such as 50 cm³, for example. Now immerse A completely in the water so that it hangs vertically below S and does not touch the sides of C (Fig. 6.6b). Note the new reading on S and the new level of water in C.

Measurements: The following measurements were taken in one experiment.

Weight of A in air = 1·2 N, weight of A in water = 0·9 N,

Original water level in C = 50 cm³, final water level = 80 cm³.

Scanned with CamScanner

Calculation Upthrust on A = weight in air − weight in water
= 1·2 − 0·9 = 0·3 N

Volume of water displaced by A = 80 − 50 = 30 cm³

∴ mass of water displaced = 30 g

∴ weight of liquid displaced = 30 × 0·01 = 0·3 N

since the weight of 1 g mass is 0·01 N in round figures (see page 40)

Conclusion The upthrust on an object immersed in water is equal to the weight of water displaced

Repeat the experiment with other liquids and see if this is always true

Archimedes' Principle

More than two thousand years ago Archimedes stated his famous principle or law about the magnitude or size of the upthrust on an object in a fluid.

Archimedes' Principle states *when a body is totally or partially immersed in a fluid (liquid or gas), the upthrust on it is equal to the weight of fluid displaced.*

Examples on Archimedes' Principle

1. An iron cube of volume 800 cm³ is totally immersed in (a) water; (b) oil of density 0·8 g/cm³; (c) oxygen gas of density 0·0015 g/cm³. Calculate the upthrust in each case, if the weight of 1 g mass = 0·01 N.

Upthrust = weight of fluid displaced

(a) Upthrust = weight of 800 cm³ of water
= 800 × 1 × 0·01 N = 8 N

(b) Upthrust = weight of 800 cm³ of oil
= 800 × 0·8 × 0·01 N = 6·4 N

(c) Upthrust = weight of 800 cm³ of oxygen
= 800 × 0·0015 × 0·01 N = 0·012 N

2. A metal cube weighs 1·0 N in air and 0·8 N when totally immersed in water. Calculate the volume of the cube and its density.

We have, upthrust = 1·0 − 0·8 = 0·2 N

So 0·2 N = weight of water displaced

$$\therefore \text{ mass of water displaced} = \frac{0{\cdot}2}{0{\cdot}01} = \frac{20}{1} = 20\text{ g}$$

∴ volume of water displaced = 20 cm³ = volume of cube

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{100\text{ g}}{20\text{ cm}^3} = 5\text{ g/cm}^3$$

3. An iron cube, mass 480 g and density 8 g/cm³, is suspended by a string so that it is half-immersed in an oil of density of 0·9 g/cm³. Find the tension (force) in the string.

$$\text{Volume of cube} = \frac{\text{mass}}{\text{density}} = \frac{480}{8} = 60\text{ cm}^3$$

∴ upthrust = weight of 30 cm³ of oil (because the cube is half immersed)
= 30 × 0·9 × 0·01 N = 0·27 N

So tension in string (apparent weight) = weight − upthrust = 4·80 − 0·27 = 4·53 N

Scanned with CamScanner

Density measurement

Archimedes' Principle helps us to measure the density of solids and liquids.

Experiments

1. Density of a solid

Weigh a large piece of metal, such as a brass weight, by attaching it with thread to a spring-balance and note the value. Then weigh the solid *fully* immersed in water in a beaker, making sure the solid does not touch the sides of the beaker.

Measurements:	Weight of solid in air, W_1/N	
	Weight of solid in water, W_2/N	

Calculation: Upthrust in water $= W_1 - W_2$

Using 0·01 N = weight of 1 g mass, then

$$\text{mass of water displaced in grams} = \frac{(W_1 - W_2)}{0{\cdot}01}$$

This is also numerically the *volume in cm³* displaced, since 1 g of water has a volume of 1 cm³. Also

$$\text{mass of solid in grams} = \frac{W_1}{0{\cdot}01}$$

So

$$\text{density of solid} = \frac{\text{mass}}{\text{volume}} = \frac{W_1/0{\cdot}01}{(W_1 - W_2)/0{\cdot}01}$$

$$= \frac{W_1}{W_1 - W_2}\ \text{g/cm}^3$$

Results: In one experiment, a solid X had a weight of 1·8 N in air and 1·6 N fully immersed in water.

Then upthrust $= 1{\cdot}8 - 1{\cdot}6 = 0{\cdot}2$ N = weight of water displaced by X

So mass of water displaced $= \frac{0{\cdot}2}{0{\cdot}01} = 20$ g. Also, density of water $= 1\ \text{g/cm}^3$

So volume of water displaced $= 20\ \text{cm}^3$. This is the *volume of X*.

Since mass of X $= 1{\cdot}8/0{\cdot}01 = 180$ g

$$\text{density of X} = \frac{180\ \text{g}}{20\ \text{cm}^3} = 9\ \text{g/cm}^3.$$

2. Density of a liquid

Weigh a large piece of metal or other solid (a) in air, W_1, (b) fully immersed in a beaker of the liquid, W_2, (c) fully immersed in a beaker of water, W_3, after drying the solid on removal from the liquid. Make sure the solid does not touch the sides of the beaker when weighed in the liquid.

Calculation: Upthrust in liquid $= W_1 - W_2$ = weight of liquid displaced.

Using 0·01 N as the weight of 1 g, then

mass of liquid displaced in grams $= (W_1 - W_2)/0{\cdot}01$

Also upthrust in water $= W_1 - W_3$ = weight of water displaced

Scanned with CamScanner

As we have just shown, the *volume* of water displaced by the object in cm^3 = $(W_1 - W_2)/0{\cdot}01$. This is also the volume of the liquid displaced by the solid. So

$$\text{density of liquid} = \frac{\text{mass of liquid displaced}}{\text{volume of liquid displaced}} = \frac{W_1 - W_2}{W_1 - W_3}$$

Measurements: In one experiment a solid weighed 0·8 N in air, 6·4 N in oil and 6·0 N in water.

Calculation: Upthrust in oil = 8·0 − 6·4 = 1·6 N = weight of oil displaced
and upthrust in water = 8·0 − 6·0 = 2·0 N = weight of water displaced

Using 0·01 N as the weight of 1 g mass, the mass of water displaced = 2·0/0·01 = 200/1 = 200 g. This has a volume of 200 cm^3. So

volume of oil displaced = 200 cm^3

But *mass of oil* displaced = 1·6/0·01 = 160 g

So $$\text{density of oil} = \frac{160\,g}{200\,cm^3} = \frac{8\,g}{10\,cm^3} = 0{\cdot}8\,g/cm^3$$

Principle of Flotation

At anchor, boats and ships are floating objects. Although only partially immersed in the water, these objects have an upthrust on them equal to the weight of water they displace, from Archimedes' Principle.

Experiment

To investigate the upthrust on a floating object, a narrow wooden rod is slightly weighted at one end and then weighed. A burette B is now partly filled with water and the level of the liquid noted (Fig. 6.7). The rod is now placed in the burette so that it floats clear of the sides. The new reading of the water level in B is taken.

Measurements: The following measurements were obtained in an experiment.

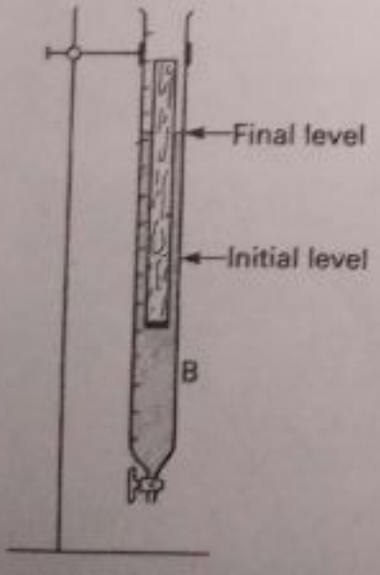


Fig. 6.7 Principle of Flotation

Weight of wood	= 0·6 N
Initial reading in burette	= 25·0 cm^3
Final reading in burette	= 85·0 cm^3
∴ Volume of water displaced	= 60·0 cm^3
∴ Mass of water displaced	= 60 g
∴ Weight of water displaced	= 0·6 N

Conclusion: The weight of the floating wood is equal to the weight of water displaced.

Scanned with CamScanner

This experimental result also follows from simple reasoning. Any floating object is in equilibrium under two opposing forces: (a) its weight, (b) the upthrust of the liquid. So *the weight of liquid displaced* (the upthrust) *is equal to the weight of the object*. This is called the *Principle of Flotation*.

Some objects floating in water are shown in Fig. 6.8. In each case the weight of the object is equal to the weight of water displaced.

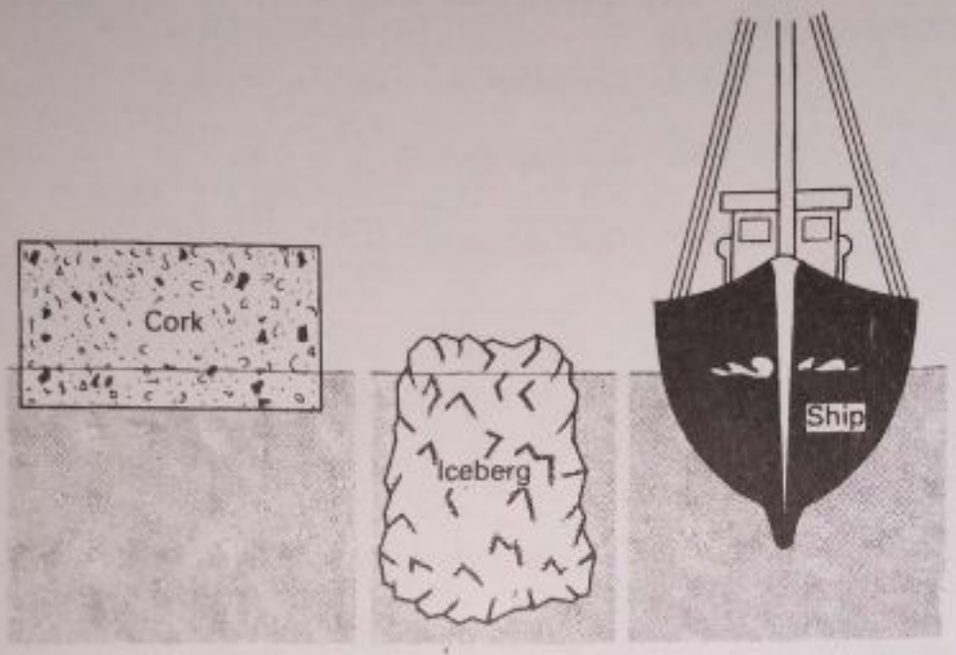


Fig. 6.8 Floating objects of different densities

Examples on Archimedes' Principle and floating objects

1. A piece of cork of volume 100 cm^3 is floating on water. If the density of cork is 0·25 g/cm^3, (a) calculate the volume of cork immersed in the water. (b) What force is needed to immerse the cork completely? (c) Ice has a density about 0·9 g/cm^3. What fraction of the volume of an iceberg is submerged in water? (Assume mass of 1 g has a weight of 0·01 N.)

(a) Mass of cork = volume × density = $100 \times 0{\cdot}25 = 25$ g

So, from Principle of Flotation,

weight of water displaced (upthrust) = weight of 25 g mass

$\therefore$ mass of water displaced = 25 g

Since 1 g of water has a volume of 1 cm^3,

volume of water displaced = 25 cm^3 = volume of cork immersed

(b) When the cork is pushed down by a force and is submerged completely,

upthrust = weight of liquid displaced = weight of 100 cm^3 water

= weight of 100 g mass = 1 N

From (a), weight of cork = weight of 25 g mass = $25 \times 0{\cdot}01$ N = 0·25 N

downward force needed = upthrust − weight

= 1 − 0·25 = 0·75 N

(c) The density of cork is 0·25 g/cm^3 and, from (a), 25/100 or 0·25 of its volume is immersed when it floats. Similarly, since the density of ice is 0·9 g/cm^3, then 0·9 or 9/10 of its volume is submerged when the iceberg floats (see Fig. 6.8).

Scanned with CamScanner

2. A hollow metal sphere of mass 5 kg is tied to the bottom of the sea-bed by a rope. The tension or force in the rope is 60 N. Calculate the volume of the sphere. (Assume density of sea water = 1100 kg/m³ and weight of 1 kg is 10 N.)

$$\text{Weight of sphere} = mg = 5 \times 10 = 50\ \text{N}$$

$$\text{downward total force on sphere} = \text{tension} + \text{weight}$$
$$= 60 + 50 = 110\ \text{N}$$

Since the sphere is in equilibrium,

$$\text{upthrust on sphere} = 110\ \text{N}$$

From Archimedes' Principle,

$$\text{weight of liquid displaced} = \text{upthrust} = 110\ \text{N}$$

$$\therefore \text{mass of liquid displaced} = \frac{110}{10} = 11\ \text{kg}$$

$$\text{volume of liquid displaced} = \frac{\text{mass}}{\text{density}}$$

$$= \frac{11}{1100} = 0{\cdot}01\ \text{m}^3 = \text{volume of sphere}$$

Ice melting in water

Suppose some ice is floating on water inside a vessel such as a measuring cylinder. If the mass of the ice is 9 g, then the volume of water displaced by the ice is 9 cm³, from the principle of flotation. If the ice density is 0·9 g/cm³ the volume of ice itself is 10 cm³; 1 cm³ is above the water line.

When all the ice melts, 9 g of water is formed. This has a volume of 9 cm³. So the water formed takes up the space left by the melted ice. Consequently the level of water in the measuring cylinder remains unchanged.

For this reason, if a glass of water, with some ice floating on top, fills the glass to the brim, the water does not overflow as the ice melts but remains at the same level.

Upthrust in gas Balloons

Balloons used for weather soundings in the upper atmosphere, or for cosmic ray investigations, are filled at ground level with a small amount of a light gas such as helium or hydrogen. So the balloon rises into the air when released and the gas inside it expands. At some height where the density of air is much less than at ground level, perhaps as high as 30–40 km, the upthrust is equal to the total weight of the balloon. The balloon then stops rising and drifts sideways with the wind. As an illustration, consider the following calculation, which assumes the weight of 1 kg mass is 10 N (page 40).

Example on upthrust in gas

A meteorological balloon has a volume on the ground of 10 m³ and the fabric of the balloon weighs 80 N. It is filled with hydrogen of density 0·09 kg/m³, and the air density is 1·2 kg/m³. Estimate the maximum weight additional to that of the hydrogen and fabric which the balloon can lift.

Scanned with CamScanner

Upthrust = weight of air displaced = mg
= volume × density × g = $10 \times 1{\cdot}2 \times 10$ N
= 120 N

Now weight of hydrogen = mg = $10 \times 0{\cdot}09 \times 10$ N = 9 N

So total weight of hydrogen + fabric = 89 N

∴ maximum additional weight lifted = $120 - 89 = 31$ N



Plate 6A The Goodyear airship 'Europa' at mooring. The volume of the balloon is about 5740 m^3. It is filled with helium gas and the 'skin' is made of a synthetic fibre with a total surface area of about 2300 m^2. The maximum altitude of this airship is about 2600 m. (*Goodyear Tyre and Rubber Company Ltd*)

Why ships float

Ships float in water, although they are made from iron and steel which are materials denser than water. This is because the ship is an 'open' object, not a solid object. It is a large hollow object whose outer surfaces are coated with iron and steel. The volume enclosed by the bottom and sides of the ship is large. Thus the weight of water displaced, and hence the upthrust, is large.

The ship sinks to a level such that the upthrust is equal to the total weight of the ship.

The size of a ship is often listed as '10000 tonnes displacement', for example, since its weight is equal to the weight of water displaced. (The 'tonne' is a metric unit equal to 1000 kg.)

Scanned with CamScanner

Example on a floating ship

A barge, 40 metre long and 8 metre broad, whose sides are vertical, floats partially loaded in water. If 125000 N of cargo is added, what distance will it sink? (Assume weight of 1 kg mass is 10 N; density of water = 1000 kg/m³.)

The increased upthrust of the water = 125000 N

So, from Archimedes' Principle

weight of extra water displaced = 125000 N

$\therefore$ mass of extra water displaced = 125000/10 = 12500 kg

$$\therefore \text{increase in volume of water displaced} = \frac{\text{mass}}{\text{density}}$$

$$= \frac{12500\ \text{kg}}{1000\ \text{kg/m}^3} = 12{\cdot}5\ \text{m}^3$$

$$\therefore \text{distance barge sinks} = \frac{\text{volume of water}}{\text{area of water}}$$

$$= \frac{12{\cdot}5\ \text{m}^3}{40 \times 8\ \text{m}^2} = 0{\cdot}04\ \text{m} = 4\ \text{cm}$$

Plimsoll line

With a cargo, a ship sinks lower in the water as the weight of water displaced then increases. If it sinks too low, the ship becomes unsafe in heavy seas and may sink. Ships therefore carry a line painted on their hulls, which must be above the water. It is called the *Plimsoll line*, after the person who first suggested it.

The density of sea-water varies all over the world. As shown in Fig. 6.9 for a ship on Lloyd's Register of shipping (LR), the Plimsoll lines are TF, tropical fresh water; F, fresh water; T, tropical; S, summer; W, winter; WNA, winter in North Atlantic Ocean.

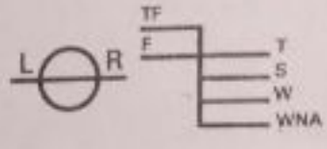


Fig. 6.9 Plimsoll line

Submarine

The submarine is another application of Archimedes' Principle. In surface trim (Fig. 6.10) the submarine floats with its conning tower and most of the deck clear

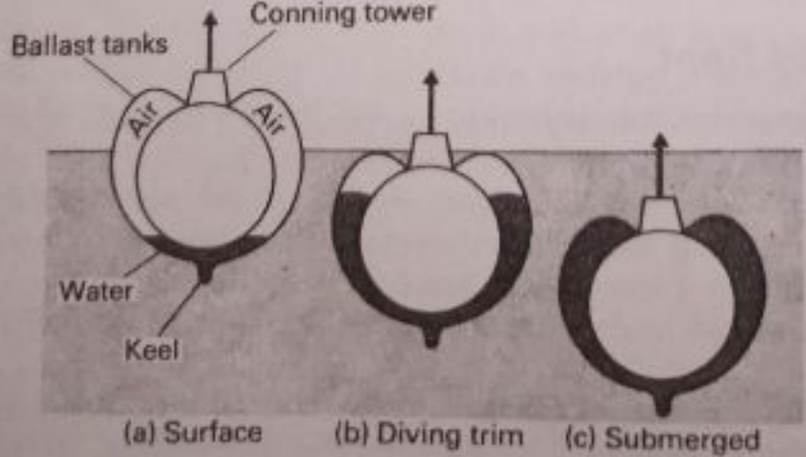


Fig. 6.10 Submarine going down

Scanned with CamScanner

of the water. The submarine has large ballast tanks which may be filled with water, thus increasing the weight so that it sinks lower. Horizontal rudders are tilted to make the submarine dive downwards when the tanks are practically full and it is ready to submerge. When the submarine is ready to surface the rudders are moved to direct the vessel upwards, and compressed air is forced into the ballast tanks to drive the water out so the submarine can rise.

The hydrometer

A *hydrometer* is an instrument for measuring the density or relative density of a liquid. It is used, for example, to measure the density of acid in a car battery (because this is related to the battery condition), the strength of spirits, or the degree of richness of milk. In dock, the density of the sea-water must be entered in a ship's log book.

A simple hydrometer can be made in the laboratory by flattening the closed end of a test-tube A and placing lead shot in the bottom to make it float upright in liquids (Fig. 6.11). Suppose the mass of A and its contents is 10 g so that the weight is 0·1 N and it is placed upright in water of density 1 g/cm^3. Then, from the Principle of Flotation, the upthrust of the water is equal to the weight, 0·1 N.

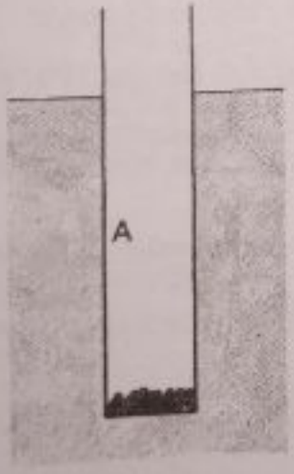


Fig. 6.11 Model simple hydrometer

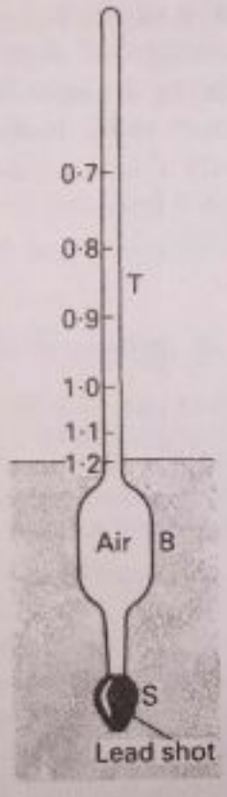


Fig. 6.12 Practical hydrometer

Now the upthrust = weight of water displaced = weight of a mass 10 g. So the volume of water displaced is 10 cm^3. If the uniform cross-section area of A is 1 cm^2, its depth in water is 10 cm.

Now suppose A is transferred to a liquid of much greater density, say 2 g/cm^3. The upthrust when A floats = weight of A = 0·1 N. So mass of liquid displaced = 10 g. The volume of this liquid = mass/density = 10/2 = 5 cm^3. So A is now

Scanned with CamScanner

only 5 cm below the surface of the dense liquid, whereas it was 10 cm below the surface in water.

Generally, A will sink more in liquids *less* dense than water and sink less in liquids *more* dense than water. We can make a 'model hydrometer' by placing A in liquids of known densities and marking the density values on a paper scale inside A on a level with the liquid surface. As shown in Fig. 6.12, the density scale of a hydrometer is irregular, that is, the 1·1 mark (for a density of 1·1 g/cm^3) is not half-way between the 1·0 and 1·2 marks.

Note also that the 0·7 mark is at the *top* and the 1·2 mark is at the *bottom* of the hydrometer stem, because the hydrometer will sink much further in a liquid of density 0·7 g/cm^3 than in one of density 1·2 g/cm^3.

Practical hydrometer

One form of practical hydrometer is shown in Fig. 6.12. It has (a) a hollow narrow glass tube or stem T; (b) a paper scale inside T graduated in densities; (c) a wide bulb B; (d) a loaded end S containing lead shot, for example, to keep it upright in liquids.

The densities that can be measured depend largely on the volume of the bulb B. For a given hydrometer weight, the bulb must sink so that the surface of the liquid comes up to the stem. The narrower the stem, the more sensitive is the hydrometer; a small change in volume when the hydrometer is taken from one liquid to another of slightly differing density then produces a large vertical difference along the stem. The density scale on the stem has unequal divisions, and the density values increase towards the bottom of the stem.

The density of a liquid varies with temperature (page 184). A temperature such as '15°C' on a hydrometer shows that the density values marked on the stem apply at 15°C.

Examples on upthrust in air, density of liquid and the hydrometer

1. A balloon of volume 2000 m^3 is filled with hydrogen of density 0·09 kg/m^3. If the mass of the fabric is 100 kg and that of the pilot is 75 kg, what will be the greatest mass of equipment that can be carried when operating in air of density 1·25 kg/m^3? How would this figure change if helium, which has twice the density of hydrogen under the same conditions, were to be used? (L)

Upthrust in air = weight of air displaced (Archimedes' Principle)
= volume × density × g
= 2000 × 1·25 × 10 = 25000 N

Mass of hydrogen in balloon = 2000 × 0·09 = 180 kg

$\therefore$ total weight = (100 + 75 + 180) × 10 = 3550 N

$\therefore$ greatest weight carried = upthrust − total weight = 25000 − 3550 = 21450 N

$\therefore$ greatest mass carried = 21450/10 = 2145 kg

If helium were used

mass of helium = 2 × mass of hydrogen
= 2 × 2000 × 0·09 = 360 kg

This is 180 kg more than the mass of hydrogen.

So new greatest mass carried = 2145 − 180 = 1965 kg

2. A body weighs 0·52 N in air. Totally immersed in water it weighs only 0·32 N, while its weight when immersed in another liquid is 0·36 N. The density of water is 1000 kg/m^3. What is the density of the other liquid? (O and C)

Scanned with CamScanner

Upthrust in liquid = weight in air − apparent weight in liquid
= 0·52 − 0·36 = 0·16 N
= weight of liquid displaced (Archimedes)

Upthrust in water = 0·52 − 0·32 = 0·20 N = weight of water displaced

Now *volume* of liquid and water displaced is the *same*. Therefore

$$\frac{\text{density of liquid}}{\text{density of water}} = \frac{0{\cdot}16}{0{\cdot}20} = 0{\cdot}8$$

$\therefore$ density of liquid = 0·8 × density of water = 800 kg/m^3

3. The diagram (Fig. 6.13) shows one form of common hydrometer used to measure the densities of liquids over the range 0·80 to 1·00 g/cm^3.

If the area of cross-section of the stem is 0·50 cm^2 and the distance between the 0·80 and 1·00 divisions is 18 cm, determine (a) the volume of the hydrometer below the 1·00 graduation, (b) the position of the 0·90 graduation. (You may assume density of water = 1·00 g/cm^3.) (L)

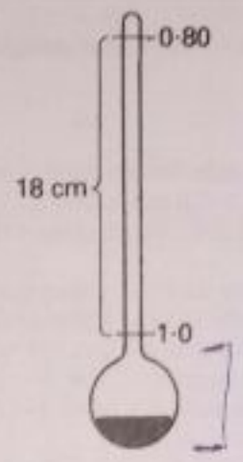


Fig. 6.13 Example

(a) Suppose V cm^3 is the volume of the hydrometer below the 1·00 mark.

Volume between 0·80 and 1·00 marks = 18 × 0·50 = 9 cm^3

$\therefore$ volume below 0·80 mark = $(V+9)$ cm^3

Now upthrust in liquid at 0·80 mark = $(V+9) \times 0{\cdot}80 \times 0{\cdot}01$ N

and upthrust in water at 1·00 mark = $V \times 1 \times 0{\cdot}01$ N

From the principle of flotation, both upthrusts are equal to the weight of the hydrometer and so are equal to each other.

$$\therefore (V+9) \times 0{\cdot}8 \times 0{\cdot}01 = V \times 1 \times 0{\cdot}01$$

Solving $V = 36$ cm^3

(b) Weight of water displaced = hydrometer weight = 36 × 1 × 0·01 = 0·36 N

So in liquid of density 0·90 g/cm^3, new volume V_1 displaced is given by

$$V_1 \times 0{\cdot}90 \times 0{\cdot}01 = 0{\cdot}36$$

$$\therefore V_1 = \frac{0{\cdot}36}{0{\cdot}009} = \frac{360}{9} = 40 \text{ cm}^3$$

$\therefore$ volume of liquid above 1·00 mark = 40 − 36 = 4 cm^3

$$\therefore \text{height of 0·90 mark above 1·00 mark} = \frac{4 \text{ cm}^3}{0{\cdot}5 \text{ cm}^3} = 8 \text{ cm}$$

Summary

1. Density = mass/volume. Unit: kg/m^3 or g/cm^3.
Density of water = 1000 kg/m^3 or 1 g/cm^3.

Scanned with CamScanner

2. Archimedes' Principle states: the upthrust on an object immersed in a fluid is equal to the weight of fluid displaced.

3. Relative density of a solid $= \dfrac{\text{weight of solid}}{\text{apparent loss of weight in water}}$

Relative density of a liquid $= \dfrac{\text{apparent loss of weight of a solid in liquid}}{\text{apparent loss of weight of same solid in water}}$

4. Principle of Flotation: when an object floats, its weight is equal to the weight of fluid displaced.

5. Practical hydrometers have a narrow stem with a scale graduated in density values which increase down the stem, a wide bulb and a loaded end.

Exercise 6 (Answers, page 640)

(Assume the weight of 1 kg mass = 10 N, the weight of 1 g mass = 0·01 N)

Density

1. A solid cube of steel has each side 10 cm long. Find its mass if the density of steel is 6·5 g/cm³. What volume of steel has a mass of 130 g?
2. 'The relative density of brass is 8·5.' What does this statement mean? Find the mass of 500 cm³ of brass.
3. A thread of mercury has a mass of 10·2 g and is in a tube of uniform cross-section 0·1 cm². Calculate the length of the thread if the density of mercury is 13·6 g/cm³.
4. 100 g of water is mixed with 60 g of a liquid of relative density 1·20. Assuming no change in volume, find the average relative density of the mixture.
5. Describe an experiment to measure the density of air. Point out any difficulties you would expect in getting an accurate result.
6. A 1 cm³ sample of tin–lead alloy has mass 8·5 g. The density of tin is 7·3 g/cm³ and that of lead is 11·3 g/cm³. Calculate the percentage by weight of tin in the alloy. You may assume that there is no change of volume when the metals form the alloy. (O and C)
7. A cube of glass, of 5 cm side and of mass 306 g, has a cavity inside it. If the density of glass is 2·55 g/cm³, what is the volume of the cavity? (O)
8. Distinguish between *density* and *relative density*. How would you attempt to determine the density of air under standard laboratory conditions?

A cylindrical beaker of mass 50 g, cross-sectional area 25 cm² and height 10 cm is filled with oil of relative density 0·8. What is the total mass? A piece of aluminium, of mass 66 g and relative density 2·2, is lowered carefully into the beaker. What volume of oil overflows? What is the final mass of the beaker and its contents after the outside has been wiped to remove over-flow liquid? (O and C)

Forces due to fluids

9. (a) A spring balance reads 12·0 N when a metal block is suspended from it and 10·0 N when the block is completely immersed in water. Calculate the upthrust on the block, the mass of water displaced, and the volume of the block.

(b) A block of wood of mass 100 g floats in water. What volume of it is below the surface? If this volume is half the volume of the wood, what is the density of the wood?

(c) Draw a diagram of a hydrometer used for measuring the relative densities of liquids in the range 0·80 to 1·10, showing the graduations 0·80, 0·90, 1·00, 1·10 on the stem. Why does the hydrometer have a weighted base and a wide bulb?

(d) A balloon of volume 100 m³ and mass 2 kg is filled with helium of density 0·18 kg/m³ at ground level. The surrounding air has a density of 1·26 kg/m³. Calculate the maximum mass which the balloon can lift at the ground.

10. Fig. 6.14 shows a buoy, B, of volume 40 litres and mass 10 kg. It is held in position in sea water of density 1·04 g/cm³ (1·04 kg/litre) by a light cable fixed to the bottom so that $\frac{3}{4}$ of the volume of the buoy is below the surface of the sea water.

Scanned with CamScanner

(a) Name the three forces that keep the buoy in equilibrium and state the direction in which each acts.

(b) Determine the tension in the cable. (L)

11. Draw a labelled diagram of a common hydrometer and explain carefully why a narrow stem leads to greater sensitivity than a wide stem.

A solid block of wood of density 0·6 g/cm³ weighs 3·06 N in air. Determine (a) the volume of the block, (b) the volume immersed when it floats freely in a liquid of density 0·9 g/cm³, (c) the minimum extra vertical force required to cause the block to submerge completely in the liquid. (L)

12. A cube of wood of side 10 cm floats in water with 4·5 cm of its depth below the surface, and with its sides vertical. What is the density of the wood? (N)

13. A solid weight 6·0 N is suspended by a string in water. The tension in the string is 4·5 N. Find (a) the volume of the solid; (b) its density; (c) its relative density.

14. Fig. 6.15 shows an iron cylinder of length 10 cm and uniform cross-section 2 cm² suspended from a spring-balance with one-half of its length immersed in paraffin oil of density 0·8 g/cm³. If the density of iron is 7·5 g/cm³, determine (a) the mass of the iron, (b) the reading of the spring-balance. (L)

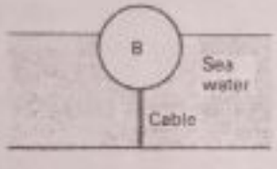


Fig. 6.14

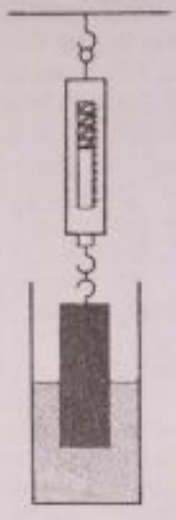


Fig. 6.15

15. (a) Draw a clearly labelled diagram of a common hydrometer which is suitable for measuring the densities of liquids varying between 1·0 and 1·2 g/cm³. Show clearly the marks indicating densities 1·0, 1·1 and 1·2 g/cm³. State the principle upon which the instrument's use depends.

(b) A metal cylinder of mass 50 g is suspended from the lower end of a vertical helical spring supported at its upper end. The unextended length of the spring is 10·0 cm. When the cylinder is in various situations, the corresponding lengths of the spring are as shown in the table.

Situation	*Length*
Cylinder in air	12·5 cm
Cylinder completely submerged in water	12·2 cm
Cylinder completely submerged in oil	12·3 cm

Calculate the density of the metal and the density of the oil. (N)

16. State the Principle of Archimedes. Explain how a submarine is made (a) to sink to a point below the surface of water, (b) to rise again to the surface. How is the weight related to the upthrust on it in both cases?

A floating crane has mass 20 600 kg and floats in sea water of density 1·03 g/cm³ (1030 kg/m³). If the base of the crane is a rectangular block 4 m square and 2·5 m deep, what is the maximum mass which can be lifted by the crane when the top of the base is just awash with water? (N)

17. Describe an experiment *or* give a theoretical argument in support of the Principle of Archimedes.

A balloon consists of a plastic bag which can crumple but not stretch. The maximum volume is 200 cubic metres but at ground level at s.t.p. the hydrogen filling occupies only half the maximum volume.

Scanned with CamScanner

(a) What is the greatest mass that the balloon can lift with this filling?
(b) What is the density of the surrounding atmosphere when the balloon, loaded with 5 kg mass, stops rising?
(c) Suggest a way of varying the height to which a balloon of this kind will rise.
Take density of air at s.t.p. as 1·29 kg/m³, of hydrogen at s.t.p. as 0·09 kg/m³. (O and C)

18. State Archimedes' Principle and describe an experiment to verify it.
A uniform closed glass tube, weighted internally at one end, has a mass of 50 g. When it floats vertically in water 10 cm of its stem is exposed. When it floats vertically in a liquid of relative density 1·25, 20 cm of its stem is exposed.
Calculate the area of cross-section of the tube. (N)

19. Describe an experiment to demonstrate the Principle of Archimedes.
A uniform rectangular block of wood, 20 cm × 10 cm × 7·5 cm and of mass 1000 g stands on a horizontal surface with its longer edge vertical. Find (a) the pressure exerted by the block on the surface, (b) the density of the wood.
The block is floated in water with its longest edge vertical. State the weight of water displaced by the block and find the depth to which the block sinks in the water.
The block is then floated in brine, of relative density 1·2, and a mass of metal, of relative density 11·2, is added to the block so that the top surface of the block lies in the surface of the liquid. Calculate the mass of metal required if it is (c) placed on the top of the block, (d) fixed to the bottom of the block. (C)

20. State *Archimedes' Principle* and define *relative density*.
Describe how the relative density of a solid substance may be measured accurately by the application of Archimedes' Principle. Show how you obtain the expression used to calculate the result.
Calculate the volume of a metal cylinder of mass 105 g which has an apparent weight of 0·925 N when fully immersed in water of density 1·00 g/cm³. When a piece of wax of mass 20 g is attached to the cylinder and both are fully immersed in water the total apparent weight is found to be 0·875 N. What is the density of the wax? (L)

21. A body weighs 0·52 N in air. Totally immersed in water it weighs only 0·32 N while its weight when immersed in another liquid is 0·36 N. The density of water is 1000 kg/m³.
What is the density of the other liquid? (O and C)

22. State Archimedes' Principle and *either* show how it may be deduced theoretically *or* describe an experiment to verify it.
A plastic block of volume 200 cm³ (2×10^{-4} m³) is suspected of having air cavities (bubbles) in it. It is tested by floating in water of density 1000 kg/m³, and it is found to float with 90 per cent of its volume immersed. The density of the plastic is 1200 kg/m³. Calculate the total volume of the air bubbles.
A man in a small boat at rest on a lake lowers an iron anchor over the side. What is the effect on the volume of water displaced by the *boat* (a) when the anchor is hanging in the water, (b) when the anchor is resting on the bed of the lake? (O and C)

Multiple-Choice Questions

23. Some ice floats in 20 cm³ of water in a measuring cylinder. Which of the statements A to E is correct?

A The mass of ice is 20 g
B the volume of ice is 20 cm³
C the density of ice is greater than that of water
D the weight of ice is equal to the weight of water displaced
E the mass of ice is equal to the weight of water displaced

24. A metal solid X is completely immersed in water by a string PQ, Fig. 6.16. The top of X is 8 cm from the bottom and the water surface is 20 cm from the bottom.

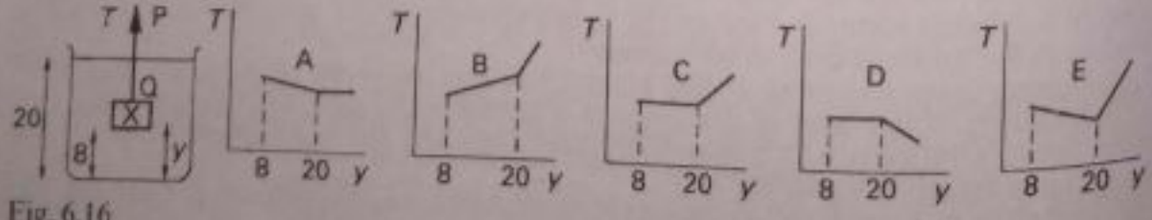


Fig. 6.16

X is now raised to the surface by the string and then pulled just clear of the surface. Which one of the graphs A to E best shows how the tension T in the string varies with the distance y of X from the bottom as X is raised.

Scanned with CamScanner

7. Pressure in Fluids

Meaning of pressure

An elephant has feet of wide area. When it walks, its large weight is therefore spread over a wide area and so its weight or force *per unit area* is a moderate value. This helps the elephant to walk over soft ground without sinking much into the ground.



Plate 7A An African elephant. The great weight of the elephant produces only a relatively small pressure below its feet because they have a large area. (*Paul Chinnock*)

On the other hand, if a heavy parcel is carried by thin string, the weight of the parcel is spread over the small area of the string in contact with the hand. So a large force per unit area is produced on the flesh. After carrying the parcel for some distance this has a painful effect.

Pressure is defined as the average force per unit area

So the average pressure p over a surface is given by

$$p = \frac{\text{total force on surface } (F)}{\text{area of surface } (A)} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Scanned with CamScanner

A unit of pressure is newton per metre^2 (N/m^2) and the name *pascal* (Pa) is given to this unit. Note carefully that pressure is *not* a force; it is a 'force per unit area'.

In Fig. 7.1 suppose the weight or force on the narrow heel of a girl is 500 N and the area of the heel in contact with the floor is 80 mm^2. Then

$$\text{average pressure} = \frac{\text{for}}{\text{area}} \quad \frac{500\text{ N}}{80\text{ mm}^2} = 6{\cdot}25\text{ N/mm}^2$$

With a wider heel of area 3200 mm^2 in contact with the floor the same force or weight of 500 N will now have a pressure given by

$$\text{average pressure} = \frac{\text{force}}{\text{area}} = \frac{500\text{ N}}{3200\text{ mm}} = 0{\cdot}16\text{ N/mm}^2$$

So the pressure on the floor with a wide heel or larger area is much less than with a narrow heel.

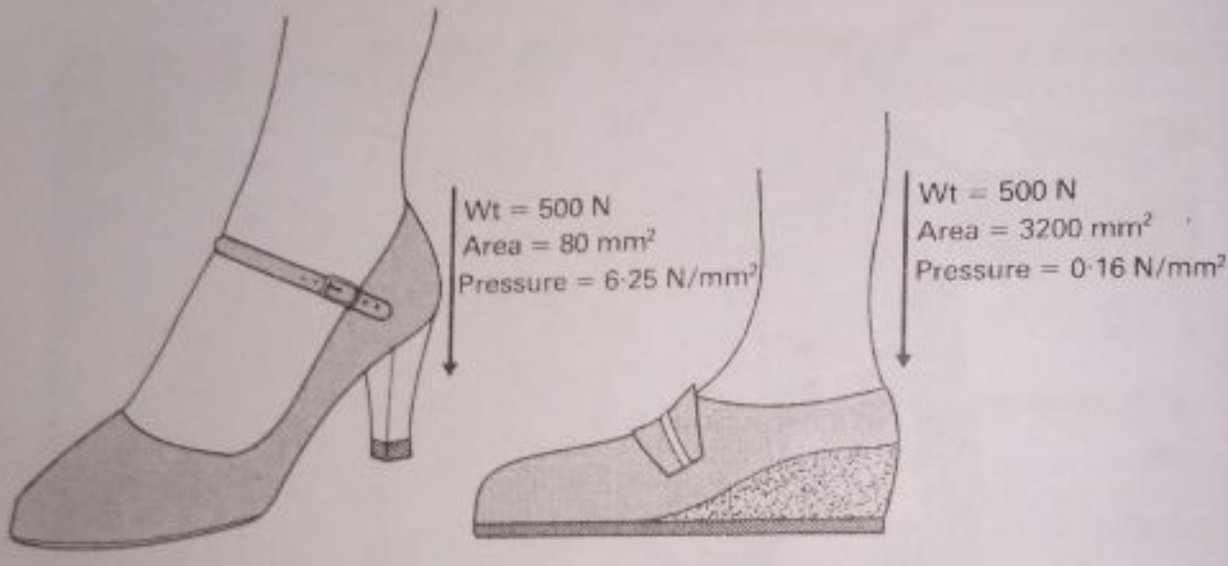


Fig. 7.1 Pressure is greater below a narrow heel

From (1), we see that the force F on a surface A, if the average pressure is p, is given by

$$F = p \times A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Suppose a block of metal produces an average pressure of 1000 N/m^2 when resting on a flat surface of area 0·5 m^2. Then, from (2),

$$\text{force on surface } F = p \times A = 1000 \times 0{\cdot}5 = 500\text{ N}$$

Some applications of pressure

A boy standing exerts a force equal to his weight on the ground below his feet. If the weight is 500 N and the area of his shoes is 100 cm^2 or 0·01 m^2, the average pressure p on ground is given by

$$p = \frac{\text{force}}{\text{area}} = \frac{500\text{ N}}{0{\cdot}01\text{ m}^2} = 50\,000\text{ N/m}^2$$

Scanned with CamScanner

If the boy lies flat on the ground his area in contact with the ground increases considerably. Suppose it is now $0{\cdot}2\,m^2$, which is twenty times the original area. The new average pressure p is then $500\,N/0{\cdot}2\,m^2$ or $2500\,N/m^2$, which is twenty times less than before. A person who goes to the rescue of someone who has fallen through ice into a pond should therefore crawl rather than walk.

Great pressure is produced by ice-skates. The blade makes such a small area of contact with the ice that the pressure below it is very high and ice melts (page 217). The skate thus moves through a thin film of water.



Plate 7B A world class skater, Robin Cousins. The narrow blade of the skate produces high pressure which helps to melt the ice. (*Skate Magazine*)

Pressure in liquids

A diver in water experiences a pressure due to the weight of water above him or her. This pressure is called *hydrostatic pressure*. We can investigate how the pressure in a liquid varies with depth or density by using the apparatus shown in Fig. 7.2.

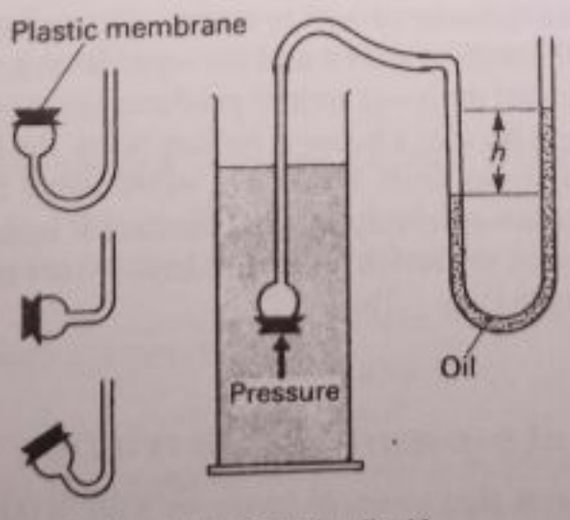


Fig. 7.2 Pressure at different depths and directions in a liquid

Scanned with CamScanner

Experiment

Thin plastic membranes or covers are tied firmly over the tops of several thistle funnels, each bent at different angles as shown. One of them is connected by rubber tubing to a U-tube containing light oil. If the plastic is pressed lightly with the finger, the air inside is compressed and the liquid levels alter. The greater the pressure on the plastic, the greater is the difference in the two levels of oil, *h*. So the U-tube acts as a convenient pressure gauge or manometer when a funnel is below the surface of a column of water, as shown in Fig. 7.2.

Results: Using this apparatus, the following results are obtained.

1. The pressure of the water increases with *depth*.
2. At a particular depth, the pressure is the *same* in different directions.
3. At a particular depth, the pressure increases when the density of the liquid increases. This is shown by replacing the water by dense copper sulphate solution and paraffin, for example.

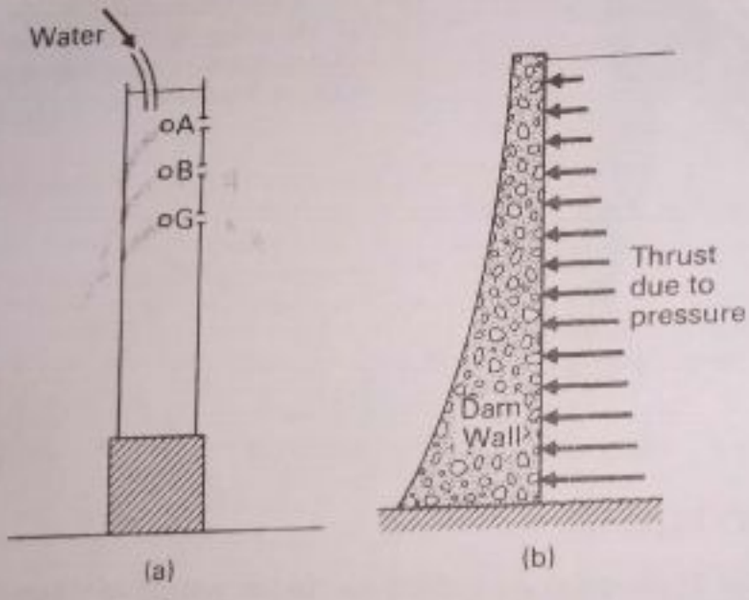


Fig. 7.3 Pressure and depth of water

Fig. 7.3a shows a simple demonstration that pressure increases with depth. It consists of a vertical tube with holes such as A, B and G. The holes are slightly staggered to avoid the collision of a jet of water with one from the hole below it. The tube is filled with water as shown and the water level is maintained by water from the tap. We see that the lower hole G produces the more powerful jets.

A tap on the ground floor of a house is further below the hot water tank in the roof than a tap on a higher floor. So the hot water comes out of the tap on the ground floor with a much greater speed than that on the higher floor. The bottom of a dam is made much thicker than the top because the pressure of the water increases with the depth (Fig. 7.3b).

Investigation of pressure–depth relation

So far we have shown that pressure increases with depth. An experiment to investigate their exact relationship may be carried out as follows.

Scanned with CamScanner

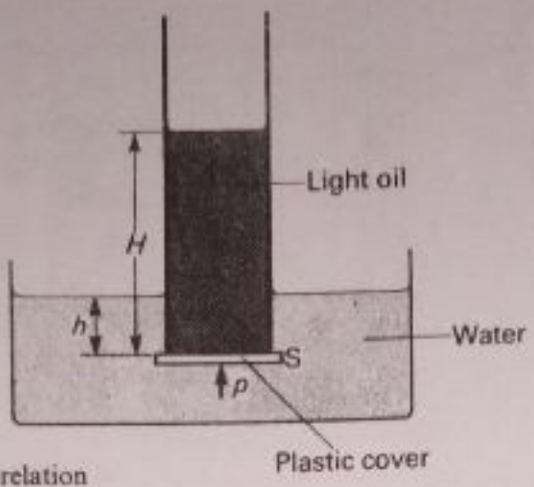


Fig. 7.4 Pressure–depth relation

Experiment

A trough is filled with water (Fig. 7.4). An open glass tube with a plastic cover S held against the lower end, is placed vertically in the water as shown. The pressure of the water on S keeps it firmly against the tube so that no water enters the tube.

The depth h of S below the water level is measured. A light oil is then slowly poured into the tube. When the weight of the oil just counteracts the force due to the pressure of the water on S the cover floats away. The height H of the oil in the tube is noted just before this happens, and is then measured.

The experiment is repeated with different depths of water h. Each time the height H of oil is measured when the cover S floats away.

Results: When a graph of the height H of the oil column is plotted against the depth of water h, a straight line graph passing through the origin is obtained.

Conclusion: The cover S floats away when the downward force due to the weight of oil in the tube is just equal to the upward force on S due to the pressure p of the water. The weight of oil is proportional to H since the area of cross-section in the tube is constant. So p, the pressure of water, is proportional to H, or H is a measure of p.

Since the graph between H (p) and h is a straight line graph passing through the origin, it follows that the pressure at a given depth in a liquid is directly proportional to that depth.

Pressure formula

A formula for the average hydrostatic pressure p at a depth h in a liquid of density ρ can be found by considering a horizontal flat surface (X) of area 1 m^2 at this depth (Fig. 7.5).

$$\text{Mass of water above X} = \text{volume} \times \text{density} = (1 \times h) \times \rho = h\rho.$$

If h is in metre, area of X is in m^2, and ρ in kg/m^3, the mass $h\rho$ is in kg. So the weight or force in newton $= h\rho g$, where g is 9·8 m/s^2.

$$\text{Average pressure } p \text{ at X} = \frac{\text{force}}{\text{area}} = \frac{h\rho g}{1}$$

$$p = h\rho g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In this formula p is in newton per metre^2 (N/m^2) or Pa if h is in m, ρ in kg/m^3 and g is in m/s^2.

Scanned with CamScanner

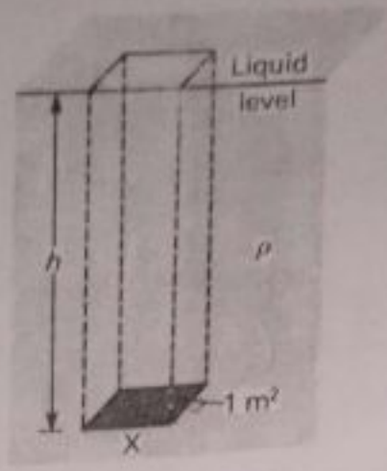


Fig. 7.5 Proof that pressure = $h\rho g$

At a depth of 50 metre in water, density = 1000 kg/m^3, we have (using $g = 10$ m/s^2 to simplify the calculation)

$$\text{pressure} = h\rho g = 50 \times 1000 \times 10$$
$$= 500\,000 \text{ N/m}^2 \text{ or Pa}$$

The pressure formula (3) shows that the pressure is directly proportional to h, the depth below the surface, and to the density ρ of the liquid.

Note carefully that *pressure is independent of the cross-sectional area of the vessel holding the liquid.* So a narrow cylinder filled with water to a height of 50 cm, has the same pressure at the bottom as a wide cylinder also filled to a height of 50 cm. There is a much greater weight of water above the bottom in the wider cylinder, but the weight is distributed over a bigger area. The weight (force) per unit area at the bottom is the same as for the narrow vessel.

So the hydrostatic pressure at a point in a liquid depends only on its depth and not on the shape of the container or its cross-sectional area.

Transmission of pressure

A rubber ball or a polythene can, punctured by many fine holes and filled with water, sprays water in all directions when squeezed. The pressure on the liquid is transmitted equally to all parts of it. The same effect is produced when a watering-can is inclined; the pressure due to the weight of water behind it forces out water through the holes in the rose of the can.

The water shoots out in a direction normal to the surface where the particular hole is situated. So the pressure of the liquid is exerted *at right angles to the surface.* For this reason water is shot out horizontally from a hole in a vertical pipe and vertically from a hole in a horizontal pipe.

Hydraulic press

The transmission of liquid pressure is used in the hydraulic press.

Fig. 7.6 shows the principle of the press. The object to be compressed, such as a bale of material or forged steel, is placed above a piston D of *large* cross-sectional area. The force is applied by means of a lever P to a piston or plunger C of *small*

Scanned with CamScanner

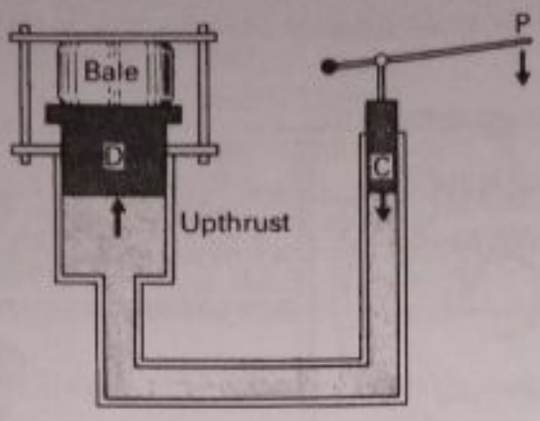


Fig. 7.6 Hydraulic press principle

cross-sectional area. C and D both fit tightly inside a vessel filled with liquid, as shown.

Suppose the downward force due to C on the liquid is 200 N and the area of C is $20\,cm^2$. The pressure exerted on the liquid is thus $200\,N/20\,cm^2 = 10\,N/cm^2$. This pressure is transmitted to the bottom of D. If D has an area of $1000\,cm^2$, then the upward *force* on D = pressure × area = $10 \times 1000 = 10000$ N. Thus a large force is obtained for compression by using only a small force.

If the piston C falls a distance x and the piston D rises a distance y, then, since the volume of fluid transferred is the same, $x \times$ area of cross-section (A_1) of C = $y \times$ area of cross-section (A_2) of D.

So
$$\frac{x}{y} = \frac{A_2}{A_1}$$

But x/y is the *velocity ratio* of the machine (see page 82).

$$\therefore \text{ velocity ratio of press} = \frac{A_2}{A_1}$$

In practice, owing to frictional forces, the mechanical advantage of the machine is less than the velocity ratio (page 82). The larger the ratio of the area of cross-sections, however, the greater is the mechanical advantage of the press.

Car brakes

The modern car requires a braking system which slows down each wheel equally. This reduces the dangers of skidding or locking the wheels when the brakes are applied. The best way is to operate the brakes hydraulically (Fig. 7.7). When the brake pedal is pressed the increased pressure on it is transmitted through oil to the brake shoes, which are then forced apart. They now press on a revolving brake drum to which the wheel is bolted, so the wheel is made to stop. When the pressure on the brake pedal is released a return spring connected to the two brake shoes pulls them together again. The drum is then released and the wheel is free to turn.

The so-called slave cylinders on all four wheels are connected by oil-filled pipes and are operated by a master cylinder connected by levers to the brake pedal. The

Scanned with CamScanner

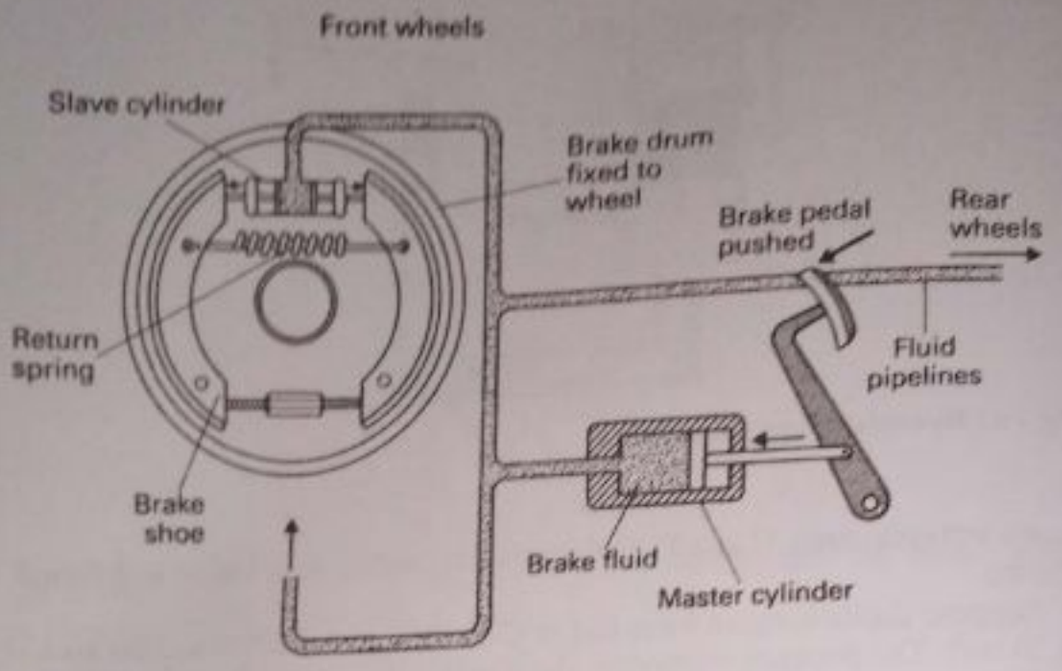


Fig. 7.7 Brake system in a vehicle



Plate 7C Disc brake: when the brake pedal in the car is pressed, small pads grip the disc which turns with the wheel. (*Vauxhall Ltd*)

amount of braking force applied to each wheel is exactly equal, since the same pressure is transmitted through the oil or brake fluid.

Examples on pressure and hydraulic press

1. A rectangular solid iron block has sides 1·0 m × 0·6 m × 0·5 m and rests on a horizontal flat surface. If the density of the iron is 8000 kg/m^3, calculate the maximum and minimum pressure which the block can exert on the surface. (Assume 10 N = weight of 1 kg mass.)

Mass of block = volume × density = $(1{\cdot}0 \times 0{\cdot}6 \times 0{\cdot}5) \times 8000 = 2400$ kg

$\therefore$ weight of block = 2400×10 N = 24000 N

Maximum pressure This is obtained when the smallest area of the block rests on the surface. The area = $0{\cdot}5 \times 0{\cdot}6 = 0{\cdot}3$ m^2. So

$$\text{maximum pressure} = \frac{\text{force}}{\text{area}} = \frac{24000\text{ N}}{0{\cdot}3\text{ m}^2} = 80000\text{ N/m}^2$$

Scanned with CamScanner

Minimum pressure This is obtained when the largest area of the block rests on the surface. The area $= 1{\cdot}0 \times 0{\cdot}6 = 0{\cdot}6\,\text{m}^2$. So

$$\text{minimum pressure} = \frac{\text{force}}{\text{area}} = \frac{24000\,\text{N}}{0{\cdot}6\,\text{m}^2} = 40000\,\text{N/m}^2$$

2. A hydraulic press has a large circular piston of radius 0·8 m and a circular plunger of radius 0·1 m. A force of 200 N is exerted by the plunger.

(a) Find the force exerted on the piston. State *one* reason why the weight of the load just raised by the piston is much less than this force. (b) If the plunger is moved through a distance of 0·64 m in exerting its force, through what distance is the piston raised?

(a) Area of circular plunger $= \pi r^2$, where r is the radius

$$\therefore \text{ pressure due to plunger} = \frac{\text{force}}{\text{area}} = \frac{200}{\pi \times 0{\cdot}1^2}\,\text{N/m}^2$$

This pressure is transmitted to the piston, area $\pi \times 0{\cdot}8^2$

$$\therefore \text{ force on piston} = \text{pressure} \times \text{area}$$

$$= \frac{200}{\pi \times 0{\cdot}1^2} \times \pi \times 0{\cdot}8^2$$

$$= \frac{200 \times 0{\cdot}8^2}{0{\cdot}1^2} = \frac{200 \times 0{\cdot}64}{0{\cdot}01} = 200 \times 64$$

$$= 12800\,\text{N}$$

The load raised by the piston is much less than 12800 N because of frictional forces.

(b) When the plunger moves through a distance of 0·64 m,

volume of liquid moved = area × distance $= (\pi \times 0{\cdot}1^2) \times 0{\cdot}64\,\text{m}^3$

This volume is transferred to the piston side of the press. So

$$\text{distance moved by piston} = \frac{\text{volume}}{\text{area}}$$

$$= \frac{\pi \times 0{\cdot}1^2 \times 0{\cdot}64}{\pi \times 0{\cdot}8^2} = \frac{0{\cdot}01 \times 0{\cdot}64}{0{\cdot}64}$$

$$= 0{\cdot}01\,\text{m}$$

Atmospheric and gas pressure

Earth's atmosphere

The earth's surface is surrounded by a thick layer of air called the atmosphere. Air is a mixture of gases, containing about four-fifths by volume of nitrogen, about one-fifth oxygen and a small percentage of carbon dioxide, together with very small quantities of neon, krypton, xenon, helium, hydrogen, ozone and water vapour. At considerable heights traces of ozone are formed by the action on oxygen of the sun's radiation. This ozone layer plays a vital part in shielding the earth from excessive ultra-violet radiation, which is harmful to life.

The atmosphere is most dense at ground- or sea-level, estimated at about $1{\cdot}2\,\text{kg/m}^3$; it extends to a height of about 80 to several hundred kilometres above sea level. The higher one goes, the less dense is the air, since there is then less weight of air above to compress it. At an altitude of 2000 m the density falls to about $1\,\text{kg/m}^3$; at 10000 m the density is about $0{\cdot}4\,\text{kg/m}^3$. Above the earth's atmosphere, in interstellar space, there is practically no gas. Only a very small number of molecules, estimated at about a thousand per litre, exist there, whereas at ground-level the number of air molecules is of the order of $10^{25}/\text{m}^3$.

Scanned with CamScanner

Evidence of air pressure

We cannot see the air because it is so 'thin', but air pressure is practical evidence of the existence of its molecules. Sailing boats are driven by air pressure, windmills rotate owing to winds and an aeroplane is kept up in flight by air pressure. In the laboratory a convincing demonstration of air pressure is as follows.

Experiment

Find a thin-walled can, such as a paraffin can, and insert in the opening a tight-fitting bung with a tube passing through (Fig. 7.8). Connect a vacuum pump to the tube and switch on the pump. The can collapses after a short time. When there is little air in the can, the external air pressure on the can is no longer counterbalanced by the pressure inside, causing the can to collapse.

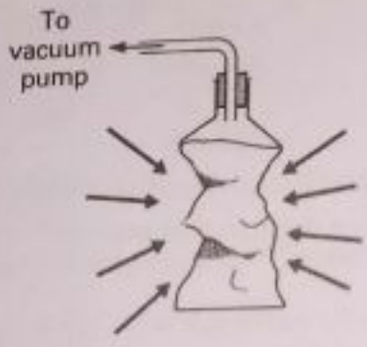


Fig. 7.8 Air pressure effect on a metal can

We also see evidence of the air's pressure when the air inside two thick metal hemispheres is completely removed. A very large force is needed to pull them apart against the air pressure outside (Fig. 7.9).

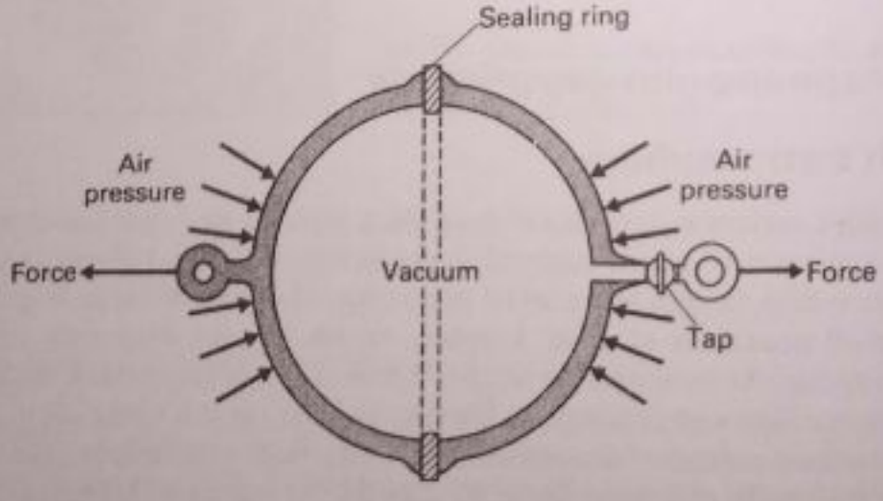


Fig. 7.9 Air pressure demonstration with the Magdeburg hemispheres

Simple barometer

If a long burette is completely filled with water to exclude all air, and then inverted in a beaker of water, the column of water inside the burette stays up. The

Scanned with CamScanner

air pressure outside the burette, which is transmitted through the water, easily supports the water column.

If we need a column of liquid of reasonable height to counterbalance the air pressure, a much denser liquid than water is required. Over 300 years ago Torricelli used mercury to measure the atmospheric pressure and so made the first *barometer*. Mercury is 13·6 times as dense as water.

A simple barometer can be made as follows.

Experiment

Warning This should only be done as a demonstration by the teacher. Mercury is very poisonous and should only be used in a well ventilated room.

A clean, dry thick-walled tube, about one metre long, is filled with clean mercury nearly to the top. A finger is placed over the top and the tube is inverted, so that a large air-bubble runs up and collects all the tiny air-bubbles which may have been trapped in the mercury. The tube is now turned round again, the finger is removed, and the top of the tube is *completely* filled to the brim with mercury to exclude all air.

A finger is now placed over the top of the mercury. The tube is inverted, then placed with the end under the surface of mercury in a dish or reservoir, and the finger is removed. The instant the finger is removed, the mercury drops in the tube to the height h shown in Fig. 7.10. Since air was completely excluded, the space now at the top of the tube is a *vacuum*; it is often called a 'torricellian vacuum'. The atmospheric pressure is numerically equal to the pressure at the base of the column of mercury of height h above the outside level X of mercury. Measurement shows that h is about 760 mm or 0·76 m.

It should be noted that h is the *vertical* height above the outside level X of the mercury surface in the dish. In making a simple barometer, then, a metre rule should be placed (a) vertically, (b) a small distance from the tube, where surface tension (see page 162) has no effect and, (c) with its zero end just touching the free mercury surface.

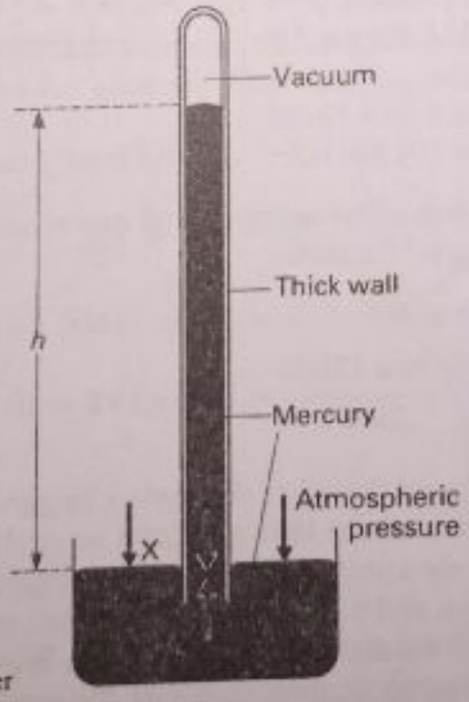


Fig. 7.10 Mercury barometer

Testing the vacuum

If the vacuum is faulty and contains air or water-vapour, the barometer reads less than the true atmospheric pressure. The vacuum can be tested by inclining the

Scanned with CamScanner

tube. The level of the top of the mercury should then keep the same height above the free mercury surface in the trough, as shown at A and B (Fig. 7.11). When the tube is inclined so that the vertical height of its closed end is less than 70 cm, for example as at C, the mercury should fill the whole tube. If the vacuum is faulty an air-bubble is seen at the top, as at D.

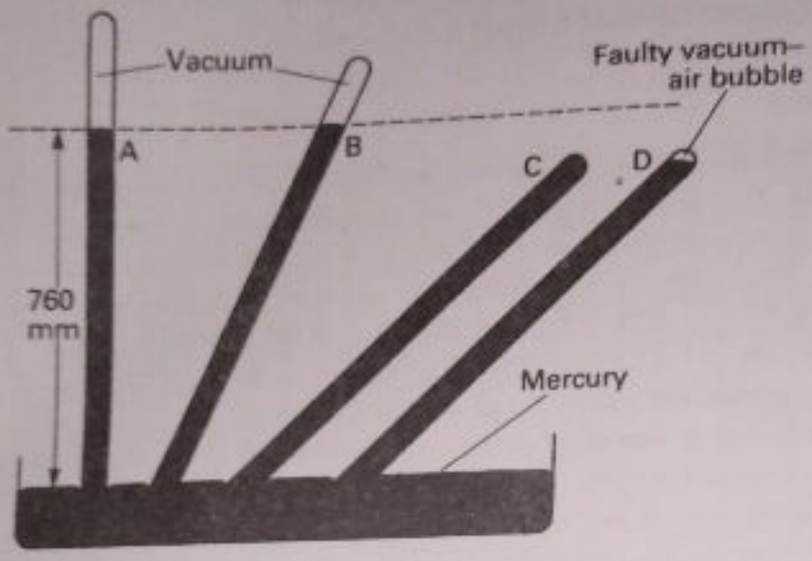


Fig. 7.11 Testing the vacuum in a barometer

Magnitude of atmospheric pressure

A pressure unit sometimes used is *one atmosphere* or *standard pressure*. It is defined as that pressure that supports a column of mercury 760 mm high.

It is important to note that pressure is the force per unit area and not a length.

From page 123, the pressure $p = h\rho g$, where $h = 760\,\text{mm} = 0{\cdot}76\,\text{m}$, $\rho =$ density of mercury $= 13600\,\text{kg/m}^3$ and $g = 9{\cdot}8\,\text{m/s}^2$. So

$$p = 0{\cdot}76 \times 13600 \times 9{\cdot}8$$
$$= 101300\,\text{N/m}^2 = 1{\cdot}013 \times 10^5\,\text{N/m}^2$$

We can find the height h of *water* supported by one atmosphere. In this case, as $p = h\rho g$ and $\rho = 1000\,\text{kg/m}^3$ for water,

$$h \times 1000 \times 9{\cdot}8 = 0{\cdot}76 \times 13600 \times 9{\cdot}8$$

So
$$h = \frac{0{\cdot}76 \times 13600}{1000} = 0{\cdot}76 \times 13{\cdot}6 = 10{\cdot}34\,\text{m}$$

One atmosphere of pressure therefore supports a height of about 10 m of water.

Torricelli's experiment tells us that a column of air 1 m² in cross-section and extending to the top of the atmosphere is equal in weight to a column of mercury of the same cross-section and 0·76 m high. The density of the air decreases as we go up because it is less compressed by the air above. We can make an estimate of the height h of the atmosphere *assuming* that the density of air is constant at about $1{\cdot}2\,\text{kg/m}^3$, as at sea-level. Then, if h is in metres

$$\text{atmospheric pressure} = h \times 1{\cdot}2 \times 9{\cdot}8 = 0{\cdot}76 \times 13600 \times 9{\cdot}8$$
$$\therefore h = \frac{0{\cdot}76 \times 13600}{1{\cdot}2} = 8600\,\text{m} = 8{\cdot}6\,\text{km}$$

Scanned with CamScanner

So a height of about 9 km of air with a uniform density of 1·2 kg/m³ would exert a pressure equal to that of 760 mm mercury (otherwise known as 760 mmHg) at its base. In practice, the height of the atmosphere extends well above this figure. The total weight of air pressing down on the earth's surface is of the order of a million million million newtons!

Under normal conditions, the pressure of the blood in our bodies counteracts the external atmospheric pressure. Deep-sea divers must return slowly to the surface however, as a sudden decrease in pressure when they come up from deep down in the water is very painful. Conversely, pilots operating at great heights must have protective headgear to prevent nose-bleeding as the external pressure is then considerably lower than normal blood pressure.

Units of pressure

As we have seen, atmospheric pressure which supports a column of mercury 760 mm or 0·76 m high is calculated by

$$p = h\rho g = 0{\cdot}76 \times 13\,600 \times 9{\cdot}8 \text{ N/m}^2 = 101\,300 \text{ N/m}^2$$
$$= 1{\cdot}013 \times 10^5 \text{ N/m}^2 = 0{\cdot}1013 \text{ N/mm}^2$$

1 N/m² is also called 1 *pascal*, symbol Pa. In meteorology a unit of pressure called the *bar* (b) is used. This is defined as 10^5 (100 000) N/m² or Pa. So

$$1 \text{ millibar (mb)} = \frac{1}{1000} \text{bar} = 100 \text{ N/m}^2 \text{ or } 100 \text{ Pa}$$

Atmospheric pressure of 760 mm mercury is about 1·013 b or 1013 mb, a value which may be seen on the 'isobars' of weather charts, which are lines of equal pressure.

Aneroid barometer

Fig. 7.12 illustrates an aneroid barometer. It has no liquid and is widely used in the home for showing weather changes. The basic feature is a partially evacuated

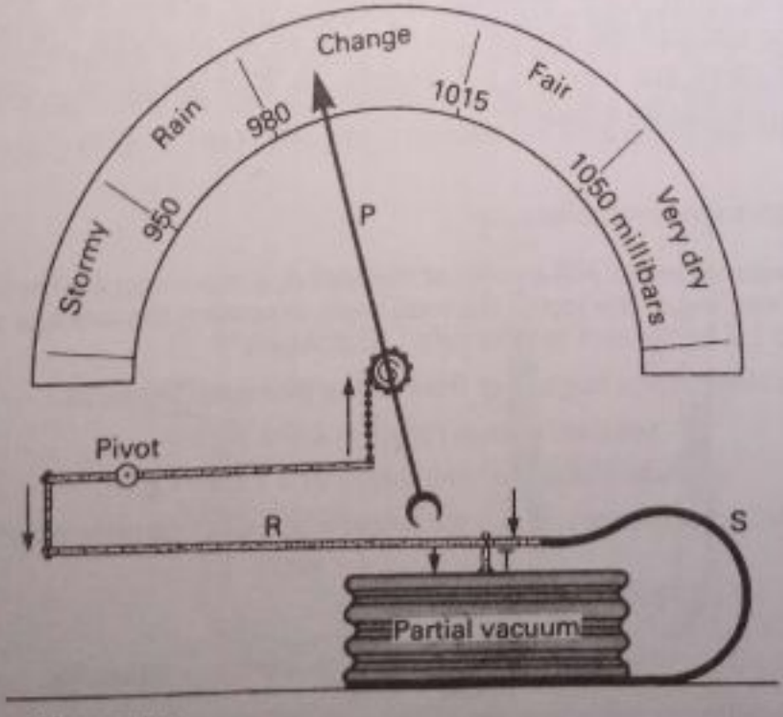


Fig. 7.12 Aneroid barometer

Scanned with CamScanner

steel corrugated box (that is, one containing a partial vacuum). The top surface T is held by a strong spring S, which stops it collapsing.

If the air pressure rises, the side T is pressed inwards; if the air pressure decreases, T moves outwards. The small movement of T is magnified by using a rod R and a connected system of jointed levers, with a small chain at the end passing round a grooved wheel. When the pressure changes, a pointer P is deflected. The scale is often calibrated in millibars; higher air pressure indicates better weather. Violent storms have occurred in the Caribbean at 920 mb pressure.

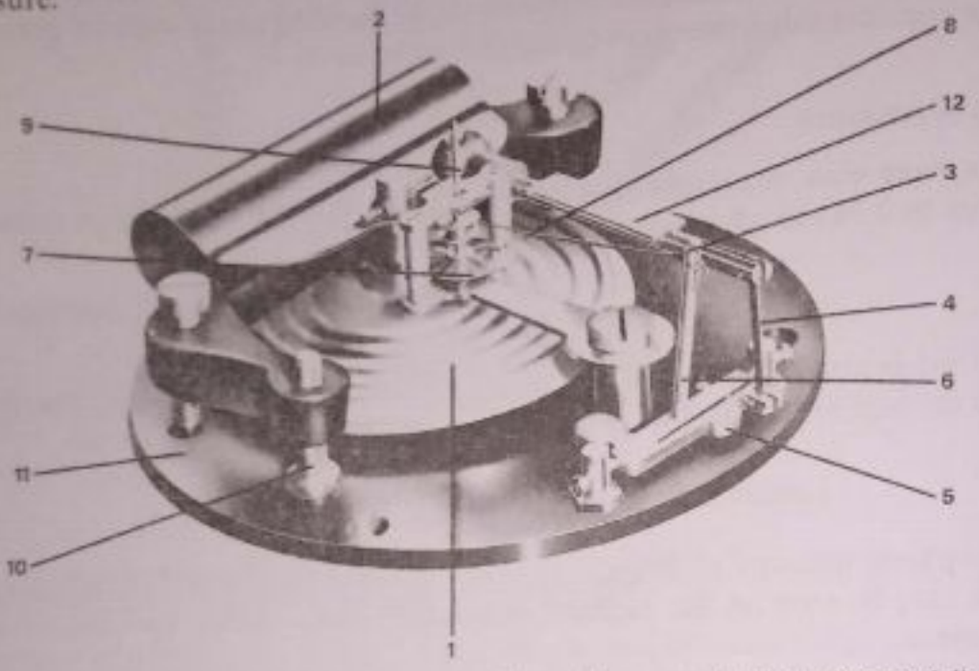


Plate 7D Aneroid barometer, showing 1. capsule with partial vacuum, 2. spring, 3. arm, 4. link, 5. rocking bar, 6. arm of bar, 7. hair spring, 8. fine chain, 9. spindle, 10. zero adjustment screw, 11. base plate, 12. bimetal for temperature compensation (*Negretti and Zambra Ltd*)

As already discussed, the pressure of the air decreases with height. At 650, 1300, 2500 and 5000 m the pressure is respectively about 940, 875, 750 and 550 mb. So the aneroid barometer can be used as an altimeter, which measures heights. Altimeters are used in aeroplanes. In hilly countries, cars carry an aneroid barometer as an altimeter.

Example on atmospheric pressure

A mercury barometer reads 760 mmHg at the foot of a mountain 440 m high. What is the barometer reading at the top of the mountain, assuming the average value for the density of air is 1·2 kg/m^3 and for mercury 13 200 kg/m^3?

The pressure *change* for a height h of fluid $= h\rho g$ (the usual formula)

So pressure change for air $= 440 \times 1{\cdot}2 \times g$

and pressure change for mercury $= H \times 13\,200 \times g$

where H is the height of mercury. If H is the height which gives the same pressure change as the air, then

$$H \times 13\,200 \times g = 440 \times 1{\cdot}2 \times g$$

So $$H = \frac{440 \times 1{\cdot}2}{13\,200} = \frac{1{\cdot}2}{30} = 0{\cdot}04\text{ m} = 40\text{ mmHg}$$

$\therefore$ barometer reading at mountain top $= 760 - 40 = 720$ mmHg

Scanned with CamScanner

Siphons

Petrol tanks in cars, and other fixed vessels which are not easy to empty, can be emptied by using a *siphon* method.

The principle is shown in Fig. 7.13. A is a fixed tank filled with liquid. To empty it, a piece of flexible tubing B is filled with the liquid, and one end is pushed below the surface S in the tank. Liquid then pours out of the other end D when it is lower than the end in the liquid, so emptying the tank A.

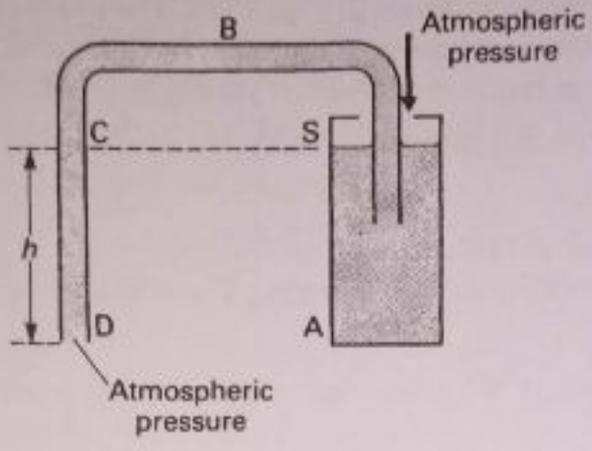


Fig. 7.13 Principle of the siphon

An accurate explanation of the siphon action requires a knowledge of the laws of hydrodynamics. Here we can only give a simplified explanation. The pressure at C is roughly atmospheric, since C is on the same horizontal level as the surface S. The pressure at level D in the liquid is greater than the atmospheric pressure outside D by the height CD or *h*. This forces the liquid out of the tube. Note that an empty tube B will not work as a siphon. In this case the pressure inside B will be atmospheric, the same as at D outside B.

Gas pressure measurement

A *manometer* is a piece of equipment for measuring gas pressure. The domestic gas pressure can be measured by using a *water manometer*. This consists simply of water in a U-tube open on one side to the atmosphere, with the other side joined to the gas supply (Fig. 7.14a). The gas is turned on, and when the water levels are

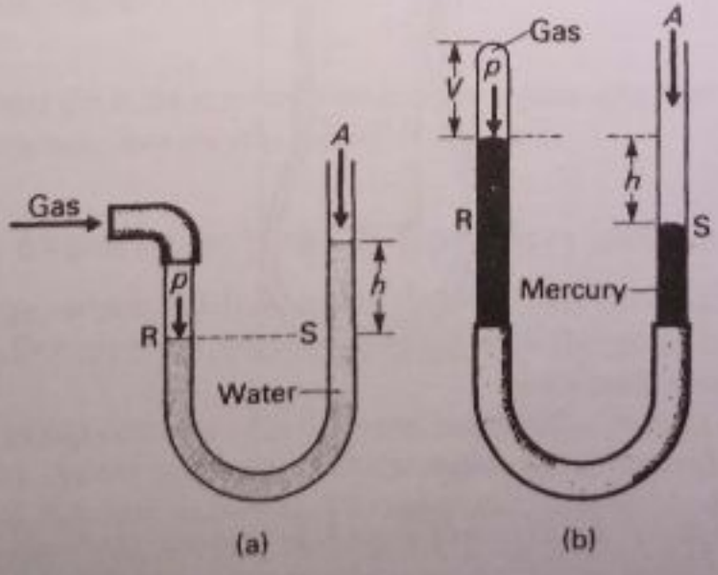


Fig. 7.14 Measurement of pressure by a manometer

Scanned with CamScanner

steady the pressure p of the gas at R = the pressure in the liquid at S = $A+h$, where A is the atmospheric pressure and h is the pressure due to the head of water above S.

Suppose h = 10 cm = 100 mm water. Since the relative density of water and mercury is 1/13·6 (approx.), 100 mm water exerts the same pressure as a height of mercury = 100/13·6 = 8 mmHg approximately. So if A = 758 mmHg,

$$\text{gas pressure } p = 758+8 = 766 \text{ mmHg}$$

A mercury manometer is shown in Fig. 7.14b. Here some dry gas is present in the closed space above a mercury column in a U-tube, which has the other end open to the air. The pressure at S, which is atmospheric, A, equals the pressure at R on the same level of mercury = $p+h$, where p is the gas pressure. So

$$p+h = A$$
$$\therefore\ p = A-h$$

Thus if A = 760 mmHg and h = 20 mmHg, p = 740 mmHg for the volume V of gas.

Boyle's law

Reducing the volume of a gas increases its pressure. The relationship between the pressure and volume of a gas at constant temperature was first investigated experimentally about 1660 by Robert Boyle.

A simple laboratory apparatus with air as the gas is shown in Fig. 7.15. It consists of dry air in a glass tube of uniform cross-section, attached by tubing to a mercury reservoir. The experiment can be carried out as follows.

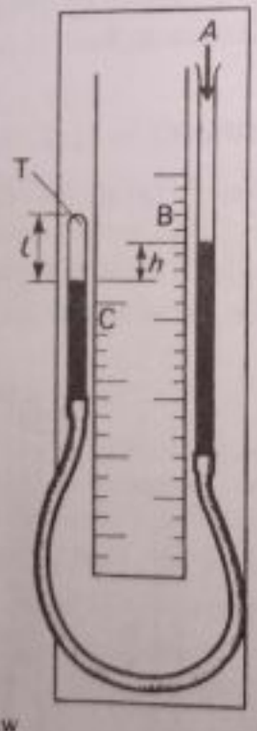


Fig. 7.15 Investigation of Boyle's law

Experiment

Observe the length l of the air column in the tube T from the scale beside it. This is a measure of the *volume* V of air, since T has a uniform cross-section. Observe the difference

Scanned with CamScanner

in levels, h, of the mercury at C and B. Record the atmospheric pressure A from a Fortin barometer. The pressure p of the air in T is then $(h + A)$ mmHg.

Increase the pressure by screwing down clips round the tubing or by raising the open tube B of the reservoir. Observe the new difference in level h and the new reading of l, which is a measure of the gas volume V. Repeat with several measurements of pressure and volume, for example taking three readings with B above C (pressure greater than atmospheric pressure) and three readings with B below C (pressure less than atmospheric pressure).

Measurements: Atmospheric pressure = ... mmHg

p/mmHg	l/mm ($\propto V$)	$1/l$ ($\propto 1/V$)

Graph: (a) Plot p against V. (b) Work out $1/V$, the reciprocal of V, and enter the values in the measurements table, as shown. Plot p against $1/V$.

Result: (a) The graph of p against V is a curve which approaches the $p-$ and $V-$axes (Fig. 7.16a). (b) The graph of p against $1/V$ is a straight line which passes through the origin (Fig. 7.16b).

Conclusion: From graph (b), p is directly proportional to $1/V$, or p and V are *inversely* proportional. This means that the product pV is constant.

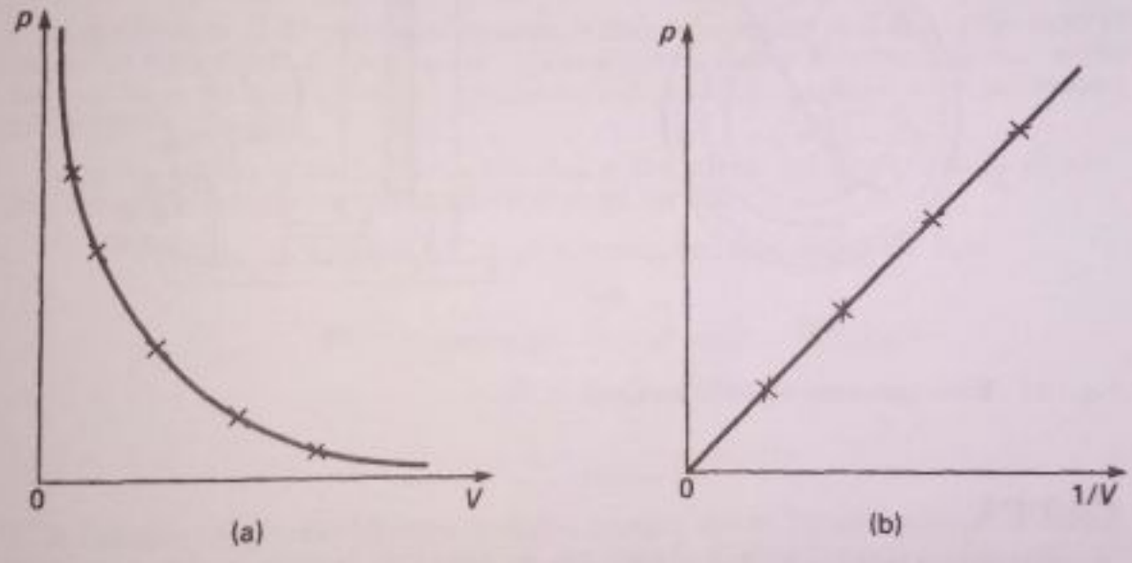


Fig. 7.16 Graph of (a) p against V, (b) p against $1/V$

Boyle's law states if p is the pressure and V is the volume of a fixed mass of gas at a constant temperature, then the product pV is constant.

Boyle's law experiment using Bourdon gauge

A Bourdon gauge, which reads pressure directly, can be used to investigate Boyle's law. The Boyle's law apparatus together with the gauge is shown in Fig. 7.17

Basically, the gauge consists of a flattened metal tube N bent into a circular shape (Fig. 7.17a). As the pressure on it increases, the tube tends to straighten and the closed end A therefore moves. The movement is magnified by a lever and gear train and recorded by a pointer moving round a scale previously calibrated in pressure units such as kN/m^2.

Scanned with CamScanner

Experiment

The air is trapped above the oil in the tube T and its volume V is read from the scale beside the tube (Fig. 7.17b). The pressure p of the air is read directly from the Bourdon gauge. The tap is opened and air pumped in from a pump. This increases the pressure on the oil in the reservoir, which is transmitted to the air in T, and so this air is compressed. The tap is then closed and when the oil level in T becomes steady, the pressure p and volume V are both read. This process is repeated four times so that other values of p and V are obtained.

Graph and conclusion: As in the experiment on page 135, graphs of (a) p against V, and (b) p against $1/V$ are plotted. The same conclusions come from the results.

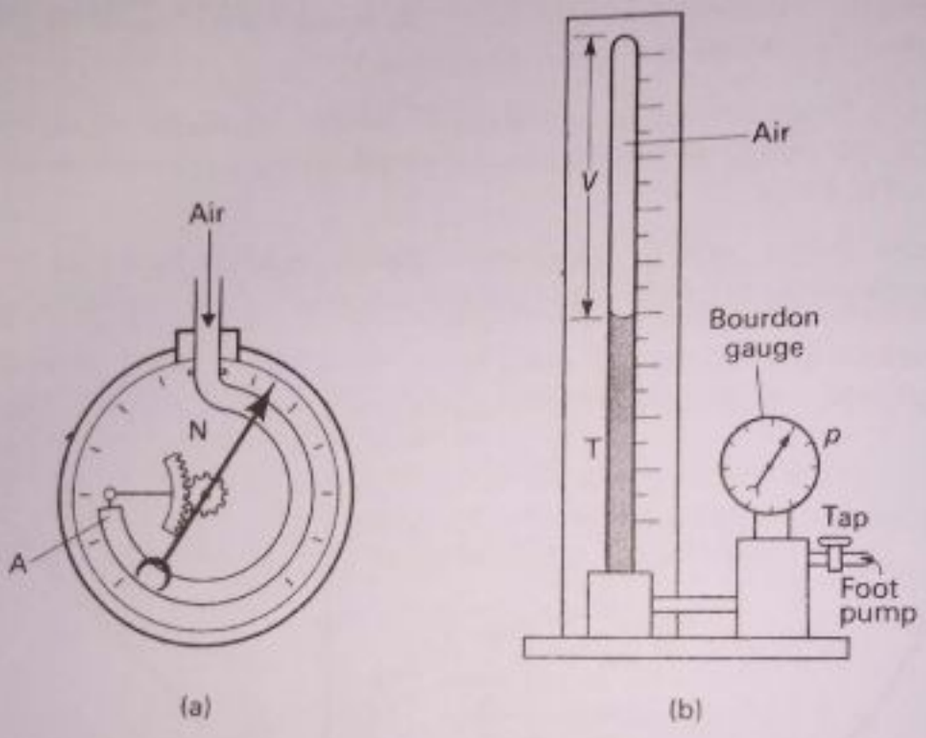


Fig. 7.17 Boyle's law using the Bourdon gauge

Pumps

Air pumps can be constructed either to compress or force air into a given space or to pump out air.

A bicycle pump is a simple form of compression pump which has a flexible leather washer at the end of a piston or plunger inside the barrel (Fig. 7.18a). It is connected to the tyre, which has a rubber valve in it. When the piston is drawn out the air below it expands and the pressure is reduced below atmospheric pressure. Air from outside the pump then flows past the leather washer into the barrel. At the same time the tyre valve remains closed.

When the piston is pushed down the compressed air below it forces the leather washer against the sides of the barrel so that no air enters, and the air is then forced into the tyre through the valve, which now opens (Fig. 7.18b). In this way air from outside is forced into the tyre with each stroke of the piston.

Fig. 7.18b shows how the valve in the tyre stops the air from leaving but allows more air to enter. As more air enters the tyre, the pressure inside increases. It then becomes harder to force air through the tyre valve; the valve only opens when the pressure of the air in the barrel of the pump is greater than that in the tyre.

Scanned with CamScanner

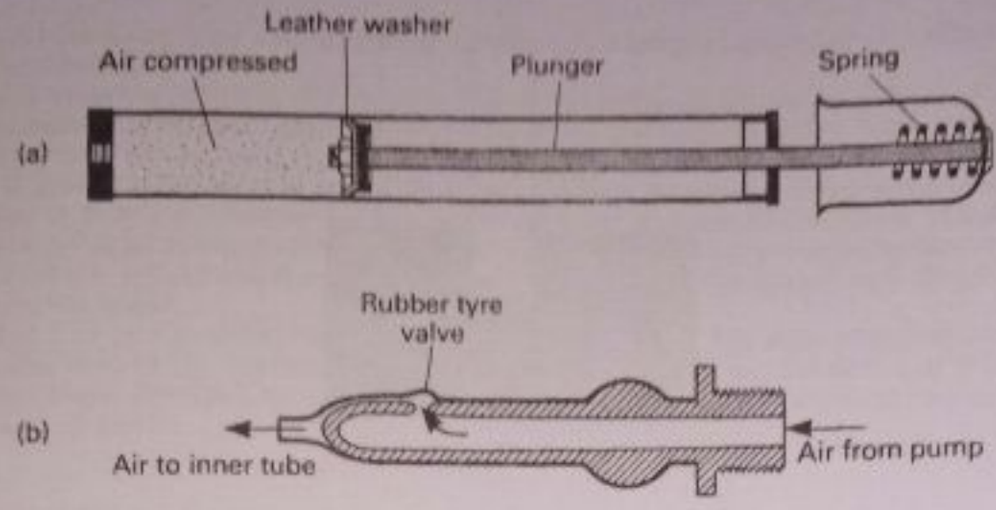


Fig. 7.18 Principle of bicycle (compression) pump. (a) The pump. (b) The valve.

Examples on Boyle's law

1. Explain why a bubble of air increases in volume as it rises from the bottom of a pond to the surface. If the volume as it reaches the surface is double that at the bottom of the pond, estimate the depth of the pond. (Assume that the water temperature is uniform.) Take the pressure at the water surface to be 10^5 N/m^2 and the density of water to be 1000 kg/m^3. (L)

At the bottom of the pond, the pressure is atmospheric plus that due to the head of water. At the surface, the pressure is only atmospheric and is therefore less than at the bottom. So as the bubble rises the pressure on it decreases. From Boyle's law, the volume of the bubble increases.

Since the volume at the bottom is half that at the surface, it follows from Boyle's law that the pressure at the bottom is *twice* that at the surface.

$$\text{Pressure due to depth } h \text{ of pond} = \text{atmospheric pressure} = 10^5 \text{ N/m}^2$$
$$= h\rho g$$
$$\text{Assuming } g = 10 \text{ m/s}^2 \text{ and } \rho = 1000 \text{ kg/m}^3$$
$$10^5 = h \times 10 \times 1000$$
$$\therefore\ h = \frac{10^5}{1000 \times 10} = 10 \text{ m}$$

2. A cylinder of volume 10 litres contains oxygen gas at 20 atmospheres pressure. When the tap is opened, 40 litres of the gas at 1 atmosphere is obtained, the temperature of the gas remaining constant.
Calculate (a) the volume of oxygen at 20 atmospheres escaping from the cylinder, (b) the pressure of the gas remaining in the cylinder.

(a) From Boyle's law, pV = constant. So 40 litres at 1 atmosphere has a volume V at 20 atmospheres given by

$$20 \times V = 40 \times 1$$
$$\therefore\ V = \frac{40}{20} = 2 \text{ litres}$$

(b) The volume of gas left in the cylinder is (10 − 2) or 8 litres at 20 atmospheres. Since this gas fills the 10 litre cylinder, the new pressure p of this gas is given, from Boyle's law, by

$$p \times 10 = 20 \times 8$$
$$\therefore\ p = \frac{20 \times 8}{10} = 16 \text{ atmospheres}$$

3. A uniform tube, 1·0 m long and closed at its upper end X, is pushed vertically downwards into mercury until the liquid rises 0·2 m inside the tube (Fig. 7.19).

Scanned with CamScanner

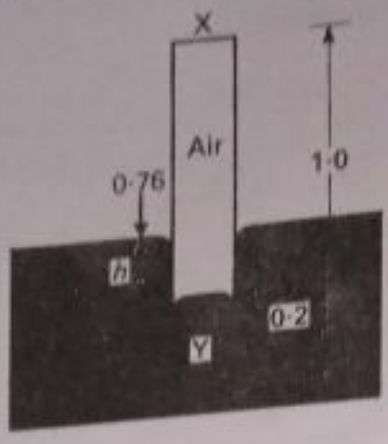


Fig. 7.19 Example

Calculate the depth of the open end Y below the mercury surface if the atmospheric pressure is 0·76 m mercury.

Suppose A is the cross-sectional area of the tube XY in m^2, so that its volume is $1{\cdot}0 \times A$. Then initially the air inside the tube has a pressure of 0·76 mHg, and a volume of $1{\cdot}0\,A\ m^3$.

When the tube is pushed into the mercury, the air is compressed and now has a volume of $0{\cdot}8\,A\ m^3$ and a pressure of $(0{\cdot}76 + h)$ mHg, where h is the depth of the top of the mercury inside the tube below the surface outside.

From Boyle's law (pV = constant) for the mass of air in the tube

$$(0{\cdot}76+h) \times 0{\cdot}8A = 0{\cdot}76 \times 1{\cdot}0A$$

$$\therefore\ 0{\cdot}76+h = \frac{0{\cdot}76 \times 1{\cdot}0}{0{\cdot}8} = 0{\cdot}95$$

$$\therefore\ h = 0{\cdot}95 - 0{\cdot}76 = 0{\cdot}19\ \text{m}$$

$$\therefore\ \text{depth of open end Y} = h + 0{\cdot}2 = 0{\cdot}39\ \text{m}$$

Summary

1. Pressure = force per unit area. Unit: newton per metre^2 (N/m^2) or Pa.
2. Pressure in liquid = $h\rho g$, where h is depth and ρ is density.
3. Pressure is the same on the same horizontal level of a liquid and acts equally in all directions at a given place.
4. Hydraulic press. Velocity ratio $= \dfrac{\text{area of cross-section of large piston}}{\text{area of cross-section of small piston}}$
5. Atmospheric pressure supports a column of mercury about 760 mm high.
   One atmosphere = 760 mmHg pressure $= 0{\cdot}76 \times 13600 \times 9{\cdot}8\ \text{N/m}^2$ (approx.) $= 101300\ \text{N/m}^2$. 1 bar $= 10^5\ \text{N/m}^2$.
6. Boyle's law states for a fixed mass of gas at a constant temperature, the volume is inversely proportional to the pressure, or pV is a constant.

Exercise 7 (Answers, page 641)

(Assume $g = 10\ \text{m/s}^2 = 10\ \text{N/kg}$)

1. (a) A rectangular box rests on a flat horizontal surface and has a weight of 100 N. Calculate the pressure on the surface if the base of the box has an area of 2 m^2.
   (b) A weight W is placed on top of the box. If the pressure at the base is increased to 60 N/m^2, find W.
   (c) Calculate the pressure in N/m^2 at the bottom of a pool of water 10 m deep due to the water above it. (Density of water = 1000 kg/m^3.)
   (d) A cylinder contains a volume V of oxygen gas at a pressure of 3 atmospheres. If this gas has a volume of 30 litres at 1 atmosphere when the temperature is constant, what is V? State the law used in your calculation.

Scanned with CamScanner

2. (a) Describe how you would set up and use a simple barometer to measure the atmospheric pressure.

(b) State the principle of moments and define pressure. Study Fig. 7.20. Calculate the pressure of steam in the cylinder which would just raise the valve, if the area of the valve in contact with the steam is 2 cm^2. Atmospheric pressure is 10^5 N/m^2. (N)

3. Explain briefly the principle of the simple mercury barometer. What test would you make to find out whether such a barometer had any gas in the space above the mercury? If, by accident, some air entered and was seen to lower the barometric height by 2·00 cm, why would it be unsatisfactory to correct for this by adding 2·00 cm to all future readings?

Fig. 7.21 (not to scale) represents a mercury barometer. The areas of the tube cross-section and of the surface of the mercury in the bowl are 0·5 cm^2 and 5·5 cm^2 respectively. What will happen to the two levels if 12 cm^3 of mercury is poured into the bowl? (O and C)

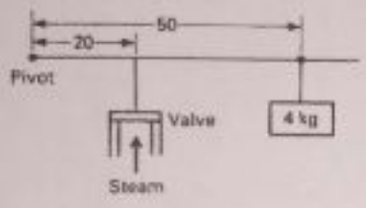


Fig. 7.20

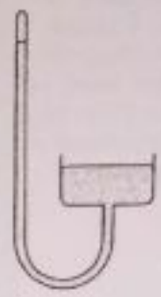


Fig. 7.21

4. Draw a labelled diagram of a simple form of mercury barometer. What experiment would you do in your laboratory to show that the mercury column is supported by the pressure of the atmosphere?

Estimate the fall in the height of the mercury column when the barometer is taken vertically upwards through a height of 1 kilometre. (For this rough calculation, take the density of mercury to be 13 g/cm^3 (13000 kg/m^3) and the density of air to be $1{\cdot}3 \times 10^{-3}$ g/cm^3 (1·3 kg/m^3). (C)

5. A rectangular solid block has sides $4 \times 10 \times 20$ cm and a density of 8 g/cm^3 or 8000 kg/m^3. If it rests on a horizontal flat surface, calculate the minimum and maximum pressure it can exert in N/m^2.

6. Describe how you would set up and use a simple barometer to measure the atmospheric pressure.

What are the objections to the use of water as the barometric liquid?

If the reading of a mercury barometer is 75·58 cm at the base of a mountain and 66·37 cm at the summit, what is the height of the mountain?

Density of mercury = 13600 kg/m^3; average density of air = 1·25 kg/m^3.) (L)

7. (a) Draw a diagram illustrating the principle of the hydraulic press and explain its action. Calculate the pressure transmitted if a load of 10000 N is supported on a circular ram of diameter 1 metre. (Neglect friction.)

(b) Describe carefully how you would use a siphon to remove the major part of the water contained in a large fixed tank, open at the top. Draw a diagram and use it to explain the action of the siphon. (L)

8. A uniform vertical tube, 40 cm long, sealed at the upper end, is lowered into mercury until the length of the enclosed air column is 35 cm. Find the depth of immersion of the tube if the atmospheric pressure is 77 cm of mercury. (N)

9. With the aid of labelled diagrams explain the action of the following (a) an aneroid barometer; (b) a bicycle pump.

10. 1 cm^3 of air at atmospheric pressure, 76 cm of mercury, is introduced into a simple barometer and the level of the mercury in the barometer is lowered by 12 cm. What is (a) the pressure exerted by the air in the tube; (b) the volume of the space above the mercury now occupied by the air? (N)

11. State Boyle's law and describe an experiment to verify it.

Indicate the graphs that should be obtained if (a) p were plotted against $1/V$, (b) pV were plotted against p, where p and V refer to the pressures and volumes obtained from the experiment.

A uniform tube, 96 cm long, sealed at one end, is lowered vertically with its open end downwards into mercury until the length of the enclosed air column is 84 cm. Find

Scanned with CamScanner

the depth of immersion of the tube in the mercury, given that the atmospheric pressure at the time of the experiment is 77 cm of mercury. (L)

12. Draw a diagram of a hydraulic press showing how an object, such as a bale of cotton, can be compressed. State the hydrostatic principle on which the press functions and describe an experiment which illustrates the principle.

The cylindrical piston of a hydraulic press has a diameter of 30 cm and the plunger a diameter of 5 cm. Calculate (a) the upward thrust of the piston produced by a thrust of 250 N on the plunger; (b) the distance moved by the piston when the plunger moves 60 cm. (O and C)

13. What is meant by the term *pressure*? Explain the fact that when someone uses his thumb to push a drawing-pin into a block of wood, the pressure on the wood is greater than the pressure on the thumb. Calculate the pressure on the thumb when the force exerted is 20 N and the top of the pin is square, the length of each side being 10 mm.

Fig. 7.22 shows a bottle, height 1 m, half full of water and containing air at atmospheric pressure; the bottle is fitted with a tap at the bottom and is closed with a bung at the top. Explain the following effects, which occur when the tap is opened: (a) water flows out; (b) the rate of flow decreases with time; (c) the flow ceases before the water level reaches the tap. (Take atmospheric pressure as equal to a column of water 10 m high.)

How would the flow of liquid differ if the bottle had contained mercury in place of water? Give reasons. (C)

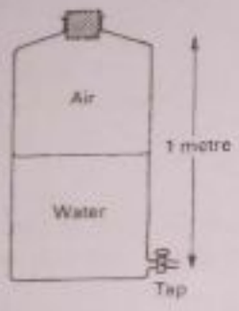


Fig. 7.22

14. Describe how you would establish experimentally the relationship between the pressure and the volume of a fixed mass of dry air maintained at room temperature. Show how you would use your results to demonstrate this relationship.

A mercury barometer, with a tube of uniform diameter and length 850 mm above the mercury level in the reservoir, reads 750 mm on a day when the atmospheric pressure is 760 mm of mercury. What will be the atmospheric pressure on a day when it reads 730 mm?

If the barometer tube had twice the diameter, would you expect to get the same readings? Give a reason for your answer. (Assume the temperature to be the same throughout.) (L)

Multiple-Choice Questions

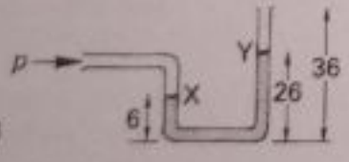


Fig. 7.23

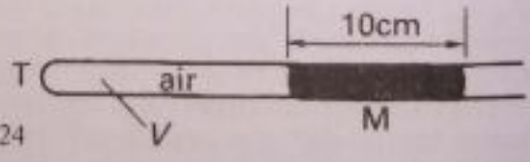


Fig. 7.24

15. Fig. 7.23 shows a water manometer used to measure the domestic gas supply. The heights of the water levels X and Y and the top of the open side are all measured from the bottom of the manometer.

When the pressure p of the supply changes, the height of level Y rises 2 cm. Which statement of A to E is now true?

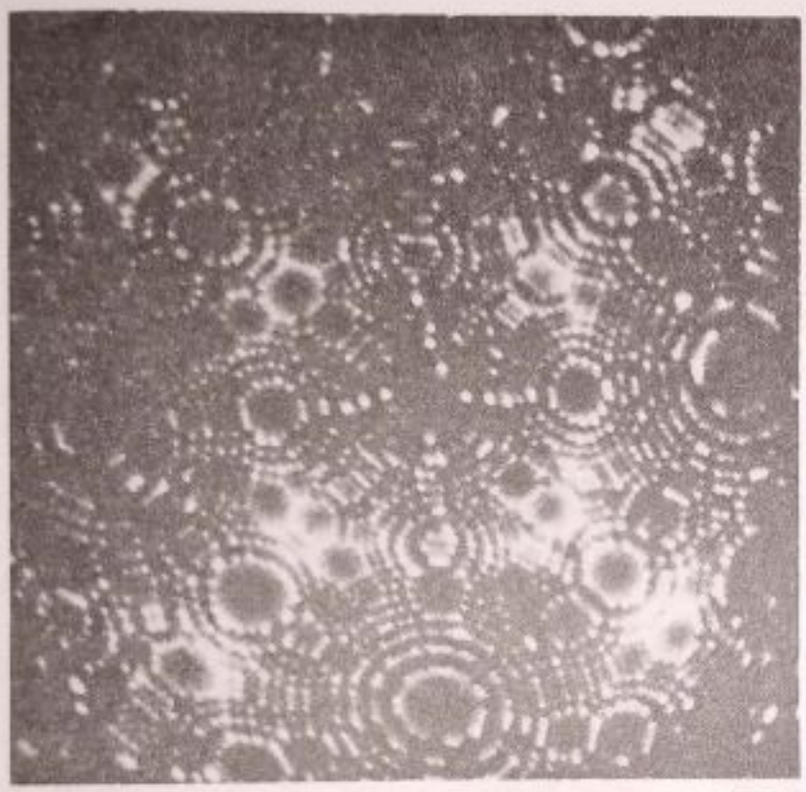
A p is 2 cm of water above atmospheric pressure
B rises from 26 cm to 28 cm of water above atmospheric pressure
C p decreases by 4 cm of water
D p rises from 20 cm to 24 cm above atmospheric pressure
E the pressure on X is equal to the pressure on Y

16. Fig. 7.24 shows a horizontal capillary tube with a closed end T and with a volume of air trapped by a small mercury thread M. When the tube is now turned vertically with T at the top,

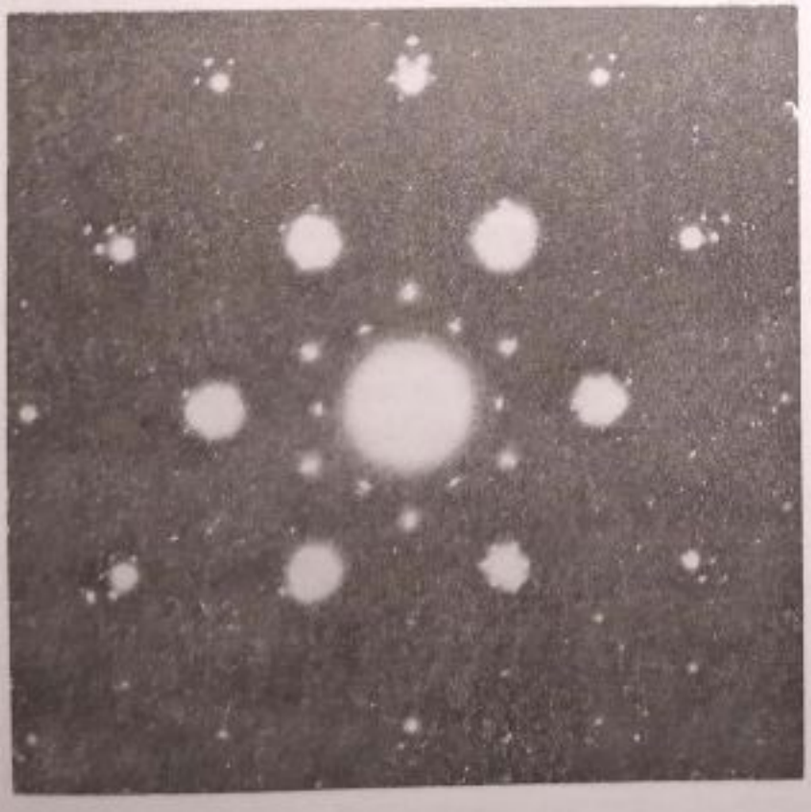
A M falls out B the air pressure decreases C the air pressure increases
D the volume V decreases E the volume V stays constant

Scanned with CamScanner

MOLECULES AND MATTER



Micrograph of pure iridium. Each white spot is an image of an individual atom (*Dr H. N. Southworth*)



Diffraction pattern of gold atoms from a single crystal of gold covered with a thin layer of copper. The pattern shows the symmetry of the atoms which build up the crystal. (*Dr R. P. T. Hills*)

Scanned with CamScanner

8. Molecules and Matter

Solid, liquid and gaseous states

Water is one of the most common substances on our planet, the Earth. All over the world, we find it in the solid, liquid and gaseous states or phases. In polar regions it is found as ice, the solid state. Rainwater is an example of the liquid state. *Water vapour* or gas exists in the air. Pools of water on the ground after rain gradually disappear; they evaporate slowly. Water vapour or steam is produced when a kettle boils.

We can see a solid or a liquid. We cannot see a gas such as air. But the air makes the leaves on trees move. Some gases are coloured – chlorine gas, for example, is greenish-yellow.

All matter, that is anything which has mass or weight, is found in the solid, liquid or gaseous states. Some differences between the three states are as follows.

1. Solids always have a constant volume and a constant shape. They cannot be poured.
2. Liquids have constant volumes but they take the shape of the containing vessel. They can be poured.
3. Gases have no fixed shape but always occupy the whole volume of their container. They can be poured.

Molecules

If you look at the corner of this page it looks very smooth. Touch it with your finger and test the smoothness. Using a microscope, however, we see that the paper actually has a 'grain'. The surface is not perfectly smooth.

If you scrape the surface of a piece of chalk, thousands of very tiny particles flake off and can be seen floating through the air. There is a limit to the size of particles the eye can see.

Dalton suggested the existence of *molecules* in matter. These are very tiny particles which together make up the whole of a solid or a liquid or a gas. The tip of a needle has many millions of molecules of the metal.

Molecules themselves are made up of *atoms*. As we discuss later in the book, gifted scientists with imagination, such as Sir J. J. Thomson and Lord Rutherford, were able to make inspired guesses or theories about the particles inside atoms from experiments, although these particles can never be seen.

Size of molecules

A rough estimate of the small size of a molecule was first made by Benjamin Franklin about two hundred years ago. During a sea journey, he became

Scanned with CamScanner

interested in the way oil calmed the waves. About 1770, he poured a teaspoonful of oil on to the middle of the lake at Clapham Common, London. He saw that the oil spread over a large area of water and estimated that it covered about 2000 m^2. The oil film is extremely thin. It cannot be less than one molecule thick because molecules are the smallest part of a substance that can exist on their own in normal circumstances.

Suppose the volume of oil was about 5 cm^3. The area of 2000 m^2 is 20 million cm^2 or 2×10^7 cm^2. Hence the thickness of the oil film

$$= \frac{\text{volume}}{\text{area}} = \frac{5\,\text{cm}^3}{2 \times 10^7\,\text{cm}^2} = 2{\cdot}5 \times 10^{-7}\,\text{cm}$$

So the size of a molecule would not be bigger than $2{\cdot}5 \times 10^{-7}$ cm.

About a hundred years after Franklin's experiment, Lord Rayleigh suggested that certain oils spread over water in a film one molecule thick. An experiment to estimate roughly the size of an oil molecule is shown in Fig. 8.1.

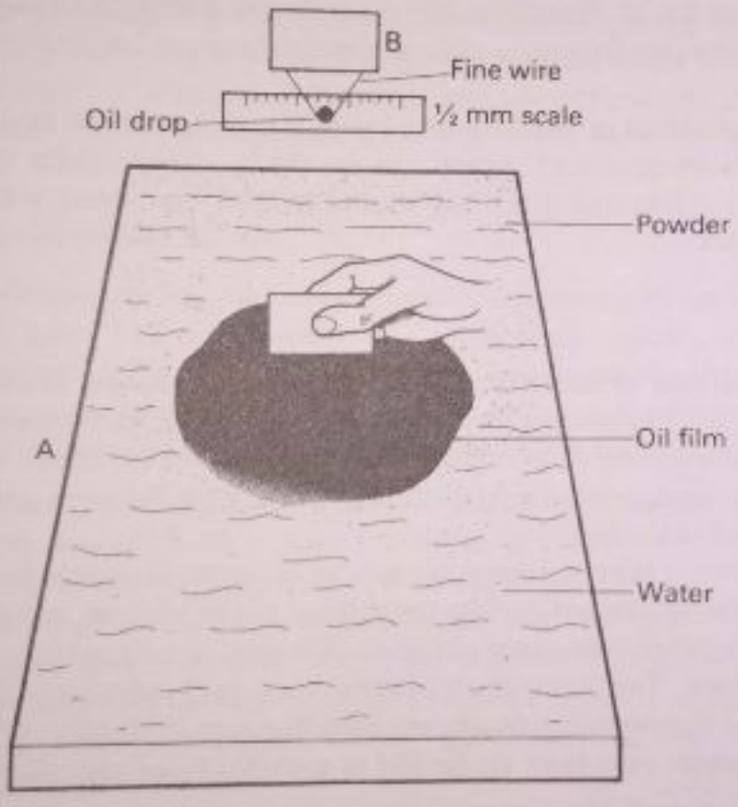


Fig. 8.1 Oil film and molecular size

Experiment

A is a large clean shallow tray, set up level, whose base is coated with blackened wax. The tray is filled to the brim with water and the surface cleaned carefully by dragging a waxed metal rod (or 'boom') across the surface. When the surface is still, some lycopodium powder is sprinkled lightly on it. A drop of olive oil is then picked up on a fine wire attached to a card B (Fig. 8.1). With the aid of a half-millimetre scale placed behind the drop, as shown, and a hand lens, the oil drop is made larger or smaller by using a fine wire until its diameter is exactly 0·5 mm.

The drop is now placed gently in contact with the water surface. The drop immediately spreads across the water into a circular film. As it spreads, the powder moves back. The

Scanned with CamScanner

clear space on the surface is the oil film, as shown. Its diameter is now quickly measured with a ruler.

Calculation: Suppose the diameter of the film is 200 mm, so that the radius is 100 mm. The area of the circular film is then $\pi r^2 = \pi \times 10^4\ \text{mm}^2$.

The volume of the drop is that of a sphere which is given by $4\pi r^3/3$, where r is the radius. If the diameter of the drop is 0·5 mm, the radius r is 0·25 mm. So

$$\text{volume} = \frac{4}{3}\pi \times 0{\cdot}25^3\ \text{mm}^3$$

$$\text{thickness of film} = \frac{\text{volume}}{\text{area}} = \frac{4\pi \times 0{\cdot}25^3}{3 \times \pi \times 10^4}\ \text{mm}$$

$$= 2 \times 10^{-8}\ \text{mm} = 2 \times 10^{-9}\ \text{m}$$

On extremely clean water, olive oil is considered to spread in a film one molecule thick, a so-called *monomolecular layer*. Accurate experiments show that the film is about 2×10^{-9} m thick. The oil molecule is long. It has a chain of about 18 carbon atoms standing up at right angles to the water surface and forming the film.

Another indication of the very small size of a molecule can be obtained from measurements on gold-leaf, which can be made exceptionally thin by skilled craftsmen. The thinnest gold-leaf manufactured has been estimated to be 6×10^{-8} m thick.

Osmosis

The very small size of molecules can also be demonstrated in *osmosis*. In this phenomenon, a 'semi-permeable membrane' such as parchment or cellophane is used. The semi-permeable membrane has very tiny pores, which would usually mean that only molecules of liquid could pass through but not molecules of some solids dissolved in the liquid.

Fig. 8.2 shows a simple demonstration of osmosis. A thistle funnel T, with a semi-permeable membrane such as parchment at the bottom, is partly filled with a fairly concentrated cane sugar solution. It is then placed upright in a beaker of water, as shown. The level of the liquid rises in T, showing that the water molecules pass through the membrane into the sugar solution. When the liquid level in T remains constant, its height is a measure of the so-called 'osmotic

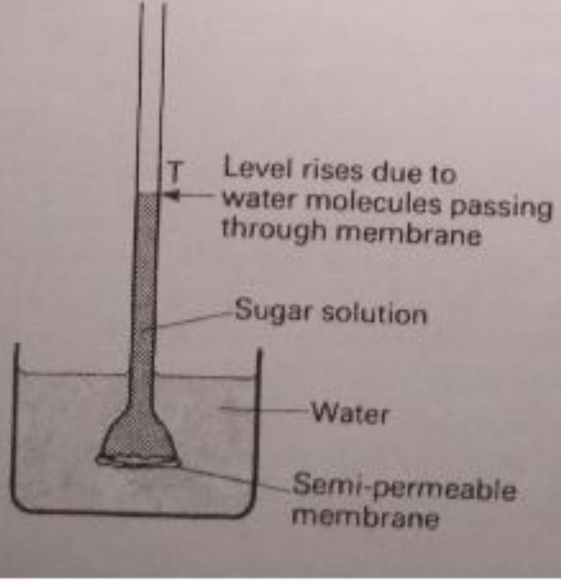


Fig. 8.2 Osmosis

Scanned with CamScanner

pressure' of the sugar solution. Here water molecules are passing through the membrane but sugar molecules are not.

Applications of osmosis

Osmosis is not limited to sugar solutions. It occurs whenever any solution is separated from more of its solvent by a semi-permeable membrane, or when a dilute solution is separated from a more concentrated one.

Many fruits have skins which act like semi-permeable membranes, and they have sugary solutions inside. Consequently dried fruit swells when placed in water–the molecules of water pass through their skins. A dried bean swells when placed in water for a similar reason.

Many plants obtain moisture by the process of osmosis across the semi-permeable membrane of the root cells. The human body also contains semi-permeable membranes which play an important part in the transfer of liquid from one part to another.

A large water purification plant, using the principle of *reverse osmosis*, is used in America to produce many thousands of gallons of pure water per day. Impure or salt water in a tank is subjected to pressure, and the water molecules are forced through a special membrane and collected as pure water on the other side.

Ideas of molecular size

Pictures of molecules under extremely powerful microscopes, and calculations based on gases, show that the diameter of a molecule is actually of the order of 10^{-10} m or one ten-millionth millimetre. The tiny diameter of a molecule is almost impossible to imagine because we do not live in a microscopic world. Some idea of what it means may perhaps be gained from the following.

1. If a fine hair is magnified until its thickness is that of a wide street, a molecule in the hair would then look like a speck of dust in the street.
2. Millions of molecules are on the tip of a needle or pin.
3. In any part of the human body, such as the brow or the hand, there are many thousands of pores. When we sweat, only a tiny amount of liquid is obvious. This has come to the surface of the skin by oozing through millions of pores; a minute amount must have come through each tiny pore, showing that a molecule of liquid must be extremely small.
4. Two grams of hydrogen contain 6×10^{23} molecules. If the whole population of the world were to count such an enormous number of molecules individually at the rate of five per second it would take 100 years to count them all.
5. If a cricket ball was magnified to one kilometre in diameter a molecule in the ball would then be magnified to about one two-thousandth of a centimentre.

Motion of molecules in solids

In a solid, for example a metal, the molecules vibrate (oscillate) continuously about an average position. So, on the average, the molecules in a solid are 'anchored' to one position while they vibrate. Fig. 8.3 shows a simple 'model' of a solid such as a metal. The balls represent molecules and the springs represent the forces or bonds between the molecules. When the balls are pulled or displaced slightly and left, they vibrate or oscillate about an average position.

Scanned with CamScanner

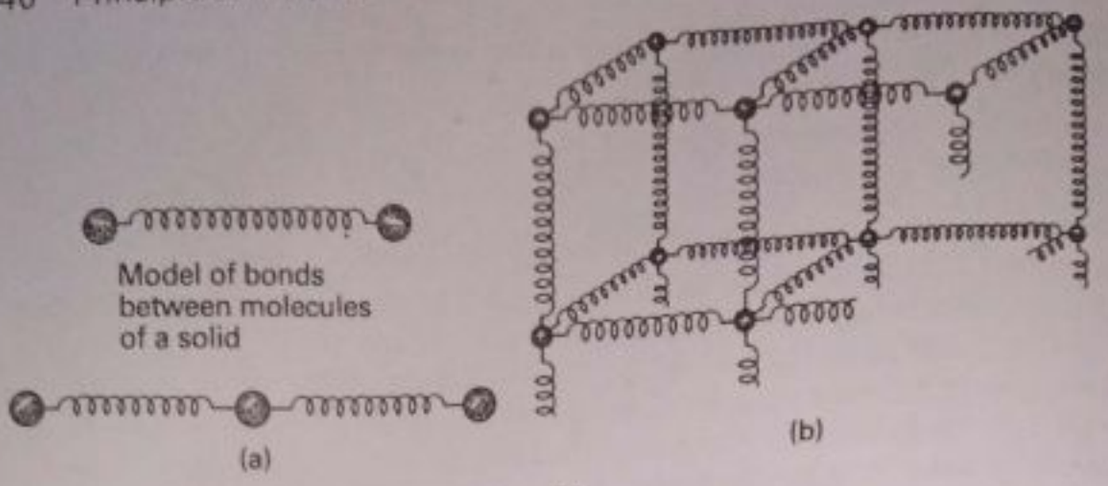


Fig. 8.3 Model of forces between molecules in solid

When a solid is heated so that its temperature rises, the molecules vibrate through greater distances, that is, their *amplitude* increases. (See plates 8A and 8B.) On the average, the molecules are then slightly further apart, that is, the solid *expands*. The increase in amplitude is small and so the expansion of a solid is generally small.

At room temperature, some molecules in solids such as moth balls (naphthalene) or camphor have sufficient energy to escape from the molecular bonds into the air. That is why these solids have strong smells.

Types of Motion

The motion of molecules just discussed is *oscillatory motion*.

In a liquid or gas, molecules move up, down or sideways across the container. Motion in one direction is called *translational motion*. Cars moving in a constant direction have translational motion.

The wheels of a moving car have *rotational motion*. So do the rotating blades of an electric fan. The Earth spins about its axis and so has rotational motion. A gas molecule such as oxygen, which has two atoms, has both rotational and translational motion.

So a moving object may have translational energy, or rotational energy or oscillatory energy or several of these energies.

We shall shortly meet the *random motion* of molecules (p. 148). In this case molecules keep changing their direction and have no definite path.

Molecules in liquids · Diffusion of liquids

Unlike in a solid, the molecules of a liquid are not anchored to one position. They move about inside the liquid, continually exchanging neighbours. In spite of their freedom, the liquid has a fixed volume.

The motion of molecules in liquids can be demonstrated as follows.

Scanned with CamScanner

Experiment

Three layers of liquid are carefully run into a large test-tube with a pipette, which is cleaned each time (Fig. 8.4). S is a layer of colourless sugar solution, on top of it is a layer C of blue copper sulphate solution and the layer W is water. Two distinct boundaries are seen. But after a time, the boundaries become indistinct. The blue colour of the copper sulphate solution has moved a little way into the colourless layers above and below it. The blue colour is also less strong. As time goes on, no boundaries can be seen and the liquids all mix.

Since the denser liquids move upwards against the pull of gravity, we conclude that the molecules are *constantly in motion in all directions.*

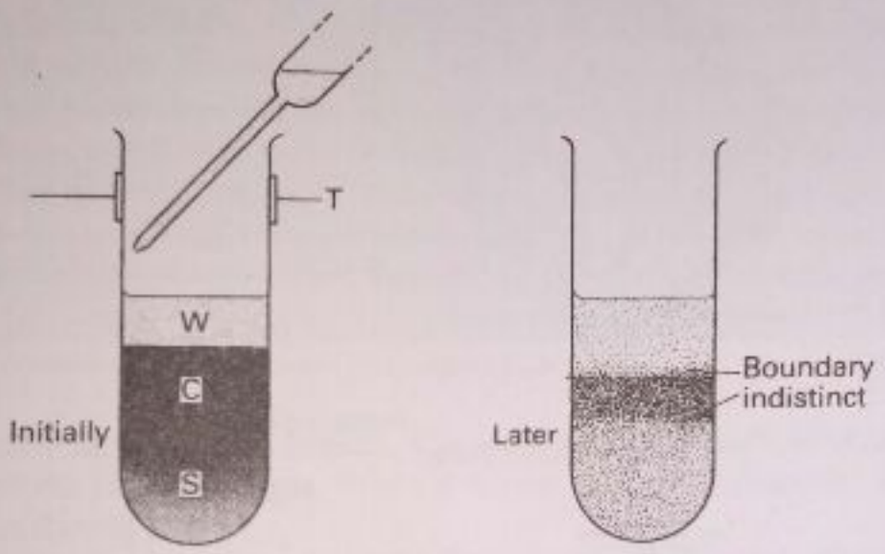


Fig. 8.4 Molecular motion in liquids: diffusion

Molecules in gases · Diffusion of gases

The molecules of a gas in a container move about in different directions throughout the whole volume. The forces or bonds between the molecules are so weak that the molecules can move independently of each other. So, unlike liquids, a gas can spread out to fill the whole volume of any container.

Perfume, or flowers such as roses, can be detected by smell in different parts of a small room. This shows that the molecules of a *gas* are in constant motion so the smell spreads across the room.

Experiment

To demonstrate the movement of gas molecules, a gas-jar of brown nitrogen dioxide gas is collected and sealed with a cover-slip. This gas is denser than air. An inverted gas-jar of air is now placed on top of the cover-slip, which is then removed. A brown colour is soon seen in the top jar containing the air. We therefore conclude that the nitrogen dioxide gas molecules move upwards against the pull of gravity and mix with the air molecules in the top jar.

Bromine capsules in a sealed glass apparatus can also be used to show the movement of gas molecules. These capsules are broken and brown bromine vapour is then seen to spread quickly throughout the apparatus.

Scanned with CamScanner

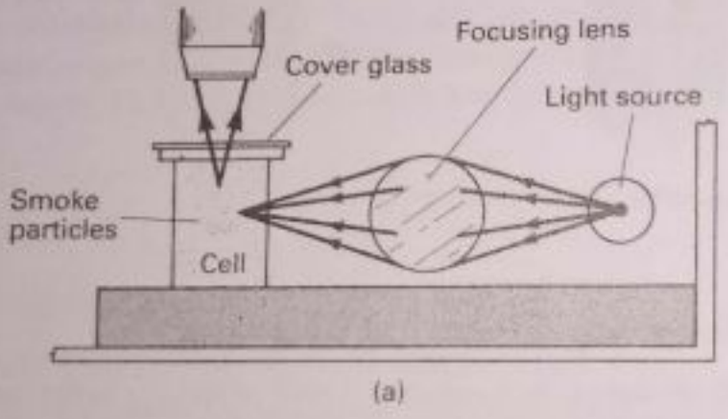
Brownian motion

During an experiment many years ago, the botanist Brown detected an effect produced by the random or haphazard movement of molecules. It is called Brownian motion.

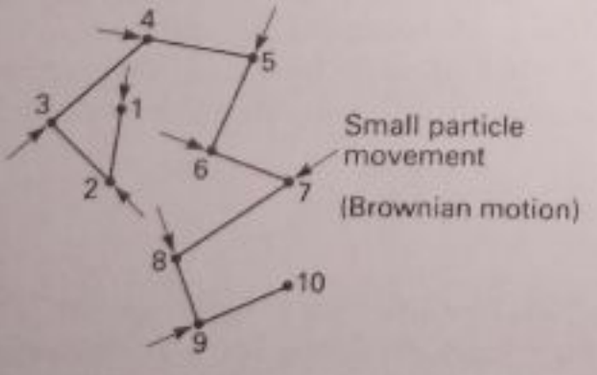
Experiment

Brownian motion can be demonstrated with the apparatus shown in Fig. 8.5a. Plate 8C shows the actual apparatus. It consists of a small transparent cell with a cover, which must be strongly illuminated from the side by a straight filament heated electrically. The cell is placed on the stand below a low powered microscope.

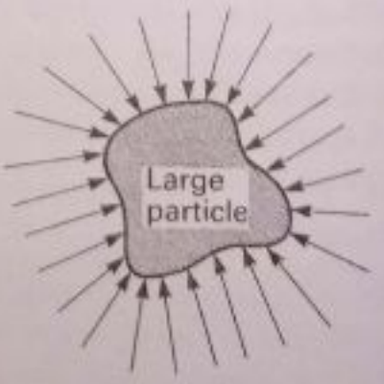
A piece of cord is set smouldering and some of the smoke, which contains tiny particles of carbon or cord, is then collected by a syringe and transferred to the cell. The cover is quickly replaced and the focus of the microscope is now adjusted until the fine particles come into view clearly. They are seen as bright dots, which move about irregularly like a drunken man staggering about. The particles dart from one place to another very suddenly, some going out of focus, others coming into focus, but always in motion. Fig. 8.5b shows a typical irregular path resulting from movement of a particle, from points labelled 1 to 10, which is in Brownian motion.



(a)



(b)



(c)

Fig. 8.5 Molecular motion in gas: Brownian (random) motion

Explanation of Brownian motion

The irregular motion is due to the movement of air molecules, which bombard the particle from all sides. The particle is so small that the number of molecules of air hitting one side is not balanced by an equal number of molecules hitting the

Scanned with CamScanner

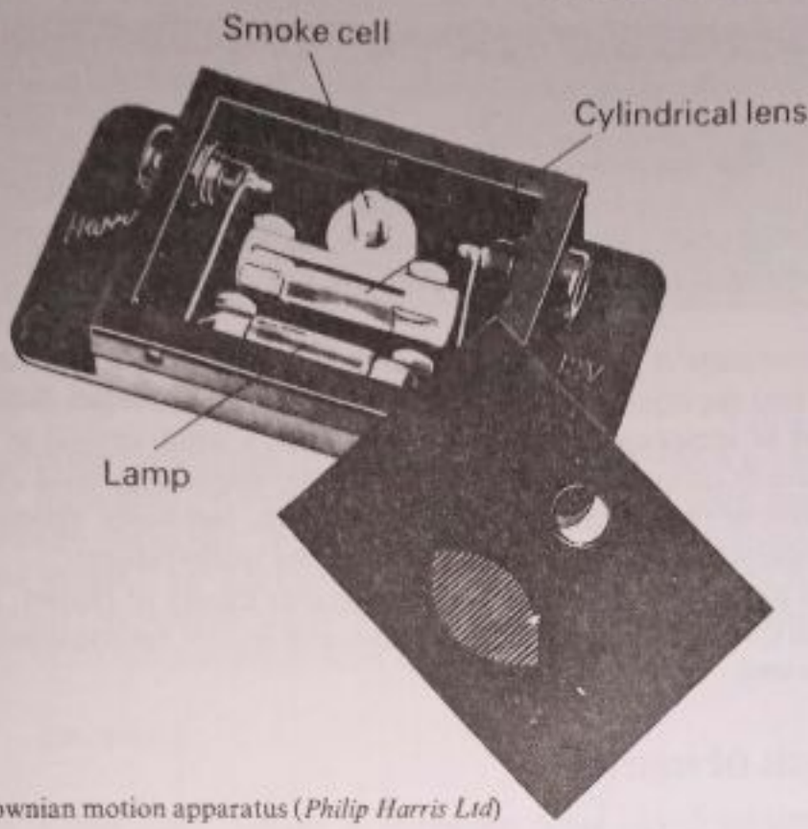


Plate 8C Brownian motion apparatus (*Philip Harris Ltd*)

opposite side at the same instant. Consequently, the particle moves in the direction of the resultant force. When it moves to another place the same thing happens, as shown.

The irregularity of the motion and the incessant movement of the particles shows that the air molecules move rapidly in all different directions and never stop. Brownian motion cannot be due to convection (see page 239). If this were the case, the particles would move upwards continually and not zig-zag from side to side.

If the particles in the air were big they would remain fairly still, although they would be bombarded on all sides by air molecules. This time the large number of molecules hitting one side is not relatively much greater than the number hitting the other side at the same instant (Fig. 8.5c).

The difference shows up much more when the number of bombarding molecules is small, as in the case of a very small particle. For this reason, a table-tennis ball (a large 'particle') suspended in air by a thread does not move. You yourself are being bombarded on all sides by molecules of gas in the air; but since you are so large, a few more bombardments on one side than on the other has no appreciable effect.

Brownian motion in liquids

Brownian motion was first detected in liquids. A demonstration can be arranged as follows.

Experiment

A few well-diluted drops of *aquadag* (finely divided graphite particles suspended in water) or *photopake* (a similar suspension used for blacking negatives) are placed on a clean microscope slide and the liquid is covered with a cover-slip. An image of the slide is

Scanned with CamScanner

then projected on to a screen by a microprojector so that the particles can be seen. Alternatively, the apparatus with a microscope shown in Plate 8C is used to look at the illuminated particles.

The graphite particles are then seen to be moving about in the liquid in an irregular way, showing Brownian motion in a liquid. The irregular motion of the graphite particles is due to their bombardment by the surrounding water molecules, which are constantly moving about in different directions.

If the temperature is raised, the Brownian (irregular) motion becomes faster. This shows that the liquid molecules now move faster. The effect of temperature on the speed of molecules can also be seen when a small crystal of potassium permanganate is dropped into two beakers, one containing cold and one hot water. The colour spreads more quickly through the hot water, showing that the crystal molecules in solution move faster at higher temperatures.

Brownian motion is important in the molecular theory of matter. It provides experimental evidence that molecules of gases and liquids are continually moving in a random way.

Properties of matter

We now consider briefly some properties of matter. They are: 1. elasticity of metals, 2. friction and viscosity, 3. surface tension.

Elasticity of metals

The metals used in steel bridges, aeroplanes and lift cables, for example, are always under tension due to the forces which act on them. These forces stretch or compress the metal. When the force is removed, the metal should go back to its initial length, that is, the metal should be 'elastic'. Otherwise the metal may be permanently strained and then crack or break up. The elasticity of metals is thus an important subject.

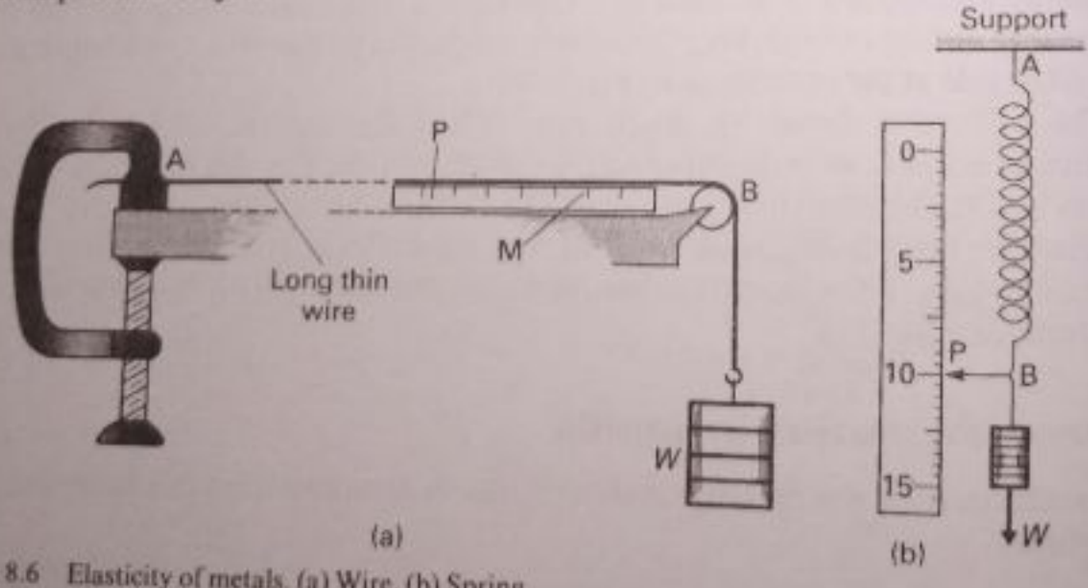


Fig. 8.6 Elasticity of metals. (a) Wire. (b) Spring

Experiment

Fig. 8.6a shows a simple way of investigating the stretching of a wire under a force or tension. A long thin metal wire, about the length of the laboratory, is clamped at one end A

Scanned with CamScanner

so that it cannot move there. The other end passes over a pulley-wheel at B, and weights *W* can be suspended from the wire as shown. A pointer P is fixed to the wire near B. Its position can be read from a millimetre scale M below it.

As weights are added, the wire stretches. The extension from its original length, before weights were added, is found each time from the readings of the scale P. Fig. 8.6b shows a similar experiment on the stretching of a metal spiral spring AB, fixed at A. The extension of the spring for loads *W* is read from the vertical scale.

Extension against load · Elastic limit

Fig. 8.7 shows the typical results obtained when the extension is plotted against the load *W*, from zero on each axis. At small loads, a straight line graph OX, passing through the origin, is obtained. So along OX, the extension is *directly proportional* to the load. Further, up to the load represented by E, the wire or spring returns to its original length when the load is removed. So the metal is said to be *elastic* up to this particular load E.

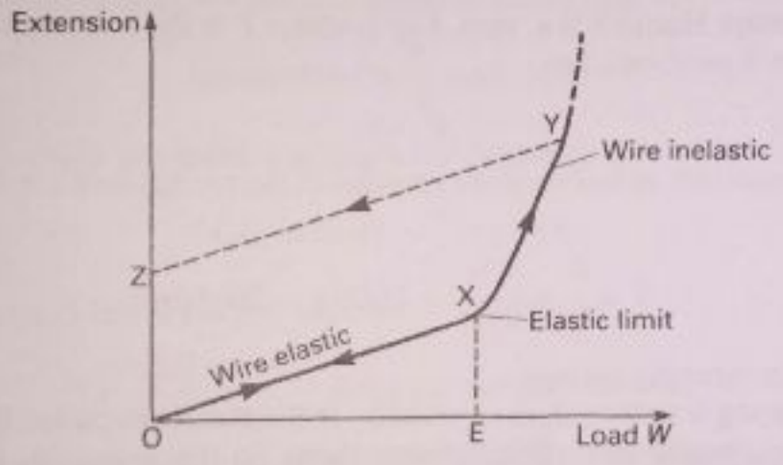


Fig. 8.7 Extension–load

Beyond X, however, the wire begins to extend rapidly along a curve XY. If the load is completely removed at Y, the wire does not now return to its original length. It has a permanent strain represented by OZ in Fig. 8.7, where W = 0. So X is the *elastic limit* of the wire.

Shortly beyond X, when the wire stretches rapidly, the metal is said to reach the *yield point* and it becomes 'plastic'.

The metals used in steel bridges or in aeroplanes, for example, must have an elastic limit well above the maximum force acting on them, otherwise they may break up.

Hooke's law

Hooke was the first person to investigate the stretching of metals. Hooke's law states *provided the elastic limit is not exceeded, the extension of a spring is directly proportional to the load.* This is shown by the straight-line graph OX in Fig. 8.7.

The spring balance uses Hooke's law. Loads greater than the elastic limit must not be added to the spring balance otherwise the spring becomes permanently strained and is then useless.

Scanned with CamScanner

Example on Hooke's law

A spring-balance has a scale-pan of 0·4 N weight attached to it. A load of 2·0 N placed on the pan extends the spring 24 mm from its original unstretched length. Calculate the load on the scale-pan when the extension is 16 mm. (Assume the elastic limit is not exceeded.)

Total force stretching spring = load + pan weight = 2·0 + 0·4 = 2·4 N.
From Hooke's law, the force F stretching the spring 16 mm is given by

$$\frac{F}{2{\cdot}4\text{ N}} = \frac{16\text{ mm}}{24\text{ mm}} = \frac{2}{3}$$

$$\therefore 3F = 2 \times 2{\cdot}4 = 4{\cdot}8\text{ N}$$

$$\therefore F = \frac{4{\cdot}8}{3} = 1{\cdot}6\text{ N}$$

$$\therefore \text{load} = F - \text{pan weight} = 1{\cdot}6 - 0{\cdot}4 = 1{\cdot}2\text{ N}$$

Force constant of spring

If a spring obeys Hooke's law, then $F \propto e$, where F is the force applied and e is the extension it produces. So

$$F = ke,$$

where k is a constant called the *force constant* of the spring. So if e is 4 cm when F is 8 N, then

$$k = \frac{F}{e} = \frac{8\text{ N}}{4\text{ cm}} = 2\text{ N/cm} = 200\text{ N/m}$$

k increases for stronger springs.

When a spring is stretched, the molecules in the metal are pulled farther apart against the molecular attraction between them. So the molecules gain energy. The molecular energy in the spring = the work done in stretching the spring against the forces *in the spring*. Before the force F is applied, the force in the spring is zero. When the force F is applied, the force in the spring grows steadily and quickly to the value F and the extension is then e.

$$\therefore \text{average force in spring} = \tfrac{1}{2}(0 + F) = \tfrac{1}{2}F$$

$$\therefore \text{work done} = \text{average force} \times \text{distance} = \tfrac{1}{2}F \times e = \tfrac{1}{2}Fe$$

Also, $F = ke$.

$$\therefore \text{energy in spring} = \tfrac{1}{2}ke \times e = \tfrac{1}{2}ke^2 \qquad (1)$$

Generally, if x is the extension of a spring and Hooke's Law is obeyed, then energy in stretched spring = $\frac{1}{2}kx^2$. We use this result later (p. 590).

Example on spring

A helical spring is stretched 2 cm by a load of 6 N. Calculate (a) the force constant, (b) the energy in the stretched spring.

(a) From $F = ke$,

$$k = \frac{F}{e} = \frac{6\text{ N}}{2\text{ cm}} = 3\text{ N/cm} = 300\text{ N/m}$$

Scanned with CamScanner

(b) Since 2 cm = 0·02 m,

$$\text{energy in spring} = \tfrac{1}{2}ke^2 = \tfrac{1}{2} \times 300 \times 0{\cdot}02^2$$
$$= 0{\cdot}06\ \text{J}$$

Young modulus

Steel bridges and aeroplanes are two cases where loads or forces in their structure produce extensions in the metals used for making them. Samples of the metals are therefore sent to laboratories, where tests are made to find out their extension under loads and their proportional or elastic limits.

Samples of the metal under test may have different lengths and different cross-sectional areas. Young therefore suggested calculating

(a) the *force per unit area*, called *tensile stress* and
(b) the *extension per unit length*, called the *tensile strain*,

when experiments were made on different samples of the same metal. So if F is the force applied to stretch a metal of cross-sectional area A,

$$\text{tensile stress} = \frac{F}{A}\ (\text{units N/m}^2)$$

If e is the extension produced in a length l of the metal,

$$\text{tensile strain} = \frac{e}{l}\ (\text{no units})$$

When Hooke's law is obeyed, experiment shows that

$$\frac{\text{tensile stress}}{\text{tensile strain}} = E, \text{ a constant for the metal}$$

E is known as the *Young modulus* for the material. The value of E for different materials used in industry is listed in Tables for reference. Measurement shows that $E = 2{\cdot}0 \times 10^{11}$ N/m^2 for mild steel and $E = 1{\cdot}2 \times 10^{11}$ N/m^2 for copper.

Example on Young modulus

A metal bar of length 1·0 m and uniform cross-sectional area 10^{-5} (0·00001) m^2 extends by 1·5 mm when a force of 200 N is applied.

If the elastic limit is not exceeded, calculate (a) the tensile stress, (b) the tensile strain, (c) the Young modulus for the metal.

(a) Tensile stress $= F/A = 200\ \text{N}/10^{-5}\ \text{m}^2 = 2 \times 10^7\ \text{N/m}^2$

(b) tensile strain $= e/l = 1{\cdot}5\ \text{mm}/1{\cdot}0\ \text{m} = 1{\cdot}5\ \text{mm}/1000\ \text{mm} = 0{\cdot}0015$

(c) $\therefore$ Young modulus $E = \dfrac{\text{stress}}{\text{strain}}$

$$= \frac{2 \times 10^7}{0{\cdot}0015} = 1{\cdot}3 \times 10^{10}\ \text{N/m}^2$$

Static friction · Laws of friction

A simple experiment to investigate frictional forces can be carried out as follows.

Scanned with CamScanner

Experiment

A block of wood A is placed on a table and connected by a light string over a pulley to a scale-pan S hanging vertically (Fig. 8.8). Increasing weights are added to S. At first, the block does not move, but at one stage the block starts to slip. The weights on S and the weight of S itself are then together equal to the *limiting frictional force F* on the block. The 'limiting' frictional force is the maximum frictional force which can act on the block, when it just starts to slip.

To see if the limiting frictional force F depends on the area of the block in contact with the table, the block is now turned on one side and the experiment repeated. The same value of F is obtained when weights are added to S.

The reaction R of the table on the block is equal to the weight of the block pressing down on the table. So R is varied by placing loads such as 500 g and more on the block, as shown in Fig. 8.8. Different values of F are then obtained when weights are added to S. The ratio F/R is found from each pair of values of F and R.

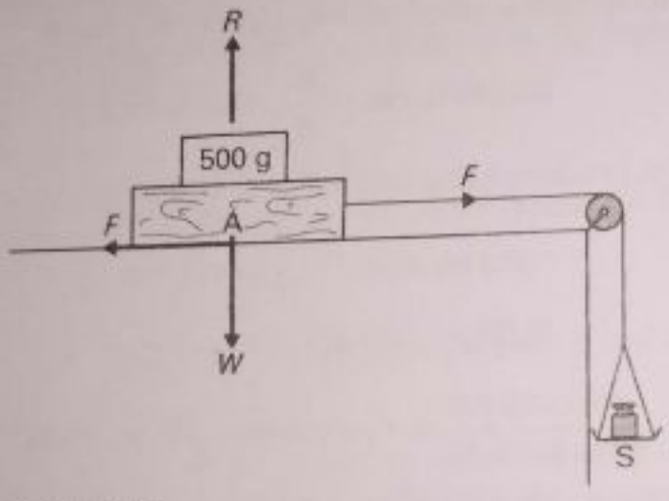


Fig. 8.8 Static friction experiment

From this experiment we can verify roughly the laws of solid friction. These are

1. Friction opposes motion.
2. The limiting frictional force is independent of the area in contact.
3. The ratio F/R is practically constant, where F is the limiting frictional force and R is the normal reaction between the surfaces. So $F = \mu R$, where μ is a constant called the coefficient of static friction.

Kinetic (dynamic) friction

A frictional force also occurs when a surface moves over another surface with a constant velocity. This can be investigated by placing a block of wood A on a table and again adding weights to a scale-pan S as shown in Fig. 8.8. This time, however, the block is given a slight push each time a weight is added. At one point the block continues to slide along the table with constant velocity. The frictional force F is now called kinetic or dynamic friction.

Experiment shows that, for a given value of R, the kinetic or dynamic frictional force is always less than the limiting static frictional force.

When the brakes of a bicycle are applied the kinetic frictional force between the wheels and the brake blocks slows down the bicycle.

Scanned with CamScanner

Fluid friction or viscosity

It is more difficult to wade through water than to walk the same distance in air. Water has a greater frictional resistance or *viscosity* than air. Treacle pours much more slowly than water and the viscosity of treacle is greater than that of water. Liquids and gases are both *fluids*.

When a liquid flows along a tube or a gas through a pipe, two adjacent layers in the fluid have different velocities. *Viscosity is the force which opposes the relative velocity between the layers.* In solid friction, as we have seen, the frictional force opposes the relative velocity between two solid surfaces in contact.

Objects falling in liquids

We can investigate the effect of liquid friction on a falling object in the following way.

Experiment

Fill a tall gas-jar nearly to the top with water (Fig. 8.9a). Then place two thin elastic bands, A and B, 2 cm apart round the gas-jar near the top of the liquid; two similar bands, C and D, 2 cm apart in the middle; and two more bands, E and F, 2 cm apart near the bottom.

Gently place a few styrocell beads, which are small and light, in the middle of the water surface. Keep your eye on one falling bead and using a stopwatch or stopclock, time how long it takes to fall each of the three equal distances AB, CD, and EF. Repeat with other beads.

Result: As it falls from the top of the liquid, the bead increases in velocity to a maximum and then continues to fall at this maximum velocity.

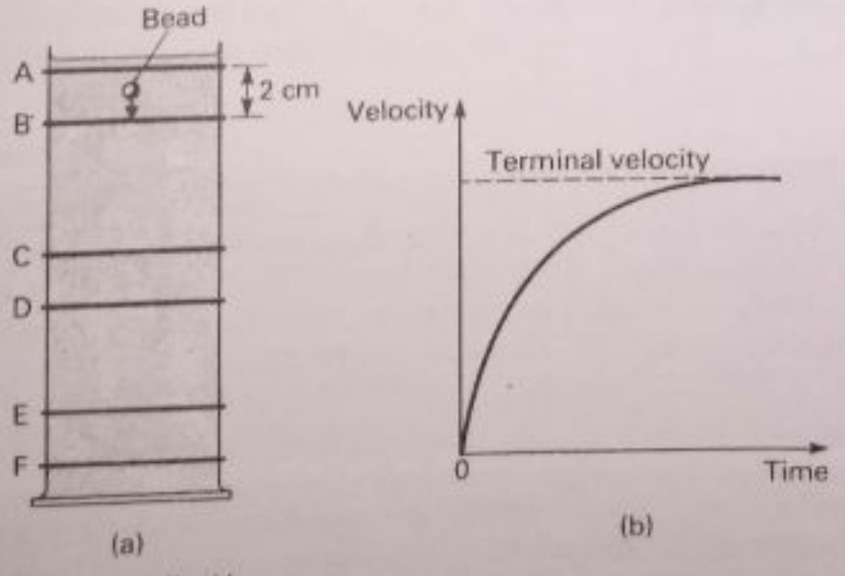


Fig. 8.9 Motion in viscous liquid

The result of this experiment can now be explained.

When it is first placed in water, the weight of the bead less the upthrust on it, which we shall call the 'apparent weight', is greater than the opposing frictional force due to the liquid. So the bead accelerates downwards under a resultant force. Now unlike the frictional force between two moving solids, which is fairly independent of their relative velocity, the liquid frictional force increases as the

Scanned with CamScanner

velocity of the bead increases. When this force becomes *equal* to the apparent weight of the bead, there is no resultant force on it and therefore no acceleration. So the bead now continues to move at a constant velocity. It is called the *terminal velocity* of the falling object.

Fig. 8.9b illustrates how the velocity of a falling object in a fluid rises to its maximum or terminal velocity. As we see shortly, air has a frictional resistance to falling objects. An object released from an aeroplane at a great height will therefore reach a terminal or maximum velocity after falling some distance.

Experiment to compare liquid viscosities

The viscosities of liquids can be roughly compared by filling three tall cylindrical vessels with a different liquid, such as water, engine oil and glycerine. A small steel ball-bearing of the same radius is now placed gently on the surface of each liquid.

We see that the ball-bearing falls quickly in water, a little slower in engine oil, and slowly in glycerine. So the liquid viscosities increase in the order water, engine oil and glycerine.

Lubrication

The subject of viscosity plays an important part in lubrication of engines and other machines. Here it is essential to keep metal surfaces from rubbing against each other. If this is not done effectively the metals wear quickly and become hot, and the surfaces seize or weld together. The oil used forms an extremely thin film (of molecular dimensions), yet it is sufficiently strong to bear a heavy load. The viscosity of a liquid decreases fairly rapidly with rise of temperature, but engine oils called 'viscostatic' oils are designed to be used in both summer and winter without changing them.

Air friction or viscosity

Stones of different sizes reach the ground at practically the same time when released at the same height. This is because the frictional force due to the air on each is relatively very small compared to their weights. Air friction can be demonstrated by using a light object such as paper.

Experiment

1. Release flat sheets of paper of different area so that they fall several metres and note which falls to the ground more slowly.

We conclude that the frictional force is larger when the area of the paper is greater.

2. Take a sheet of paper of exercise-book size and observe its fall to the ground. Now 'streamline' the paper by turning up the edges or sides to form a tray, using sticky tape to stick the corners. Observe the fall of the tray.

We see that the streamlined paper tray falls to the ground faster than the flat sheet of paper.

Scanned with CamScanner

Trains and racing cars are streamlined by making their surfaces smooth. This reduces the air resistance especially at high speeds. Seals and whales have very smooth skins which reduces the frictional force on them when they move through water. Gases are much less viscous than liquids. The hovercraft travels smoothly on a cushion of air below it and has therefore much less frictional force on it than if it moved through the water.

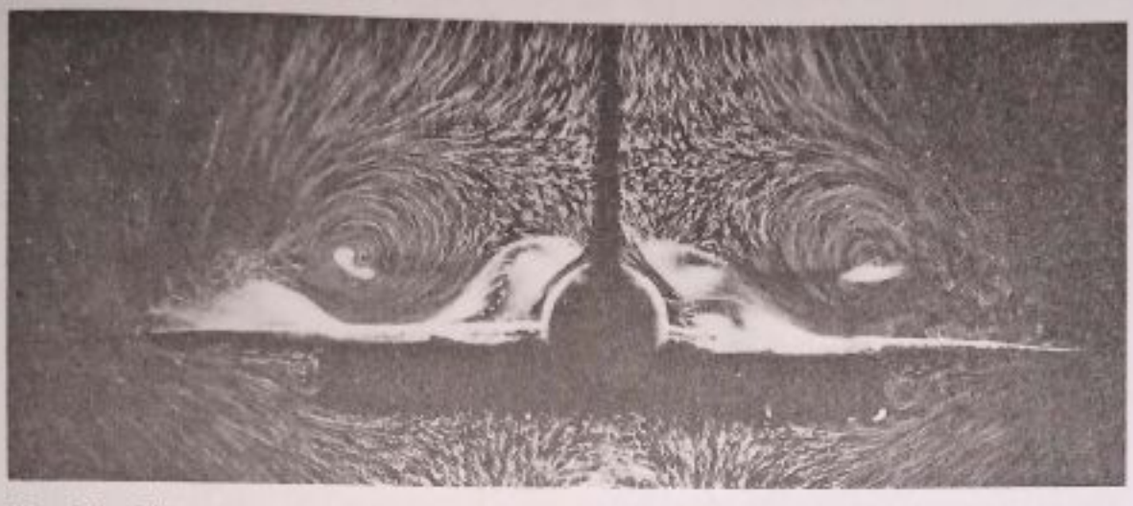


Plate 8G Flow paths of air round a model aeroplane in a wind tunnel to show effects of streamlining

Air friction is important in aerodynamic design, where turbulence must be avoided. Plate 8G shows for comparison the results of experiments on an aerofoil in a wind tunnel, where the frictional force of air is studied.

Surface tension

We now investigate the *surface* of a liquid. As we shall see, this has properties which are different from the bulk of the liquid.

Experiment

Fill a beaker of water completely to the brim. Place a dry needle on a small piece of blotting paper and gently push the paper from the rim of the glass towards the middle of the water surface. The paper becomes wet and sinks. We see that the needle, however, remains floating on the water.

Conclusion: The surface acts like a 'skin', covering the liquid, which is able to support the weight of the needle.

The property of the water surface which enables it to support the needle is called its *surface tension*. As shown in Fig. 8.10a, the weight of the needle is balanced by forces in the surface, which act upwards.

An insect called a pond skater can walk across a water surface without sinking (Fig. 8.10b). The surface is slightly pressed down by the legs, showing that the surface acts like an elastic 'skin'.

A drop of water forming slowly in a tap, Fig. 8.10c, or droplets of mercury spilled on a clean glass plate, Fig. 8.10d, show that the surface can support the weight of liquid above it, further demonstrating surface tension.

Scanned with CamScanner

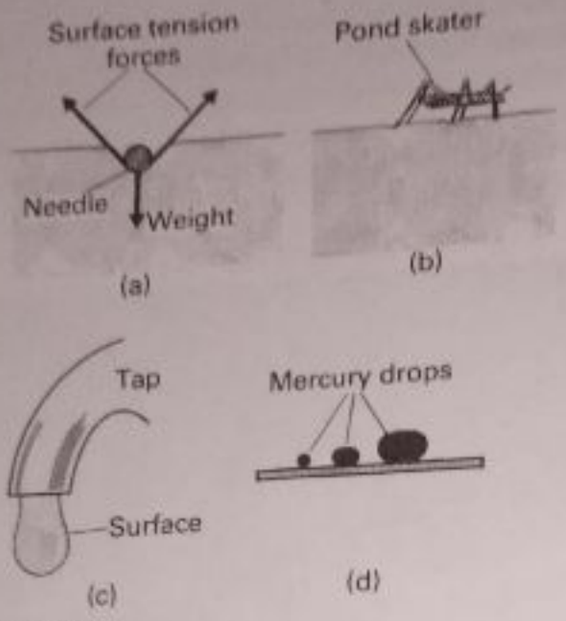


Fig. 8.10 Effects of surface tension

Surface tension forces

The existence of the surface 'skin' is due to the forces of attraction between liquid molecules. Below the surface, a molecule such as A (Fig. 8.11) is attracted by molecules all round it. The most powerful forces come from those molecules

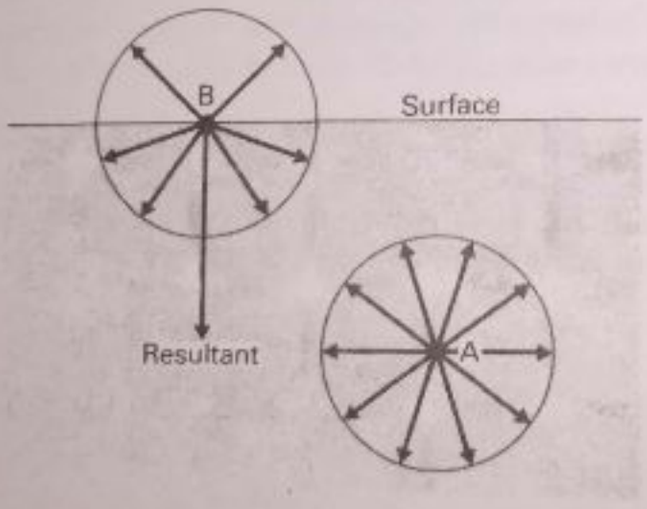


Fig. 8.11 Molecular attraction in liquids

inside a very tiny surrounding sphere whose radius is of molecular dimensions. Other molecules outside this sphere have negligible attraction and can be ignored. In any direction there are as many pulling forces on A one way as there are the opposite way. Thus the resultant force is zero.

This is not the case for a molecule such as B *in the surface*. There are far more molecules of the liquid below the surface than there are above in the vapour. So there is a resultant pull on B into the liquid. This is the case for all molecules which happen to be in the surface. The downward pull has the effect of making the surface area as small as possible. So the surface is under tension, and acts like a 'skin' covering the liquid.

The forces *in the surface itself* are called surface tension forces. Their effect may be shown in the following way.

Scanned with CamScanner

Experiment

Some bare copper wire is bent round into a circular frame and the ends are twisted to make a handle (Fig. 8.12a). The frame is dipped into a strong soap solution and removed, so that a soap film fills the frame as shown. The frame is held horizontally and a light thread in a continuous loop is placed gently in the middle of the film. By using a sharp point, the film *inside the thread* is now pierced.

We see that the thread is now pulled into a circle (Fig. 8.12b).

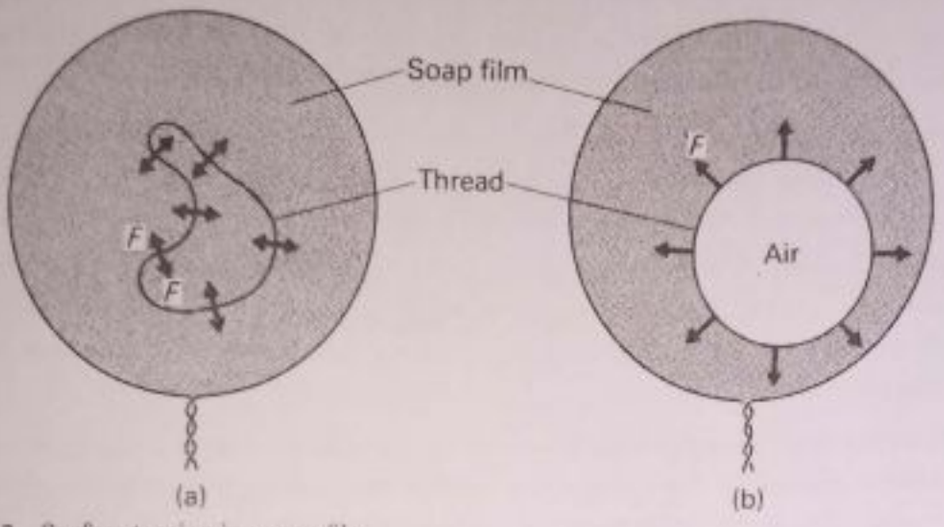


Fig. 8.12 Surface tension in a soap film

Explanation: Before the film is pierced, the surface tension forces such as F act on both sides of the thread and counter-balance each other (Fig. 8.12a). When the film is pierced, the force F is now acting only on one side of the thread, as shown in Fig. 8.12b. So the thread is pulled into a circle.

Another simple effect of surface tension is shown in Fig. 8.13a. When a paint brush is dipped into clean water, the hairs spread out as shown. When the brush is removed, the surface tension forces, in the film between the hairs, pull the hairs together tightly (Fig. 8.13b).

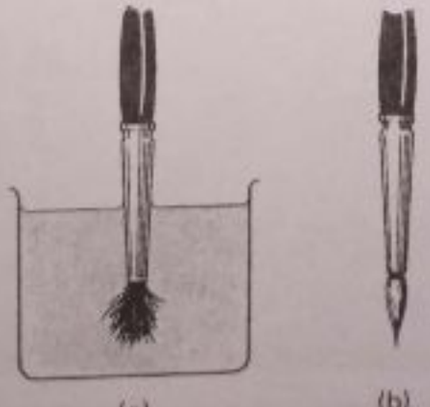


Fig. 8.13

Scanned with CamScanner

Effect of impurities

The size of the surface tension forces is affected by impurities.

Experiments

1. As described on page 158, float a needle on a water surface in a beaker. Now carefully add a little soap solution or detergent to the water surface. You will see that the needle sinks.

Explanation: The surface tension of water is reduced by soap or detergent. The surface tension forces are now unable to support the weight of the needle.

2. Drop some very small pieces of camphor on to the surface of water in a large beaker or trough. We see that the pieces move about rapidly in different directions.

Explanation: Camphor decreases the surface tension of water where it dissolves. However, it does not dissolve equally at every part. So there is always a resultant horizontal force on it, which changes direction continually.

3. Sprinkle some lycopodium powder very lightly on to the surface of water in a large beaker or trough. Dip one end of a glass rod into some soap solution or detergent, and touch the middle of the lycopodium powder. We see that the powder moves away from the end of the rod.

Explanation. Soap reduces the surface tension of water. So there is a resultant force away from the soapy end of the rod where it touches the water, as shown by the powder movement. The purpose of the powder is to show movements in the surface layer.

The reduction of surface tension also accounts for the movement of a toy boat across water when a piece of camphor is attached below it. The camphor dissolves and lowers the surface tension of the water below it. The resultant force across the camphor boundary with the clean water drags the light boat along.

Detergents are chemicals which are mixed with water to increase its cleaning power. They lower the surface tension of water. This increases the 'wetting' power of the water or its ability to float small dirt particles away from the articles being washed (Plate 8I). Detergents also contain special chemicals to combine with oils and fats to make them soluble in water. They can then be 'wetted' and removed from the material.

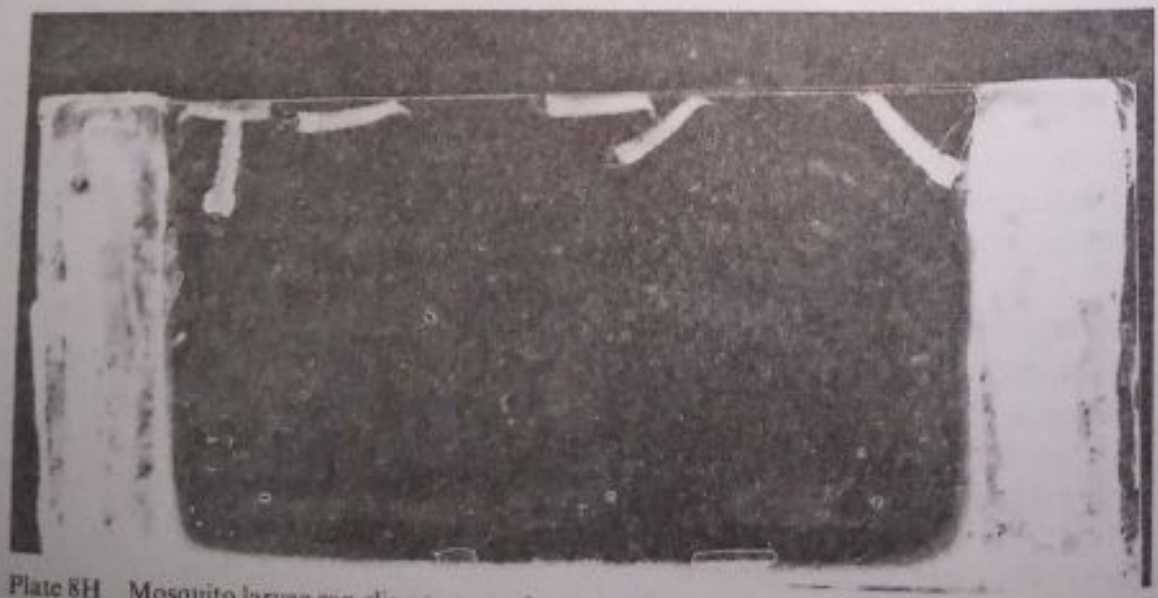


Plate 8H Mosquito larvae can cling to water because of surface tension (*Shell Photographic Unit*)

Scanned with CamScanner

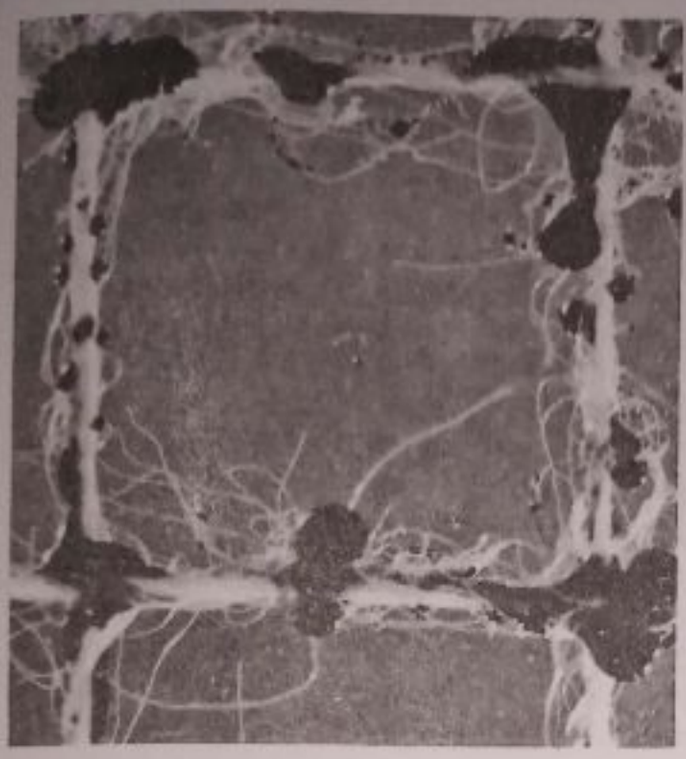


Plate 8I Detergents in solution removing black oil from fibres (*Unilever Ltd*)

Capillarity

Experiment

A number of capillary (fine-bore) tubes of different radii is dipped into a beaker of water so that the insides are wetted. They are supported upright with the bottom of each tube dipping into the water (Fig. 8.14a). We see that the liquid rises up each tube. The height of the water column above the outside water level is greatest for the smallest radius tube.

The experiment is repeated with soap solution in place of water. Although the solution rises up each tube in a similar way, the height of the liquid columns are generally less than those of the water columns.

This phenomenon is called *capillarity*. The reverse effect is seen when capillary tubes dip into mercury (Fig. 8.14b). This time the liquid level inside the tube is *lower* than that outside.

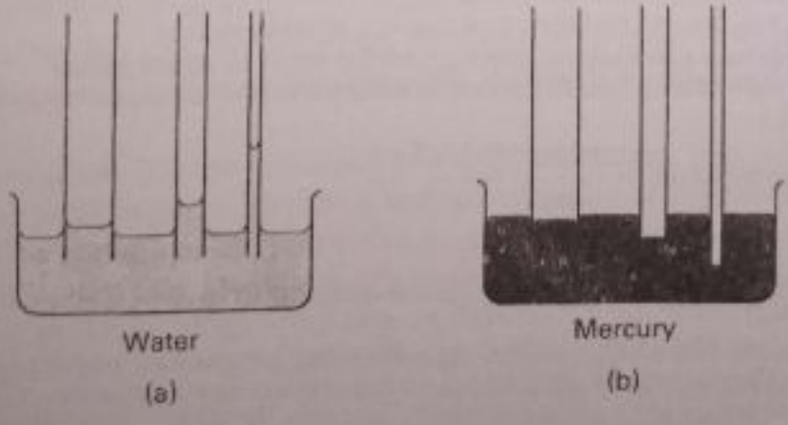


Fig. 8.14 Levels of liquids in capillary tubes

Scanned with CamScanner

Blotting paper has many fine pores, which act as tiny capillary tubes. The ink rises into the blotting paper through the fine pores. The fibres of the wick of a candle act like capillary tubes of fine bore. The melted wax is drawn up into the wick by capillary action.

Bricks are porous–water can seep up them from the ground by capillary action. *Damp courses*, layers of non-porous material such as slate, are therefore necessary at the bases of houses to protect them from damp.

Cohesion and adhesion

The attractive force between two similar molecules, such as two mercury molecules, is called a *cohesive* force. The attractive force between two unlike molecules, such as mercury and glass molecules, is called an *adhesive* force.

Fig. 8.15 shows the non-wetting of a glass plate by mercury and the wetting of it by water. This is because the cohesion between the two mercury molecules is greater than the adhesion between a mercury molecule and a glass molecule. The reverse is the case for water and glass molecules. Here the adhesion between a glass molecule and a water molecule is greater than the cohesion between two water molecules. So water spreads on clean glass.

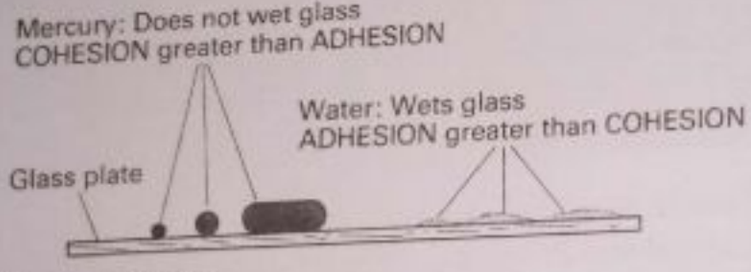


Fig. 8.15 Cohesion and adhesion

For the same reason, the surface of water rises where it meets the glass edges of its beaker; that is, the meniscus is curved upwards. In a glass beaker, however, the meniscus of mercury is curved downwards at the edges.

As we saw on page 143, an olive-oil film spreads over a water surface. This is because the adhesive force between water and an oil molecule is greater than the cohesive force between two oil molecules.

Examples on molecules and surface tension

1. (a) An oil drop of volume $10^{-9}\,m^3$ spreads out on water to form a film of area $0{\cdot}2\,m^2$. Estimate the length of an oil molecule from this information. What assumption have you made in your calculation?

Describe what could be done to make the film of oil clearly visible in this experiment. What precautions should be taken to achieve a reliable result?

(b) Explain how a gas exerts a pressure on the walls of its container.

(c) Outline very briefly *two* situations in which the phenomenon of surface tension is exhibited. (L)

(a) Length of molecule $= \dfrac{\text{volume}}{\text{area}} = \dfrac{10^{-9}\,m^3}{0{\cdot}2\,m^2}$

$= 5 \times 10^{-9}\,m$

The calculation assumes that the oil film is one molecule thick, which may not be a correct assumption.

To make the oil film clearly visible, sprinkle some lycopodium powder lightly on the surface before allowing the oil drop to fall on the surface. Some precautions necessary for a reliable result are (i) use a clean container, (ii) clean the surface carefully, (iii) level the container with a spirit level.

Scanned with CamScanner

(b) A gas exerts a pressure on the walls of its container because the molecules bombard the walls continually. A molecule hitting the wall bounces back with a *change in momentum* normal to the wall. The average momentum change per second produced by all the molecules striking the wall repeatedly produces a force. The pressure is the average value of the force per unit area of the wall.

(c) Two situations in which the surface tension phenomenon is shown are (i) a needle floating on the surface of water and (ii) the rise of water in a capillary tube when the tube is dipped into water and withdrawn a little.

2. (a) The thin sheet of aluminium shown in Fig. 8.16 will float on water in a bowl, and if a small piece of camphor is placed as shown the aluminium will move about on the surface of the water. Explain these observations.

(b) Explain why it is not sensible to rub the canvas of a tent in wet weather. (L)

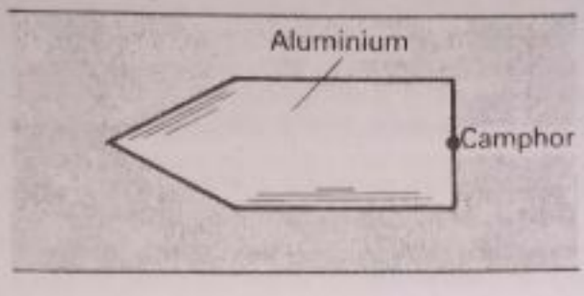


Fig. 8.16 Example

(a) The thin aluminium sheet floats on water because the water surface acts like an elastic skin and the surface tension forces round the edges act upwards and support the weight of the aluminium.

When camphor dissolves in water, it lowers the surface tension of the water. The horizontal forces on the back edge are then less than the horizontal forces on the front edges. The resultant force makes the aluminium move to another part of the water. The same thing happens here and so the aluminium moves about on the surface.

(b) In wet weather, the rain drops form a water film across small holes or pores in the canvas and this stops water entering the tent. The film is held in position by surface tension forces. Rubbing the canvas will break the film and rain can then pass through the tent to the inside.

Summary

1. Solids have volume and shape; liquids have volume but no shape; gases have no definite volume or shape.
2. The densities of solids and liquids are about 1000 times as great as those of gases. The packing of the molecules is correspondingly greater.
3. The molecules of gases and liquids are constantly in irregular motion–as shown by the phenomenon of Brownian movement and by diffusion.
4. The diameter of a molecule is of the order of several times 10^{-8} cm. A rough estimate of the size of a molecule may be obtained by measuring the thickness of an oil-film on water. Estimation from the thickness of gold-leaf gives a higher value.
5. The forces between the molecules of a solid are strong; they are less strong in liquids and weak in gases. The force may be attractive or repulsive.
6. The molecules of a solid vibrate about a particular position inside it–they have vibrational energy; the molecules of a liquid move freely through the liquid and continually exchange neighbours; the molecules of a gas move freely in the space in which they exist and the attractive forces between the molecules are very weak.
7. Elasticity: Hooke's law states that, provided the elastic limit is not exceeded, the extension of a metal is directly proportional to the tension or load applied.

Scanned with CamScanner

8. Friction: In *solid* friction, the limiting frictional force is (a) independent of the surface area in contact for a given object, (b) roughly proportional to the normal reaction of the surface. *Fluid frictional forces*, or viscous forces, increase in proportion to the area of the solid inside. Lubrication separates metal surfaces by a thin film of oil.

9. In liquids the phenomenon of surface tension is due to the attractive force on molecules in the surface by molecules below; drops of liquid under surface tension forces have a spherical shape. *Capillarity*, the rise or fall of liquids in narrow tubes, is due to surface tension.

Exercise 8 (Answers, page 641)

1. (a) If a force of 12 N extends a spring by 8 mm, calculate the force to extend it by 2 mm. What assumption did you make in your calculation?

(b) A block of wood of weight 4 N rests on a rough inclined plane which is 30° to the horizontal. Using the component of the weight, find the frictional force on the block.

(c) Seen in a microscope, pollen particles in water move about irregularly. Choose one of the statements from **A** to **E** which you think is correct:

This phenomenon is due to **A** the microscope lenses, **B** the density of the water, **C** the density of the pollen, **D** the surrounding molecules of water, **E** the surface tension of water.

(d) When a little olive-oil is placed in the middle of clean water, the oil spreads into a circular film. Why does this happen? What quantity can be estimated in this experiment?

2. Describe the apparatus required and how it would be set up in order to observe Brownian motion in *either* a liquid *or* a gas.

Describe briefly what is observed and explain what causes the observed motion. (L)

3. How would you demonstrate that, within some elastic limit, a metal, such as copper, is perfectly elastic? Describe briefly what is observed if the elastic limit is exceeded. (L)

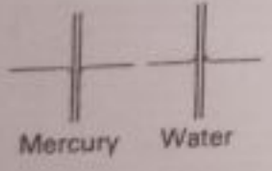


Fig. 8.17

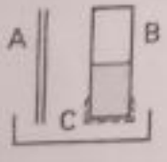


Fig. 8.18

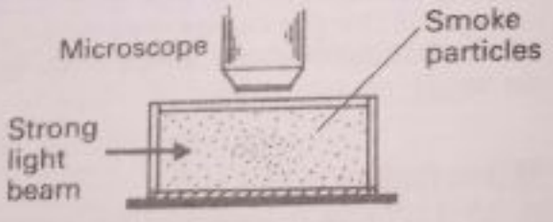


Fig. 8.19

4. Fig. 8.17 shows capillary tubes open at both ends dipping into (a) mercury, (b) water. Show on diagrams the approximate positions and shapes of the liquid menisci in each tube. (N)

5. Fig. 8.18 shows a capillary tube A of small bore and a very much wider tube B with its lower end C covered with a semi-permeable membrane and containing sugar solution. Describe with the aid of a diagram what you would expect to happen if water is poured into the dish until the lower ends of the tubes are covered.

What would you expect to happen in tube B if the water in the dish were replaced by sugar solution rather stronger than the solution in the tube? (C)

6. Fig. 8.19 shows a cell containing very fine smoke particles in air, which is illuminated from the side by a strong beam of light, and viewed from above through a microscope.

(a) Describe the motion of an individual particle.

(b) How can you tell which are the larger particles and which are the smaller particles?

(c) Give a short explanation of the motion of the particles. (O)

7. Two identical capillary tubes are placed vertically, one partially submerged in a liquid A and the other partially submerged in a liquid B. Both liquids rise up the tubes. The liquid A reaches a smaller height but takes a longer time to reach it than the liquid B.

Scanned with CamScanner

Draw a diagram showing the liquids in the tubes in their final positions. Explain this behaviour in terms of surface tension and viscosity. (C)

8. An olive-oil drop was found to have a diameter of 0·05 cm when viewed against a $\frac{1}{2}$ mm scale. When it was dropped on to a water surface it spread into a circle of diameter 10 cm. Estimate the upper limit for the diameter of an oil molecule.

Write down a rough value for the actual diameter of an oil molecule.

9. Describe how you would use a given spiral spring as a spring balance. How would you calibrate this balance? State, with reasons, *two* conditions under which the balance would *not* be suitable for use. (N)

10. What is meant by the viscosity of a liquid?

When a ball-bearing is allowed to fall through glycerine it soon attains a steady velocity. (a) What forces are acting on the ball-bearing?

(b) What relationship exists between these forces? (O)

11. Explain the following observations and name the physical effects involved.

(a) A drop of water remains for a time on the surface of a dry cloth before being absorbed, but a similar drop on a damp cloth is rapidly absorbed.

(b) A ball-bearing falling in motor oil soon attains a constant velocity.

(c) A few days after a crystal of copper sulphate has been placed at the bottom of a beaker of water a blue colour is observed throughout the water. (C)

12. Show graphically the relation between the extension of a wire and the load attached to it. Label the axes of your graph and point out the main features shown by the curve. (L)

13. State Hooke's law and describe how you would test its validity for a spiral spring. Explain briefly how the action of a spring balance depends on this law.

A light rod AB, 20 cm long, is supported horizontally by two exactly similar vertical springs attached to its ends. A load of 2 N is hung from a point on the rod 15 cm from the end A, and the upper end of the spring at B has to be raised 2 cm in order to make AB horizontal again. Find the force needed to extend each of the springs by 1 cm. (O)

14. Describe one experiment in each case to show that molecules have

(a) motion; (b) size; (c) irregular motion.

State and explain the behaviour of (a) a tiny particle, (b) a large particle suspended in a liquid medium. What effect is observed in each case when the temperature of the liquid is raised?

15. (a) Describe a simple experiment which illustrates the diffusion of gases. How is diffusion explained in terms of simple molecular theory?

(b) When deducing an upper limit of molecular size by the surface-film method, it is assumed as a rough approximation that the molecules are small cubes. Supposing that an oil molecule is a cube of side 10^{-9} m, and that the mass of a single molecule is 4×10^{-25} kg, estimate the mass of an oil drop which is capable of forming a circular film one molecule thick and of radius 0·2 m.

(c) Describe briefly the measurements and calculations that you would make, in a practical experiment, in order to determine the mass of the oil forming such a film. (O)

16. A continuous loop of cotton L rests on a soap film bounded by a flat wire frame. The soap film exists both inside and outside L. Describe what happens when that part of the film inside L is pierced and account for the new shape of the loop. (O and C)

Multiple-Choice Questions

17. When an oil-drop is placed gently on a clean water surface of area 100 cm^2,

A the drop spreads until the film is 50 molecules thick
B the drop spreads to fill exactly an area of 100 cm^2
C the volume of the drop decreases as it spreads
D the oil film formed is not less than one molecule thick
E the oil film area is 50 cm^2

18. A jar X of brown bromine gas is placed over a jar Y of air which is less dense than bromine gas and the cover slips of the jars are taken away. After a time, the jars X and Y have the same colour. This is because

A bromine molecules are heavier than air molecules
B bromine and air molecules move randomly
C air molecules are faster than bromine molecules
D bromine molecules are faster than air molecules
E air has a smaller density than bromine gas

Scanned with CamScanner

19. When illuminated by a light beam, a smoke particle in oxygen gas can be seen moving in different directions all the time when viewed in a microscope. This is because

A the light beam intensity is not constant
B the smoke particle is affected by gravity
C oxygen molecules hit the smoke particle in different directions
D the smoke particle has a high speed in oxygen
E oxygen molecules keep attracting the smoke particle

20. In a spring balance used for weighing, the manufacturer states that the balance must not be used for loads of more than 10 newtons. This is because

A the spring may break for loads greater than 10 newtons
B the extension of the spring may decrease after this load
C the extension may be doubled for greater loads
D the spring may stick for greater loads
E Hooke's law may not be obeyed for loads greater than 10 N

Scanned with CamScanner

HEAT



A scientist at work on research into solar heaters at the Solar Energy Laboratory of the Institute of Meteorological Physics at the University of Dakar, Senegal (*Almasy*)

Scanned with CamScanner

9. Temperature and Thermometers

Temperature

If a tiny amount of hot water is poured into a cup of warm water the water in the cup becomes slightly warmer. Heat passes from the hot to the warm water, although the volume of hot water is much less than that of the warm water. *Temperature* is a property of an object which decides which way heat will flow when it is placed in contact with another object. Since heat flowed from the hot to the warm water we say that the temperature of the hot water was higher than that of the warm water. Temperature must not be confused with heat itself. Heat is a transfer of energy due to temperature difference.

Temperature measurement has wide application. For example, the temperature of the body provides an indication of good health or illness. The refrigerator units in cargo ships must provide a suitable low temperature. Central heating supplies in houses must provide a suitably high temperature for rooms in winter.

Mercury-in-glass thermometer and fixed points

The sense of touch is not very reliable for estimating or measuring temperature. For example, warm water feels cool if your finger is transferred to it from hot water. The thermometer is a much more reliable instrument for measuring temperature. In the mercury-in-glass thermometer, which is a common one in everyday use in school laboratories, the volume of mercury changes when its temperature alters. All thermometers use a physical property which changes with temperature.

In making a thermometer, two constant temperatures or *fixed points* must first be marked on it. The 'lower fixed point' was first chosen as the melting point of ice. The higher or 'upper fixed point' was the temperature of steam in contact with water boiling at standard atmospheric pressure, 760 mm mercury (760 mmHg). Today, in modern thermometry, other fixed points are used.

Over two hundred years ago Celsius suggested that: (a) the temperature of the melting-point of ice should be given the number '0' and the temperature of steam at 760 mm mercury pressure the number '100'; (b) there should be 100 equal parts or *degrees* between the two fixed points. This is called the *centigrade* scale of temperature ('centi' indicating 100) or the Celsius scale. So on this scale:

Melting-point of ice $= 0^{\circ}C$
Steam temperature at 760 mmHg $= 100^{\circ}C$

In good health, the temperature of a human being is about 37°C.

To mark the lower fixed point, the thermometer A is placed upright in pure melting ice (Fig. 9.1a). When the mercury level is constant, a mark is made at that

Scanned with CamScanner

level on the glass. To obtain the upper fixed point a hypsometer B is used (Fig. 9.1b). The thermometer C is placed in the steam (*not* in the boiling water, because its temperature is affected by dissolved impurities in the water). A water manometer M is used to measure the pressure of the steam; if it is not 760 mmHg, an adjustment is made to the upper fixed point. Again, when the mercury level in C is constant a mark is made on the glass at that level.

Note that temperature is a number which depends on the scale used. It has no property of direction and is therefore a *scalar* quantity (see page 28).

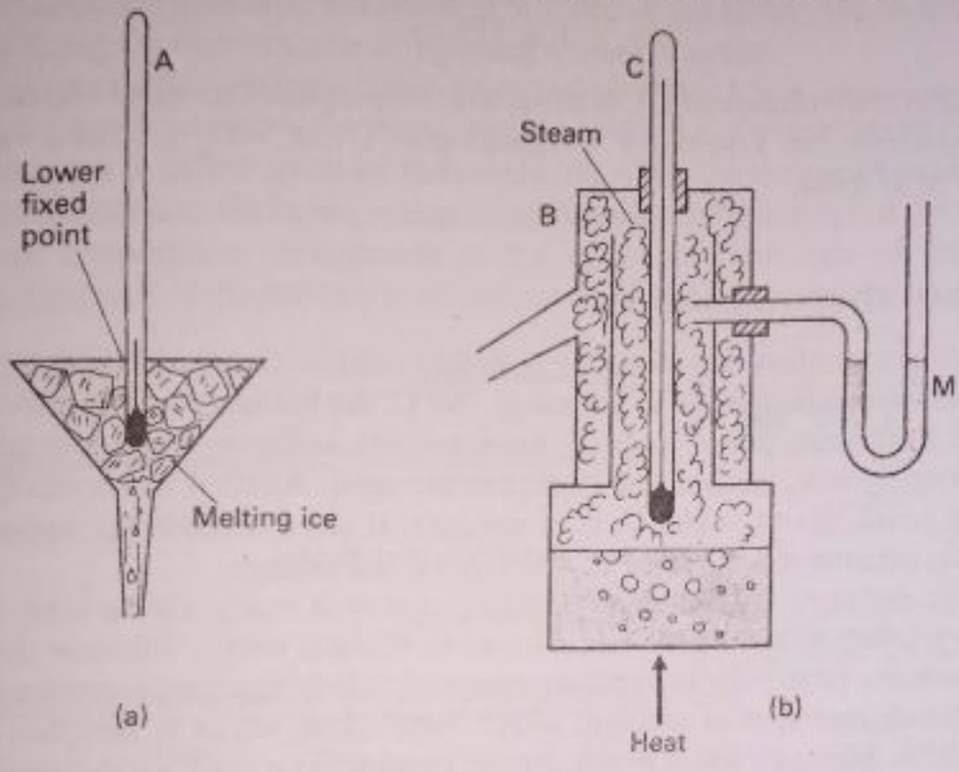


Fig. 9.1 Finding the lower and upper fixed points on a thermometer

Thermodynamic or Kelvin temperature scale

The temperature scale used in the SI system of units is the *absolute* or *thermodynamic scale*. On this scale, temperature is measured in 'kelvin' after Lord Kelvin who suggested the scale. The symbol is K (Fig. 9.2).

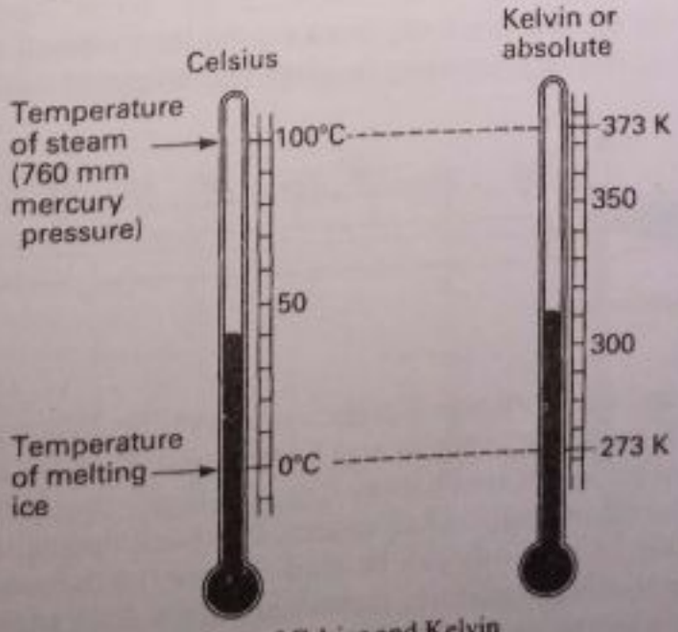


Fig. 9.2 Temperature scales: comparison of Celsius and Kelvin

Scanned with CamScanner

The lower fixed point is the 'absolute zero', which is given the value 0 K (see page 191). The upper fixed point is the temperature of the 'triple point of water', the temperature at which ice, water and water-vapour are all in equilibrium. This is given the value 273·16 K. Approximately

$$-273°C = 0\,K, \text{ or } 0°C = 273\,K$$

So we can see that the absolute temperature T in K corresponding to a temperature t in °C is given by

$$T = 273 + t$$

A temperature change of 1 °C is equal to a temperature change of 1 K. So a heat quantity which has a value of '40 joules per °C', or '40 J/°C', has a value of '40 J/K' in SI units.

Alcohol thermometer

The mercury-in-glass thermometer is widely used in chemical laboratories for temperature measurements in the range 360 °C, the boiling point of mercury, to −39 °C, its freezing point. In Arctic polar regions, however, where temperatures are extremely low, *alcohol* thermometers are used. Alcohol has a much lower freezing point, about −112 °C, than mercury. It has a much lower upper limit, however, because it boils at 78 °C, which is a disadvantage.

Unlike mercury, whose silvery surface makes it easily visible even in fine capillary tubes, alcohol must be coloured to make it visible. Mercury does not 'wet' glass (see page 162), and so responds quickly to falling temperature, whereas the concave meniscus of alcohol, which 'wets' glass, sticks to the glass as the liquid falls. Mercury has a much greater conductivity than alcohol, and so it reaches its new temperature quickly on contact with a hot body. Alcohol, however, expands about six times as much as mercury for the same temperature rise.

Clinical thermometer

The temperature of the human body is measured by a *clinical thermometer*. As shown in Fig. 9.3, this is a mercury-in-glass thermometer with a short range,

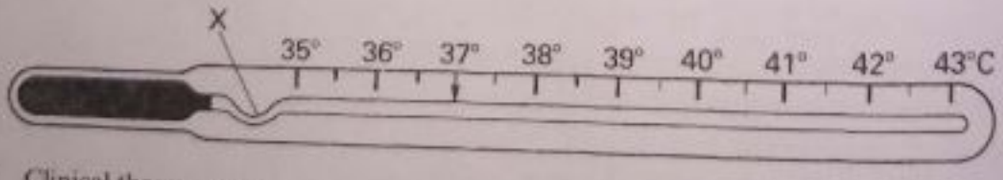


Fig. 9.3 Clinical thermometer

about 35–43 °C, and a *constriction* X in the capillary tube. When placed in contact with the body, the mercury expands and pushes past X owing to the large force exerted in expansion. When taken away from the body, the thread of mercury to the right of X remains in position as it cannot flow back through the constriction. So the temperature of the body can be read. To use the thermometer again it is given a jerk. This forces the mercury thread through X back to the mercury in the bulb. The average temperature of the human body in good health is about 37 °C.

Scanned with CamScanner

Maximum and minimum thermometers

It is often necessary to record the maximum and minimum temperatures during each day in studies of the weather.

A *maximum thermometer* uses mercury. This liquid has a *convex* surface or meniscus which pushes along a small piece of steel called an *index* when the temperature rises and the mercury expands (Fig. 9.4a). When the temperature falls, the index stays in position. At the end of the day, therefore, the maximum temperature corresponds to the lower or left side of the index. The index can be reset by tilting the thermometer or by using a small magnet.

A *minimum thermometer* uses the liquid alcohol, which has a *concave* meniscus. This time the index is below the meniscus (Fig. 9.4b). Thus when the temperature falls, the index is pulled down by the meniscus; when the temperature rises, the alcohol expands past the index, which stays in position. At the end of the day the minimum temperature corresponds to the upper or right side of the index (contrast the case of the maximum thermometer index).

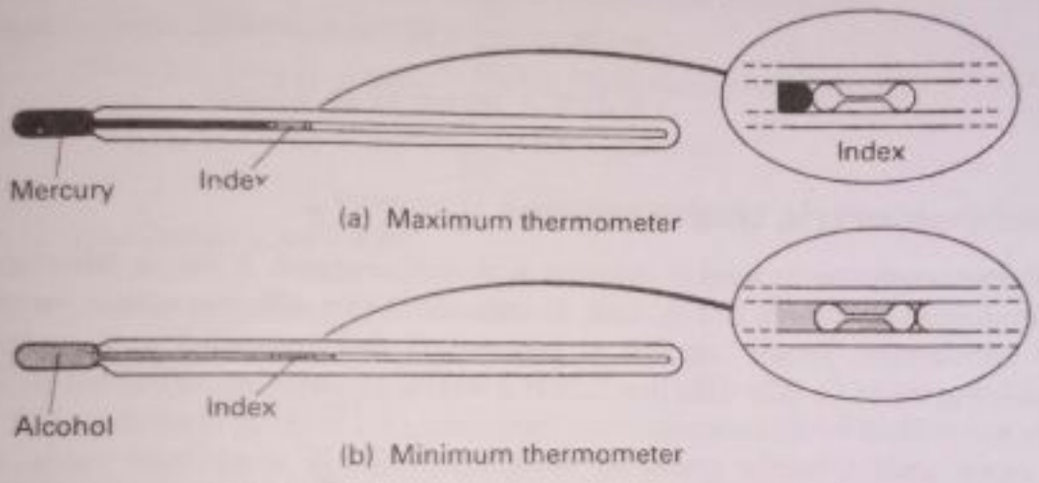


Fig. 9.4 Maximum and minimum thermometers

Principle of gas thermometer

Mercury-in-glass thermometers are not used for very accurate measurement of temperature. Apart from the disadvantage of a relatively small range of temperature–the freezing point of mercury is $-39°C$ and its boiling point is $360°C$–the glass also expands and its expansion is irregular.

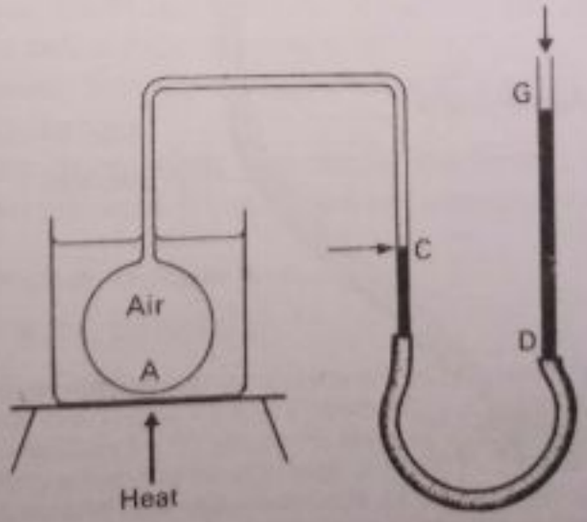


Fig. 9.5 Simple gas thermometer

Scanned with CamScanner

Gas thermometers are used for very accurate temperature measurement. A large volume change of gas occurs when its temperature is altered, so that the glass expansion is negligible. A simple form of constant volume gas thermometer consists of a large glass bulb A containing air and connected by a narrow (capillary) tube to a mercury manometer (Fig. 9.5). When A is warmed to a particular temperature, the right side GD of the manometer is moved up or down to bring the mercury level on the other side to a fixed mark C. In this way the volume of the gas is kept constant. The pressure of the gas is then read from the manometer.

As an illustration of calculating the temperature, suppose: the pressure was 600 mmHg at the ice point, 0°C; 820 mmHg at steam point, 100°C; and 655 mmHg in liquid at a temperature, t°C. Then

difference between fixed points $= 820 - 600$ mm $= 220$ mm
$= 100$°C rise above 0°C

pressure rise above 0°C $= 655 - 600$ mm $= 55$ mm
$= \text{rise of } \frac{55}{220} \times 100$°C above 0°C
$= 25$°C $=$ temperature of liquid

Thermoelectric thermometer

A *thermocouple* can be used to measure high temperatures. A simple laboratory thermocouple is shown in Fig. 9.6a. It consists of two different metals, copper and constantan, joined together at one end H and connected to a current-measuring meter G. When the junction H is warmed by immersing it in hot water,

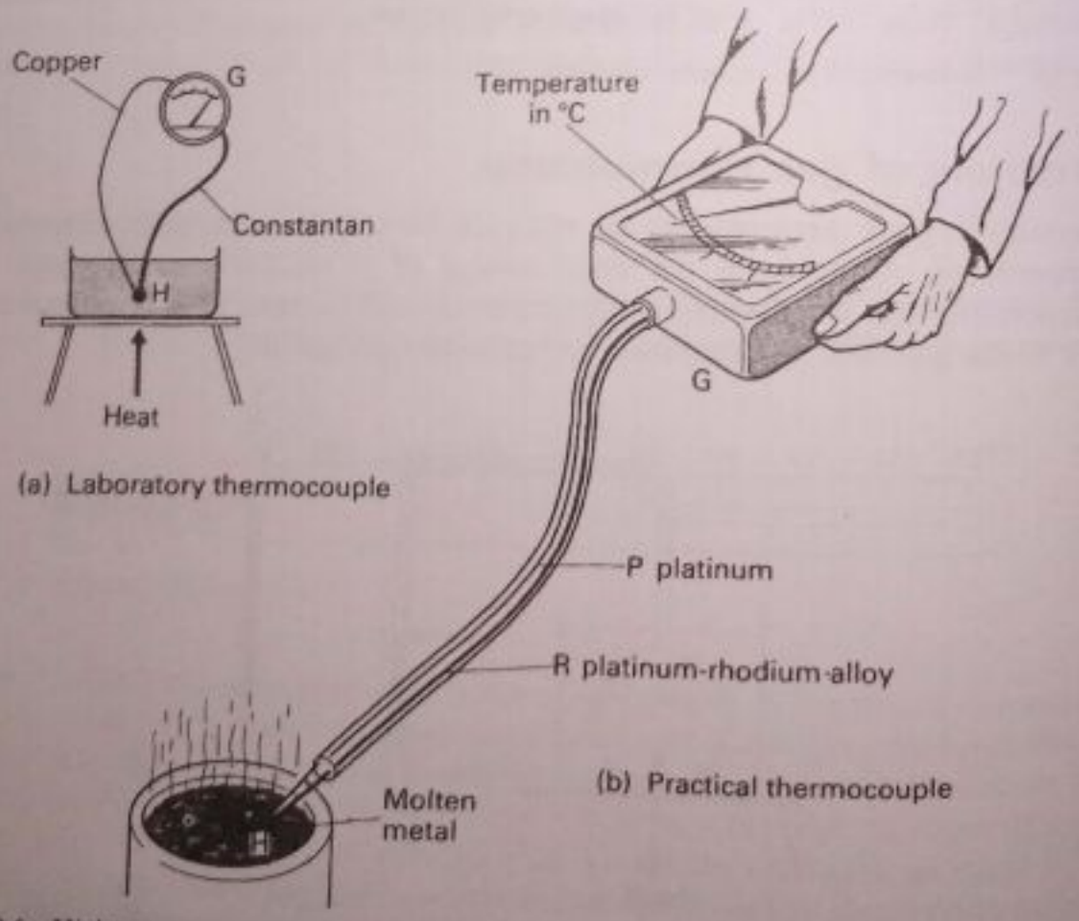


Fig. 9.6 High temperature measurement with a thermocouple

Scanned with CamScanner

for example, an electric current flows in G. The size of the current depends on the temperature of H, so the scale of G can be calibrated to read off the temperature.

Thermoelectric thermometers, which are used in industry for measuring high temperatures, use thermocouples made from metals of high melting points, for example, platinum (P) and a platinum–rhodium alloy (R) (Fig. 9.6b). The junction H is placed in molten metal, for example, when the temperature is required, and this is read from G.

The thermoelectric thermometer is particularly useful for rapidly changing temperatures as there is only a small mass of metal at the measuring point to warm up.

Example on the thermometer

In an uncalibrated mercury thermometer, the length of the mercury thread above the bulb, in the capillary, is 18 mm at a temperature of melting ice and 138 mm at a temperature of steam at 760 mmHg pressure. When placed in a hot liquid the length of the mercury thread is 118 mm. Calculate the temperature of the liquid.

Length of thread at 0°C = 18 mm, at 100°C = 138 mm

So $138 - 18 = 120\,\text{mm}$ = mercury rise for 100°C temperature increase

Thus $118 - 18 = 100\,\text{mm}$ = rise for $\frac{100}{120} \times 100°\text{C}$ increase

= rise for 83°C increase (approx.)

So liquid temperature = 83°C

Summary

1. The lower fixed point of a thermometer is the temperature of melting ice, and the upper fixed point is the temperature of steam above boiling water at 760 mmHg pressure.
2. Temperature is a scalar quality.

   Celsius temperature scale: lower fixed point = 0°C, upper fixed point = 100°C

   Kelvin (thermodynamic) scale (an SI unit): absolute zero = 0 K.

   $T(\text{K}) = 273 + t(0°\text{C})$

   Change of 1°C = change of 1 K.
3. The clinical thermometer has a short range such as 35°–43°C and a constriction in the capillary tube near the bulb.
4. Maximum and minimum thermometers use mercury and alcohol respectively, with steel pointers to indicate the temperatures.
5. The gas thermometer uses the volume change of a gas at constant pressure when its temperature changes.
6. The thermoelectric thermometer uses the current flowing in a circuit when a junction of two metals is heated. It has a quick response to changing temperature.

Exercise 9 (Answers, page 641)

1. The temperature of the melting point of ice and that of steam above water boiling at 760 mmHg pressure are marked as 40 and 120 respectively on a certain thermometer. (b) What is (a) the temperature in °C when the reading on this thermometer is 60, the thermometer reading when the temperature is 60°C?
2. State the advantage and one disadvantage of using alcohol and mercury for thermometers. Give three reasons why water is unsuitable as a thermometric liquid.

Scanned with CamScanner

3. In checking the lower fixed point of a thermometer, *melting ice* is used; in checking the upper fixed point, a *water manometer* is used. Explain the reasons for the words in italics.

4. What is *one* advantage of using a constant volume gas thermometer for temperature measurement? In such a thermometer, the gas pressure at 0°C is 200 mmHg and at 100°C it is 270 mmHg. What is the room temperature if this produces a pressure of 214 mmHg?

5. Fig. 9.7 represents an unfinished drawing of a *hypsometer* used for checking the upper fixed point of a mercury thermometer. Complete the diagram as it should have been drawn, together with the thermometer fixed correctly in position. (L)

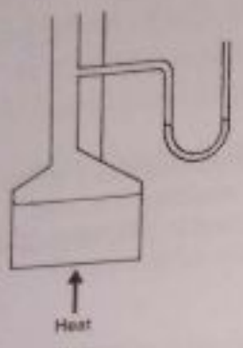


Fig. 9.7

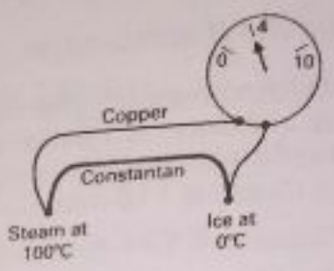


Fig. 9.8

6. Describe how you would check the accuracy of the fixed points on a mercury-in-glass thermometer.

Briefly explain why the reading of a mercury thermometer with its bulb enclosed in damp muslin may differ from that of a similarly exposed but dry thermometer.

State *two* ways in which the design of a thermometer may be modified so as to increase its sensitivity. (N)

7. State four properties of a liquid which make it suitable for a thermometer to be used in a school laboratory and explain why each property is desirable.

Draw a fully labelled diagram of a thermometer suitable for recording the highest and lowest temperatures recorded in a period of time. (C)

8. Fig. 9.8 shows the junctions of a thermocouple thermometer connected to a millivoltmeter which can read up to 10 mV. When one junction is in ice at 0°C and the other in steam at 100°C, the millivoltmeter reads 4·0 mV.

(a) What is the maximum temperature which this arrangement can measure (the cold junction remaining at 0°C)?

(b) If the hot junction in Fig. 9.8 is removed from the steam, the millivoltmeter reading *very quickly* falls to *1·0 mV*. Give an explanation of this with reference to the items printed in italics. (C)

9. Explain as far as you can the difference between heat and temperature, and state two properties of matter which would enable it to be used for measuring temperatures.

Give the reasons for the following features of a clinical thermometer: (a) the bulb is small; (b) it is a long cylinder and not spherical; (c) the bulb is made of thin glass; (d) the capillary is of very small cross-sectional area; (e) there is a constriction in the capillary. (O and C)

10. (a) Give three reasons why mercury is a suitable liquid for use in a thermometer.

(b) State one way in which the design of a thermometer may be modified so as to increase its sensitivity. (N)

11. Draw a labelled diagram of a minimum thermometer. Name the liquid, indicate the temperature range and show where the minimum temperature is read. (N)

Scanned with CamScanner

10. Thermal Expansion of Solids and Liquids

Expansion of solids

Temperature changes affect metals and other solids. Everyday experience shows that most metals expand when warmed and contract when cooled. For example, the metals used for telephone wires sag more in summer than in winter, showing they expand with temperature rise.

The expansion of a solid with temperature rise is so small that special apparatus is needed to demonstrate it. Fig. 10.1 shows a simple laboratory demonstration of linear expansion. A long metal pipe B, with rubber tubing at one end, is gripped in a vice at that end. The other end passes over a steel knitting needle K. A long drinking straw P, or a long thin piece of balsa wood, is attached to the end of K as a pointer. On blowing down B, the pointer P is seen to move over a graduated scale S on a card fixed to the bench. Hot breath has warmed the tube B which has then expanded slightly. The expansion turns the needle and so P moves.

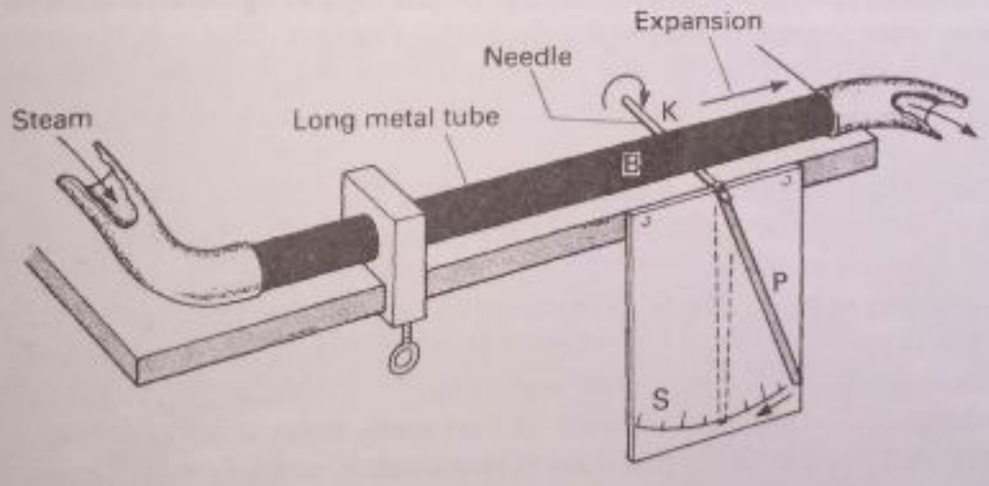


Fig. 10.1 Linear expansion demonstration

When steam is passed through tube B, the pointer P turns considerably. On allowing B to cool, or pouring cold water over it, P returns to its original reading, showing the metal has contracted or shortened.

Force due to expansion

When metals are prevented from expanding or contracting, very large forces are exerted.

Scanned with CamScanner

Experiment

This is demonstrated by a 'bar-breaker' experiment (Fig. 10.2). A thick steel bar B has a cast-iron pin P at one end and is tightened by a large nut K against a metal frame. The bar is heated by a bunsen burner for a time. As B expands, tighten the nut continually so that the pin P is still tightly held against the frame at the other end. The flame is then taken away and B cools. After a while the cast-iron pin breaks because the contracting bar is trying to pull it through the small gap in the frame. So a large force is exerted when a metal is prevented from contracting. A large force is also exerted when a metal is prevented from expanding.

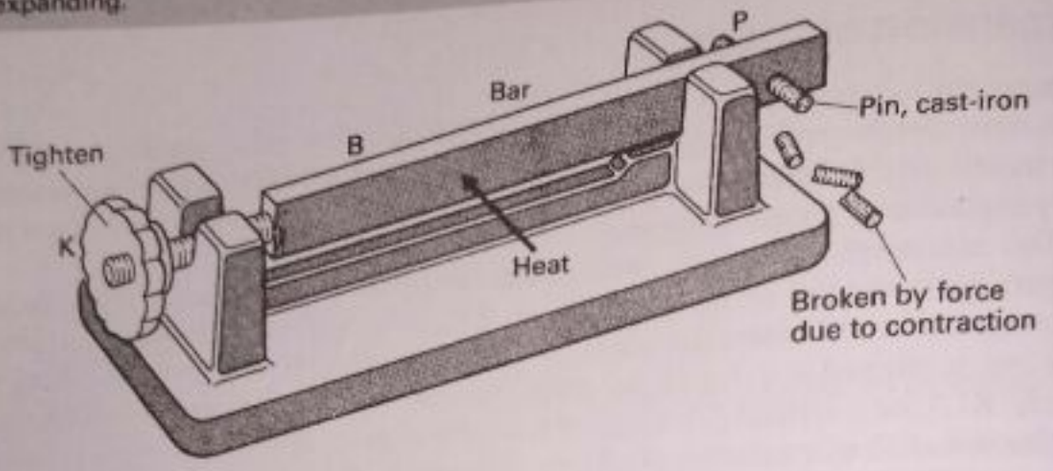


Fig. 10.2 Force of expansion and contraction

Some consequences of expansion

In hot weather, bridges fixed at both ends would expand against the fixtures and the large force exerted would make the bridge fracture. This would eventually lead to the break-up of the bridge. To prevent it, the bridge rests on rollers at one or both ends so that the metal is free to expand and contract in summer and winter.

A thick glass bottle or drinking-glass is likely to crack if hot water is poured into it. Glass is a poor conductor of heat and so the outer, colder glass does not expand as much as the hot glass on the inside. To prevent the cracking, only a small quantity of hot water at a time should be poured in. The expansion is then small and there is time for the glass outside to become warmer and expand too.

Railway tracks have gaps to make allowance for expansion due to temperature rise, otherwise the rails would buckle. In past years, many miles of railway tracks have been replaced with long sections of *continuously welded track*. The old track had fish-plate junctions at fairly short intervals. A 'clickety-click' effect was then felt as the train passed over the jointed parts. The newer and safer track has long rail lengths joined by rigid welds. These are flat with the rail surfaces to give a much smoother train ride (Fig. 10.3).

The expansion of metal which can be expected from a long length of track is correspondingly great. An expansion of up to about 30 cm may be expected for a length of about 1 kilometre. Special expansion joints are therefore used with these tracks (Fig. 10.4). These joints are made by tapering the ends of the adjacent rail lengths, as shown, and placing them in line. The actual gap w is small and may be only about a centimetre. But the total expansion d allowed in the direction of the rails is much larger than w, because of the gradually tapering long slope. If a conventional fish-plate were used, this could allow only for expansions up to

Scanned with CamScanner

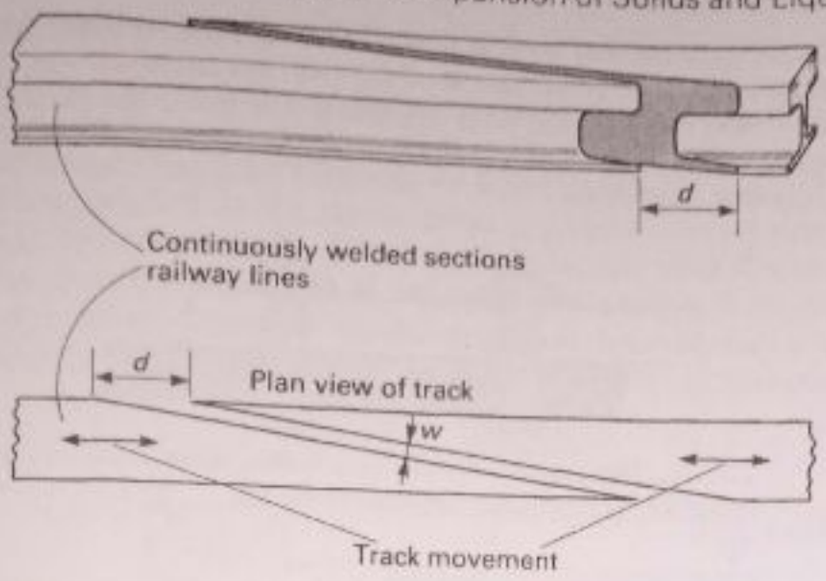


Fig. 10.3

Fig. 10.4 Continuously welded track

about 2 cm. Modern continuously-welded track could be very long indeed without needing expansion joints since the rail is pulled and put into tension when installed.

Applications of expansion

Rivets

Steel plates such as those used in shipbuilding or in large boilers are usually riveted together using red-hot rivets. Holes are made in the overlapping plates, A



Plate 10A Measuring the diameter of a large cast iron pin after immersing it in a tank of liquid nitrogen, boiling point −196 °C. The pin is shrunk in this way so it fits very tightly into its metal housing. Note the ice formed by condensation of water vapour in the air. (*British Oxygen Company Ltd*)

Scanned with CamScanner

and C (Fig. 10.5), a red-hot rivet is pushed through and its head H held tightly against one plate A. The other end B of the rivet is hammered tight against the other plate C. On cooling, the rivet contracts and holds the plates even more tightly together. This provides a good seal against the sea for ship plates and against steam in large boilers.

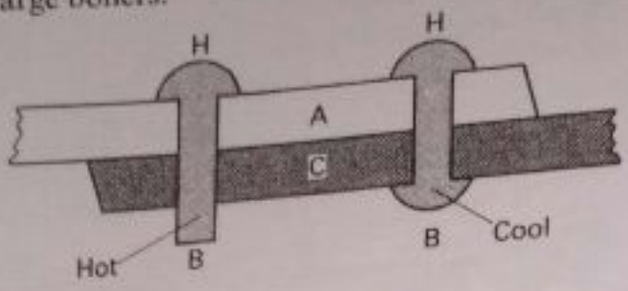


Fig. 10.5 Rivets

In contrast to riveting, in some engine casings nuts and bolts are secured very tightly by first cooling the bolt in liquid air, until the nut just fits on. When the bolt reaches air temperature, the large force due to expansion of the bolt makes a very tight fit with the nut.

Bimetal strips

Bimetal strips are widely used in many different appliances.

A bimetal strip can be made by placing two strips of different metals (e.g. brass and iron) side by side and welding them together along their entire length (Fig. 10.6a).

If the bimetal strip is heated, it curves as shown (Fig. 10.6b). This shows that *brass expands more than iron* for the same temperature rise. The bimetal strip straightens when it cools to room temperature. If it were cooled below room temperature the brass would contract more than the iron. So the strip would curve with the iron on the outside of the curve (Fig. 10.6c).

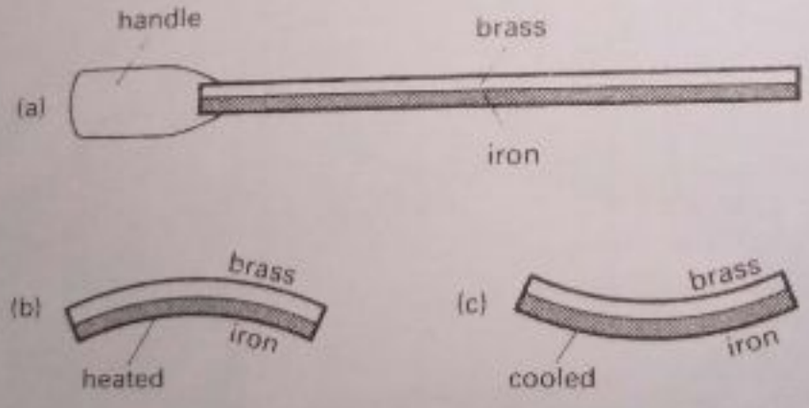


Fig. 10.6 Bimetal strips

A straight length of thin bimetal, made of brass and invar, is very sensitive to temperature changes. It curves when one end is warmed by a match. Invar is a steel–nickel alloy which expands very little when warmed. It expands only about one-millionth of its length per °C rise in temperature. So the bimetal strip curves with brass on the outside when heated because the brass expands much more. If the strip is cooled, it curves the other way because the brass also contracts much more than the invar.

Scanned with CamScanner

Electric thermostats

Fig. 10.7 shows the principle of an electric thermostat of the type used in electric blankets for keeping a steady temperature. The blanket contains a length of insulated resistance wire sewn between two layers of blanket material. When the blanket is first switched on the current flows through a bimetal strip to a fixed metal block through contacts, X. As the temperature rises the bimetal strip curves away from the block, and at one stage the contacts X are broken. The current is then cut off. When the bimetal strip cools down contact is again made at X. Current then flows once more to warm up the blanket as shown in Fig. 10.7.

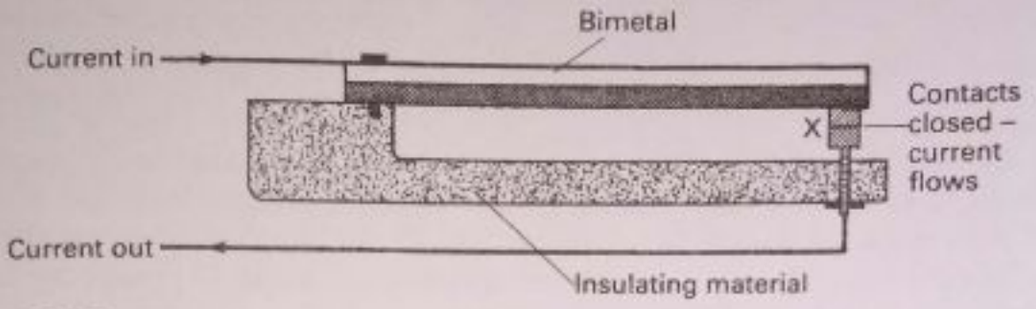


Fig. 10.7 Temperature control by a bimetal strip

Gas thermostats

Bimetals are used in making a thermostat for gas cookers, so that the temperature can be controlled.

Fig. 10.8 shows the principle of a gas thermostat. A brass tube Y, projecting into the oven, encloses an invar rod X attached to it at one end. A valve is attached at the other end of X and this allows gas from the mains to flow through to the burners, as shown. When the oven temperature rises, the brass tube Y expands and moves to the left, but the invar rod X is practically unchanged. So the valve moves to the left and partly closes. This cuts down the gas flow.

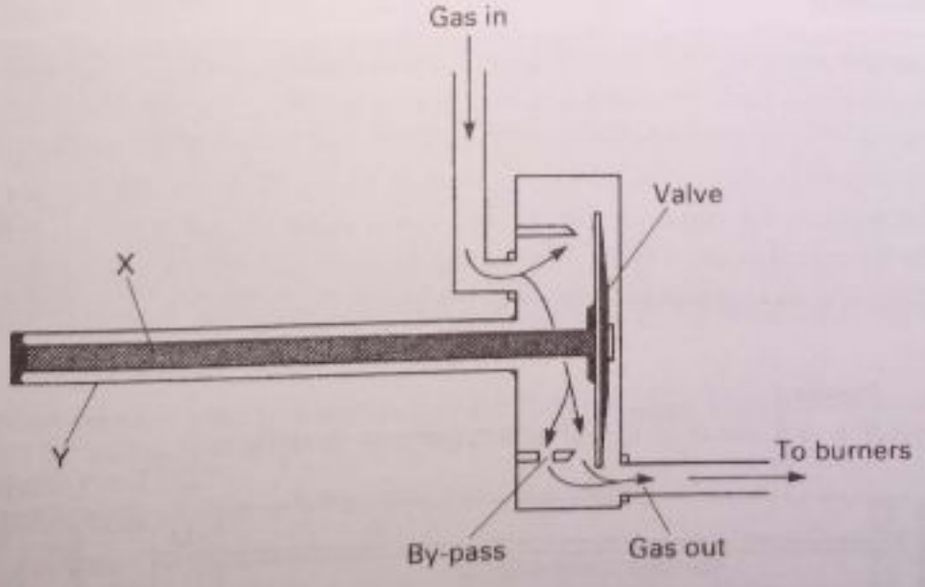


Fig. 10.8 Gas thermostat with a bimetal

If the oven temperature decreases, the brass tube contracts a little and the valve moves to the right. The gas flow is then increased. The oven temperature is set by turning a knob on the regulator. This adjusts the valve opening so that the gas supply is reduced at the required temperature.

Scanned with CamScanner

Linear expansivity

To make allowances in designing engines for the expansion of metals due to temperature change, we need to know the values of the *linear expansivity* of metals.

The linear expansivity α of a solid is defined as *the increase in length per unit length of the solid when its temperature rises by one degree.* So if 100·00 cm of steel increases by 0·11 cm when its temperature rises by 90°C

$$\text{linear expansivity} = \frac{0{\cdot}11\text{ cm}}{100{\cdot}0\text{ cm} \times 90^\circ\text{C}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$= 0{\cdot}000012\text{ per }^\circ\text{C}$$

$$= 12 \times 10^{-6}/^\circ\text{C}$$

Observe carefully that since the unit 'cm' appears in both the numerator and denominator in (1), it cancels out. So the unit of linear expansivity is 'per °C', or in SI units 'per K', since 1°C = 1 K (page 170). The linear expansivity α is always an average or mean figure for a wide range of temperature. In this range one may state that the mean value is given by

$$\alpha = \frac{\text{increase in length}}{\text{original length} \times \text{temperature rise}}$$

Measuring the linear expansivity of metal

The expansion of a solid when warmed is usually so small that a special gauge or method is needed to measure it accurately. On the other hand, little percentage error will be made if a ruler is used to measure the original length of a rod provided it is long.

Experiment

The apparatus is set up as shown in Fig. 10.9. The length of the rod R of metal used is first measured with a metre rule and it is then placed in the jacket J with one end fixed at A. A thermometer hangs from the rod by two short wire loops. The micrometer screw gauge G is turned until the end of the screw just touches the end O of the rod, and the gauge reading is noted. The micrometer screw is now turned back enough to make sure that the end of the rod can expand freely. The reading on the thermometer is noted and steam is now passed through the jacket for several minutes.

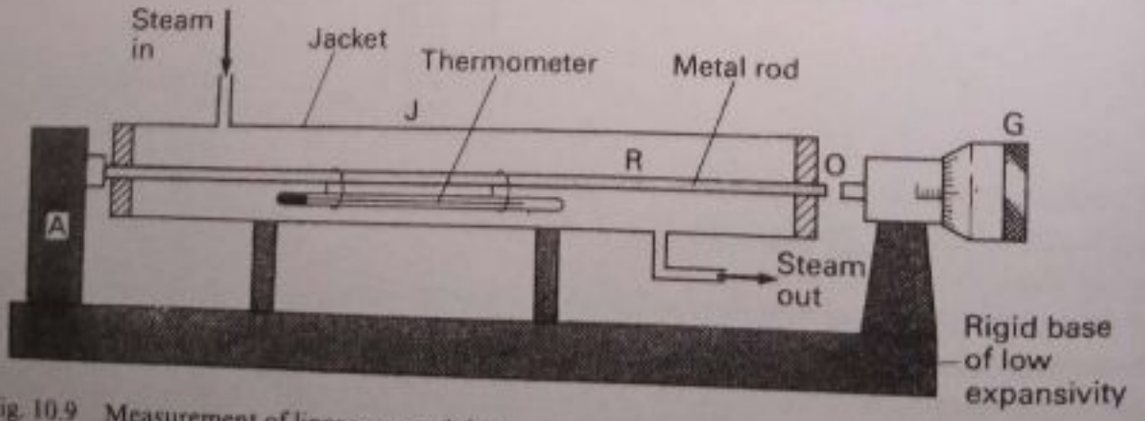


Fig. 10.9 Measurement of linear expansivity

Scanned with CamScanner

The micrometer is then screwed up to the rod, the gauge reading is noted and the screw is then turned back again to allow for any further expansion of the rod on passing more steam through. This is repeated until a constant reading is obtained. The reading is then recorded and the temperature of the rod is noted from the thermometer.

Calculation: If l is the original length of the rod, e is the expansion due to a temperature rise from t_1 °C to t_2 °C, then

$$\text{linear expansivity} = \frac{e}{l(t_2 - t_1)}$$

Results: Suppose the following measurements are obtained.

Original gauge reading = 1·365 cm. Final gauge reading = 1·515 cm

Original length of rod = 93·3 cm

Original temperature = 16°C. Final temperature = 99°C

Then

$$\text{linear expansivity} = \frac{\text{increase in length}}{\text{original length} \times \text{temperature rise}}$$

$$= \frac{1{\cdot}515 - 1{\cdot}365}{93{\cdot}3 \times (99 - 16)} = \frac{0{\cdot}15}{93{\cdot}3 \times 83} = 0{\cdot}000\,19/^\circ\text{C}$$

$$= 19 \times 10^{-6}/^\circ\text{C or } 19 \times 10^{-6}/\text{K}$$

The mean linear expansivity for brass is 18×10^{-6}/K, for iron 11×10^{-6}/K, and for Pyrex 3×10^{-6}/K

Formulae

Since α, the linear expansivity, is the increase in length of 1 m for 1°C temperature rise, it follows that the increase (or decrease) in length of l m for t°C temperature change is given, by proportion, by

$$\text{change in length} = \alpha l t$$

$\therefore$ the new or increased length, $l_t = l + \alpha l t = l(1 + \alpha t)$

The same formulae apply when the lengths are measured in other units, such as cm or mm.

Examples on linear expansion

1. A telegraph wire has a length of 30·00 m and is made of metal of linear expansivity 0·000 02 (20×10^{-6})/K. Find the change in length from a hot day, temperature 30°C, to a very cold day, −5°C.

Decrease in length of 1 m for 1°C change = 0·000 02 m

$\therefore$ decrease of 30 m for 35°C change

$$= 0{\cdot}0002 \times 30 \times 35 \text{ m} = 0{\cdot}000\,02 \times 30 \times 35 \times 100 \text{ cm}$$

$$= 2{\cdot}1 \text{ cm}$$

2. Part of a steel tape used by a surveyor is 20·00 m at 12°C. What is the overall length measured by using this part of the tape one hundred times on a warmer day corresponding to 22°C?
[Linear expansivity of steel = 0·000 011 (11×10^{-6})/K]

Using the formula

$$\text{increase in length} = \alpha l t = 0{\cdot}000\,011 \times 20 \times (22 - 12)$$

$$= 0{\cdot}000\,011 \times 200 = 0{\cdot}0022 \text{ m}$$

$\therefore$ new length of tape = 20·0022 m

Hence, if used 100 times, overall length measured = 2000·22 m.

(*Note*. Surveyors' measuring tapes, calibrated accurately by the maker at a particular temperature, are thus subject to error when used at a different temperature.)

Scanned with CamScanner

3. A steel tyre diameter is 150·0 cm at 10°C and is to be fitted on to a train wheel of diameter 151·0 cm. To what temperature must the tyre be heated to just fit the wheel? (Linear expansivity of steel $= 11 \times 10^{-6}$/K.)

Increase in tyre diameter required $= 151 - 150 = 1$ cm
Using the formula for increase in length, αlt,

$$11 \times 10^{-6} \times 150 \times t = 1$$

$$\therefore t = \frac{1}{11 \times 10^{-6} \times 150} = \frac{10^6}{11 \times 150} = 606°C$$

$$\therefore \text{Temperature required} = 606 + 10°C = 616°C$$

Expansion of liquids

We now investigate the effect of temperature rise on the expansion of liquids. The measurement of the expansion is always complicated by the expansion of the container itself, which depends on the material used for making the container.

Experiment

The expansion of liquids can be compared by filling three large test-tubes (or small flasks) of equal volume with say alcohol A, paraffin oil B, and water C (Fig. 10.10). Narrow tubes of equal diameter are pushed through rubber bungs on the tubes until the levels of the three liquids in the tubes are the same. Their positions or initial levels are marked by loops of thread round the tubes.

The test-tubes are kept together by a tight rubber band and are then placed inside a bath of hot water. This is sufficiently deep to cover the test-tubes fully. At first the levels of the liquids are seen to *fall* slightly below the loops of thread. This is because glass is a poor conductor and so the glass expands before the heat reaches the liquids. Soon, however, the liquid levels rise above the thread. The alcohol is seen to expand more than the paraffin oil, which expands more than the water.

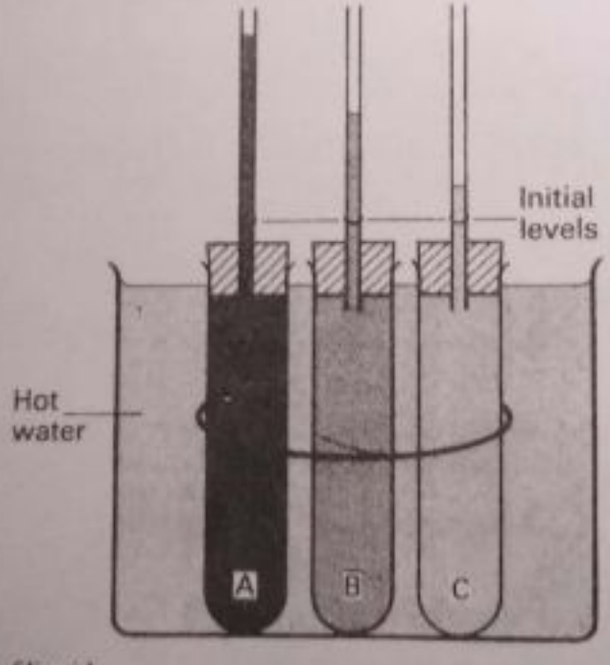


Fig. 10.10 Expansion of liquids

Alcohol and mercury are both used in common liquid-in-glass thermometers. Experiment shows that alcohol expands about six times as much as an equal volume of mercury for the same temperature rise.

Scanned with CamScanner

The experiment also shows that liquids expand much more than glass of equal volume when heated through the same temperature rise. Generally, liquids expand about ten times as much as solids in these conditions.

Cubic expansivity

The *apparent cubic expansivity* of a liquid, γ, is defined as the *increase in volume per unit volume per unit temperature rise* relative to that of the material of the vessel. If $\gamma = 0{\cdot}0003/\text{K}$ for a particular liquid, this means that for 1 K temperature rise, 1 cm^3 of the liquid expands by 0·0003 cm^3 or 1 m^3 expands by 0·0003 m^3

Suppose mercury in a glass thermometer has an apparent cubic expansivity of 0·000 15/K. If the volume of mercury in the bulb up to the 0°C graduation is 0·5 cm^3, then the volume in the stem from the 0°C to the 100°C graduations is the increase in volume of 0·5 cm^3 when its temperature rises 100 K. Now 1 cm^3 of mercury for 1 K temperature rise expands 0·000 15 cm^3. So the mercury volume in the stem from 0°C to 100°C

$$= 0{\cdot}000\,15 \times 0{\cdot}5 \times 100 = 0{\cdot}0075\,\text{cm}^3$$

Density of water · Hope's experiment

An experiment carried out by Hope about 200 years ago showed that *the density of water varies with temperature*. He placed some water inside a tall vessel X and surrounded the middle with a freezing mixture, ice and salt (Fig. 10.11a). A thermometer A measures the water temperature near the surface and a thermometer B measures the temperature near the bottom.

As shown in Fig. 10.11b, the reading on the lower thermometer B begins to fall at first, while the reading on the upper thermometer A remains constant at say 14°C. After some time, however, the temperature on B becomes constant at about 4°C, and the temperature on A begins to fall. After some time the temperature on A reaches 0°C and ice then begins to form on the water surface.

Hope's experiment shows that the *maximum density of water is at about 4°C*, since this is the temperature at the bottom of the water. When the temperature falls below 4°C the density *decreases*, and so this colder water rises to the surface. When the water at the surface reaches 0°C, ice begins to form and thicken downwards. Substances usually contract when they become colder.

This exceptional (anomalous) expansion of water below 4°C is because water molecules bond together differently above and below 4°C.

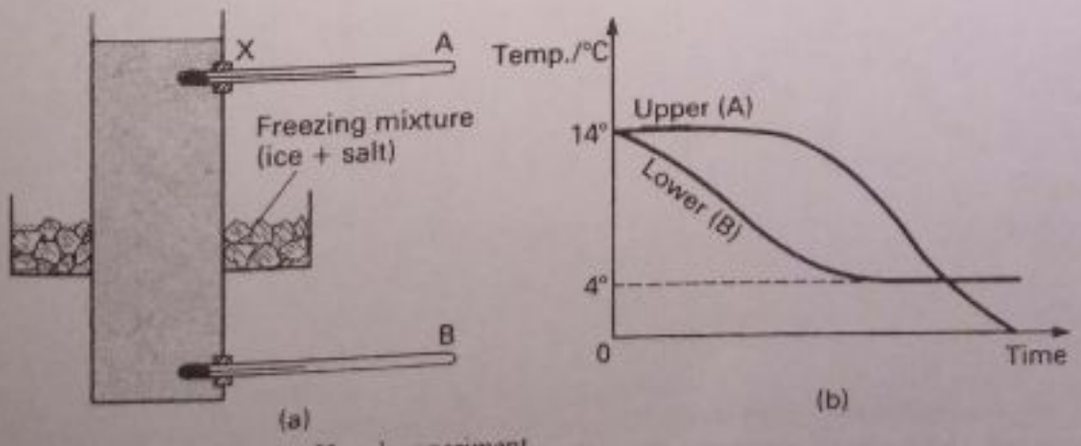


Fig. 10.11 Density of water: Hope's experiment

Scanned with CamScanner

Volumes of ice, water and steam

Ice (a solid) expands slightly when warmed from $-5°C$ say to $0°C$; at $0°C$ it forms water and contracts (Fig. 10.12a). When the water is warmed it *contracts*

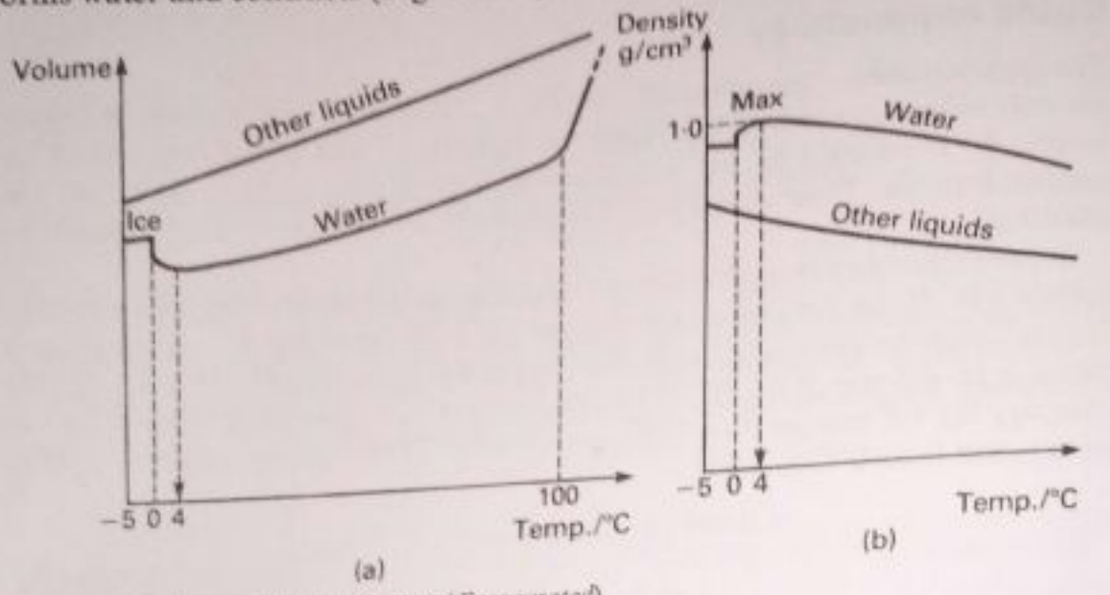


Fig. 10.12 Volume and density changes (*Exaggerated*)

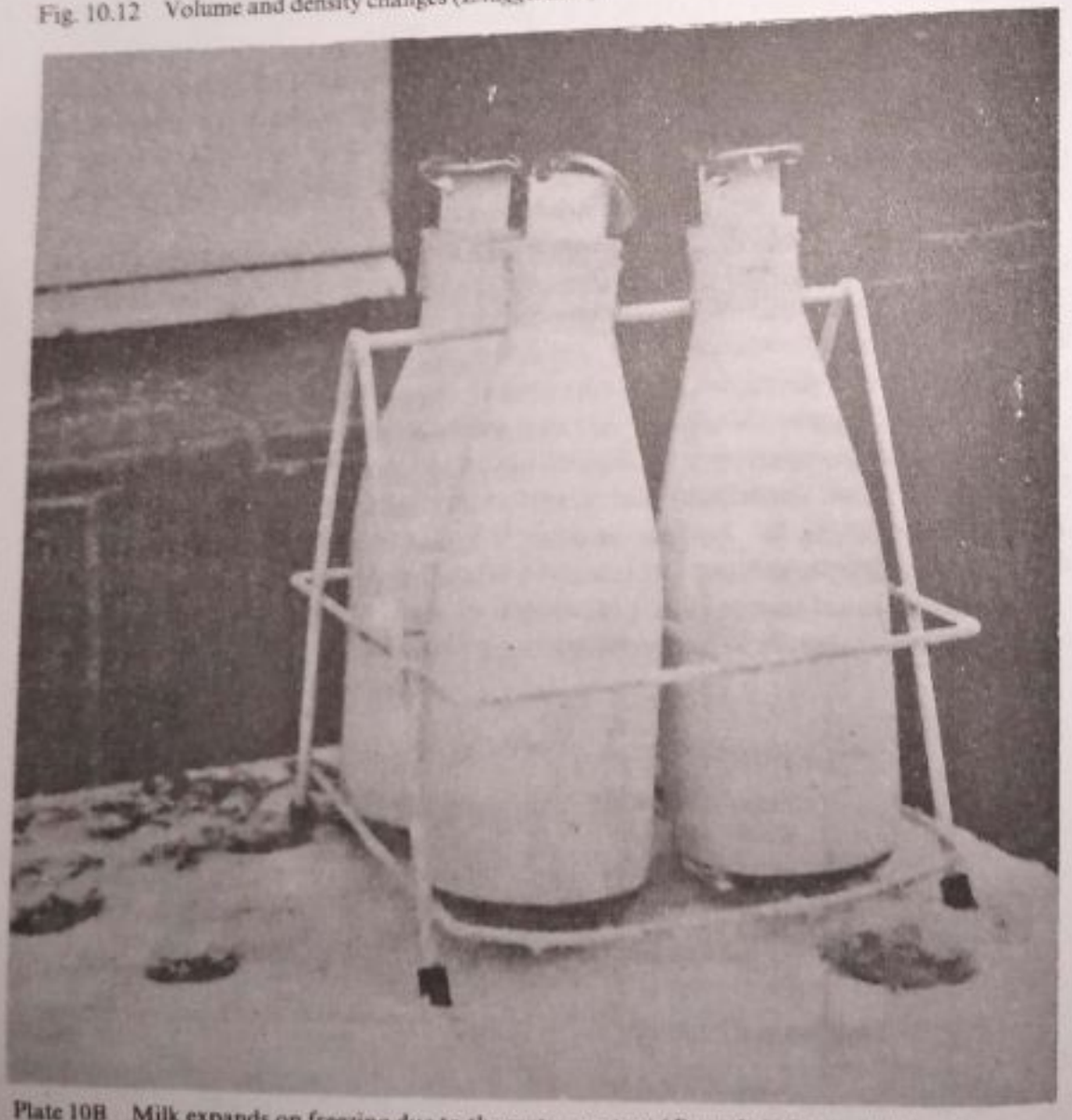


Plate 10B Milk expands on freezing due to the water content (*Barnaby's Picture Library*)

Scanned with CamScanner

from 0°C to 4°C, which is exceptional, but from 4°C to 100°C it expands, behaving now like most liquids. When it changes into steam at 100°C the volume increases by 1600 times. Water is thus exceptional or 'anomalous' in the range 0°C to 4°C. Since density is mass/volume, and the mass is unaltered by warming, *water has a maximum density at about 4°C*, as Hope found.

Fig. 10.12b shows the density–temperature variation of water compared with most liquids. Most liquids increase in volume as their temperature rises from 0°C and so their density diminishes continuously.

In Arctic regions the water below the ice at the top is at 0°C but at the bottom it is at 4°C. So marine and plant life survives in severe winters at the bottom of a lake frozen at the top.

Examples on thermal expansion

1. Explain why a platinum wire can be sealed through glass and maintain this seal over a wide temperature range, but a copper wire cannot.
A clinical thermometer needs to be an accurate maximum thermometer. Explain briefly how these two basic requirements are achieved. (L)

Platinum has about the same linear expansivity as glass over a wide temperature range. So when the glass is heated, the glass and platinum expand by the same amount and the seal is maintained. Similarly, the seal is maintained when the glass is cooled because the glass and platinum contract by the same amount.

Copper has a much greater linear expansivity than glass. So if the copper is sealed into the glass at a high temperature, it contracts more than the glass when both are cooled and the seal no longer holds.

A clinical thermometer is a fairly accurate thermometer because the liquid used is mercury, which expands uniformly and is a good conductor of heat. The volume of mercury in the bulb is small so the mercury quickly reaches the temperature to be measured. The stem has a narrow bore and is capable of measuring small changes in temperature, which makes it more sensitive.

This thermometer is a maximum thermometer because it has a constriction (narrowing of the bore) just above the bulb. When the thermometer is used, the force of expansion pushes the mercury past the constriction until it reaches the maximum temperature. On removal, the mercury cannot pass the constriction (unless it is shaken). It therefore remains, indicating the maximum temperature. To re-set the thermometer the mercury must be shaken vigorously to make it pass the constriction.

2. Explain why the cooling unit (freezer) inside a refrigerator is placed near the top but an electric immersion heater in a water tank should be near the bottom. (L)

The air in the refrigerator cabinet in contact with the freezer at the top is colder than the air below. It is therefore denser and sinks to the bottom. The less cold air rises to the top and this in turn becomes colder and sinks. The air in the refrigerator cabinet is thus kept cold with the help of the freezer.

When the immersion heater is placed at the bottom of the water tank, the water here becomes warmer, and therefore less dense, than the water above it. The warm water rises and the colder water sinks to the bottom to be heated. In this way all the water in the tank becomes hot in a short time.

Summary

1. The bimetal strip has two metals such as brass and invar which have considerably different expansivities. When warmed, the strip curves; the metal of greater expansivity is on the outside of the curve.
2. The linear expansivity (α) of a solid is the increase in length per unit length for 1°C rise in temperature. Unit: /K.

Scanned with CamScanner

3. The increase in length of a solid $= \alpha l t$, where l is the original length, t is the temperature rise and α is the linear expansivity. The new length $l_t = l(1 + \alpha t)$.
4. The cubic expansivity of a liquid is about ten times that of a metal. When heated, the densities of most liquids decrease because their volumes increase and their masses remain constant. Water is an exception.
5. Water contracts when warmed from 0°C to 4°C and then expands. The maximum density of water is thus at about 4°C.

Exercise 10 (Answers, page 641)

Solids

1. Fig. 10.13 shows a compound strip, AB, of brass and iron which is clamped at A. Draw the appearance of the strip after it has been heated to a high temperature. Account for its shape. (L)

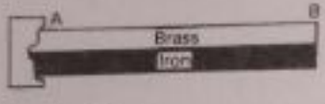


Fig. 10.13

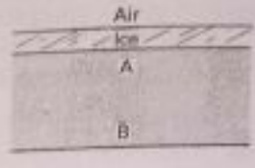


Fig. 10.14

2. Describe, with a diagram, the action of a practical device which works on the principle that different metals have different linear expansivities. (N)
3. A metal rod 50·00 cm long at 0 °C becomes 50·06 cm long at 100 °C. Find the linear expansivity of the metal. (N)
4. In an experiment to determine the linear expansivity of a metal: (a) why is it impracticable to heat the rod directly with a bunsen burner? (b) state in words the formula from which this expansivity is calculated. (N)
5. Explain the statement that the linear expansivity of steel is 0·000 012 /K.

The Forth bridge has a steel span of 1000 m between the main piers. Calculate in centimetres the change in length of the span when the temperature changes from −10° to 30 °C.

Describe a method of finding the linear expansivity of a metal. (N)

6. (a) Describe one experiment in each instance to show the expansion of a solid, a liquid and a gas on heating.

(b) A compound strip of brass and iron is straight at room temperature. Give a labelled diagram to show its appearance when it is cooled. Describe one use of such a strip.

A compound strip of brass and iron, 10 cm long at 20°C, is held horizontally with the iron uppermost. When heated from below with a bunsen the temperature of the brass is 820 °C and of the iron 770 °C. Calculate the difference in lengths of the iron and the brass. (Linear expansivity of brass is 0·000 019 and of iron 0·000 012 /K.) (L)

Liquids

7. The temperature of the air in contact with a thin layer of ice which has formed on the surface of a pond is just below 0°C (Fig. 10.14).

(a) State the probable temperatures of the water at the positions A and B.

(b) What fact concerning the density of water do these temperatures indicate?

(c) Why will it take a long time for the pond to freeze even if the air temperature falls well below 0°C? (L)

8. How does the volume of a given mass of water change: (a) as the water is cooled from 10° to 0°C; (b) when the water is frozen to ice at 0°C? Sketch a rough (not to scale) volume–temperature graph for the range between +10° and −10°C. (L)
9. Describe how you would check the accuracy of the fixed points on a mercury thermometer.

What special features in the construction of a thermometer make it sensitive to small temperature changes?

The bulb and stem of a thermometer contain 0·4 cm^3 of mercury up to the zero mark.

Scanned with CamScanner

If the bore has an area of cross-section of 0·03 mm², what is the length of a degree on the scale?

(Apparent cubic expansivity of mercury in glass = 0·00015/K.) (N)

10. Describe a simple experiment which shows that a liquid expands more than glass when its temperature is similarly increased.

A solid floats in a liquid so that it is totally immersed. Describe and explain what happens when the system is heated so that the temperatures of the solid and of the liquid are always equal.

An iron rod is 600 cm long at 0°C. It is mounted alongside a copper rod and both are always maintained at the same temperature. When they are heated to 100°C it is found that the difference in their lengths is the same as it was at 0°C. Find the length of the copper rod at 0°C.

(Linear expansivity of iron = 0·0000109/K. Linear expansivity of copper = 0·0000168/K) (C)

Multiple-Choice Questions

11. Which substance does NOT contract if cooled from 2°C to 0°C?

A iron B pure oil C pure water D brass E silver

12. Telegraph wires high above the ground must not break in summer or winter. If they are renewed in summer, they must be arranged to

A sag B be exactly horizontal C curve slightly upward
D be in groups of two E be wrapped round each other

13. A copper disc X has a circular hole Y in it. When the disc is heated

A the area of X increases, the area of Y decreases
B the area of X decreases, the area of Y increases
C X and Y have the same area as before
D the area of X and the area of Y both increase
E the area of X increases, the area of Y stays the same

14. Which statement in A to E is best correct?

In a clinical thermometer C and an ordinary mercury-in-glass thermometer T,

A C and T have the same temperature range
B C and T both have uniform capillary tubes throughout
C C and T both show that mercury expands more than glass
D the mercury in C falls after a temperature is taken but the mercury in T stays up
E C and T can both be used to measure the temperature of boiling water

15. A cube is made with equal lengths of thin copper *wire*. When the cube is uniformly heated from 20°C to 40°C

A it expands the same amount as a solid copper cube of the same size
B the shape changes to a rectangular one
C the lengths of wire are doubled
D the area of each side of the cube is doubled
E the cube will twist

Scanned with CamScanner

11. Thermal Expansion of Gases

The expansion of gases is used in engines of all descriptions, for example, in aeroplane and car engines.

In the jet engine, as we see later, hot gases are blown out with high velocity through nozzles at the rear. This gives the engine a considerable forward push or thrust. Sir Frank Whittle, the inventor of the jet engine, carried out many experiments investigating the changes in pressure, volume and temperature of the gases before he designed the engine. Rocket engines, used to lift satellites and send them into orbit round the earth or moon, also blow hot gases from behind to give them a forward thrust.

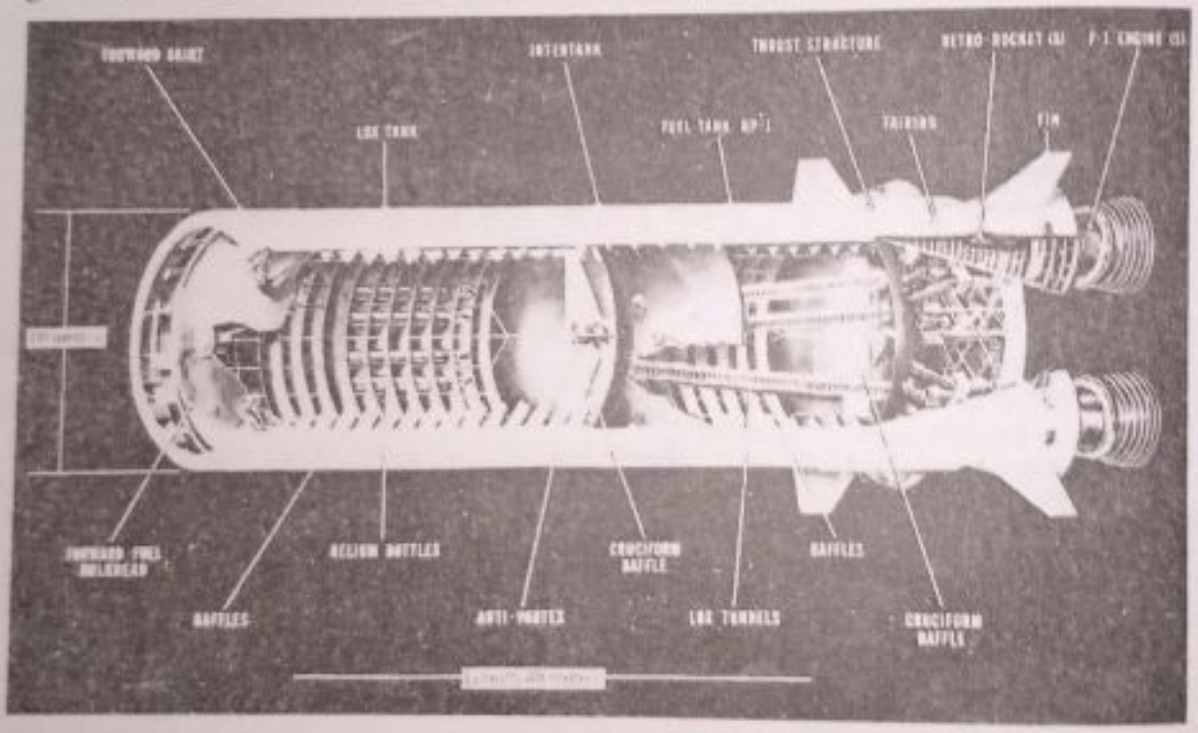


Plate 11A Saturn V rocket engine (first stage) used to launch manned flights to the moon. Burning fuel produces hot gases from nozzles at the rear (marked F–1 engine) which have sufficient energy to overcome the downward pull of gravity on the rocket. (*NASA*)

The commonly used car engine is the internal combustion engine. It uses a mixture of air and petrol vapour which is ignited by a spark from a spark plug inside the engine. The expanding hot gases force down a piston inside the engine. This piston movement is transformed by a crankshaft to rotary motion and in this way the expanding gases are made to turn the wheels of the car.

The pressure, volume and temperature of a gas may all change appreciably when any one of these quantities is altered. In contrast, the volume of a solid or a liquid is only very slightly affected by pressure change, and we therefore usually ignore the effect due to pressure in studying the expansion of solids and liquids.

Scanned with CamScanner

A given mass of gas, then, has three variables: *volume*, *pressure*, *temperature*. To investigate the laws relating them, one can be kept constant and the other two varied.

Volume expansion at constant pressure

A simple apparatus for a class experiment to investigate the expansion of a gas at constant pressure is shown in Fig. 11.1.

Experiment

A column AB of air is trapped by a length of sulphuric acid, which dries the air, or by a length of mercury, in a capillary tube T of uniform diameter. This is sealed at the lower end and is attached to a half-metre rule S by rubber bands.

Place the whole arrangement inside a deep vessel and fill this with enough water to cover most of the tube T as shown. Warm the water with a burner, stirring gently with the thermometer, and after a suitable temperature rise, record the temperature t and the length AB of the air column from the reading on the rule S, which is proportional to the volume V of the air. Obtain more readings of t and V by heating the water further to higher temperatures.

Measurements	t/°C	V

Graph: Plot a graph of volume V against temperature t°C, using zero (0°C) on the temperature axis as in Fig. 11.2.

Result: As shown in Fig. 11.2, a straight line graph AB is obtained. During the experiment the air was heated under conditions of *constant pressure*, equal to the atmospheric pressure plus that due to the height of the liquid column in Fig. 11.1. We therefore conclude that when the pressure of a fixed mass of gas is kept constant, the volume increases *linearly* as the temperature rises.

(Keep the results of this experiment, (a) to calculate later α, the cubic expansivity of air at constant pressure (page 190), and (b) to verify later the relation V/T = constant at constant pressure, where T is the 'absolute temperature' (page 191).)

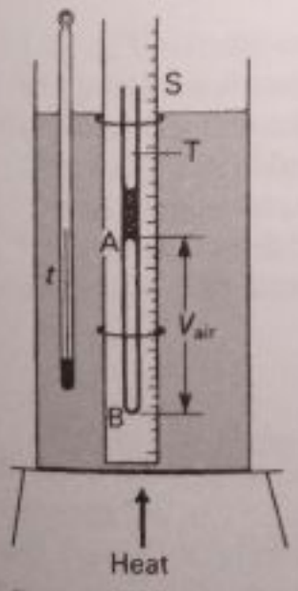


Fig. 11.1 Investigation of volume–temperature change (constant pressure)

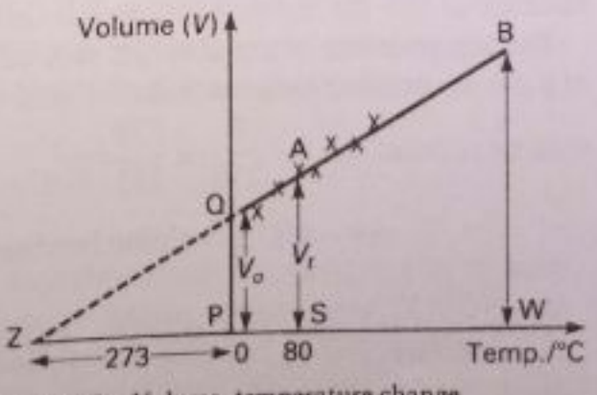


Fig. 11.2 Volume–temperature change graph (constant pressure)

Scanned with CamScanner

The *density* of a constant mass of gas is inversely proportional to its volume. The same experiment can thus be used to investigate the variation of density with temperature by plotting $1/V$ against t°C. The graph obtained is a *curve* in this case; the density *decreases* with increasing temperature because the volume has increased.

Cubic expansivity of gas (constant pressure)

The cubic expansivity (α) of a gas at constant pressure is defined as *the increase in volume per unit volume at 0°C per °C temperature rise, when the pressure is constant.*

So
$$\alpha = \frac{\text{increase in volume from } 0^\circ\text{C}}{\text{original volume at } 0^\circ\text{C} \times \text{temperature rise}}$$

This is the same definition as for the cubic expansivity of a liquid except that the original or initial volume of the gas must always be taken at 0°C because the volume of a gas is very sensitive to temperature changes.

The cubic expansivity α can be calculated by producing the line BA to Q on the volume axis (Fig. 11.2). V_0, the volume of the gas at 0°C, is thus read from the volume axis, since $PQ = V_0$. Suppose $V_0 = 10{\cdot}4$ units and the volume at a temperature such as 80°C is given by $V_t = SA = 13{\cdot}5$ units. Then, from above

$$\alpha = \frac{V_t - V_0}{V_0 \times t} = \frac{13{\cdot}5 - 10{\cdot}4}{10{\cdot}4 \times 80}$$
$$= 0{\cdot}0037/^\circ\text{C} = 0{\cdot}0037/\text{K}$$

Careful experiments, first carried out by the French scientist Jacques Charles, show that for all gases $\alpha = 0{\cdot}00366$ or $1/273$/K (approx.). So Charles' law says *for a fixed mass of gas at constant pressure, the volume increases by 1/273 of its volume at 0°C per degree Celsius temperature rise.*

From your graph of Fig. 11.2, calculate a value for the cubic expansivity α of air as above. (This value may differ appreciably from $\alpha = 1/273$ which is obtained with more elaborate apparatus.)

Absolute temperature, *T*

Suppose the fixed mass of gas at constant pressure in the experiment in Fig. 11.1 is reduced in temperature below 0°C. The volume then decreases along the straight line BQZ in Fig. 11.2. If the gas did not liquefy as the temperature was lowered further, then, theoretically, the volume of the gas would fall to zero at Z. Because of this the temperature at Z is called the *absolute zero* of temperature.

The temperature of the absolute zero can be found from Charles' law. If 1 cm^3 of a gas at constant pressure is at 0°C and its temperature is reduced to -1°C the volume reduces to $1 - \frac{1}{273}$ or $\frac{272}{273}$ cm^3.

At -4°C its volume becomes $1 - \frac{4}{273}$ or $\frac{269}{273}$ cm^3

In this way it can be seen that at -273°C the volume of the gas $= 1 - \frac{273}{273} = 0$.

This is true for any volume of gas at constant pressure, because it would shrink from 0°C by 1/273 of its volume for every degree Celsius temperature fall.

Scanned with CamScanner

Consequently, the absolute zero on the Celsius scale is about $-273°C$. Temperatures very close to the absolute zero have been reached.

Volume and absolute temperature

The absolute zero is called 0 K, after Lord Kelvin, who first suggested using the idea of absolute temperature. The temperature of 0°C is 273° from the absolute zero, or 273 K; the temperature of 100°C is 373 K (Fig. 11.3). Generally

$$t°C = (273 + t)\,K$$

The absolute temperature is usually given the symbol T, and we shall use t for temperature in degrees Celsius. Thus $T = 273 + t$.

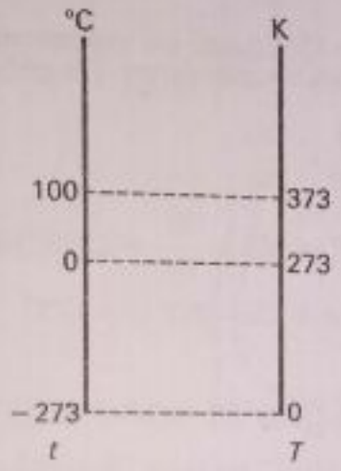


Fig. 11.3 Temperature in °C(t) and K(T)

Suppose V_0 is the volume of a gas at 0°C, represented by PQ in Fig. 11.2, and V_t is the volume at 80°C, represented by SA. From the diagram, it can be seen that, by similar triangles, the ratio SA/PQ = the ratio SZ/PZ. So

$$\frac{V_t}{V_0} = \frac{273+80}{273} = \frac{353}{273}$$

Generally, if V_t is the volume at t°C, it follows that

$$\frac{V_t}{V_0} = \frac{273+t}{273}$$

But $273 + t = T$, the absolute temperature, and V_0 and 273 are constants.

$$\therefore\ V_t \propto T$$

So we arrive at the important result that *the volume of a given mass of gas at a constant pressure is directly proportional to its absolute temperature*, or

$$\frac{V}{T} = \text{constant}$$

From your results on page 189 in the experiment on the variation of V with temperature in t°C, calculate the absolute or kelvin temperature $T(= 273 + t)$ and then work out the ratio V/T for each result and see if this ratio is fairly constant. Alternatively, plot a graph of V against T with a zero origin and see if a straight-line graph passing through the origin is obtained.

Scanned with CamScanner

Examples on gas volume and temperature change at constant pressure

A gas has a volume of 100 cm^3 at 15°C.
1. Calculate the new volume if it is warmed at constant pressure to 30°C.
2. Calculate the new temperature of the gas if its volume decreases to 80 cm^3 at constant pressure when it is cooled.

1. At 30°C, kelvin temperature $T = 273 + 30 = 303$ K
At 15°C $T = 273 + 15 = 288$ K
Since $V \propto T$ at constant pressure, the new volume V at 30°C is given by

$$\frac{V}{100} = \frac{303}{288}$$

$$\therefore V = \frac{303}{288} \times 100 = 105\,cm^3 \text{ (approx.)}$$

2. If the temperature falls from 15°C until the volume of the gas at constant pressure decreases to 80 cm^3, the new *absolute* temperature T is given by

$$\frac{80}{100} = \frac{T}{288}$$

$$\therefore T = 288 \times \frac{80}{100} = 230\,K \text{ (approx)}$$

$$\therefore \text{temperature} = 230 - 273 = -43°C$$

Pressure changes of gas at constant volume

We next investigate the pressure changes of a fixed mass of gas at constant volume when its temperature varies.

Experiment

Fig. 11.4 shows one form of apparatus. It consists of a large bulb F filled with dry air; a mercury manometer with one side BC open to the atmosphere, is connected to F by a very narrow tube D.

The bulb F is placed inside a large vessel and completely covered with water as shown. The initial temperature t is recorded and the open side BC of the manometer is raised or lowered until the top of the mercury on the other side reaches the level L. The height h of the mercury column above L is then recorded, together with the atmospheric pressure A.

The air in the bulb F is then warmed, by heating the water round the bulb and stirring gently; at a suitable temperature rise of 15–20 C the burner is taken away and BC is raised until the mercury level on the other side again reaches L. The new difference in levels, h, between B and L is noted, together with the temperature t. The experiment is repeated with higher temperatures for the air; each time BC is raised until the mercury level on the other side is at L *so that the volume of the air is kept constant*, and h and t are noted.

Measurements Atmospheric pressure, A = . . . mmHg

t/°C	h/mmHg	$p(=A+h$ or $A-h)$

Graph: Plot values of the air pressure p at constant volume against its temperature t°C. If B is above L, as in Fig. 11.4, $p = A + h$; if B is below L, then $p = A - h$.

Result: As shown in Fig. 11.5, a straight line graph AB is obtained. So at constant volume, the pressure of a gas varies uniformly or linearly with temperature.

Scanned with CamScanner

Keep the results of this experiment (a) to calculate later α, the pressure expansivity of air at constant volume, and (b) to verify later the relation p/T = constant at constant volume, where T is the absolute temperature.

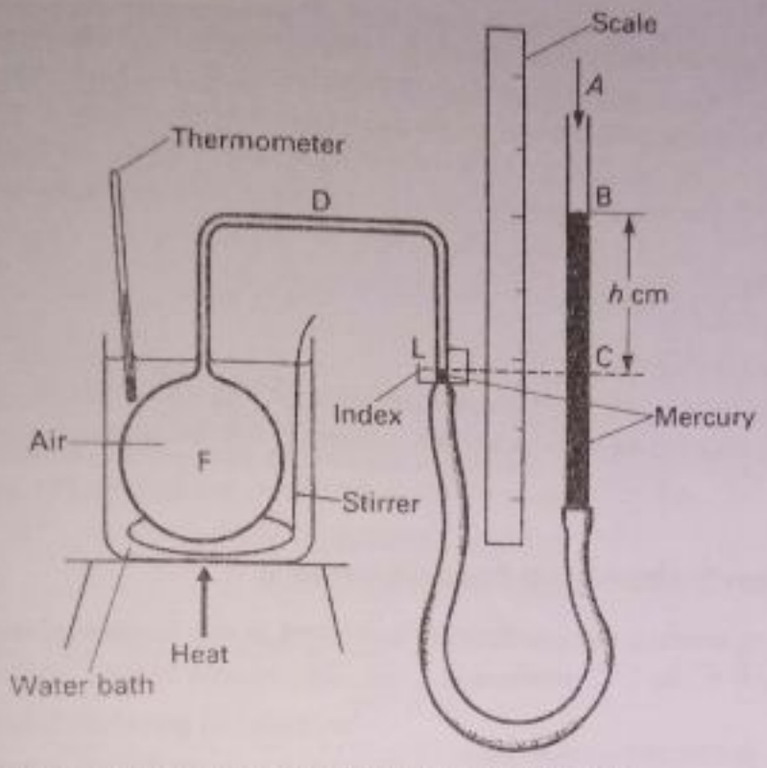


Fig. 11.4 Pressure–temperature change apparatus (constant volume)

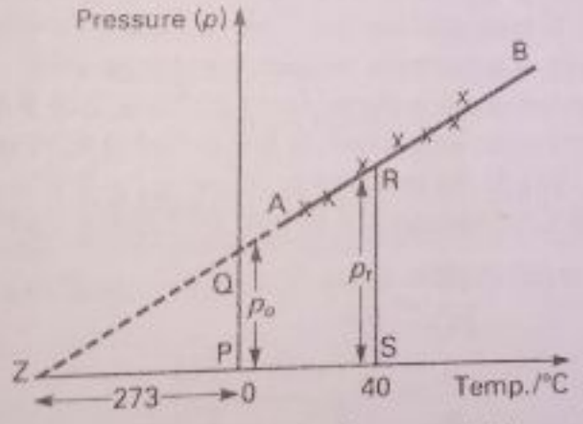


Fig. 11.5 Graph of pressure against temperature (volume constant)

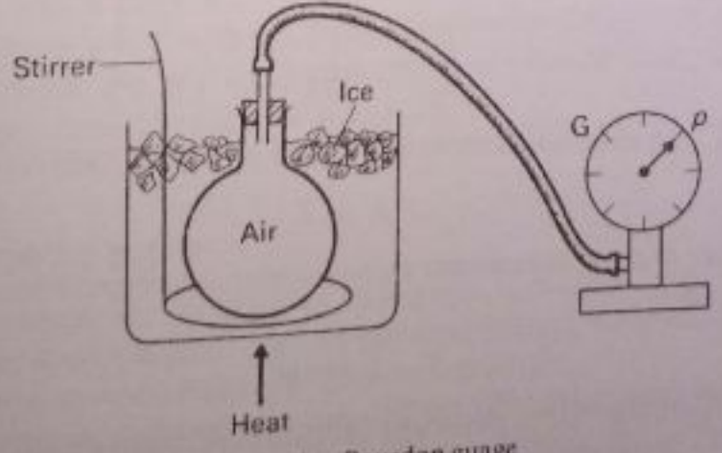


Fig. 11.6 Pressure–temperature change using Bourdon guage

Scanned with CamScanner

The Bourdon gauge (page 135) measures pressure directly and can be used to investigate the change in pressure with temperature of air at constant volume.

Experiment

Place an open large round-bottomed flask in a large container of ice-cold water. After a while connect it to a Bourdon gauge G as shown (Fig. 11.6), and observe the pressure reading and the temperature. Now warm the ice and water. At suitable rising temperatures until the water boils, record the corresponding pressure p inside the flask and the temperature t°C. While taking readings reduce the flame to keep the temperature steady.

Measurements

p	t/°C

Graph: Plot p against t°C.

Result: As shown in Fig. 11.5, a fairly straight line graph AB is obtained. So, at constant volume, the pressure of a gas varies uniformly or linearly with temperature.

Pressure and absolute temperature

The *pressure expansivity* (or coefficient) is defined as the *fractional increase of the gas pressure at 0°C for 1°C temperature rise*, the volume being constant.

$$\text{So} \qquad \text{pressure expansivity} = \frac{\text{increase in pressure from 0°C}}{\text{original pressure at 0°C} \times \text{temperature rise}}$$

Results show that the pressure of all gases increases by about 1/273 of its pressure at 0°C for 1°C temperature rise. This is the same numerical result as that for the volume change of a gas with temperature (page 190).

So when the temperature is *reduced* from 0°C, the line BA passes through a point Z on the temperature axis which is about −273°C (Fig. 11.5). This is the absolute zero (page 191). If the pressure p_0 of the gas at 0°C is represented by PQ, and pressure p_t at 40°C is represented by SR, we can see that

$$\frac{\text{SR}}{\text{PQ}} = \frac{\text{ZS}}{\text{ZP}}$$

So

$$\frac{p_t}{p_0} = \frac{273+40}{273} = \frac{313}{273}$$

Generally, if p_t is the pressure at t°C, then

$$\frac{p_t}{p_0} = \frac{273+t}{273}$$

But $273 + t = T$, the absolute temperature, and p_0 and 273 are constants.

$$\therefore \; p_t \propto T$$

So *the pressure of a gas at constant volume is directly proportional to its absolute temperature* T, or

$$p/T = \text{constant}$$

From your results in the experiment on pressure change with temperature at constant volume, see if the ratio p/T is fairly constant for all the values obtained.

Scanned with CamScanner

Alternatively, plot a graph of p against T with zero origin and see if a straight-line graph passing through the origin is obtained.

Example on gas pressure and temperature

A car tyre is pumped to a pressure of 0·20 N/mm² in the morning at 10°C. Calculate the new pressure when the temperature rises to 18°C later.

Since $p \propto T$ at constant volume, the new pressure p is given by

$$\frac{p}{0{\cdot}20} = \frac{273+18}{273+10} = \frac{291}{283}$$

$$\therefore\ p = 0{\cdot}21\ \text{N/mm}^2\ \text{(approx.)}$$

Ideal gas law

We can now write down a general gas law, in which pressure, volume and absolute temperature may all vary.

From page 191 and above, we have, for a given mass of gas,

$$p \propto T \text{ (volume constant)}$$
$$V \propto T \text{ (pressure constant)}$$

So if p, V and T all vary, we deduce that

$$pV \propto T$$

We can check this is a correct relation by making the volume V constant in it, when we have $p \propto T$, and then making the pressure p constant in it, when we have $V \propto T$.

Since $pV \propto T$

$$\frac{pV}{T} = \text{a constant value}$$

the value depending on the mass of gas used and its nature. So if a *constant mass* of gas has (a) a volume V_1 at a pressure p_1 and absolute temperature T_1, and then (b) a volume V_2 at a different pressure p_2 and a different absolute temperature T_2

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

If the temperature $T_1 = T_2$, that is the temperature is constant, then $p_1V_1 = p_2V_2$. This is Boyle's law (page 135).

The relationship 'pV/T = constant' is a law for an ideal gas. It assumes that the attraction between the gas molecules and the volume they occupy are both negligible, which is not the case for a real gas.

Conversion to s.t.p.

In order to compare the volumes of two gases, it is convenient to convert both volumes to standard temperature and pressure (s.t.p.), which is taken as 0 C and 760 mmHg pressure (or $1{\cdot}013 \times 10^5$ N/m² pressure).

As an illustration, suppose a volume of (a) 100 cm³ of gas, measured at 10 C and 780 mmHg pressure, is compared with a volume of (b) 120 cm³ measured at

Scanned with CamScanner

50°C and 700 mmHg pressure. To calculate the volumes, V_2, of both at s.t.p., we have

(a) $V_1 = 100\,\text{cm}^3$, $p_1 = 780\,\text{mm}$, $T_1 = 273 + 10 = 283\,\text{K}$
$p_2 = 760\,\text{mm}$, $T_2 = 273\,\text{K}$

From $$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\frac{780 \times 100}{283} = \frac{760 \times V_2}{273}$$

$$\therefore V_2 = \frac{780 \times 100 \times 273}{283 \times 760} = 99\,\text{cm}^3$$

(b) $V_1 = 120\,\text{cm}^3$, $p_1 = 700\,\text{mm}$, $T_1 = 273 + 50 = 323\,\text{K}$
$p_2 = 760\,\text{mm}$, $T_2 = 273\,\text{K}$

From $$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$\frac{700 \times 120}{323} = \frac{760\,V_2}{273}$$

$$\therefore V_2 = \frac{700 \times 120 \times 273}{323 \times 760} = 93\,\text{cm}^3$$

It can now be seen that the measured volume of $120\,\text{cm}^3$ ($93\,\text{cm}^3$ at s.t.p.), is actually a smaller volume at s.t.p. than the measured volume of $100\,\text{cm}^3$ ($99\,\text{cm}^3$ at s.t.p.).

Suppose the mass of gas of volume $120\,\text{cm}^3$ at 50°C and 700 mmHg pressure is 0·12 g. From the calculation above, the volume at s.t.p. is $93\,\text{cm}^3$. Since the mass remains constant,

$$\text{density at s.t.p.} = \text{mass/volume} = \frac{0{\cdot}12}{93}\,\text{g/cm}^3$$
$$= 0{\cdot}001\,29\,\text{g/cm}^3 = 1{\cdot}29\,\text{g/litre}$$

Kinetic theory of gas laws

The effects of temperature on the volume and pressure of a gas can be explained by the kinetic theory of gases.

This theory considers the molecules of a gas to be like elastic spheres. Each time one of the molecules strikes the wall of the container, it rebounds (Fig. 11.7a). So its momentum changes. The force produced on the wall by a molecule is the momentum change per second (page 45). So the gas pressure due to all the bombarding molecules is proportional to their average total momentum change per second normal to the walls.

Pressure and temperature, volume constant

Heat is a form of energy, and when a gas is heated the molecules gain kinetic energy and move about faster. Consequently, for a *fixed volume*, the momentum change produced by the molecules at the walls per impact is greater. The number of impacts per second of each molecule also increases since the speed increases.

Scanned with CamScanner

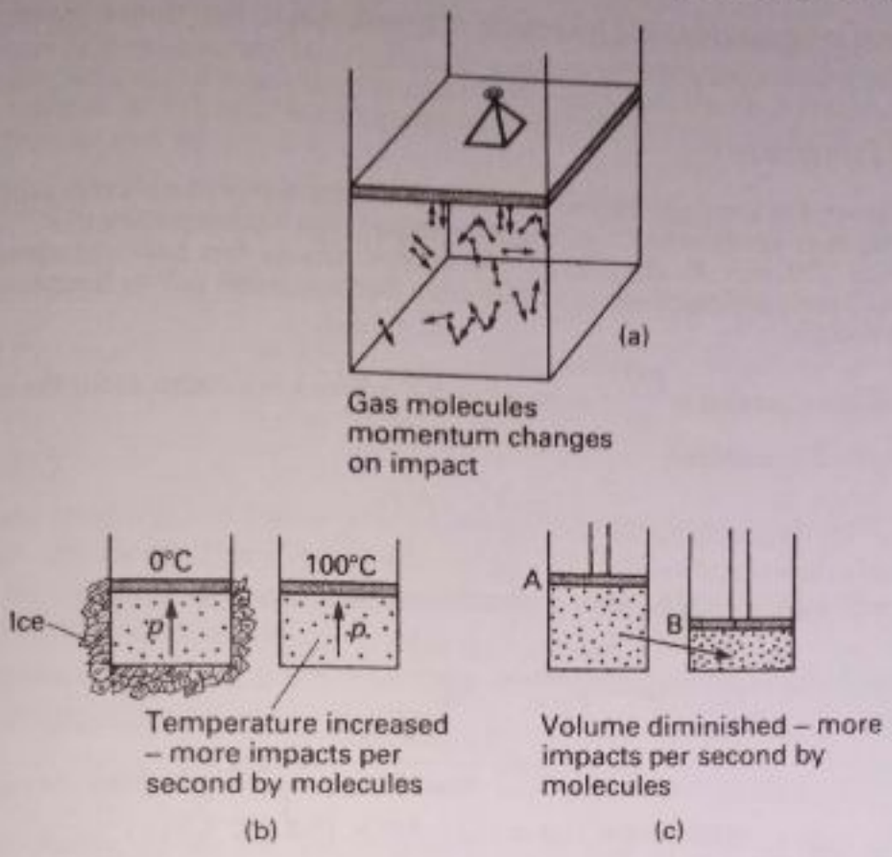


Fig. 11.7 (a) Pressure and momentum (b) pressure and temperature (c) pressure and volume

So the pressure increases with temperature rise when the volume is constant (Fig. 11.7b).

Volume and temperature, pressure constant

When the gas is heated at *constant pressure*, part of the energy supplied is used in pushing back slowly the piston shown in Fig. 11.7 while the pressure is kept constant, and the rest of the energy makes the gas molecules move faster. So the volume has increased and the temperature of the gas becomes higher. Although the molecules are moving faster, the momentum change per second at the piston is the same because the molecules have further to travel between successive impacts. The pressure is therefore constant.

Pressure and volume, temperature constant

When the volume of a gas at a given temperature is decreased, by pushing a piston down from A to B, for example (Fig. 11.7c), the molecules take less time to travel between the piston and the wall opposite. Consequently, more impacts per second are made. Thus the pressure of a gas at a given temperature increases when the volume decreases. Conversely, if the piston is raised so that the volume of the gas increases, the molecules take a longer time to move across to the piston and there are fewer impacts per second. The pressure of the gas is now less. If the volume is doubled, for example, the number of impacts per second is halved. Since the speed of the molecules is unchanged at constant temperature, we see that the pressure of the gas is halved when the volume is doubled.

Two hundred years before Maxwell proposed the kinetic theory of gases, Robert Boyle had discovered experimentally in 1660 that the pressure p of a gas

Scanned with CamScanner

at constant temperature was inversely proportional to its volume V, or $p \propto 1/V$. This is *Boyle's law* (page 135).

Examples on gas law

1. Write down the ideal gas equation. At the start of a stroke of a bicycle pump, the air in the pump is at a pressure of 1 atm (atmosphere), has a temperature of 27°C and has a volume of 100 cm³. At the end of one stroke, this air has been compressed to a pressure of 3 atm, and has a volume of 35 cm³. By how much will its temperature have risen? (O and C)

The ideal gas equation is $\frac{pV}{T} = k$, or $pV = kT$, where k is a constant for the amount of gas. From pV/T = constant

$$\frac{p_2V_2}{T_2} = \frac{p_1V_1}{T_1}$$

Since $T_1 = 27 + 273 = 300$ K, the new absolute temperature T_2 is given by

$$\frac{3 \times 35}{T_2} = \frac{1 \times 100}{300}$$

$$\therefore\ T_2 = \frac{3 \times 35 \times 300}{1 \times 100} = 315\ \text{K}$$

$$\therefore\ \text{temperature rise} = 315 - 300 = 15\ \text{K (or °C)}$$

2. (i) Fig. 11.8 shows a gas enclosed in a cylinder by a piston P. Account for the pressure exerted on the piston by the molecules of the gas.

State the changes which occur in this pressure when (a) the piston is pushed to the left, so that the gas now occupies only half the volume it occupied originally, but the temperature of the gas is unaltered, (b) the piston remains fixed in position, but the temperature of the gas is increased. Account for these changes in the pressure exerted by the molecules.

(ii) Draw a fully labelled diagram of an apparatus which could be used to investigate how the volume of a mass of gas changes with temperature at constant pressure. Indicate clearly how the constancy of pressure is achieved.

A mass of gas which is initially at a pressure of 760 mmHg has its volume halved, and at the same time its temperature increases from 300 K to 400 K. What does the pressure become? (C)

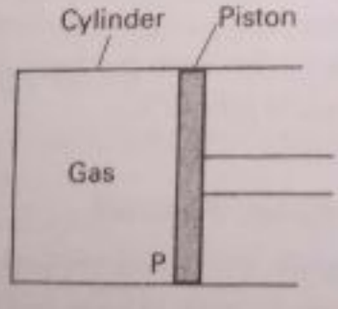


Fig. 11.8 Example

(i) When a molecule hits the piston P, it rebounds and there is a momentum change normal to the piston. Other molecules also have a momentum change normal to the piston when they strike it and rebound. The average force on the piston is the average momentum change per second normal to the piston produced by all the molecules, which hit the piston repeatedly. The pressure on the piston is the average force per unit area of the piston.

(a) When the gas occupies half the volume at constant temperature then, by Boyle's law, the pressure is *doubled*. In this case the molecules strike the piston twice as frequently because they now have only half the distance to travel between successive impacts.

(b) When the temperature is increased at constant volume, the heat supplied increases the kinetic energy of the gas molecules and hence their speed. So the momentum change on striking the piston is increased, and the rate of striking the piston is increased. The pressure is therefore increased.

Scanned with CamScanner

(ii) For an apparatus, see Fig. 11.1 (page 189). A barometer close to the apparatus should be read before and after the experiment to ensure pressure is constant.

Suppose the volume is V when the pressure is 760 mmHg and the temperature is 300 K. When the volume is $V/2$ and the temperature is 400 K, the new pressure p is given, since pV/T = constant, by

$$\frac{p \times \frac{V}{2}}{400} = \frac{760 \times V}{300}$$

Cancelling V,

$$p = \frac{760 \times 400}{\frac{1}{2} \times 300} = 2027 \text{ mmHg}$$

Summary

1. Absolute temperature $T\,\text{K} = 273 + t$, where t is the temperature in °C.
2. Charles' law states: For a given mass of gas at constant pressure, the volume increases by 1/273 of its volume at 0°C for each °C rise in temperature. $V \propto T$ (absolute) or V/T = constant.
3. For a given mass of gas at constant volume, the pressure is proportional to the absolute temperature, or p/T = constant.
4. For a given mass of gas, $\frac{pV}{T}$ = constant. This is an ideal gas law.
5. *Kinetic theory of gases.* The pressure of a gas is due to the continual bombardment by molecules of the walls of the vessel. The number of collisions per second is proportional to the velocity, and the momentum change at each collision is proportional to the velocity. The molecules have different velocities and the average gas pressure is proportional to the average total momentum change per second.
6. In the kinetic theory of gases, increasing the temperature at constant volume increases the speed of the molecules, and hence the number of collisions per second and the momentum change at each collision. The pressure therefore increases.

When the gas is heated at constant pressure it expands, and the molecules move faster because they have more energy.

Exercise 11 (Answers, page 641)

1. (a) A gas at constant pressure is heated so that it expands from a volume of 100 cm³ at 27°C to a volume V at 87°C. Calculate V.

(b) A gas in a cylinder has a pressure of $2{\cdot}8 \times 10^5$ N/m² at 7°C. Find the new pressure if the temperature rises to 87°C.

(c) 5·0 g of gas has a volume of 4·50 litres at 20°C and 750 mmHg. Calculate the volume at s.t.p. (0°C and 760 mmHg). What is the density of the gas at s.t.p.?

2. A vessel contains 1 litre of a gas at 30°C and at 1 atmosphere pressure. Calculate the temperature at which the volume will, without change in pressure, be 3 litres. (C)

3. The volume of a given mass of air at 27°C and 75 cm pressure is 152 cm³. Find its volume at 76 cm pressure, the temperature being −23°C. (N)

4. State the law which indicates how the pressure of a fixed mass of air varies with its temperature as recorded on a mercury thermometer, when the volume remains constant.

Describe an experiment to verify the law.

80 cm³ of hydrogen are collected at 15°C and 75 cm of mercury pressure. What is its volume at s.t.p.? (L)

5. How does the *density* of a gas depend on (a) its pressure when the temperature is kept constant, (b) its temperature when the pressure is kept constant?

Describe an experiment to verify one of these statements.

Scanned with CamScanner

A fixed mass of gas occupying 6·0 litres at 27°C is compressed at constant temperature until the pressure is doubled and is then cooled at that pressure until the volume is 2·0 litres. What is the new temperature? (O and C)

6. Describe as fully as you can the kinetic picture of a gas, and show how it accounts for the change in pressure when the volume of a gas is increased, as in a Boyle's law experiment.

What explanation can be given, in terms of kinetic theory, of the rise in temperature of a gas when it is compressed, as in a bicycle pump?

The pressure inside a bicycle tyre can be increased either (a) by pumping, or (b) by heating alone. What explanation can you give to account for the rise in pressure in each case? (O and C)

7. Describe an experiment by which an approximate value for the density of air may be obtained. Show how the measurements made are used to obtain the final result.

By what fraction and in what way would the density be altered if such an experiment, first performed in winter at an air temperature of 12°C, were then repeated in summer at the same pressure but at an air temperature of 27°C?

Explain briefly how the molecules of air exert a pressure on the walls of a closed container and why the pressure increases when the volume of gas in the container is decreased while its temperature remains constant. (L)

8. Using the ideas of the kinetic theory of matter, (a) distinguish between solids, liquids and gases, (b) explain why a gas exerts pressure. Briefly explain how it is possible for a steel needle to float on water. (N)

9. Make a diagram of an apparatus which enables you to use the expansion of a gas as a means of measuring temperature.

Explain how you would calibrate it to enable you to do this. Mention one advantage of using a gas instead of a liquid as the expanding substance.

The gas in a gas thermometer occupies 54·0 cm^3 at 0°C and 74·0 cm^3 at 100°C. What are the temperatures when it occupies (a) 62·0 cm^3, (b) 46·0 cm^3, if its pressure is constant throughout? (O and C)

10. Describe an experiment to determine the absolute zero of temperature using a mass of air *either* at constant volume, *or* at constant pressure.

In an experiment the pressure of a mass of air at constant volume was found to increase from 75 cm of mercury at 15°C to 91 cm of mercury at 75°C. From these results what would be: (a) the pressure of the air at 0°C; (b) the coefficient of increase of pressure of air at constant volume? (N)

11. Draw a labelled diagram of the apparatus you would use to investigate the expansion of a sample of air heated at constant pressure. State how you would try to ensure that the sample of air was uniformly heated and how you could know whether its pressure remained constant during the experiment.

State the gas equation and state what the symbols in it represent. Calculate the density of air at −100°C and at a pressure of 20·0 atmospheres, given that the density of air at 0°C and at a pressure of 1·00 atmospheres is 1·29 g/litre. (C).

Multiple-Choice Questions

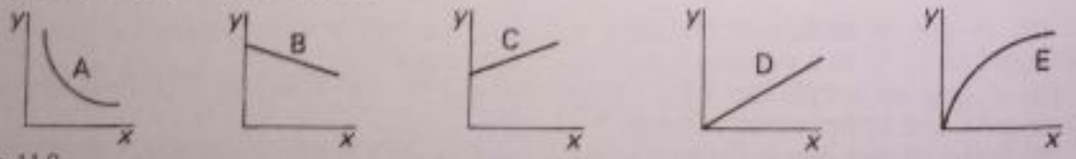


Fig. 11.9

In Fig. 11.9, which graph A to E would best represent how:

12. The volume of a gas (y) varies with temperature in °C (x) at constant pressure.

13. The pressure of a gas (y) varies with temperature in kelvin (x) at constant volume.

14. The pressure of a gas (y) varies with volume (x) at constant temperature.

15. When a car is driven for some time, the air in the tyres has a higher temperature. This is because the molecules

A occupy a smaller volume
B make more collisions with each other
C gain heat from the road
D have greater speeds
E have a greater volume

Scanned with CamScanner

12. Heat capacity · Heat energy

Heat and energy

In early days it was thought that heat was a fluid called caloric, which flowed from one body to another at a lower temperature. However, Benjamin Thompson, or Count Rumford as he was later known, showed that *heat is a form of energy*. In one experiment, carried out in about 1800, he placed a metal borer inside a gun-barrel and put the whole arrangement in a large water tank. The borer was then rotated and after a time the water boiled. The heat was produced from the mechanical energy used in rotating the borer. He therefore concluded that heat was a form of energy. About 50 years later careful experiments by J. P. Joule showed that this was correct.

We now consider that a transfer of heat from a metal A to a metal B increases the *internal energy* of B (that is, the energy of all its molecules is increased) and decreases the internal energy of A.

Quantity of heat and units

Since heat is a form of energy, the joule J, is used as the unit of *quantity of heat*, symbol Q. About 4·2 J is the quantity of heat needed to raise the temperature of 1 g of water by 1°C, or 4200 J is needed to raise the temperature of 1 kg of water by 1°C. (The calorie, an older unit, is the heat needed to raise the temperature of 1 g of water by 1°C.)

When we use the term 'heat' in measurements, we shall mean 'quantity of heat', symbol Q. Commercial gas companies in Great Britain use a unit of heat known as the *therm*. This is about 100 million J or 100 megajoules (MJ).

Calorific or energy values

The calorific or energy value of one grade of coal is approximately 28 million J/kg. The human body is a machine which needs food as fuel to keep working efficiently, and some approximate energy values, in kilojoules per 100 g, of a variety of foods are given below. Diets to reduce weight exclude chocolate and sugar, for example, and include lean meat and green vegetables.

Proteins		Carbohydrates		Fats		Others	
Cheese	1680	Chocolate	2300	Butter, Margarine, Olive oil, Fat meat	2900	Peas	420
Lean meat	1200	Sugar	1600			Boiled potatoes	340
Eggs	700	Wholemeal bread	1000			Milk	300
Liver	600					Fresh fruit	200
White fish	300					Green vegetables	150

Scanned with CamScanner

Heat capacity

The heat to raise the temperature of a substance by 1°C is defined as its *heat capacity*.

A large mass of water such as the sea requires a large amount of heat to raise its temperature 1°C; it has a large heat capacity. A small piece of metal, such as the junction of a thermo-couple, has a small heat capacity. The temperature of the metal rises appreciably when it receives a small amount of heat, which is an advantage of the thermocouple for measuring the temperature at a particular part of a hot metal surface (page 172). Heat capacity, symbol C, has the unit J/K (J per K) or J/°C (J per °C).

Specific heat capacity

The *specific heat capacity*, c, of a substance is defined as *the quantity of heat which raises the temperature of unit mass by 1°C (1 K)*. The specific heat capacity of water is about 4200 J/kg K (4200 J per kg per K) or 4·2 J/g K (4·2 J per g per K). This means that 4200 J of heat raises the temperature of 1 kg of water by 1°C, or 4·2 J raises the temperature of 1 g of water by 1°C. The specific heat capacity of copper is about 400 J/kg K, and that for aluminium about 1100 J/kg K. Because of the relatively high specific heat capacity of water the sea does not become unbearably hot in normal summers or frozen in normal winters.

Since the specific heat capacity of copper is 400 J/kg K, a block of copper of mass 3 kg needs 3×400 J to raise its temperature by 1 K or 1°C. So 1200 J/K is the heat capacity of that block. We see that generally

$$\textit{heat capacity} = \textit{mass} \times \textit{specific heat capacity}$$

or specific heat capacity is the heat capacity per unit mass of the material. You should remember that specific heat capacity is measured in J/kg K whereas heat capacity is measured in J/K.

Formula for quantity of heat, Q

Suppose the temperature of a mass of 0·5 kg of copper rises by 10°C. If 400 J/kg K is the specific heat capacity of copper, then, since a quantity of heat 400 J raises the temperature of 1 kg by 1°C or 1 K, the quantity of heat Q supplied to the 0·5 kg mass is $400 \times 0{\cdot}5 \times 10$, or 2000 J.

Similarly, if 2 kg of aluminium, specific heat capacity 1100 J/kg K, cools from 50°C to 30°C, a temperature change of 20°C, the quantity of heat lost Q is $1100 \times 2 \times 20$, or 44000 J. From these calculations we see that the heat Q gained or lost by a mass m of specific heat capacity c when its temperature *changes* by t°C is always given by

$$Q = mct$$

Scanned with CamScanner

Experiment to measure specific heat capacity of a metal

Apparatus: Solid aluminium cylinder A of mass 1 kg (or other suitable mass), with holes for electrical heater H and thermometer T; 50 W (or other suitable power) electrical heater; thermometer; felt jacket B; stopclock or stopwatch; connecting leads.

Method: Place the electrical heater H and thermometer T inside the holes bored in the block (Fig. 12.1). A little oil in each hole helps to make good thermal contact with the block. Note the initial temperature t_1.

Switch on the heater and start the clock. After a measured time t (seconds) when the temperature rise is at least 15° C, switch off the heater and watch the thermometer until it has passed its maximum temperature. Record the maximum temperature t_2.

Measurements: Initial temperature, t_1
Final (maximum) temperature t_2
Time of heating t
Power of heater
Mass of block m

Calculation: If the time t is in seconds,

heat supplied by 50 W heater $= 50t$ joules.

Assuming no heat losses, then if c is the specific heat capacity of the block

$$mc(t_2 - t_1) = 50t$$

$$\therefore c = \frac{50t}{m(t_2 - t_1)}$$

Results: In one experiment a rise in temperature of 14° C was obtained in 300 s. The mass of the block was 1 kg, and a 50 W heater was used. Then

$$c = \frac{50 \times 300}{1 \times 14} \approx 1100 \text{ J/kg K}$$

In this experiment, errors may be due to loss of heat to the surroundings and to lack of thermal contact between the heater and block, or thermometer and block.

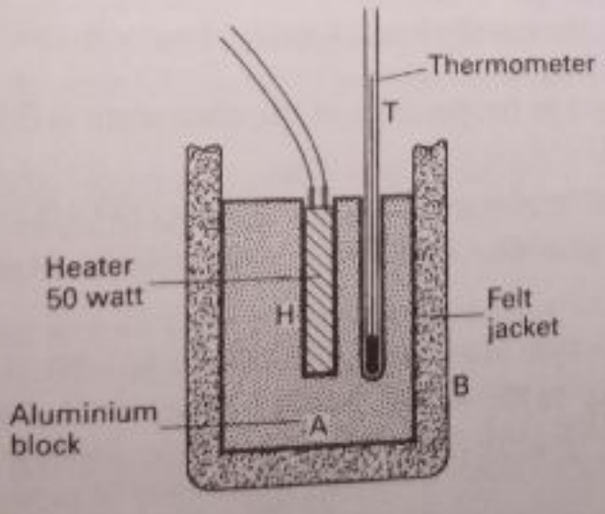


Fig. 12.1 Finding specific heat capacity of a solid by electrical heating

Scanned with CamScanner

Experiment to measure specific heat capacity of a liquid

Apparatus: 50 W (or other) electrical heater; container for liquid (a large polystyrene beaker or lagged calorimeter); thermometer; balance or measuring cylinder; stopclock or stopwatch; stirrer; connecting leads.

Method: Place a weighed or measured quantity of liquid such as water into the container, sufficient to cover the immersion heater completely. Note the initial temperature t_1.

Switch on the heater for a measured time t (seconds) to give a temperature rise of at least 15°C. Keep stirring gently while the liquid is being heated and note the final or highest temperature t_2 after the heater is switched off.

Measurements: As in previous experiment, except that m is now the mass of liquid.

Calculation: As before, if m is in kg and the time t in seconds

$$c = \frac{50t}{m(t_2 - t_1)}$$

Results: In one experiment a 50 W heater produced a temperature rise of 17·5°C in 0·5 kg of water in a polystyrene container after a time of 750 s. So

$$c = \frac{50 \times 750}{0{\cdot}5 \times 17{\cdot}5} = 4300 \text{ J/kg K (approx.)}$$

The error due to heat lost to the surroundings can be reduced by cooling the water with ice to say 8°C *below* the temperature of the room and then heating the water until it just reaches a temperature 8°C above the room temperature. The average water temperature is then the room temperature, so, on the average, very little heat is lost to the surroundings when it is heated. Another source of error is that we have neglected the heat capacity of the container in the calculation.

Heat exchanges

When a hot object is placed in thermal contact with a cold object, heat flows from the hot to the cold object until they are at the same temperature. As an example of *heat exchanges* between two substances, consider an aluminium block of mass 0·2 kg at 85°C dropped into 0·1 kg of water at 15°C.

To find the final steady temperature, x°C say, we note that, assuming no heat losses to the air,

heat lost by hot solid = heat gained by water . . . . . (1)

Let us assume that the specific heat capacity of water is 4200 J/kg K and that of aluminium is 1100 J/kg K.

Since the change t in temperature of the aluminium is $(85 - x)$°C and of the water is $(x - 15)$°C,

$$\text{Heat lost by aluminium} = mct = 0{\cdot}2 \times 1100 \times (85 - x) \text{ J}$$

$$\text{Heat gained by water} = mct = 0{\cdot}1 \times 4200 \times (x - 15) \text{ J}$$

So, from (1),

$$0{\cdot}2 \times 1100 \times (85 - x) = 0{\cdot}1 \times 4200 \times (x - 15)$$

$$\therefore \; 18\,700 - 220x = 420x - 6300$$

$$\therefore \; 420x + 220x = 18\,700 + 6300 = 25\,000$$

$$\therefore \; x = \frac{25\,000}{640} = 39°\text{C}$$

Scanned with CamScanner

The calorimeter

Experiments to measure specific heat capacity (or heat capacity) can be done by a method of heat exchanges. The exchange between a hot metal and cold water, for example, is often made inside a metal vessel called a *calorimeter*.

The calorimeter is usually made of copper, a good conductor, so that it reaches the same temperature as its contents as soon as possible. To reduce heat losses by radiation to a minimum the vessel is polished both inside and out. To eliminate heat losses by convection and conduction, the calorimeter is placed in a larger container and surrounded by layers of lagging, such as cotton waste, which is a poor conductor (Fig. 12.2). An insulating lid is sometimes used to cover the calorimeter. This prevents evaporation of the liquid inside.

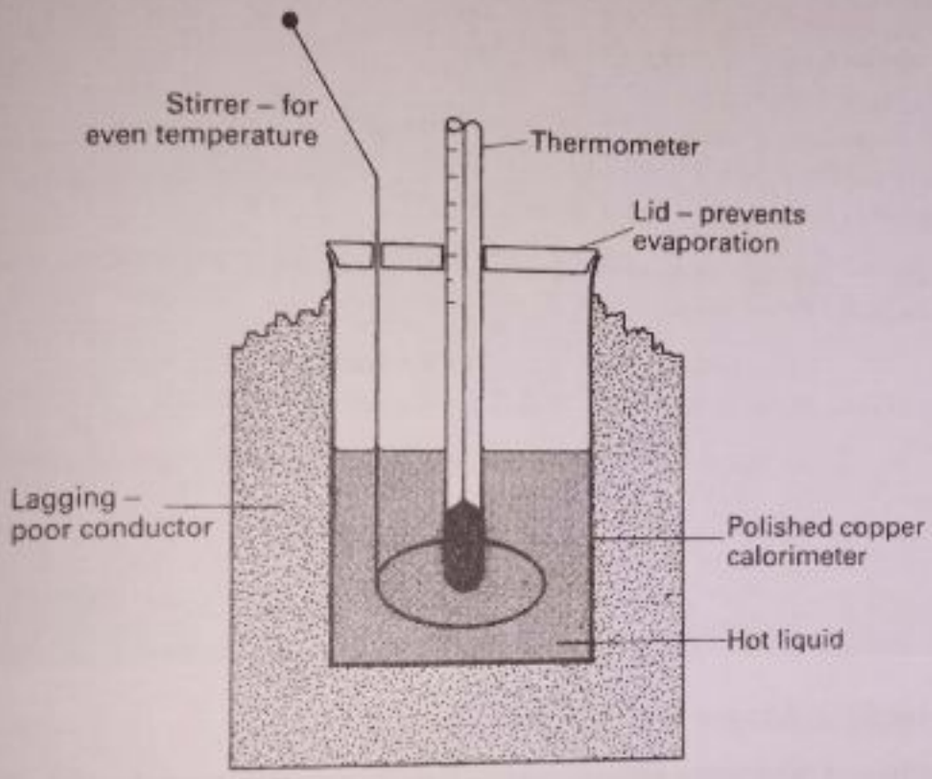


Fig. 12.2 Cross-section of a calorimeter

Cups called Aerocups, made of expanded polystyrene, are very good insulators, and may be used in place of calorimeters in elementary experiments without any need for lagging.

Suppose the copper calorimeter has a mass of 0·1 kg and the specific heat capacity of copper c is 400 J/kg K. If the calorimeter contains 0·2 kg of water of specific heat capacity 4200 J/kg K, and the temperature of the water rises 5°C, then

$$\text{heat gained by calorimeter} = mct = 0{\cdot}1 \times 400 \times 5 = 200\,\text{J}$$

$$\text{and} \qquad \text{heat gained by water} = mct = 0{\cdot}2 \times 4200 \times 5 = 4200\,\text{J}$$

$$\text{So} \qquad \text{total heat gained} = 200 + 4200 = 4400\,\text{J}$$

A *fuel calorimeter* is used for measuring the quantity of heat produced by burning fuel or food. The fuel is placed inside a sealed container surrounded by water and burning is started by an electric heating coil in the fuel. The heat produced is found from the temperature rise and heat capacity of the surrounding water and its vessel, and the calorific value, or heat per unit mass, of the fuel can then be calculated.

Scanned with CamScanner

Experiment to measure c by heat exchange

To measure the specific heat capacity of a block of metal by heat exchange, first weigh a copper calorimeter, fill it half to two-thirds full of cold water (sufficient to cover the metal) and reweigh. Replace the calorimeter in its lagged jacket. Record the water temperature. Measure the mass of the metal.

Attach a thread to the metal, place it inside water in a beaker and heat the water to boiling point. After a few minutes transfer the hot metal *quickly* to the calorimeter and water, shaking the metal to remove any surplus water on it. Stir the water, and note its final steady maximum temperature.

Measurements. As an illustration, suppose the measurements are as follows.

Mass of metal = 0·3 kg
Mass of calorimeter = 0·25 kg (c = 400 J/kg K)
Mass of water = 0·5 kg
Initial temperature of water = 12·0 C (c = 4200 J/kg K)
Final temperature of water = 20·0 C
Temperature of hot metal = 100·0 C

Calculation. Heat lost by metal = $0{\cdot}3 \times c \times (100-20) = 24c$ J where c is the specific heat capacity of the metal.

Heat gained by water = $0{\cdot}5 \times 4200 \times (20-12) = 16\,800$ J
and heat gained by calorimeter = $0{\cdot}25 \times 400 \times (20-12) = 800$ J.

Now heat lost by metal = heat gained by water and calorimeter, neglecting any heat losses. So

$$24c = 16800 + 800 = 17600$$
$$\therefore\ c = \frac{17600}{24} = 730 \text{ J/kg K (approx.)}$$

Temperature of furnace or flame

One method of measuring approximately the unknown temperature of a furnace (or a flame) is to place a small aluminium block X inside the furnace, allow it to reach the furnace temperature, and then transfer the hot solid quickly to cold water inside a calorimeter. The final steady water temperature is measured.

As an example, suppose the following measurements were taken.

Mass of block X = 0·2 kg
Mass of empty calorimeter = 0·1 kg
Mass of water = 0·3 kg
Initial water temperature = 15·0°C
Final water temperature = 55·0°C

Suppose the specific heat capacity of the aluminium and the calorimeter material are respectively 1100 and 400 J/kg K, and the furnace temperature is x°C. Now ignoring heat losses

heat lost by block X = heat gained by water and calorimeter
$$\therefore\ 0{\cdot}2 \times 1100 \times (x-55) = [0{\cdot}3 \times 4200 \times (55-15)] + [0{\cdot}1 \times 400 \times (55-15)]$$
$$\therefore\ 220\,(x-55) = 50400 + 1600 = 52000$$

So
$$x - 55 = \frac{52000}{220} = 236, \text{ or } x = 291°\text{C}$$

Scanned with CamScanner

Measurement of *c* using mechanical energy

So far in laboratory experiments heat has been produced by burners, which change chemical energy to heat energy, or electrical energy to heat energy. We now consider an experiment which changes mechanical energy by friction to heat (internal) energy in metals. This enables the specific heat capacity of a metal to be measured, as below.

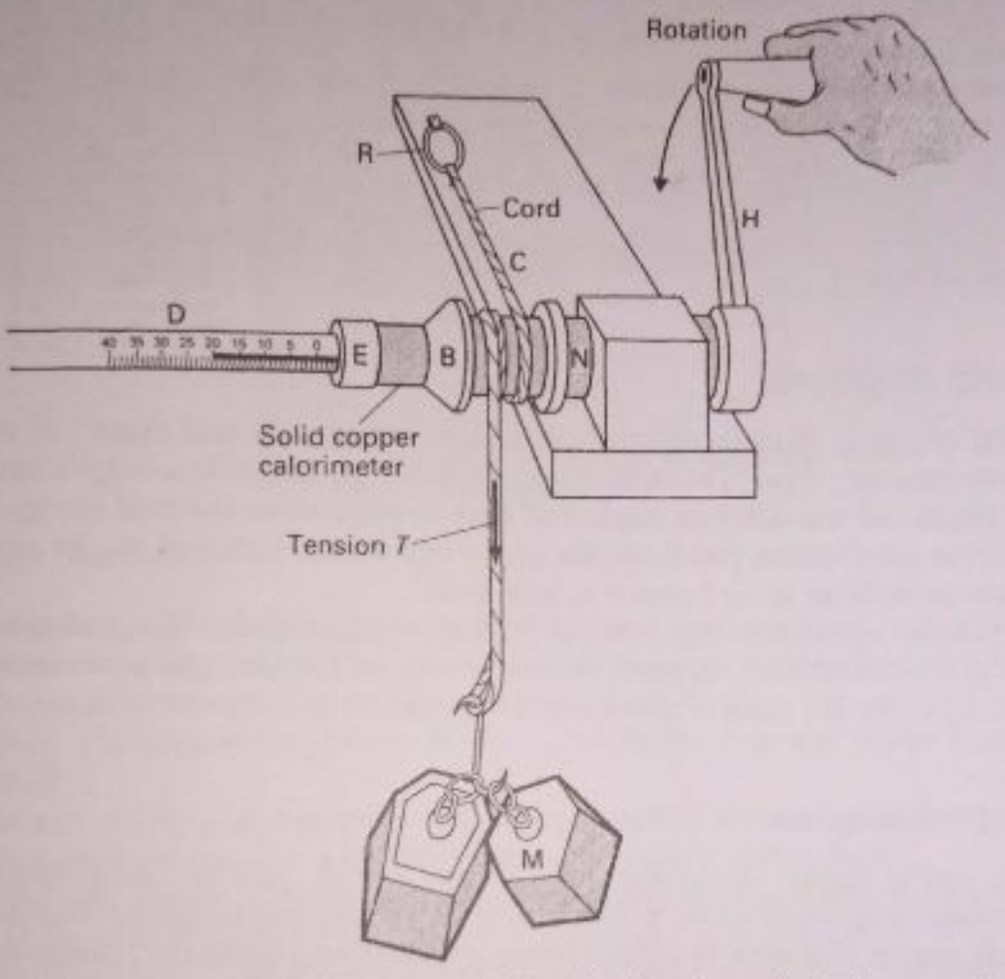


Fig. 12.3 Measurement of specific heat capacity of metal by changing mechanical energy to heat

Experiment

The apparatus consists of a solid cylindrical metal calorimeter B, mounted horizontally on a nylon frame N to provide insulation from the rest of the apparatus (Fig. 12.3). N can be turned by a crank H. A short-range thermometer D is fixed into a hole bored axially in the calorimeter and held firmly by a plastic collar E. One end of a flexible cord C is attached to a rubber band R and then wound several times round B as shown. Masses M of about 5 kg are attached to the other end of C.

The crank H is turned steadily. The surface of the calorimeter rubs against the cord and the heat produced is transferred to the metal, whose temperature rises. After a number of revolutions to obtain a suitable temperature rise, the final or maximum temperature is noted.

Results: The following results were obtained in one experiment.

Initial calorimeter temperature = 15·5 C
Final temperature = 32·5 C
Number of revolutions = 330

Scanned with CamScanner

Mass of calorimeter = 0·184 kg
Diameter of calorimeter = 2·54 cm
Mass on cord = 5 kg

Calculation: Work done against frictional force between cord and metal = work done against tension (force) in cord

= tension × distance moved
= 5 × 9·8 (N) × π × 2·54 × 330/100 (m) = 1290 J

Assuming all the mechanical energy is transferred to heat, since heat $Q = mct$, where c is the specific heat capacity of the metal of the calorimeter

$$0{\cdot}184 \times c \times (32{\cdot}5 - 15{\cdot}5) = 1290$$

$$\therefore\ c = \frac{1290}{0{\cdot}184 \times 17} = 410\ \text{J/kg K (approx.)}$$

Heat engines

Heat energy is transformed into mechanical energy in a *heat engine*, of which there are many types. The internal combustion engine, the diesel engine and the jet engine all use different sources of heat to produce mechanical energy. The internal combustion petrol engine makes car wheels turn and the jet engines make aeroplanes move forward at high speed.

Engines are always less than 100% efficient; that is, the energy obtained is always less than that supplied. In the jet engine, for example, some energy is wasted in the hot gases of the jet as they expand.

Petrol engine

The petrol engine, commonly used in motor cars, gets its energy from an exploded mixture of air and petrol vapour. It is sometimes called the 'four-stroke cycle' engine because four piston strokes or movements inside the cylinder repeat themselves continuously. The piston strokes are in the order *intake*, *compression*, *power*, *exhaust*. Fig. 12.4 shows the valve positions in the cylinder, the piston movements and the state of the gases during each stroke.

Intake As the piston moves down the cylinder the inlet valve opens and a mixture of air and petrol vapour is drawn into the cylinder.

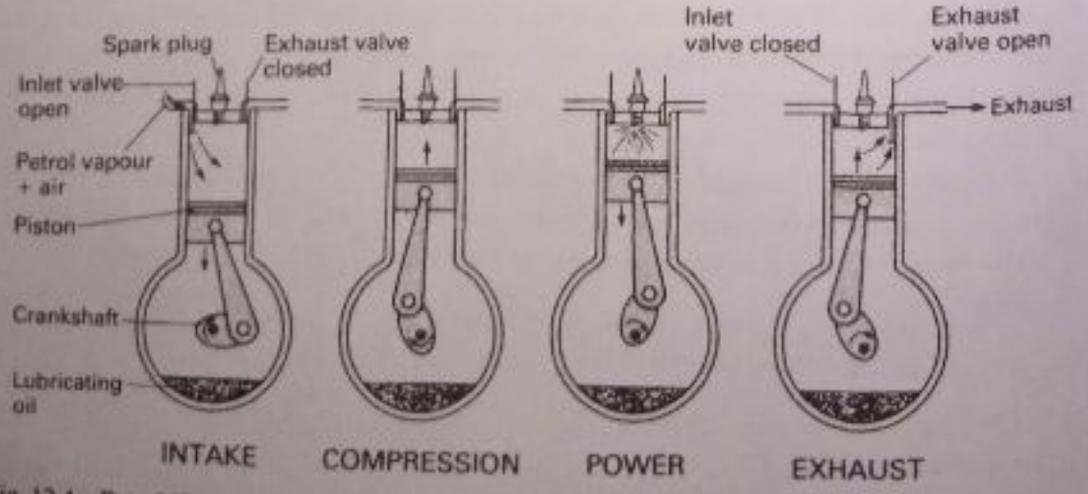
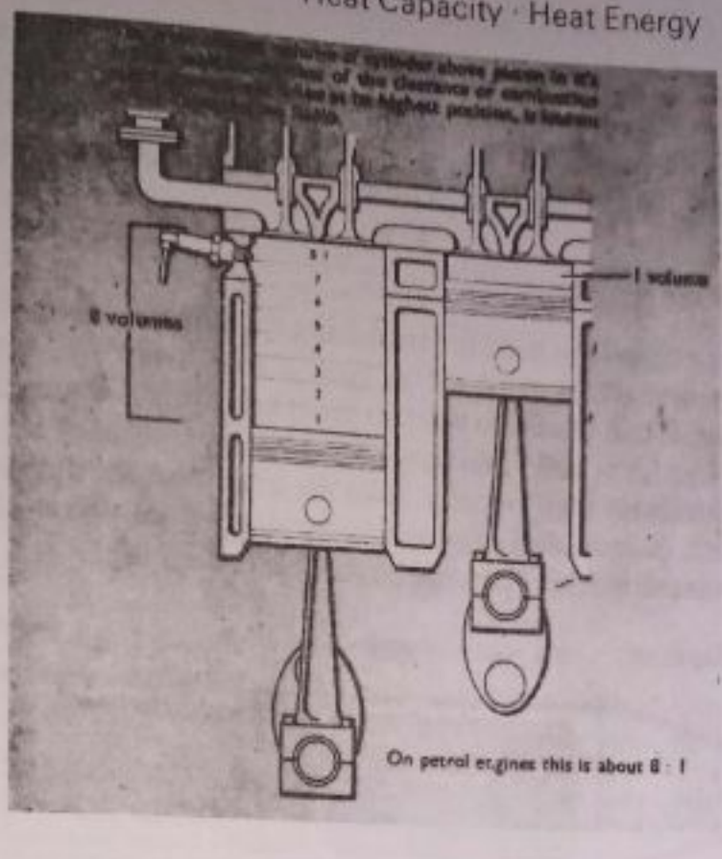


Fig. 12.4 Petrol (four-stroke) engine

Scanned with CamScanner



PETROL

In a petrol engine, a mixture of fuel (petrol) and air in the correct proportions, is drawn into the cylinder during the inlet stroke, and then compressed on the compression stroke. As the piston reaches approximately the top of the stroke, the mixture is ignited by means of a spark caused by an electrical discharge across the plug points. Rapid expansion of the gases, forces the piston downwards on its power stroke.

Plate 12A Petrol engine

Compression The piston moves up and compresses the mixture to about one-seventh of its volume. At the top of the stroke the mixture is exploded by a spark which passes between the electrodes of a spark plug.

Power The expanding gases force the piston down. A power stroke is now obtained.

Exhaust As the piston moves up, the exhaust valve is opened and the burnt or exhaust gases are expelled from the cylinder. The cycle is then repeated.

In the four-stroke engine a power stroke is obtained in a cylinder once every four strokes, which corresponds to two revolutions of the crankshaft. With an engine of say four cylinders, the power strokes in each cylinder are arranged to follow each other so that the engine works smoothly.

Jet engine

In a *turbo-jet engine*, air is drawn in at the front and compressed (Fig. 12.5). Then, after mixing with fuel, it is burnt in a combustion chamber and the hot gases are ejected with very high velocity through an exhaust nozzle at the rear. From

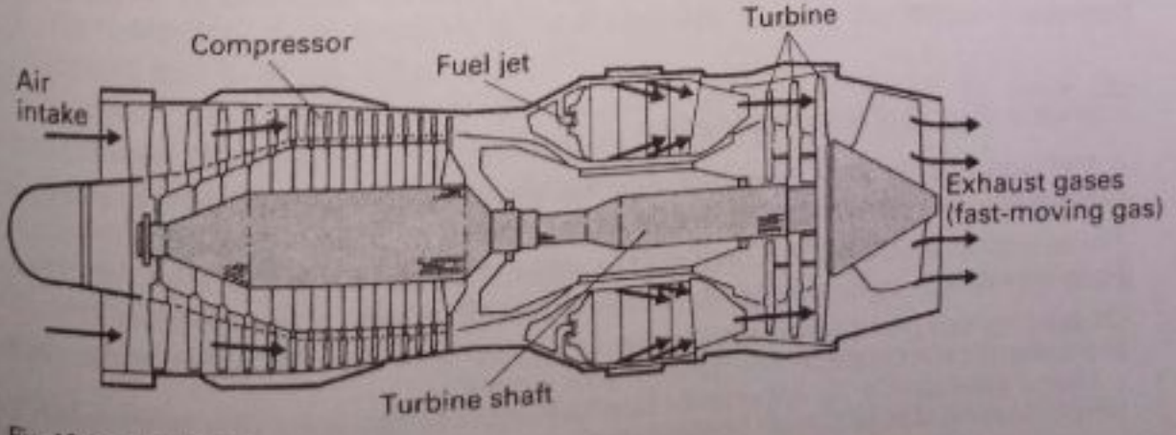


Fig. 12.5 Jet engine

Scanned with CamScanner

Newton's Law of Action and Reaction, the thrust on the engine is equal and opposite to that on the gas molecules. This thrust can drive an aeroplane forward at very high speeds. Jet aeroplanes can fly at very high altitudes, where the efficient engine takes in very large volumes per second of thin air.

Rocket engine

Rocket engines are used for launching satellites in orbits round the earth. Unlike jet engines, which need air as fuel, a rocket engines carries its own fuel and oxidiser. It can therefore work at great heights where very little air is present (Fig. 12.6). Liquid or solid fuel may be used. Very hot gases are expelled from the rear with extremely high velocity. Like the jet engine, the upward thrust on the rocket and load is equal and opposite to the downward thrust on the burnt gases. The rocket stops working as soon as the fuel is used up.

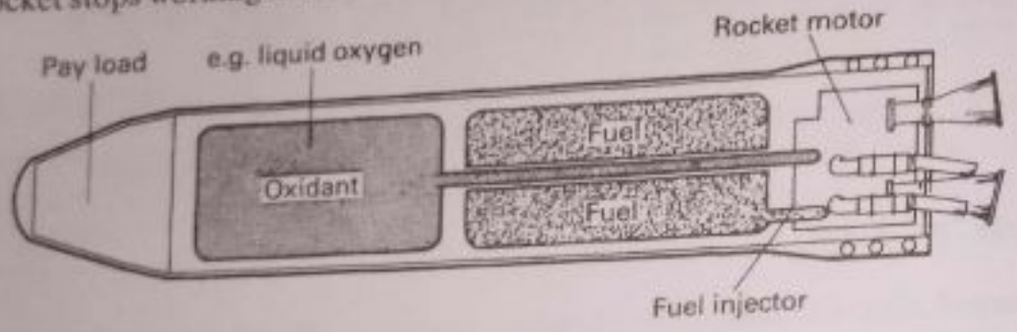


Fig. 12.6 Rocket engine

Transformation of sun's energy

The sun is the source of most of our energy. It sends out the energy in the form of radiation. On the earth this is transformed into other forms of energy by so-called natural processes.

Wind power, for example, arises from the unequal heating of land and sea, which produces convection currents. Water power, used in hydroelectric plants, comes from the water which evaporates from the sea and returns to the earth in the form of rain and creates lakes and rivers.

Oil has chemical energy derived initially from the sun. It is formed from decayed micro-organisms buried deep in the ground millions of years ago, after using energy from the sun to grow. Coal has chemical energy because it comes from decayed vegetation and wood, buried deep in the ground after using the sun's energy to grow. Today, energy from the sun is used directly to generate electrical energy in the solar battery and to generate heat energy in the solar furnace.

Example on specific heat capacity

A 180 watt heater and a thermometer were immersed in 0·5 kg of water in a copper calorimeter. The following readings were obtained.

Temperature/°C	30	36	40	45	49	54	57
Time/minutes	3	4	5	6	7	8	9

On graph paper plot a graph of temperature against time. Start the axes from the origin and draw the best straight line through your points.

Using your graph, or otherwise, find (a) room temperature (the temperature at which heating started), and (b) the specific heat capacity of water.

Scanned with CamScanner

Give two reasons why the value obtained for the specific heat capacity is more than the accepted value. State two precautions you would take in carrying out this experiment to ensure a more accurate value for the specific heat capacity. (L)

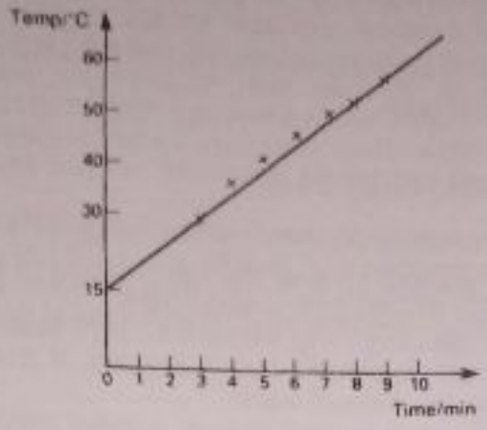


Fig. 12.7 Example

Fig. 12.7 shows the graph obtained and the best straight line through the points.

(a) Using the graph, the line through the first few points passes through 15°C when produced backwards to zero time, so this is the room temperature. (Otherwise, the average temperature rise per minute from 30°C to 40°C is 10°C for 2 min, or 5°C per minute; so 3 minutes before the 3 minute temperature of 30°C, the temperature was 3×5°C or 15°C less, which is a room temperature of $30 - 15$ or 15°C).

(b) If c is the specific heat capacity of water

$$\text{heat per min gained} = 0{\cdot}5 \times c \times 5 = 2{\cdot}5c \text{ J}$$

$$\text{But heat per min supplied} = 180 \times 60 = 10800 \text{ J}$$

Assuming no heat lost

$$2{\cdot}5c = 10800$$

$$\therefore \quad c = \frac{10800}{2{\cdot}5} = 4320 \text{ J/kg K}$$

Two reasons why c is more than the accepted value are that no allowance has been made for (a) the heat lost to the surroundings as the water temperature rose, and (b) the heat gained by the calorimeter. So in the calculation, 10800 J is more than the actual heat gained by the water to raise its temperature by 5°C/min.

Two precautions to ensure a more accurate value are (a) cool the water below room temperature and then warm it to the same temperature above room temperature; this reduces the heat lost to the surroundings, (b) lag the calorimeter, and make the surface shiny to reduce heat loss.

Summary

1. Units of quantity of heat are: joule (J), kilojoule (kJ), megajoule (MJ).
2. Heat capacity is the quantity of heat required to raise the temperature of a substance by 1°C or 1K. Unit: J/K.

Specific heat capacity, c, is the quantity of heat required to raise the temperature of unit mass of a substance by 1°C or 1K. Unit: J/kg K.

3. Heat capacity = mass × specific heat capacity.
4. Specific heat capacities of solids and liquids can be found by the method of mixtures: heat lost = heat gained: or by the method of electrical heating.
5. Mechanical energy can be transformed into heat energy by doing work against friction.

Work done = force × distance in direction of force.
Heat produced = heat capacity × temperature rise.

Scanned with CamScanner

6. Heat engines convert heat into mechanical energy. The petrol internal combustion engine explodes a mixture of air and petrol vapour by means of a spark from a sparking plug, and the order for a four-stroke cycle is intake, compression, power and exhaust. Jet and rocket engines produce a forward thrust (reaction) equal and opposite to the thrust (action) on hot gases ejected with high velocity from the rear.

Exercise 12 (Answers, page 641)

(Assume the specific heat capacity of water = 4·2 J/g K or 4200 J/kg K)

1. How many joules are required to raise the temperature of 500 g of copper from 16°C to 116°C? (*c* of copper = 0·4 J/g K or 400 J/kg K).
2. A liquid of specific heat capacity 3 J/g K or 3000 J/kg K rises from a temperature of 15°C to 65°C in 1 min when an electric heater is used. If the heater generates 63 kilojoules per min, calculate the mass of liquid.
3. An electric heater of 50 watts is used to heat a metal block of mass 5 kg. In 10 minutes, a temperature rise of 12°C is produced.
   (a) How much heat is produced by the heater in 10 minutes?
   (b) Calculate the specific heat capacity of the metal.
4. 4 g of a substance is burned completely, and the heat produced raises the temperature of 1200 g of water from 16°C to 26°C. Calculate the calorific or energy value of the substance in kilojoules per gram.
5. It was found that 0·018 m^3 of gas was consumed in raising the temperature of 1·20 litres of water from 20°C to 100°C. Find the calorific value of the gas in J per m^3. (C)
6. Explain the meaning of (a) specific heat capacity, (b) temperature, (c) quantity of heat and state how they are related.

   2 kg of lead at 100°C is dropped into a copper vessel containing 0·3 kg of water at 0°C and rapidly stirred. The final temperature attained by the vessel and its contents is 16°C. Taking the specific heat capacity of lead as 130 J/kg K, calculate the heat capacity of the copper vessel.
7. Describe an experiment to determine the specific heat capacity of a lump of metal. State the precautions that should be taken and show how the result is calculated from the measurements made.

   A small immersion heater raises the temperature of 250 g of water contained in a vacuum flask from 15°C to 55°C in 5·0 minute. When the experiment is repeated with 500 g of water in the same flask a time of 9·0 minute is required for the same range of temperature.

   Determine (a) the heat per minute given out by the immersion heater, (b) the heat absorbed by the vacuum flask for each degree C it rises in temperature. (L)
8. Describe an experiment to determine the specific heat capacity of a liquid. Point out errors which are likely to occur and the precautions you would take to minimise them.

   A machine with an input power of 2 kilowatts uses 80% of this power. If all the remaining energy appears as heat, and heats 40 kg of iron, what will be the rise of temperature of this iron in 2 min? (Specific heat capacity of iron = 0·5 J/g K or 500 J/kg K). (N)

Energy conversion (Where necessary, assume $g = 10\ m/s^2$)

9. An object of mass 80 kg falls from a height of 100 m. Calculate the heat developed at the ground.
10. A waterfall has a height of 50 metre. Considering 1 kg of water and the energy it loses, estimate the difference in temperature between the top and bottom of the waterfall. Why is the answer approximate?
11. How do you explain the rise in temperature of a bicycle pump during the action of pumping up a tyre? (L)
12. Describe, with the aid of a diagram, the action of a single cylinder four-stroke internal combustion engine. (Details of the ignition system and the carburettor are not required.) (N)
13. State the meaning of the terms mechanical energy, joule.

    A calorimeter of thermal capacity 42 J/K contains 500 cm^3 of water at 20°C. An automatic stirrer dips into the water and is driven by a $\frac{1}{4}$ h.p. motor. Assuming that all

Scanned with CamScanner

the work done by the stirrer heats the water, calculate the temperature of the calorimeter and its contents after 10 min. (Take 1 h.p. = 746 W.) (O and C)

14. Describe an experiment, based on conversion of mechanical or electrical energy, to show that the specific heat capacity of water is about 4·2 J per g per K (4200 J/kg K).

A piece of lead falls 3 m from rest, coming to rest again on the ground. Calculate the rise in its temperature, the specific heat capacity of lead being 0·13 J/g K or 130 J/kg K. State the assumptions you make in the calculation. (L)

15. Draw labelled diagrams, one for each case, to show the basic arrangement of (a) a jet engine, (b) a rocket engine. State two similarities and two differences in their operation.

A mass of 0·50 kg (500 g) of water in an aluminium container of mass 0·20 kg (200 g) both initially at 15·0°C are placed in a refrigerator maintained at −5·0°C. Calculate the quantity of heat which has to be removed from the water and from the container to reduce their temperature to that of the refrigerator. Assume the following values.

		Ice	Water	Aluminium
Specific heat capacity	in J/kg K	$2{\cdot}0 \times 10^3$	$4{\cdot}2 \times 10^3$	$0{\cdot}8 \times 10^3$
	in J/g K	2·0	4·2	0·8
Specific latent heat	in J/kg	340×10^3		
	in J/g	340		

(N)

Multiple-Choice Questions

16. A metal X and a metal Y both lose the same quantity of heat when their temperature falls from 16°C to 10°C. The specific heat capacity of X is twice that of Y. Then the ratio *mass of X/mass of Y* is

A 3/1 B 2/1 C 1/2 D 1/3 E 1/1·6

17. Dropped from a height, a metal block X becomes slightly warmer after hitting the ground. This is because

A X is a good conductor
B the molecules of C expand on hitting the ground
C X gains energy from the ground
D the molecules of X gain energy
E X has a high specific heat capacity

18. The following substances are heated from an initial temperature of 10°C. Which of A to E gains the greatest amount of heat?

A coffee-tin warmed to 80°C
B sauce-pan of milk warmed to boiling point
C hot-water tank in house warmed to 50°C
D metal of bicycle wheel warmed to 60°C
E tungsten filament of lamp warmed to 1200°C

19. If no lubricating oil is used, the metal X of a piston rubs against the metal Y of an engine casing. The amount of heat produced depends on

A the specific heat capacity of X
B the frictional force between X and Y
C the mass of Y
D the specific heat capacity of Y
E the conductivity of X and Y

Scanned with CamScanner

13. Latent heat · Vapours

Latent heat

When sufficient heat is given to a solid, it melts and passes into the liquid state. Further heat will make the liquid change into a gas.

As an example of a change of state from solid to liquid and then to a gas, suppose a block of ice is cooled by a freezing mixture to $-10°C$ and then heated steadily. At first the ice, which has a specific heat capacity of about 1700 J/kg K, rises in temperature to 0°C. It now changes into water, a liquid, but although heat continues to be supplied, *the temperature remains constant at 0°C until all the ice has changed to water*. Since the temperature was constant, the heat needed to change from a solid to a liquid was called *latent heat* ('hidden' heat). The heat given to a solid when its temperature rises is not latent heat, because there is no change of state.

When the water formed from the ice is heated further, its temperature rises until it reaches about 100°C. Here it begins to change into vapour or gas, and the temperature remains constant until *all* the water is changed to vapour. The heat is again called *latent heat* because a change of state, this time from liquid to gas, occurs. Experiment shows that the latent heat to change a given mass from ice to water is about one-seventh of the latent heat to change the same mass from water to vapour under normal conditions. After a vapour is formed, further heat causes the vapour temperature to rise once more.

Molecular explanation

We know that the molecules of a solid are kept in an average fixed position by strong forces of attraction (page 146). So the latent heat is the energy needed to break down the forces which hold the molecules of a solid in a regular pattern. The molecules then move about more freely as they are in the liquid state.

Similarly, the latent heat to change a liquid to a vapour without temperature change is the energy needed to overcome the forces of attraction between the molecules of a liquid. The molecules are then practically independent of each other and exist as a gas. The energy needed in this case is much greater than from solid to liquid, and the latent heat is therefore much greater.

Fusion

We first consider the change of state from solid to liquid, called *fusion*.

The *specific latent heat of fusion* of a substance is the quantity of heat required to change 1 kg (or 1 g) from a solid at the melting point to a liquid at the same temperature. For example, ice melts at 0°C, and its specific latent heat of fusion *l*

Scanned with CamScanner

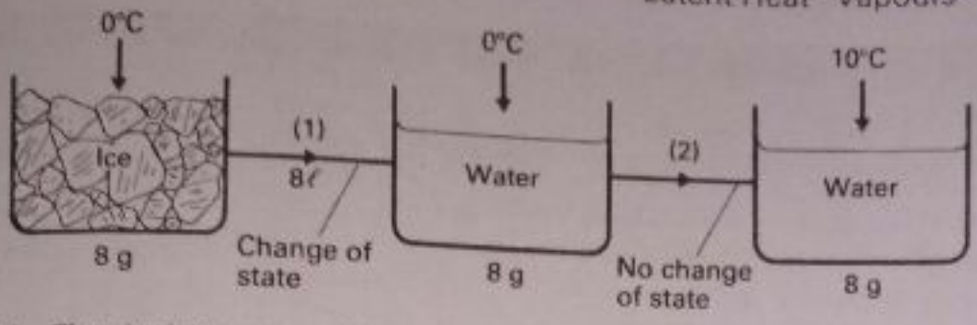


Fig. 13.1 Changing ice to water at 10°C

is the heat required to change 1 kg (or 1 g) of ice at 0°C to water at 0°C. This is about 320000 J/kg or 320 J/g.

Consider 8 g of ice at 0°C, heated so that it forms water at 10°C. As shown diagrammatically in Fig. 13.1, this change of state takes place quickly in two steps:

First, the 8 g of ice melts and forms water at 0°C (1).

Second, the temperature of this water, mass 8 g, rises from 0°C to 10°C (2).

In step (1)

heat required = latent heat = $8l$, where l is the specific latent heat of fusion of ice

$= 8 \times 320 = 2560$ J, assuming l is 320 J/g.

In step (2), the heat required = $mct = 8 \times 4{\cdot}2 \times (10 - 0) = 336$ J (note this step is *not* a change of state).

$\therefore$ total heat supplied $= 2560 + 336 = 2896$ J

Suppose a larger mass of ice, 0·1 kg, at 0°C melts and forms water at 10°C. If $l = 3{\cdot}2 \times 10^5$ J/kg, or 320000 J/kg, then

heat needed to change 0·1 kg of ice

at 0°C to water at 0°C $= 0{\cdot}1\,l$

$= 0{\cdot}1 \times 320000 = 32000$ J

and heat to raise temperature of 0·1 kg of water from

0°C to 10°C $= mct$

$= 0{\cdot}1 \times 4200 \times (10 - 0) = 4200$ J

$\therefore$ total heat needed $= 32000 + 4200 = 36200$ J

The method of heat exchanges can be used to measure the specific latent heat of fusion of ice.

Experiment to measure l of ice

Weigh a clean, dry copper calorimeter empty, together with a stirrer. Half-fill the calorimeter with water previously warmed to about 8 C above room temperature and reweigh it to find the mass of water.

Record the temperature of the water when it is say about 5 C above room temperature and then immediately add small pieces of ice, dried on blotting or other paper, to the water. Add one piece of ice at a time and stir the water until *all* the ice is melted before adding another piece. Continue until the final water temperature is 5 C *below* the room temperature. Record this temperature. (In this way we have largely compensated for the heat exchange between the calorimeter and contents and the surrounding air when they

Scanned with CamScanner

are at different temperatures.) Finally, reweigh the calorimeter to find the mass of ice melted.

Measurements: Suppose the following measurements were obtained.

Mass of calorimeter (c = 400 J/kg K) = 0·10 kg
Mass of water (c = 4200 J/kg K) = 0·20 kg
Mass of melted ice = 0·025 kg
Initial water temperature = 21·0 C
Final water temperature = 11·0 C

Calculation: The ice melts and first forms water at 0 C (latent heat of fusion gained = ml), then the temperature of this water rises to 11 C (no change of state, so heat gained = mct). Since 0·025 kg of ice was melted

heat gained by ice in forming water at 11 C
$$= 0{\cdot}025l + 0{\cdot}025 \times 4200 \times 11 = 0{\cdot}025l + 1155$$

Now heat lost by water and calorimeter
$$= 0{\cdot}2 \times 4200 \times (21 - 11) + 0{\cdot}1 \times 400 \times (21 - 11) = 8400 + 400$$

Since heat lost = heat gained
$$0{\cdot}025l + 1155 = 8400 + 400 = 8800$$

Solving
$$\therefore\ l = \frac{8800 - 1155}{0{\cdot}025} = 306\,000 \text{ J/kg}$$

Calculation using gram as mass unit: Using c = 4·2 J/g K for water and 0·4 J/g K for the copper calorimeter, then since 25 g is the mass of ice melted, the equation becomes

$$25l + 25 \times 4{\cdot}2 \times (11 - 0) = 200 \times 4{\cdot}2 \times (21 - 11) + 100 \times 0{\cdot}4 \times (21 - 11)$$

So
$$25l + 1155 = 8400 + 400 = 8800$$

Solving
$$l = \frac{8800 - 1155}{25} = 306 \text{ J/g (approx.)}$$

Melting point and measurements

The *melting point* of a substance is defined as the temperature at which the substance changes from the solid to the liquid state.

We can find the melting point of naphthalene, a white, crystalline solid obtained from coal tar, as follows.

Experiment

Fill a boiling tube with naphthalene and support the tube vertically. Using a large beaker with water as a water-bath, warm the tube in the water until the naphthalene just melts. Now put a thermometer inside the liquid naphthalene and continue heating until a temperature of about 90°C is reached (Fig. 13.2). Then take away the water-bath, leaving the tube to cool in the air, and record the temperature readings as they fall every half-minute. At a certain stage the temperature remains *constant* for a time—when a solid is forming—after which the temperature falls once more.

Graph: Plot a graph of temperature against time (Fig. 13.2). The temperature corresponding to the *flat portion* is the melting point of naphthalene. At this temperature, the latent heat of fusion, which is given *out* when the liquid changes to the solid state, just balances the heat lost to the surroundings so the temperature remains constant.

Scanned with CamScanner

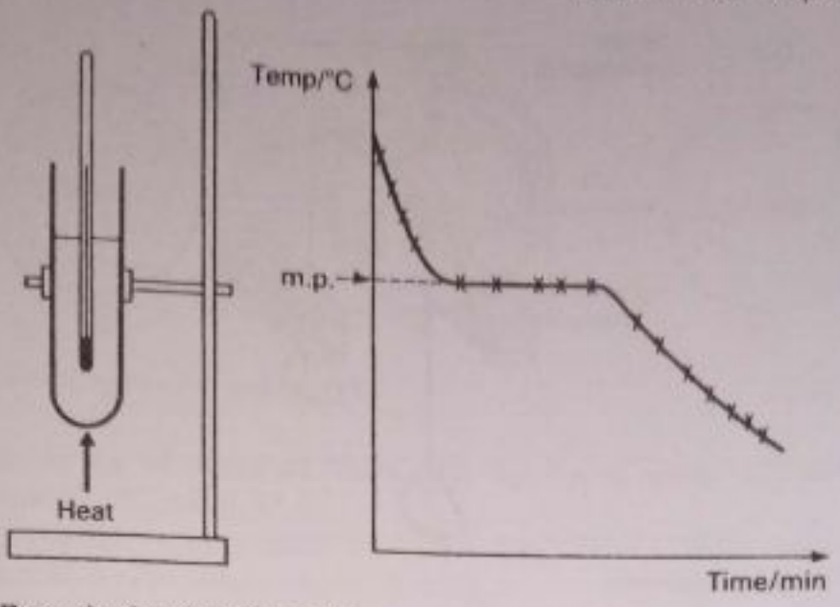


Fig. 13.2 Determination of melting point

From the molecular theory of liquids and solids, we can see why heat is given out when a liquid changes to a solid at constant temperature. In most liquids the molecules are slightly further apart than in the solid state. Thus when the material solidifies the molecules move together under attractive forces. So their potential energy decreases and this energy appears as heat.

Expansion and contraction on fusion

Because of the expansion of water on freezing, ice is less dense than water. So ice floats on water. Icebergs float and drift a long way before they melt. Only about one-tenth of the volume of an iceberg lies above the surface of the sea, the remaining nine-tenths being hidden from view below the surface. In January 1966 an iceberg five times as big as outer London ran aground near Russia's south polar base, Molodezhnaya, in Enderby Land. It is probably the biggest one ever seen, measuring about 8000 km^3. The height of the iceberg was 30 m above sea-level.

When the air temperature falls below 0°C, the water in exposed unlagged pipes outside houses becomes frozen. Since the ice formed from a given mass of water has a greater volume than the water, the expansion may crack the pipes. This is not noticeable until the temperature rises above 0°C. The ice then melts and the water leaks through the cracks in the metal pipe.

Effect of pressure on melting point

The melting point of a solid such as ice is changed by pressure. This can be shown as follows.

Experiment

A thin wire is passed round a large block of ice and two heavy weights are attached to the ends of the wire (Fig. 13.3). As time goes on, the wire slowly sinks into the ice but no cut can be seen above it. Eventually, with the block still in one piece, the wire passes through at the bottom.

Scanned with CamScanner

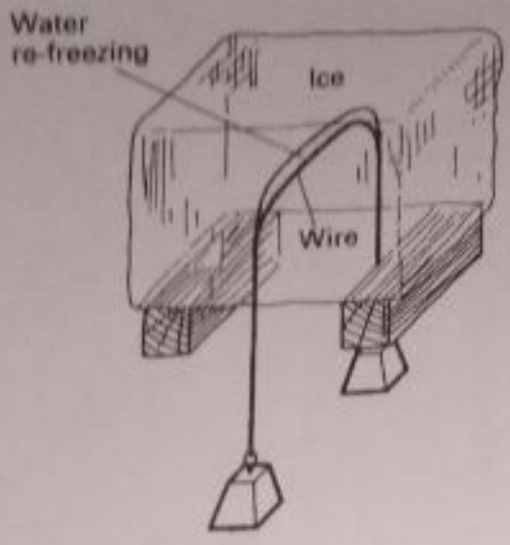


Fig. 13.3 Refreezing (regelation)

We can explain what happens as follows. The thin wire produces such a high pressure on the ice below it that the ice melts. The wire now sinks into the water formed, which immediately re-freezes above the wire because there is no extra pressure here. So the wire passes slowly through the ice, leaving the block unchanged. This is a demonstration of 'regelation' or refreezing.

We can understand why increased pressure on ice makes it melt. Increased pressure makes the volume of ice decrease. Now ice *diminishes* in volume on forming water. So increased pressure assists a change of state to water, that is, *increased pressure lowers the melting point of ice.*

Some substances, however, expand when changing from a solid to a liquid state, for example, tin, paraffin-wax and a metal alloy used in printing. Under increased pressure, the melting points of these substances rise.

Ice skates have a narrow blade. So the pressure below the blade due to the weight of the skater (pressure = weight per unit area) is high and the ice melts. Assisted by the heat produced by friction the blade of the skater moves through a film of water on the ice below it.

Two small pieces of ice, pressed one against the other, stick together. The increased pressure lowers the melting point, and those parts of the ice under high pressure therefore melt. The water formed flows away to a place where the pressure is less. Here it refreezes, so binding the two pieces of ice together.

A hard snowball is formed by compressing snow, a form of ice. The increased pressure makes some of the ice melt and this refreezes solid when the pressure is released.

Latent heat of vaporisation

The change of state from liquid to vapour or gas is called *vaporisation.*

The *specific latent heat of vaporisation l* of a substance is the quantity of heat required to change unit mass of the substance from its liquid to its vapour state at the boiling point. About 2200 J is needed to change 1 g of water at the boiling point to vapour at the same temperature. So $l = 2200\ \text{J/g}$ or $2{\cdot}2 \times 10^6\ \text{J/kg}$ for steam.

Consider the heat given up when 8 g of steam at 100°C condenses to water at 45°C. As on page 215, this occurs in two stages: (1) 8 g of steam at 100°C

Scanned with CamScanner

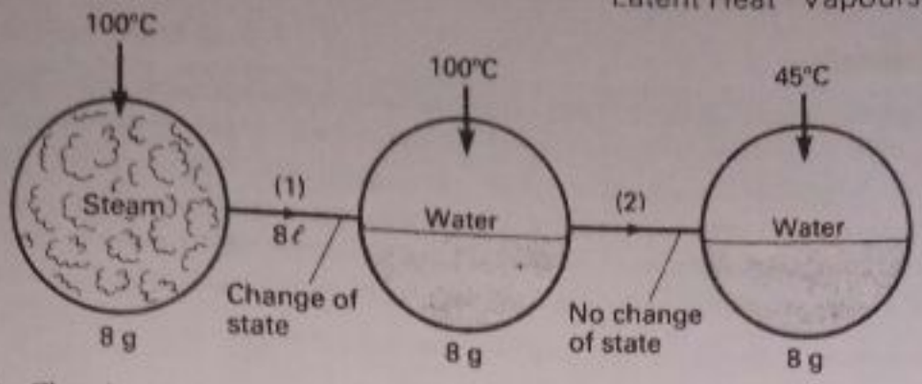


Fig. 13.4 Changing steam to water at 45°C

condenses to 8 g of water at 100°C; (2) the 8 g of water at 100°C falls in temperature to 45°C (Fig. 13.4).

In (1), the latent heat given up $= 8l = 8 \times 2200 = 17600$ J, where l is the specific latent heat of steam, about 2200 J/g.
In (2), the heat given up by the water $Q = mct = 8 \times 4{\cdot}2 \times (100 - 45) = 1848$ J; there is no change of state here.

$$\therefore \text{ total heat given up} = 17600 + 1848 = 19448 \text{ J}$$

Suppose a larger mass of steam, 0·1 kg, at 100°C condenses to water at 40°C. Then, using $l = 2{\cdot}2 \times 10^6$ J/kg and c for water $= 4200$ J/kg K

(1) heat given up when steam at 100°C condenses to water at 100°C
$= 0{\cdot}1l = 0{\cdot}1 \times 2{\cdot}2 \times 10^6 = 2{\cdot}2 \times 10^5 = 220000$ J

(2) heat given up when water cools from 100°C to 40°C
$= mct = 0{\cdot}1 \times 4200 \times (100 - 40) = 420 \times 60 = 25200$ J

So total heat given up $= 220000 + 25200 = 245200$ J

Specific latent heat of steam

The specific latent heat of vaporisation of steam can be found by passing steam into water in a calorimeter, where it condenses and gives out its latent heat. One form of apparatus is shown in Fig. 13.5. The steam is dried by a water trap S, which ensures that only steam and not liquid water enters the calorimeter by the short delivery tube T. This prevents errors in the calculation, which assumes that only steam enters. T can be lagged to stop condensation of steam.

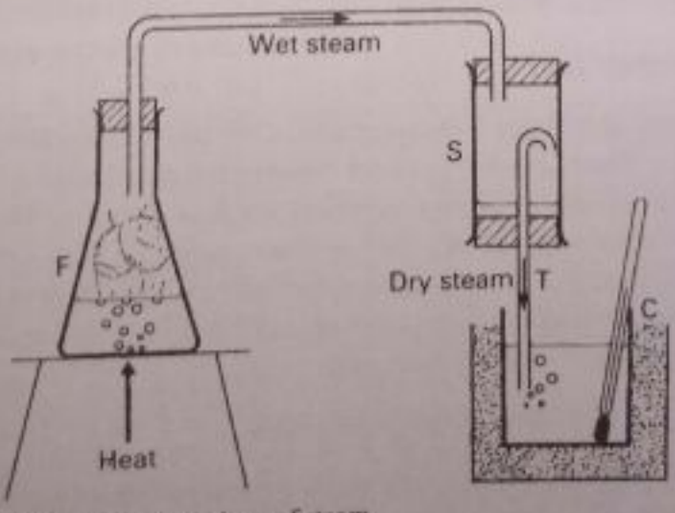


Fig. 13.5 Apparatus for finding latent heat of steam

Scanned with CamScanner

Experiment

Weigh a clean, dry calorimeter C empty, then half-fill it with water and reweigh. Replace the calorimeter and water inside its lagging material. Boil the water in the flask F. When the steam is coming out freely from the end of T, raise the calorimeter so that T passes steam into the water and condenses. After the temperature has risen about 25°C above room temperature, remove the calorimeter and note the final temperature of the water after it is stirred. Finally, reweigh the calorimeter to find the mass of steam condensed in it.

Measurements: Suppose the following measurements were obtained.

Mass of calorimeter ($c = 0{\cdot}4$ J/g K) = 60·0 g
Mass of calorimeter + water ($c = 4{\cdot}2$ J/g K) = 144·0 g
Mass of calorimeter + water + steam = 147·8 g
Initial water temperature = 17·5°C
Final water temperature = 43·0 C

Calculation: When the steam passes into the water it first changes from steam at 100°C to water at 100°C (latent heat given up $= ml$) and then drops in temperature to 43°C (no change of state, heat given up calculated from $Q = mct$). Since the mass of steam is 3·8 g from the measurements

$$\text{heat given up by steam} = 3{\cdot}8l + 3{\cdot}8 \times 4{\cdot}2 \times (100 - 43)$$

Now heat gained by water + heat gained by calorimeter, using mct,

$$= 84 \times 4{\cdot}2 \times (43 - 17{\cdot}5) + 60 \times 0{\cdot}4 \times (43 - 17{\cdot}5)$$

Assuming no heat loss, then

$$3{\cdot}8l + 910 = 9607$$

So
$$l = \frac{9607 - 910}{3{\cdot}8} = 2300 \text{ J/g (approx.)}$$

Calculation using kilogram as mass unit: Changing masses to kg, and using $c = 4200$ J/kg K for water and 400 J/kg K for calorimeter

$$0{\cdot}0038l + 0{\cdot}0038 \times 4200 \times (100 - 43) = 0{\cdot}084 \times 4200 \times (43 - 17{\cdot}5) + 0{\cdot}06 \times 400 \times (43 - 17{\cdot}5)$$

Solving
$$\therefore\ l = 2{\cdot}3 \times 10^6 \text{ J/kg (approx.)}$$

To reduce the error due to loss of heat to the surroundings when the temperature of the water rises as steam is passed in, the water can initially be cooled by melted ice until the temperature is, say, 8 C below the temperature of the room and then steam passed in until it rises about 8 C above room temperature. Compare this with the experiment to find the specific latent heat of fusion of ice, page 215.

Electrical method for l

The specific latent heat of steam can be measured roughly by using an immersion heater inside water in a plastic beaker. The heater is supported by a stand and the beaker with water is placed on the pan of a lever balance or a top pan balance.

The water is heated until it starts to boil. As boiling continues the water evaporates and the reading on the balance is noted every minute for 10 min. At the end of this time, 600 seconds, suppose the mass of water vaporised is 14·0 g and a 50 W immersion heater was used. Then if l is the specific latent heat in J/g,

$$14l = \text{heat supplied} = 50 \times 600 \text{ J}$$

$$\therefore\ l = \frac{50 \times 600}{14} = 2140 \text{ J/g} = 2{\cdot}1 \times 10^6 \text{ J/kg (approx.)}$$

Scanned with CamScanner

Examples on latent heat

1. A lump of copper of mass 0·5 kg is placed in an oven for some time and then transferred quickly to a large dry block of ice at 0°C (Fig. 13.6). When the temperature of the lump of copper reaches 0°C, 0·3 kg of ice is found to have melted. Estimate the temperature of the oven. (Take the specific heat capacity of copper as 400 J/kg K and the specific latent heat of fusion of ice as $3{\cdot}2 \times 10^5$ J/kg.)

Give *two* reasons why this estimate might be lower than the actual temperature of the oven. (L)

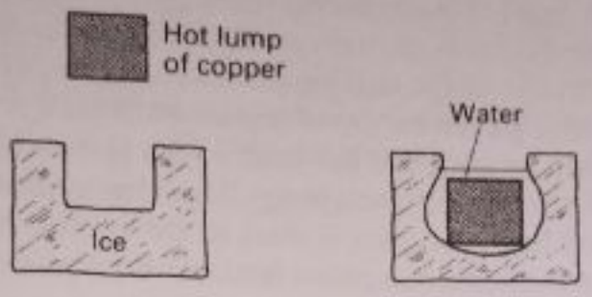


Fig. 13.6 Example

Suppose x is the temperature of the oven in °C.

Then heat lost by copper $= mct = 0{\cdot}5 \times 400 \times (x-0) = 200x$

and heat gained by ice $= ml = 0{\cdot}3 \times 3{\cdot}2 \times 10^5 = 96000$

$$\therefore\ 200x = 96000$$

$$\therefore\ x = \frac{96000}{200} = 480°\text{C}$$

Two reasons why 480° C may be lower than the actual oven temperature are (a) heat is lost while the hot copper is transferred to the ice, (b) heat is gained by the ice from the surroundings, which have a temperature higher than 0°C.

2. A copper calorimeter of mass 120 g contains 70 g of water and 10 g of ice at 0°C. What mass of steam at 100°C must be passed into the calorimeter to raise the temperature to 40°C?

(Specific latent heats: of ice 320 J/g; of steam 2200 J/g; specific heat capacity of copper is 0·4 J/g K.)

Heat given up by steam condensing to water at 40°C

$$= ml + m \times 4{\cdot}2 \times (100-40) = 2200m + 252m = 2452m\ \text{J}$$

Heat gained by water and calorimeter from 0° to 40°C

$$= 70 \times 4{\cdot}2 \times (40-0) + 120 \times 0{\cdot}4 \times (40-0)$$
$$= 13680\ \text{J}$$

Heat gained by ice in melting to water at 40°C

$$= 10 \times 320 + 10 \times 4{\cdot}2 \times (40-0)$$
$$= 4880\ \text{J}$$

Since heat lost = heat gained

$$2452m = 13680 + 4880 = 18560$$

$$\therefore\ m = \frac{18560}{2452} = 7{\cdot}6\ \text{g}$$

Evaporation

Small pools of water, formed on roads after rain, soon disappear. The change of water or other liquid from a liquid to a gaseous state is known as *evaporation*.

Some liquids evaporate easily; others do not.

Scanned with CamScanner

Pour a few drops of methylated spirit onto your hand. Wipe it away then repeat with olive oil and finally water. Each time observe if evaporation occurs and whether your hand feels cold. Note that only the spirit evaporates easily and that your hand feels cold at the same time.

Liquids, such as methylated spirit and ether, which evaporate very easily have low boiling points. They are called *volatile* liquids. Latent heat is needed to change a liquid to vapour at the same temperature, and this heat is absorbed from the hand when a few drops of a volatile liquid are placed on it. Consequently, the hand feels colder. We will see later that volatile liquids are used in refrigerators.

Another demonstration of the cooling produced by evaporation is shown in Fig. 13.7. A beaker B is placed on top of a little water on a wooden block, and some liquid ether then poured into the beaker. Air is now blown through the ether with the aid of a bellows or foot-pump. The ether evaporates quickly, and after a time it is found that the beaker is stuck to the wooden block. A thin layer of ice is formed between them. The latent heat of evaporation for the ether was taken from the water, which then turned into ice.

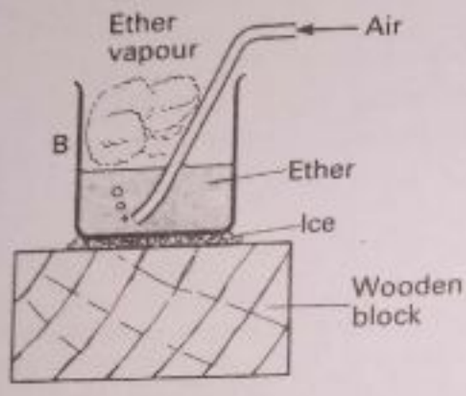


Fig. 13.7 Evaporation producing cooling

Factors affecting evaporation

Generally, evaporation depends on:

(a) *Temperature of the liquid* Wet clothes hung on a line dry quicker on a warm day. So the higher the temperature, the greater the rate of evaporation.

(b) *Area of exposed surface* A wet sheet on a line dries more quickly when opened than when left folded. So the evaporation increases as the water surface area increases.

(c) *Rate of removal of vapour* Wet clothes dry quicker on a windy day than on a calm day. This effect can also be felt when a finger is moistened and then waved about in the air.

Molecular explanation of evaporation

In the kinetic theory, evaporation is explained by the random motion of molecules in a liquid. They all have different velocities, and at any instant they are moving in many different directions. Near the surface, some liquid molecules with high velocities in an upward direction may have sufficient kinetic energy to break through the surface. Then they become molecules of vapour (Fig. 13.8). Others which escape may meet molecules of air or vapour and return to the liquid

Scanned with CamScanner

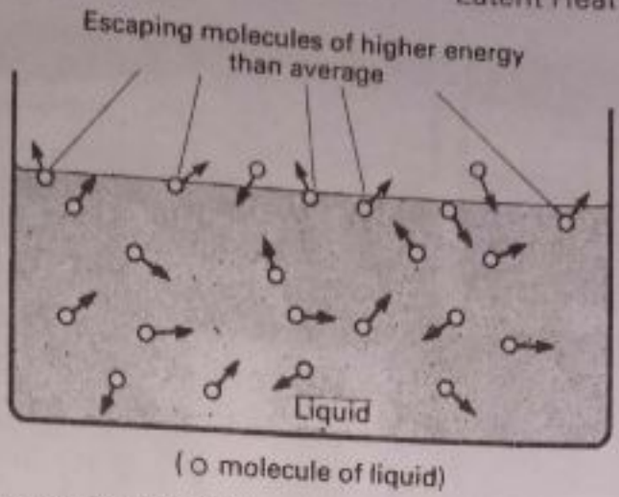


Fig. 13.8 Kinetic theory explanation of evaporation

after collision. So a wind, which removes molecules of air and vapour, helps evaporation.

It can now be seen that evaporation is due to the escape of those molecules of the liquid which have greater energy than the others. The average kinetic energy of the liquid remaining is therefore reduced, and so its temperature is lowered. This explains the cooling of a liquid due to evaporation.

Refrigerator

Fig. 13.9 shows the principle of how the domestic refrigerator works. A volatile liquid–one which evaporates at fairly low temperature–is put into the pipes surrounding the freezing chamber where food is stored. The liquid evaporates, so cooling the pipes and the surrounding air, from which it takes the necessary latent heat. The vapour formed is compressed by a pump P into a condenser Y outside the refrigerator. Here it condenses and gives out latent heat, and this is lost to the air by using metal cooling fins F round Y.

From Y, the liquid is forced into the pipes at X, where it again evaporates. In this way the refrigerator chamber is kept cold continuously. A valve (not shown) keeps X at a lower pressure than Y, so that the volatile liquid always evaporates.

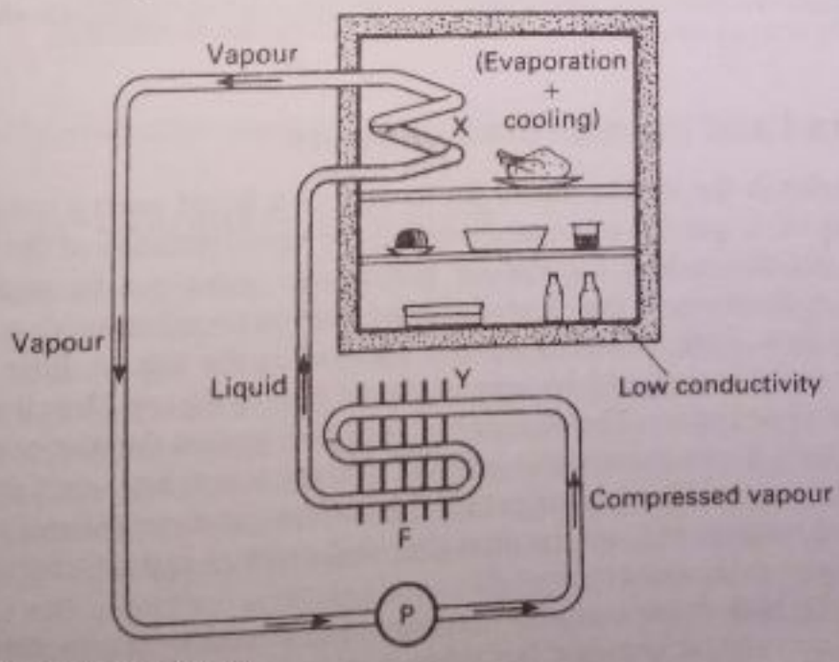


Fig. 13.9 Refrigerator principle

Scanned with CamScanner

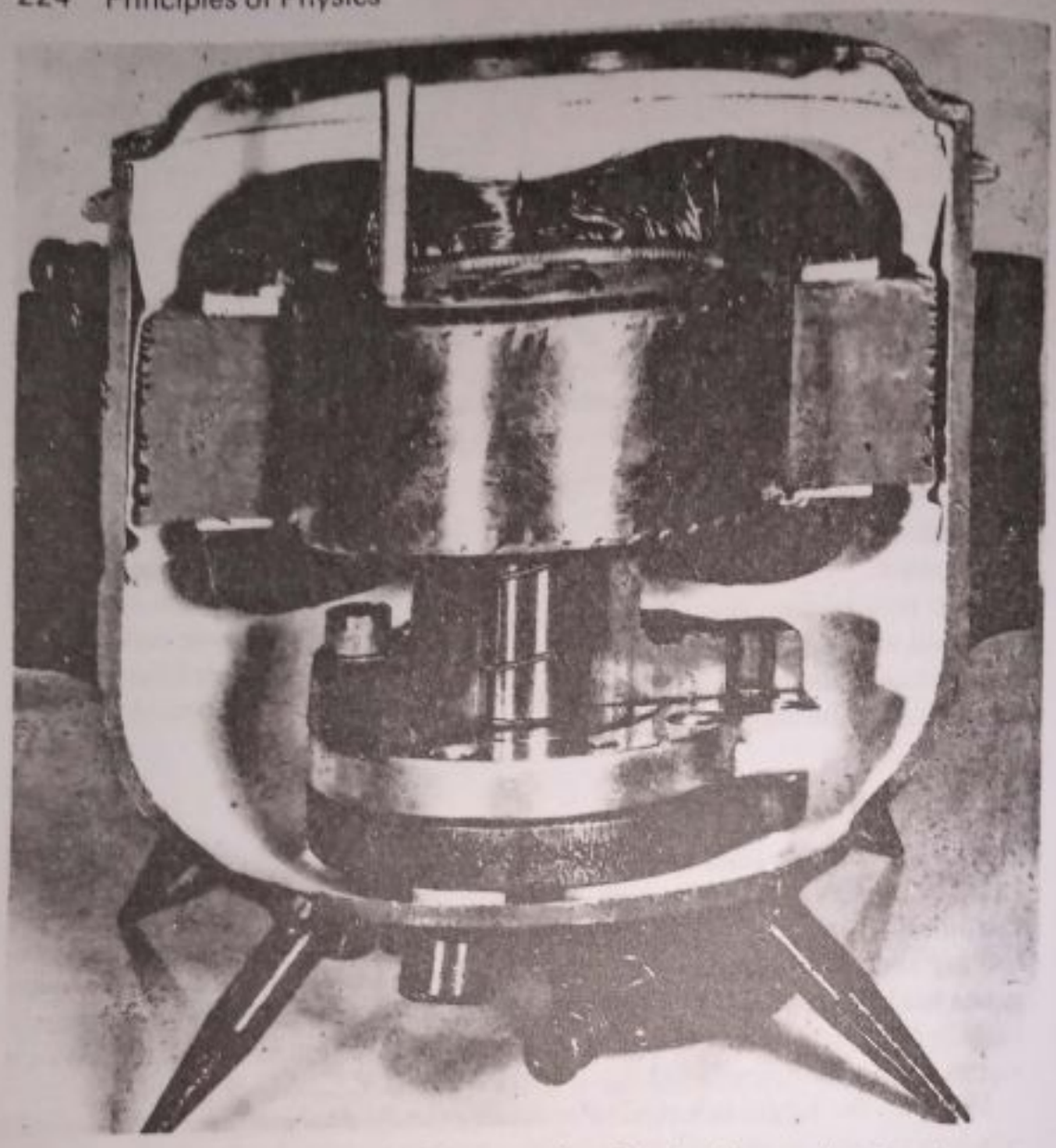


Plate 13A Refrigerator compressor unit. Cutaway section of interior shows an electric motor (magnetic field provided by coils and rotor in centre) which drives a compressor mounted at the base. The refrigerant vapour is compressed inside and then passes out into a condenser where it changes slowly to liquid. (*Frigidaire*)

Saturated and unsaturated vapours

The molecules in the vapour above the surface of a liquid exert a pressure like those in any other gas. This pressure is called the *vapour pressure* of the liquid.

Fig. 13.10a shows how the vapour pressure of water can be measured at different temperatures. A and B are inverted barometer tubes inside a mercury trough. At first, both A and B have a vacuum at the top. A little water is introduced at the bottom of A by a pipette and it rises to the top. Here it vaporises and fills the closed space. The vapour pressure now pushes the mercury column below the level R. As more water is added a stage is reached when no further vaporisation occurs. The vapour pressure now remains constant and any more water added remains at Q on the mercury. Molecules of vapour now enter and leave the water at the same rate.

The vapour in A, in contact with the water at Q, is said to be *saturated vapour*. The saturation vapour pressure (s.v.p.) in mmHg is the height RQ of mercury.

Scanned with CamScanner

The vapour at the top of the closed tube A', however, is *unsaturated* water vapour because no liquid is present here. The unsaturated vapour has a pressure equal to the height RS of mercury.

At a constant temperature the saturation vapour pressure is constant. When the temperature of the liquid bath T shown surrounding A is increased, some of the water at Q evaporates. The increased height of RQ is the s.v.p. at the new temperature. In this way, with water always present at Q, the s.v.p. of water can be measured at different temperatures.

Some values of the s.v.p. of water from 0°C to 140°C are shown in the table below.

Temp. °C	0	1	2	3	4	5	6	7	8	9	10	20	30	50	100	120	140
s.v.p. mm mercury	4·6	4·9	5·3	5·7	6·1	6·5	7·0	7·5	8·0	8·6	9·2	17·5	31·8	92·5	760	1450	2710

The considerable rise in saturation vapour pressure (s.v.p.) above 100°C explains why boilers may burst. Fig. 13.10b (not to scale) shows roughly how the s.v.p. of water varies with temperature, t°C.

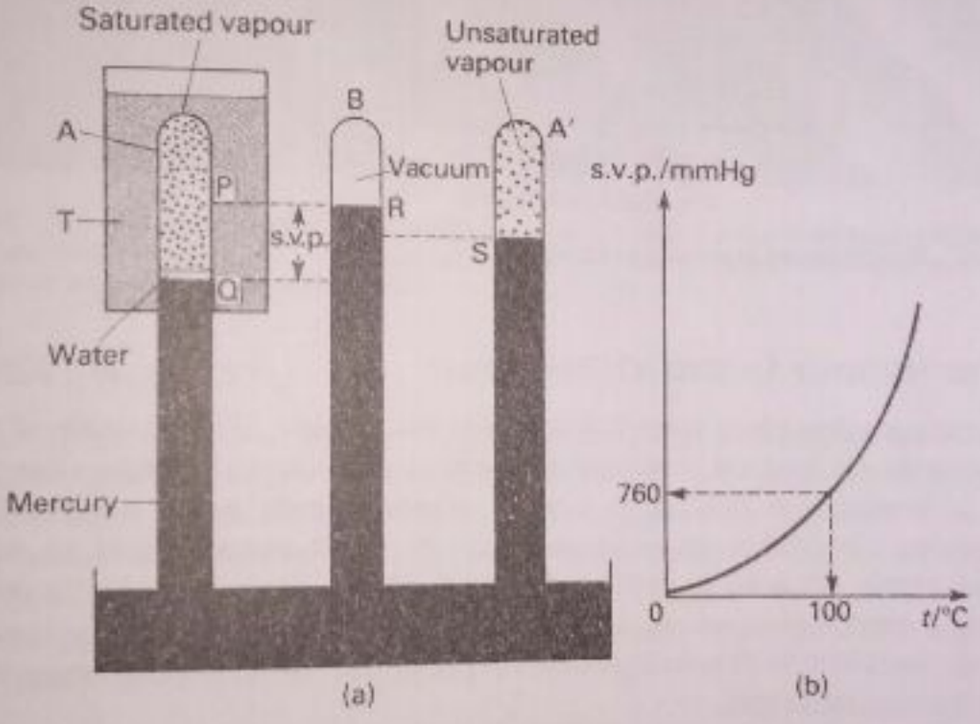


Fig. 13.10 Saturated and unsaturated vapours

Molecular explanation of s.v.p.

We can explain vapour pressure and saturation vapour pressure from the movement of molecules of the liquid. As the temperature of the liquid rises, the speed of its molecules increases and so the number of molecules escaping from the surface increases. So the vapour pressure becomes greater.

Suppose a liquid is in a closed space, such as the layer of water at the top of the tube at A (Fig. 13.10). As already explained, molecules continually leave the liquid at the surface and form molecules of vapour. Some of the molecules pass through the surface again into the liquid. So at any instant molecules are leaving the liquid and others are returning to it. When the vapour is unsaturated more molecules leave the liquid than return to it per second, and so the liquid evaporates until it all changes into vapour.

Scanned with CamScanner

When the space above the liquid becomes saturated, that is, some liquid is present, the number of molecules leaving the liquid per second is then equal to the number per second returning to it. We call this 'dynamic equilibrium'. The space above the liquid now has the maximum possible number of molecules at the particular temperature (Fig. 13.11). It is a *saturated vapour*. An unsaturated vapour is one not in equilibrium with its liquid. Its pressure is less than the saturated vapour pressure (s.v.p.). At a higher temperature the s.v.p. increases, because more molecules per second then escape from the liquid.

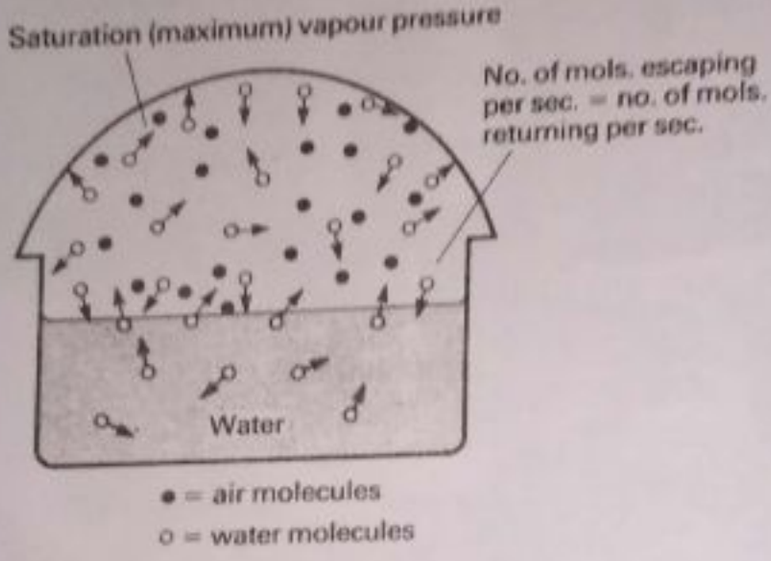


Fig. 13.11 Kinetic theory explanation of s.v.p.

Water vapour in the atmosphere

Evaporation takes place from lakes, rivers and oceans. The amount of water vapour in the air depends on the temperature and climate at the place concerned.

On a humid day the air is nearly saturated with water vapour. Little evaporation then takes place through the pores of the skin and so we feel uncomfortable. On a warm dry day, the air can hold more water vapour without becoming saturated than on a cold dry day. At night and early morning, however, moisture or dew is deposited on grass. The drop in temperature makes the air become saturated.

The *dew point* is the temperature below which the air first becomes saturated with the water vapour in it. In a warm room, a glass of ice-cold water cools the air surrounding it below the dew point. The water vapour therefore condenses on the outside of the glass surface. For the same reason, a film of moisture forms on the outside of a milk bottle after it is taken out of a refrigerator. Likewise when a hot bath is run, the bright metal surface of the cold tap has a film of moisture on it but the hot tap is unaffected.

In a warm room, condensation forms on the windows. The cold glass cools the air close to its surface to below the dew point. Double glazing, however, reduces condensation; the air between the two glass surfaces is a bad conductor and the temperature of the inside window is kept above the dew point.

After rain, evaporation from wet surfaces is more rapid when the air above contains little water vapour. For this reason pavements dry quicker in summer than in winter. If the air above a wet surface is saturated, water vapour will condense as fast as it evaporates and so the surface remains wet.

Scanned with CamScanner

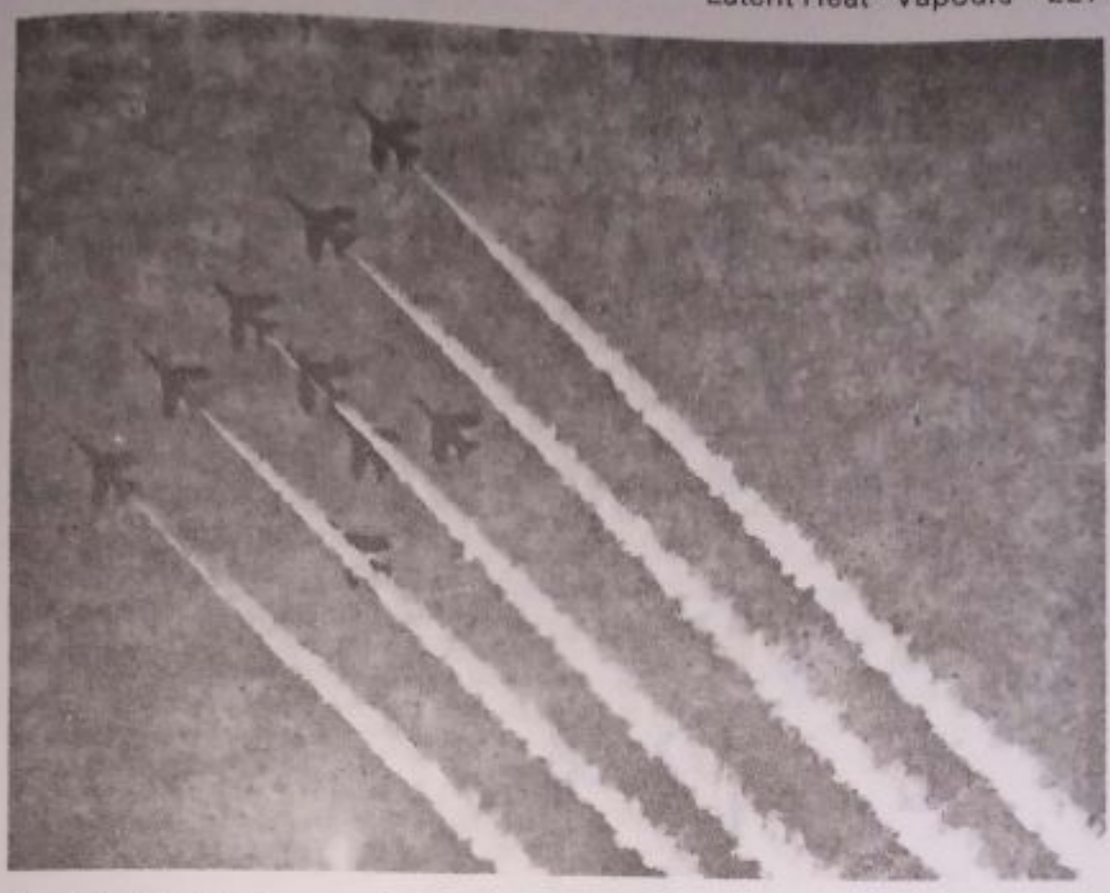


Plate 13B The Red Arrows, an air force acrobatic team. Droplets of water condense from the atmosphere round particles of hot gases emitted from the aeroplanes' engines at the back and these form vapour trails showing the flight paths. (*J. Allen Cash Ltd*)

Relative humidity

On a fine day, with a cool breeze blowing, we are conscious that the air is 'dry'; whereas on 'muggy' days the air feels damp or *humid*. This feeling of comparative wetness or dryness is due to the water vapour in the air. Since, however, the actual mass of water in the air may be more on the dry day than on the humid day, the degree of humidity in the air is not measured by the mass of water alone. A *ratio*, known as the *relative humidity*, is chosen as a measure of the degree of 'wetness' in the air, and is defined by the relation:

$$\text{relative humidity} = \frac{m}{M} \times 100\,\%$$

where m is the mass of water vapour actually present in a certain volume of the air, and M is the mass of water vapour required to *saturate* the same volume of air at the same temperature.

Since the mass of water vapour in a given volume is roughly proportional to its pressure, the relative humidity is also given by the ratio

$$\frac{\text{s.v.p. at dew point}}{\text{s.v.p. at air temperature}} \times 100\,\%$$

Wet-and-dry bulb hygrometer

A *hygrometer* is an instrument for measuring the relative humidity of the air. Very accurate measurements of relative humidity can be made with a standard

Scanned with CamScanner

hygrometer in which the dew point is carefully measured. The 'wet-and-dry bulb' hygrometer, however, can be used by an untrained person.

The instrument consists of two thermometers, A and B, standing beside each other (Fig. 13.12). A piece of muslin is wrapped round the bulb of B, and its lower end dips into water. So the bulb of B is always wet and that of A is always dry. If the two thermometers are placed outside in the air the water round the bulb of B evaporates at a rate which depends on the relative humidity. So the bulb is cooled and B usually reads a steady temperature lower than A, as shown.

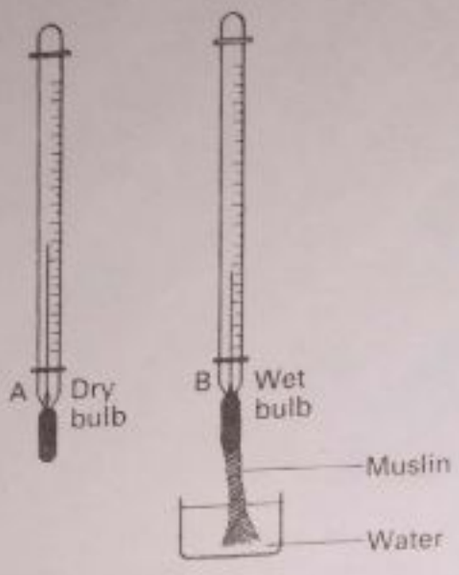


Fig. 13.12 Wet-and-dry bulb hygrometer

To take an extreme case, no water evaporates if the relative humidity is 100%, as the air is then saturated. The temperature on B is then exactly the same as the temperature on A, which records the temperature of the air. If the air were far from being saturated the water in the muslin surrounding the bulb of B would

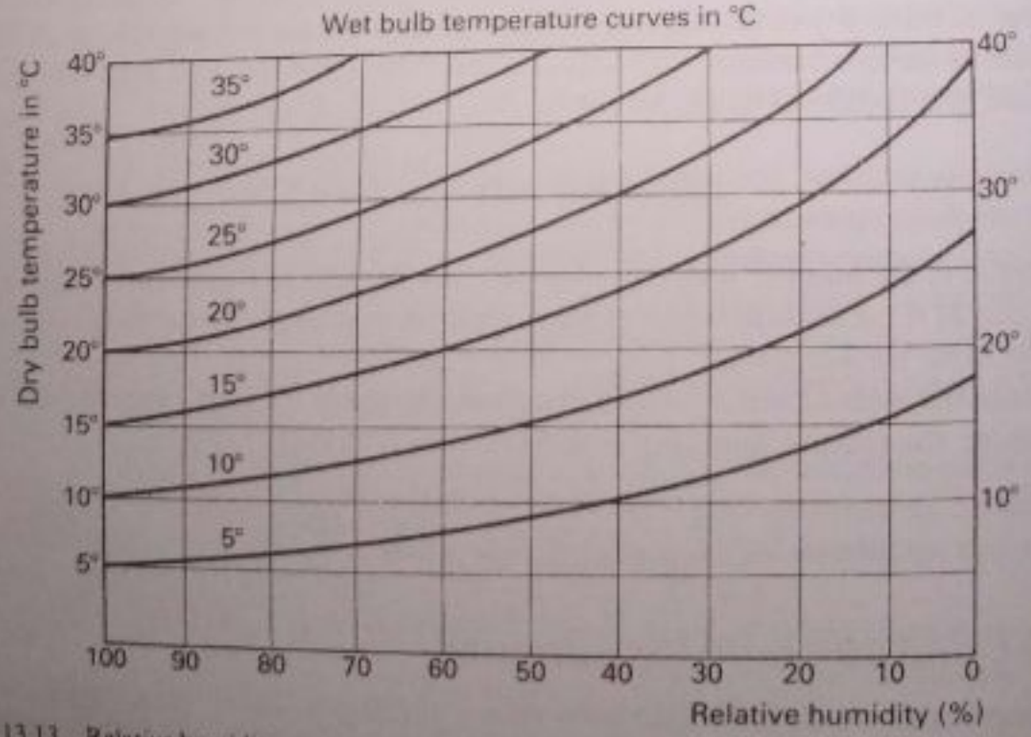


Fig. 13.13 Relative humidity curves

Scanned with CamScanner

evaporate quickly, and so the temperature on B would be much lower than that on A.

Tables have been drawn up which give the value of the relative humidity from the readings of the wet and dry bulb thermometers. The values of the relative humidity were obtained by using a standard hygrometer. Fig. 13.13 is a useful chart from which the relative humidity can be read directly.

Boiling and s.v.p.

When water in a beaker is raised to the boiling point small bubbles, containing dissolved air and saturated water vapour, form at the bottom. As they rise to the surface the bubbles expand because there is then less pressure on them. When the bubbles reach the surface, they burst open because the saturation vapour pressure (s.v.p.) in the bubbles is just slightly more than the external or atmospheric pressure (Fig. 13.14). Generally, we can say that *a liquid boils at the temperature at which its saturation vapour pressure is equal to the external pressure.*

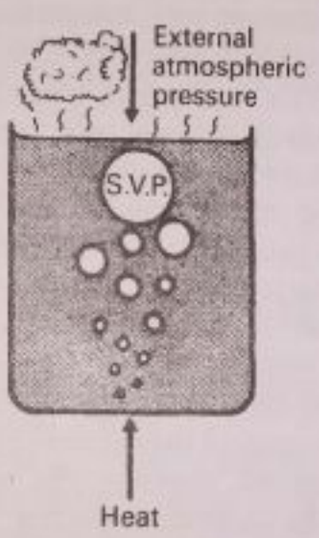


Fig. 13.14 Bubbles in a boiling liquid

At standard atmospheric pressure, 760 mmHg, water boils at 100°C.

Note that *evaporation* occurs only at the surface of a liquid and takes place at *all* temperatures. *Boiling* takes place throughout the whole volume of liquid–bubbles are formed everywhere inside it–and can only occur at one temperature, the boiling point.

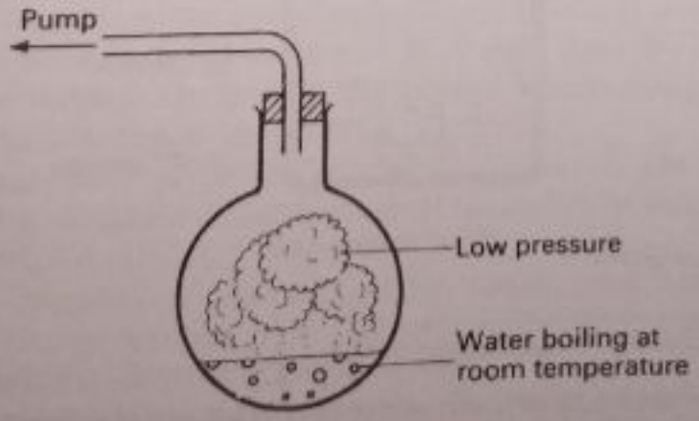


Fig. 13.15 Boiling point depends on external pressure

Scanned with CamScanner

Experiment

A demonstration of water boiling at very low pressure is easily arranged. A round-bottomed flask is filled with a little water, and a tube in the tight-fitting rubber cork is connected to a vacuum pump (Fig. 13.15). When the pump is switched on bubbles are soon seen rising through the water. So the water boils at the room temperature.

The pump lowers the external pressure above the water until it becomes equal to the s.v.p. at the temperature of the room. The water then begins to boil, as already explained.

At places higher above sea-level, the atmospheric pressure decreases. Water boils only at 90°C at about 3 kilometres above sea-level and at 80°C at about 5 kilometres above sea-level. High up in the mountains some food cannot be properly cooked in boiling water unless a pressure cooker is used, as discussed below.

Pressure rise and boiling point

Since the s.v.p. of water increases with temperature rise, it follows that water boils at a temperature above 100°C when the external pressure is greater than 1 atmosphere (760 mmHg). At a pressure of two atmospheres, for example, the boiling point is raised to about 120°C (see table, page 225).

In a pressure cooker, used in the home for rapid cooking and by the bacteriologist for sterilising culture media, water is boiled under increased pressure (Fig. 13.16). If the gauge is set for 0·1 N/mm², or about 1 atmosphere, *above* the external atmospheric pressure, the pressure inside the cooker is then increased to about 0·2 N/mm² or two atmospheres. When the water is heated its saturation vapour pressure rises. The water does *not* boil when a temperature of 100°C is reached because the saturation vapour pressure at this temperature is 1 atmosphere, and this is not equal to the external pressure now present above the liquid. At 120°C approximately, however, the saturation vapour pressure of water is 2 atmospheres. So the water begins to boil. Food cooked in water boiling at higher temperatures than 100°C takes less time to prepare. Fig. 13.6 illustrates the pressure cooker principle.

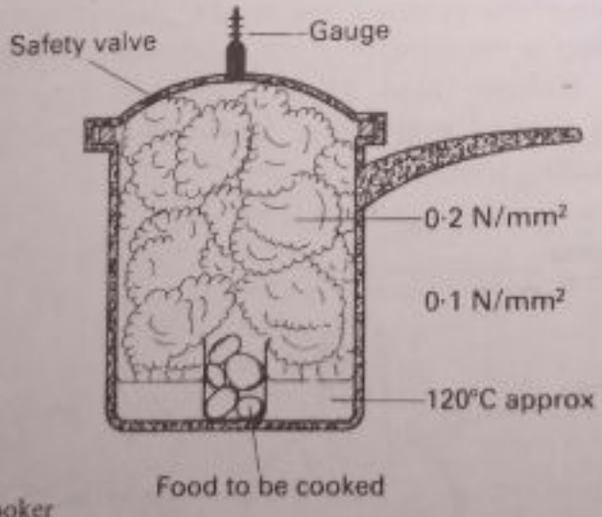


Fig. 13.16 Pressure cooker

Examples on boiling, evaporation, frost

1. Explain (a) why a kettle of water with a steady supply of heat takes a much longer time to boil dry than it does to reach its boiling point, (b) how evaporation differs from

Scanned with CamScanner

boiling. (c) how the molecular theory of matter accounts for the drop in temperature which results when rapid evaporation of a volatile liquid occurs. (L)

(a) In molecular terms, energy is needed at the boiling point to overcome the forces of attraction between molecules in the liquid state so the molecules can escape outside the liquid and form a gas. The amount of energy needed (latent heat) is much greater than the energy needed to raise the water to the boiling point, where no change of state occurs. So the water takes longer to boil dry than to reach the boiling point.

(b) Evaporation differs from boiling as (i) evaporation takes place from the liquid surface whereas boiling occurs throughout the liquid, (ii) evaporation occurs at all temperatures whereas boiling takes place at one temperature which depends on the external pressure, (iii) wind assists evaporation but has no effect on boiling.

(c) Although all the molecules in a liquid are in motion with an average kinetic energy depending on the temperature, some molecules have kinetic energy greater than the average while others have less. Evaporation occurs because the more energetic molecules near the surface escape from the liquid into the space above. This leaves the remaining molecules with an average kinetic energy lower than before. So the temperature of the liquid, which is a measure of the average kinetic energy, falls.

2. Why is frost more likely to occur on a clear night than on a cloudy night? Explain why, under these conditions, the frost is usually more severe in lower lying areas. (L)

At night the earth loses heat by radiation into outer space. On cloudy nights the clouds reflect some heat back to earth. More heat is therefore lost on clear nights and the earth becomes colder. The moist air above it may then be cooled below the dew point and condense as frost when the temperature is below 0°C.

As the air is cooled, it becomes denser and sinks. So the densest and coldest air concentrates in the areas which lie lowest, resulting in more severe frosts there.

Summary

1. Specific latent heat is the heat required to change the state of unit mass of a substance without temperature change.

Specific latent heat of fusion, l, is the heat to change unit mass of a solid at its melting point to liquid at the same temperature.

Specific latent heat of vaporisation, l, is the heat to change unit mass of a liquid at its boiling point to vapour at the same temperature.

2. In determining the specific latent heat of ice: (a) the ice must be dried; (b) the final temperature after it melts must not be too low, otherwise water vapour from the air condenses. In determining the specific latent heat of steam, a water (steam) trap must be used.

3. Melting points are determined by plotting a cooling curve. As the solid solidifies its temperature remains constant–its latent heat is given out and balances the heat lost to the surroundings.

4. Water expands on freezing. Ice contracts on melting.

5. Increased pressure lowers the melting point of ice.

6. Evaporation depends on the liquid temperature, its surface area and the wind present. It takes place from the surface of the liquid, since the more energetic molecules escape through the surface, the average kinetic energy of the liquid decreases, and hence its temperature falls.

7. A saturated vapour is one in equilibrium with its liquid. The s.v.p. of water can be measured with the aid of two barometer tubes, one of which has water at the top and the other a vacuum.

8. The relative humidity of the air can be found from tables using the temperatures of the wet-and-dry bulb hygrometer; the smaller the temperature difference, the higher is the relative humidity value.

9. *Boiling* A liquid boils when its saturation vapour pressure is equal to the external atmospheric pressure. Increased pressure increases the boiling point.

Scanned with CamScanner

Boiling occurs throughout the whole volume of liquid and occurs at one particular temperature, unlike evaporation which is a surface phenomenon.

Exercise 13 (Answers, page 641)

(Assume the specific heat capacity of water = 4·2 J/g K or 4200 J/kg K.)

1. Calculate the quantity of heat needed to melt ice at 0°C to water at 0°C if the mass of ice is (a) 2 kg, (b) 4 g. (l = 330000 J/kg or 330 J/g)

2. When heat is supplied at the constant rate of 32000 J/min, it takes 5·0 min to melt 0·5 kg of ice at 0°C to water at the same temperature. Calculate a value for the specific latent heat of ice.

3. (a) Find the heat required to evaporate completely 0·2 kg of water at the boiling point. ($l = 2{\cdot}0 \times 10^6$ J/kg)

(b) What heat is given up when 0·1 kg of steam at 100°C condenses to water at the same temperature?

4. It is better to use ice at 0°C than an equal mass of water at 0°C to cool warm water. Explain.

5. Calculate the heat needed to change 6 g of ice at 0°C to water at 10°C. (Assume l = 300 J/g and c for water = 4 J/g K)

6. What mass of ice at 0°C needs to be added to 100 g of water at 22°C to lower the temperature to 5°C? (Specific latent heat of fusion of ice = 336 J/g.)

7. A piece of copper of mass 250 g at a temperature of 900°C is quickly transferred to a vessel of negligible thermal capacity containing 200 g of water at 20°C. If the final temperature is 100°C what mass of water will boil away? (Specific heat capacity of copper = 0·4 J/g K and specific latent heat of steam = 2200 J/g.) (N)

8. Define specific heat capacity and specific latent heat of fusion.

Describe an experiment to determine the specific heat capacity of a metal, stressing the precautions that should be taken and showing how the result is calculated. Find the rate at which heat must be extracted in a refrigerator to produce 60 gram per second of ice at −6°C from water at 20°C. State why, in practice, this rate must be exceeded. (Specific heat capacity of ice = 2·1 J/g K, specific latent heat of fusion of ice = 336 J/g.) (L)

9. Define *specific heat* (specific heat capacity), *latent heat* (specific latent heat) and describe a method of finding the latent heat of steam.

50 grams of lead at its melting point of 327°C is poured into a hole in a lead block of mass 100 grams. The temperature of the block rises from 15°C to a final value of 189°C. The specific heat capacity of lead is 0·13 J/g K or 130 J/kg K. Neglecting heat losses, calculate the specific latent heat of fusion of lead. (O and C)

10. (a) Some hot water contained in an open beaker is observed to be evaporating steadily.

Describe what is believed to be the behaviour of the water molecules *below* and *above* the surface of the hot water. Name and describe the processes by which the water gradually cools to room temperature assuming that the beaker rests on a table.

(b) Describe an experiment to show what effect *either* an increase *or* a decrease in pressure will have on the boiling point of water. (L)

11. Distinguish between *evaporation* and *boiling*.

A jet delivering 0·44 gram of dry steam per second, at 100°C, is directed on to crushed ice at 0·0°C contained in an unlagged copper can which has a hole in the base. 4·44 grams of water at 0·0°C flow out of the hole per second. How many joules of heat are given out per second by the condensing of the steam and the cooling to 0·0°C of the water formed? How much heat is taken in per second by the ice which melts? Suggest why these amounts are different.

Take the latent heat of steam as 2260 J/g, the latent heat of fusion of ice as 334 J/g, and the specific heat capacity of water as 4·2 J/g K.

If the steam jet were replaced by an electric heater buried in the ice, and connected to a 240 V supply, what current would be needed to give the same heat per second to the apparatus? (O and C)

12. (a) Describe an experiment to obtain a cooling curve for naphthalene or paraffin wax when cooling from the liquid to the solid state. Sketch the curve so obtained. Indicate the melting point of the solid and give reasons for what is observed at this temperature.

A volume change often accompanies a change of state between a solid and a liquid. Describe one situation in which this is *either* an advantage *or* a disadvantage.

Scanned with CamScanner

(b) Explain briefly why (i) if you breathe gently on to the back of your hand, you will warm it, (ii) if you blow strongly on your hand you will cool it. (L)

13. With the aid of a suitably labelled diagram, explain the action of a refrigerator. (N) (L)

14. A pressure cooker will cook its contents faster than an open saucepan. Explain.

15. Describe an experiment to demonstrate the effect on the boiling point of water of a decrease of pressure. Explain clearly how the decrease of pressure is produced. State any other factor which affects the boiling point of water and say in what way the boiling point is changed.

16. A refrigerator may be described as a heat pump since it must continuously remove heat. Explain how the liquid which is used as a refrigerator is enabled to do this.

What is the function of the motor which is used in refrigeration and why must external cooling always be provided? (L)

17. Describe an experiment to measure the saturated vapour pressure of water at its normal boiling point. (N)

18. (a) Describe differences in the behaviour of the molecules of a substance in its solid, liquid and gaseous states.

Using these differences as a basis, explain why (i) energy must be removed from a liquid at its freezing point for it to solidify, (ii) evaporation of a liquid may produce a cooling of the liquid.

Explain why the pressure of an enclosed gas at a constant pressure increases when the volume is reduced.

(b) It is required to convert 0·5 kg of water at 20°C into ice using a refrigerator which can extract heat at an average rate of 20 J/s. Determine whether this is possible within a period of 2 h. (Specific heat capacity of water = 4200 J/kg K; specific latent heat of fusion of ice = 336 000 J/kg.) (L)

Multiple-Choice Questions

19.

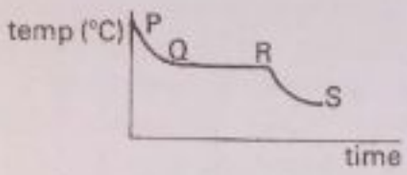


Fig. 13.17

Fig. 13.17 shows how the temperature of a molten metal X varies with time when X cools to room temperature.

The part QR of the graph is horizontal because

A it shows the boiling point of X
B the latent heat gained by X = heat lost by X to surroundings
C the latent heat at Q = the latent heat at R
D X becomes a solid along SR
E X is a liquid along PQ

20. When some water at 20°C is present in the closed space above the mercury in a barometer tube,

A the space contains a vacuum
B the water vapour temperature is above 20°C
C the vapour pressure stays constant when the tube is slightly raised
D the vapour pressure decreases when the tube is slightly raised
E the water molecules have a higher average speed than the vapour molecules

21. When 4 g of ice changes to water

A the mass of water formed is greater than 4 g
B latent heat must be removed from the ice
C the strength of the molecular bonds is increased.
D latent heat is absorbed by the ice
E the ice always decreases in temperature

22. The Table below shows the saturation vapour pressure (s.v.p.) of a liquid in centimetres of mercury:

Temp:	20°C	30°C	40°C	50°C	60°C	70°C	80°C
s.v.p.:	30	44	52	68	74	78	84

At standard (normal) atmospheric pressure the boiling point of the liquid is about

A 35°C B 40°C C 55°C D 65°C E 80°C

23. When a person perspires on a hot day

A evaporation occurs and helps to cool the body
B heat is conducted away from the body
C latent heat keeps the body warm
D the body is insulated from the warm air
E convection cools the body

Scanned with CamScanner

14. Transmission of Heat

Conduction

If one end X of an iron rod is held in a flame and the other end Y is held in the hand, the end Y soon feels warm. We say that heat is *conducted* from X to Y.

If the experiment is repeated with a thick rod of glass of similar size to the iron rod, the end held does not feel warm. So some substances conduct heat well; others are poor conductors. This can be shown by the following experiment.

Experiment to compare thermal conductivity of copper and wood

Take a long wooden rod, wind some copper foil tightly round the middle and nail the foil to the rod (Fig. 14.1). Then wind a piece of paper round the rod so that it overlaps the wood and copper and stick it so that it makes firm contact with them, excluding air. Hold one end of the rod and move it to and fro so that the paper passes repeatedly through the flame of a bunsen burner. The paper is unaffected where it covers the copper, but is charred where it covers the wood.

Conclusion: The temperature of the paper round the copper stays below its burning point because the heat of the flame is conducted away quickly by the copper. Conversely, heat is conducted away slowly by the wood because the paper becomes charred. So copper is a good conductor, wood is a bad conductor.

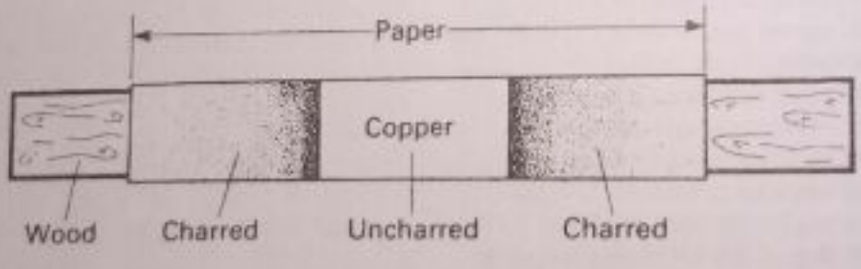


Fig. 14.1 Experiment to show copper is a good conductor, wood a bad conductor

Comparing conductivities of metals

Fig. 14.2 shows how the conductivities of metals can be roughly compared.

Experiment

A container which can be heated electrically is filled with water. Rods of metal of equal length and thickness, such as copper, lead, aluminium and iron, project vertically from the base, so that their upper ends are heated by the hot water. The rods are coated with

Scanned with CamScanner

paraffin wax. Small rings, which just slide on the rods, are also coated with wax and moved to the tops of the rods.

The heater is switched on, and the tops of the rods soon reach a high temperature. As heat is conducted along the rods the wax inside the ring melts and the rings move further down. After a while it can be seen that the ring on the copper rod has travelled furthest down and that on the lead rod least far down. This shows that copper is a much better conductor than any of the other metals and that lead is the worst conductor.

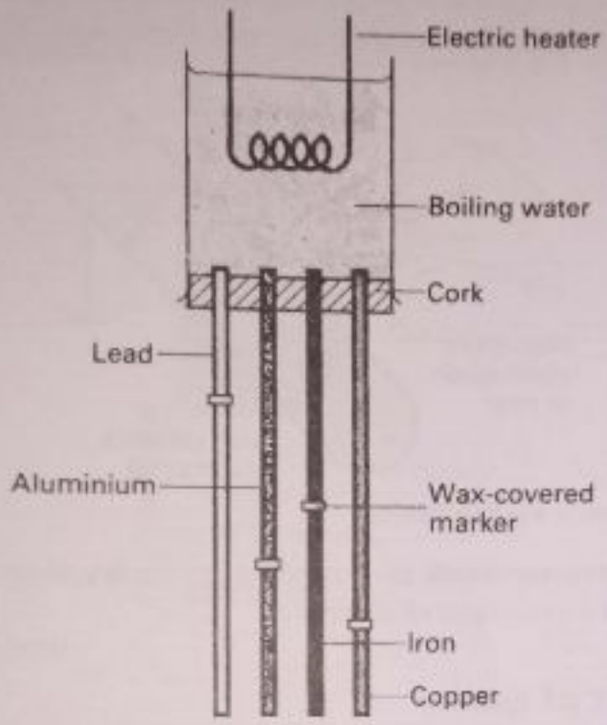


Fig. 14.2 Metals have different conductivities

Molecular explanation

Our theory of solids on page 145, tells us that their molecules are vibrating. When one end of a non-metallic solid is heated the energy supplied makes the molecules move through greater distances than before. Neighbouring molecules are therefore jostled more than before and so they have greater energy; this happens all along the solid. So thermal energy is transferred along the solid, although the *average* positions of the molecules are unchanged (Fig. 14.3).

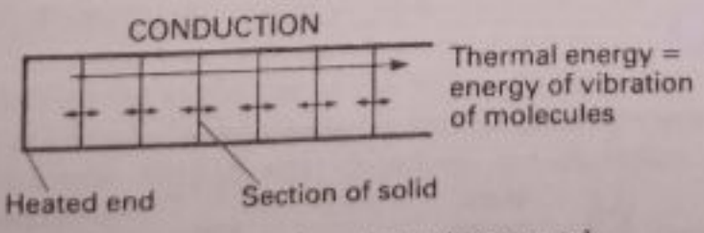


Fig. 14.3 Molecular explanation of thermal conduction in non-metals

In metals, heat is passed along by free electrons, which carry the thermal energy from the hot to the cold end. So a good electrical conductor, like copper, is also a good heat conductor.

Scanned with CamScanner

Conductivity of liquids

We can investigate the conductivity of water in the following way.

Experiment

By using a piece of metal gauze above or round it, a lump of ice is fixed at the bottom of a test-tube filled with water (Fig. 14.4). The tube is held at an angle with a paper grip and the top is heated by a bunsen flame as shown. After a time the water at the top is seen to boil, but the ice below remains unchanged. So little heat is conducted from the top to the bottom of the water, which we therefore see is a poor conductor.



Fig. 14.4 To show water is a bad conductor

Generally, substances which are liquids at ordinary temperatures are poor conductors, with the exception of mercury.

Ignition point of gas

Gases burn when their ignition temperature is reached, but if a metal, a good conductor, is placed in the gas it can stop the gas igniting.

Experiment

Place a metal gauze a short distance above a bunsen burner. Turn on the gas and light it below the gauze. The gas is seen to burn here, but there is no flame above the gauze (Fig. 14.5a).

Repeat the experiment with a cool gauze, but this time light the gas above the gauze. A flame is obtained above the gauze but not below (Fig. 14.5b).

Explanation: The metal gauze is a good conductor. It therefore conducts the heat of the flame quickly from the gas on the unlighted side, so that the gas here never reaches its ignition point. When the gauze becomes red hot this gas may be ignited

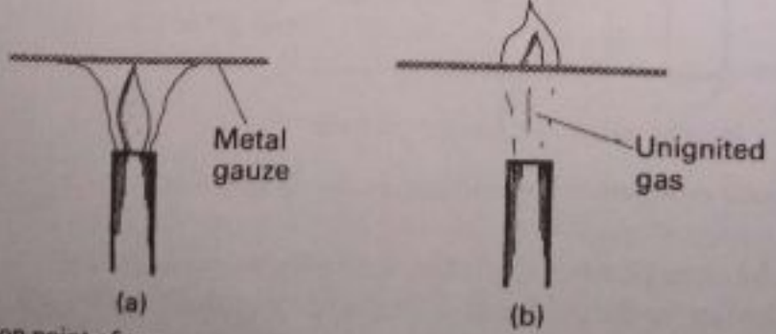


Fig. 14.5 Ignition point of gas

Scanned with CamScanner

Davy safety lamp

Sir Humphry Davy, a prominent scientist of the early 19th century, designed a miner's safety lamp shown in Fig. 14.6. Basically it has a burning wick surrounded by a metal gauze, with openings at the top of the lamp. The metal conducts the heat of the flame away from any explosive gas in the atmosphere outside the lamp (as in Fig. 14.5a). So the ignition temperature of the gas is never reached. Any small amount of gas which enters the lamp from the outside burns with a bluish flame round the white flame of the oil and can thus be recognised.

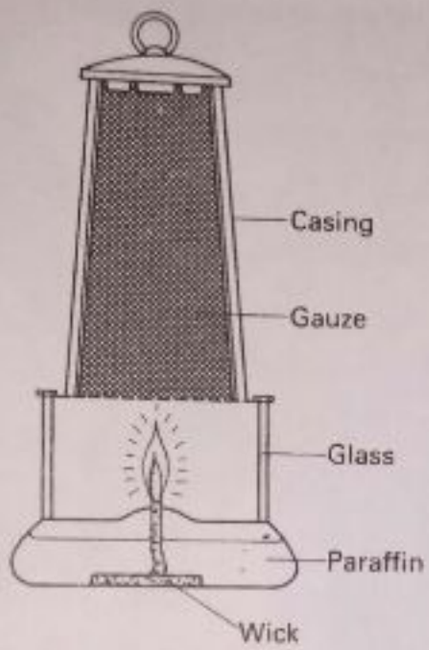


Fig. 14.6 Davy safety lamp

Good and bad conductors

In the kitchen, saucepans are made of metal such as aluminium, which is light and a good heat conductor. So heat passes quickly from the gas burner, or electric cooker filament, to the food inside. The handles of the saucepans, however, must be made of insulators, such as plastic or wood, so that they can be touched when the pan is hot. The handles of kettles and oven doors must also be made of insulators.

Air, like all gases, conducts heat extremely poorly, and is therefore a very good insulator. Pockets of air give many materials, of which wool is one, their insulation properties. In winter birds look 'fatter' than in summer. They fluff their feathers more to trap pockets of air, which help to keep them warm. If you examine the underside of some car bonnets you will find a felting. It keeps the engine warm because of the many tiny pockets of air in the material.

When large steel boilers used in industry are heated by gas flames, a thin layer of gas may be left on the underside of the boiler. As the gas is an insulator, the heat now flows at a much slower rate to the water inside. So the thermal efficiency of the boiler is lower. Periodically, therefore, the underside of the boiler is cleaned.

The inside of the boiler is also cleaned from time to time because scale, deposited from the minerals in the water, is a bad conductor. To stop further deposit of scale, the same water should be kept in a closed hot-water system, such as that used in the home for domestic heating; because if fresh water were used all the time it would bring a fresh supply of scale with it.

Scanned with CamScanner

Heat insulation in the home

Any heat which is allowed to escape from a house in winter will increase the fuel bills. Heat losses may occur through the ceiling, the loft, the chimneys, the walls, doors and windows. Fibre glass or felt is used to insulate the loft. Expanded polystyrene, a light material full of pockets of air, is an excellent insulator. Floors can be insulated by having a thick layer of felt under the carpet. Double glazing of windows provides an insulating layer of air between the outside and inside windows. This reduces heat losses appreciably.



Plate 14A A technician installing 'Ufoam' insulation from the top of a cavity wall before roofing (*ICI Insulation Service Ltd*)

Convection

Convection currents in liquids

Experiment

By using a long glass tube, a tiny crystal of potassium permanganate is dropped to the bottom of a flask filled with water (Fig. 14.7). The coloured water in the tube is removed by placing a finger over the top and withdrawing the tube. The bottom of the flask below

Scanned with CamScanner

the crystal is now heated gently, as shown. We see the coloured liquid moving upwards from the crystal, which dissolves slowly and by its colour helps to show how the liquid moves when it is warmed.

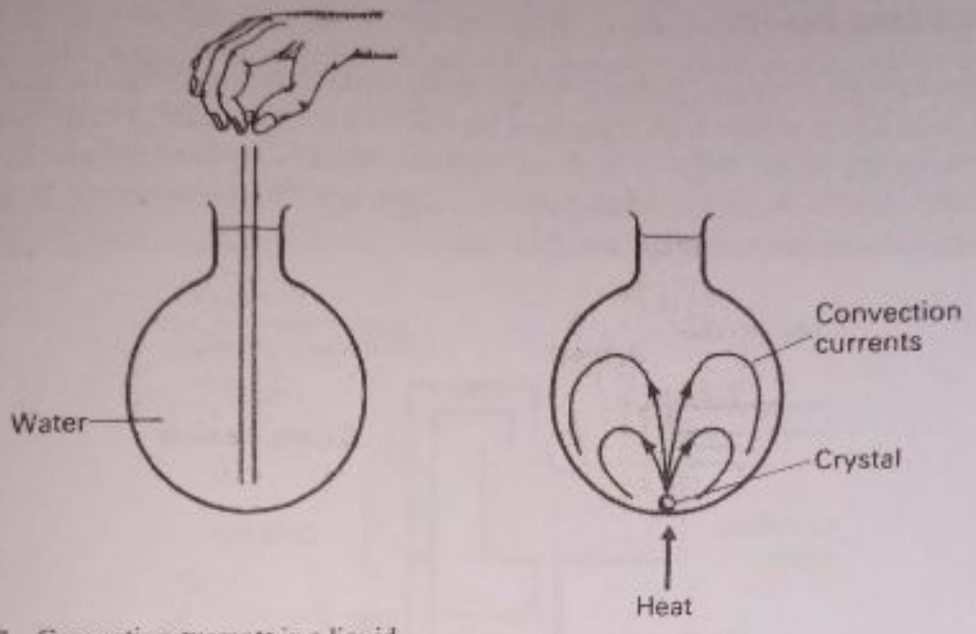


Fig. 14.7 Convection currents in a liquid

The upward movement of the coloured liquid is due to *convection*, and the circulation of liquid is called a *convection current*. It is due to the effect of heat at the bottom of the liquid. Heat makes a fluid (liquid or gas) expand. So the warm fluid at the bottom of the flask becomes less dense than the cold fluid above it, and it therefore rises. The cold (and denser) fluid above it moves down and takes its place. When this fluid becomes warmer, it also rises. In this way, the whole liquid becomes warmer.

This transfer of heat to other parts of the liquid is convection. Unlike conduction, where the average position of the molecules remains the same, the heat is carried to other parts of the liquid by movement of the warm liquid itself. In heating water in a kettle, then, convection currents circulate continually, keeping the water stirred as it is heated.

As shown in Fig. 14.8, water circulates by convection round the engine of a car and keeps it cool. Air round the radiator cools the water, and a pump helps the

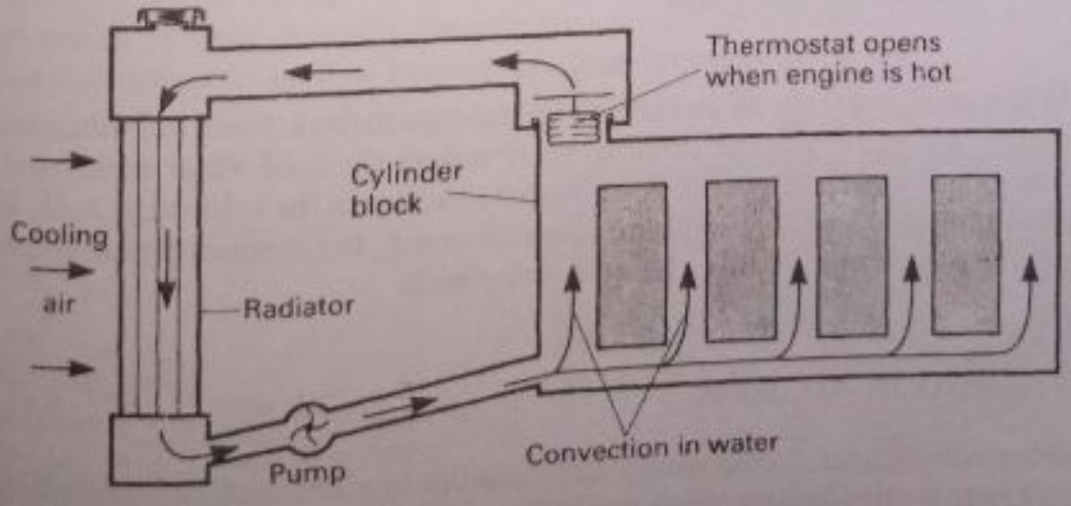


Fig. 14.8 Engine cooling by convection

Scanned with CamScanner

water to circulate. A thermostat, which opens when the engine reaches a suitable high temperature, prevents overheating.

Domestic boiler

A hot-water circulation system is used for domestic supplies (Fig. 14.9). A boiler heats cold water, which then rises up a pipe X to a storage tank. Fresh cold water returns to the boiler from a cold-water tank, usually in the roof of the house, through a pipe Y. Here it is heated and rises again to the storage tank. In this way the latter stores hot water for eventual use.

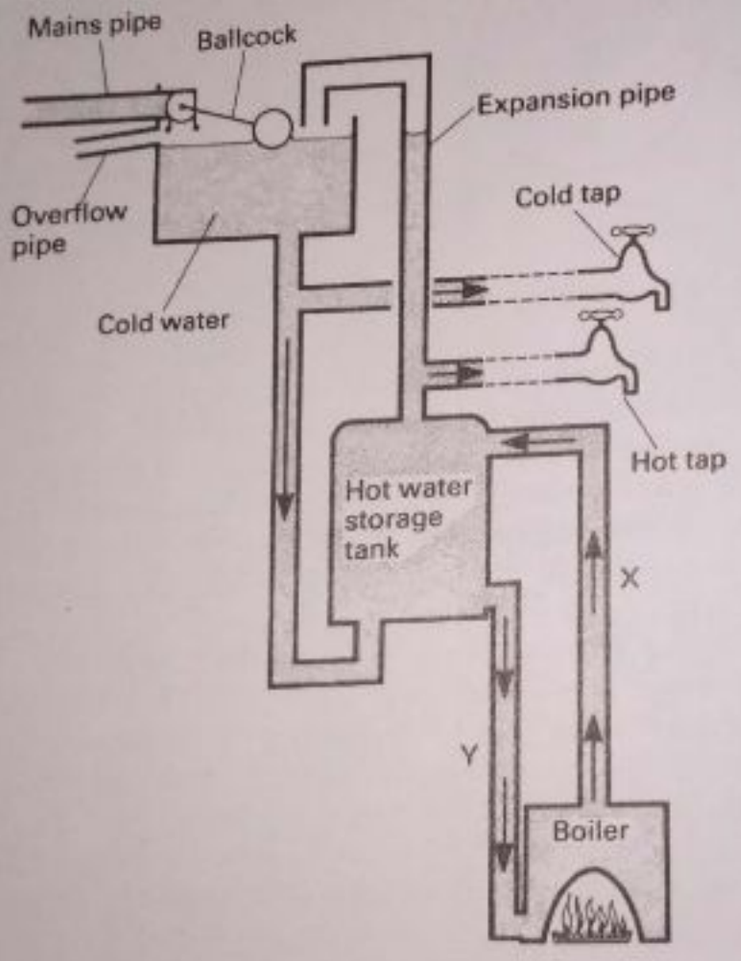


Fig. 14.9 Domestic hot-water system

When some is drawn off by turning on hot taps in the kitchen or bathroom, for example, an equal volume of cold water enters the cold-water tank from the mains. A ball-cock keeps the level of water constant in the cold-water tank. If the water in the storage tank boils and gives off steam, the expansion pipe at the top discharges it harmlessly into the cold-water tank.

Convection in gases

Convection of heat occurs much more readily in gases than in liquids because they expand considerably when their temperature rises (page 189).

Scanned with CamScanner

Experiment

Fig 14.10 shows a simple demonstration of convection currents in air. The apparatus is a closed rectangular wooden box with a plane glass front, so that the inside can be seen. Two wide glass tubes are fixed in holes at the top of the box as 'chimneys'. A candle is now lit below one tube X and a smouldering taper is held above the other tube . The smoke movement shows that an air convection current passes down through Y and up through X.

Explanation The air above the candle flame becomes hot and less dense and rises up X. Colder air then flows down Y and takes its place, giving rise to a circulating convection current

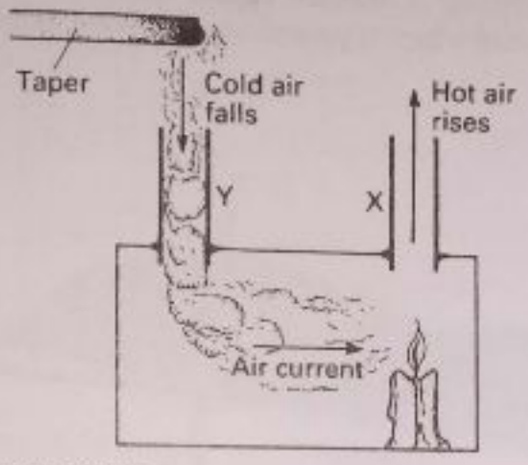


Fig. 14.10 Convection currents in air

Chimneys help to circulate fresh air in rooms in this way.

Land and sea breezes

Land and sea breezes on the coast are natural convection currents. In summer in daytime the sun warms the land to a higher temperature than the sea, because land has a lower specific heat capacity than sea-water and the sea is always moving about. The air above the land is warmed and rises, and its place is taken by cooler air above the sea moving inland (Fig. 14.11). Air higher in the atmosphere completes the circulation, and so a sea breeze blows.

At night the sea temperature drops only slightly, since it is warmed to a considerable depth during the day. On the other hand, the land temperature drops greatly at night. At night, therefore, a convection current is obtained in the opposite direction to daytime and this is a land breeze (Fig. 14.11).

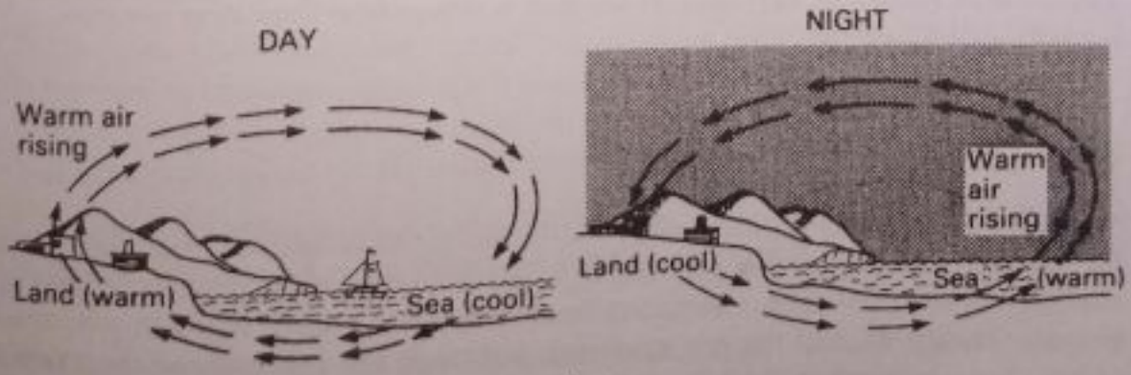


Fig. 14.11 Natural convection currents on coastlines

Scanned with CamScanner

Convection air currents

A more vigorous convection current can be seen with bonfires, when the rapid upward movement of air often carries glowing cinders upwards to considerable heights.

The sun often casts a 'shadow' of the hot air rising above a radiator in a room.

Convection air currents are useful to the glider pilot because they provide lift for the glider. Pilots call the rising air currents 'thermals'. They often look for large concrete surfaces, such as roads, or large metal surfaces, such as the roofs of factories. The air above these surfaces is warmer than the rest and rises more.

Winds are due to convection currents in the atmosphere. Near the equator hot air rises and its place is taken by cooler and denser air flowing in from the polar regions. The circulating air is affected by the earth's rotation and by the large land and sea masses over which it passes, as well as by temperature.



Plate 14B Hang glider using thermals or convection currents from the ground. The curved wings act as aerofoils, giving uplift by producing a net upward pressure when the air flows over them. (*Barnaby's Picture Library*)

Radiation

When the sun rises over the horizon in the early morning one can immediately feel its heat. The heat of the sun reaches us by *radiation*. In the transfer of heat by conduction and convection, some material substance, a solid, liquid or gas, must be present. But we feel the sun's warmth although the space above the earth's atmosphere, between us and the sun, contains practically no matter. So radiant

Scanned with CamScanner

heat or radiation can travel through empty space. Its speed here is the same as the speed of light (see page 345).

The *thermopile* is an instrument for detecting radiation. Fig. 14.12 shows rods of the metals bismuth and antimony, touching at one of their ends only, which form a thermopile. A galvanometer, a sensitive current-measuring meter, is joined to the free ends of the two metals. When radiation falls on the 'hot' junctions, as shown, the heat energy is transformed into electrical energy and a current then flows in the galvanometer. A cone is fitted over the hot junctions so that only radiation in a particular direction is received (see Fig. 14.13). The thermopile can detect radiation from a warm hand held near the hot junctions.

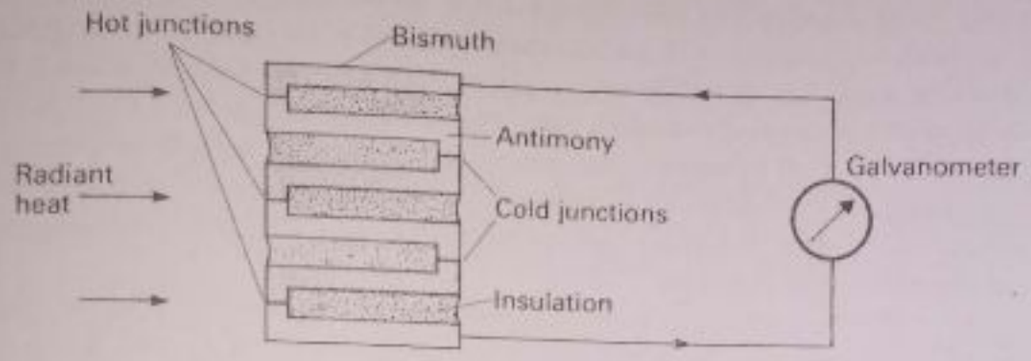


Fig. 14.12 Thermopile for radiant heat detection

Radiation from different surfaces

The amount of radiation from different surfaces kept at the same temperature can be investigated by using a hollow cube of metal, called a Leslie cube after the inventor, Sir John Leslie (Fig. 14.13). One vertical side is coated dull black by the soot from a candle flame, another side is highly polished silvery bright, and the remaining two sides are painted different colours such as grey and white.

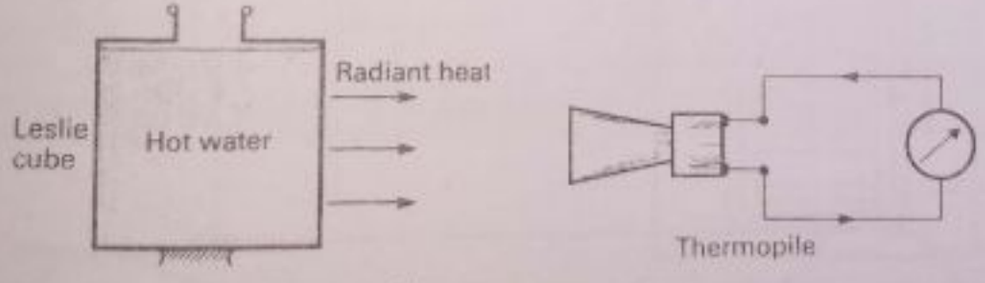


Fig. 14.13 Radiation from different surfaces

Experiment

The cube is filled with hot water. The thermopile is then placed close to the cube, so that the cone would intersect the vertical face of the cube if produced. The cube is now turned round so that each face in turn is in front of the thermopile, which collects the radiation, and the steady deflection in the galvanometer is recorded each time.

Results. The dull black surface produces the greatest deflection and the polished surface the least. The grey surface is second and the white surface third.

We therefore conclude that the radiation from a body depends on the nature of its surface and on its temperature. It also depends on its area–the larger the area,

Scanned with CamScanner

the greater is the radiation. For a given temperature, a body radiates most heat when its surface is dull black and least when its surface is highly polished.

Emission and absorption

Are good emitters of heat also good absorbers? We can investigate this in the following two experiments.

Experiment

1. Fig. 14.14 shows two air-filled bulbs, A and B, connected to a U-tube C, which is partly filled with a light oil. A is painted matt black. B is painted gloss white or a colour other than black. A small electric lamp L, placed exactly midway between A and B is switched on. It is then seen that the oil level falls on the side of A.

The air inside A has increased in temperature more than the air inside B, causing it to expand and push the oil down. Thus a blackened surface is a good absorber of radiation as well as a good emitter. Conversely, a white or highly polished surface is a poor absorber as well as a poor emitter of radiation.

2. Fig. 14.15 shows two vertical sheets of tinplate, one blackened and the other polished brightly. A cork is fixed by wax to the outside face of each sheet and a bunsen burner is placed midway between them and then lit. After a time, the wax is seen to melt first on the blackened plate and the cork falls off. So we conclude that the blackened surface absorbs radiation much better than the bright polished surface.

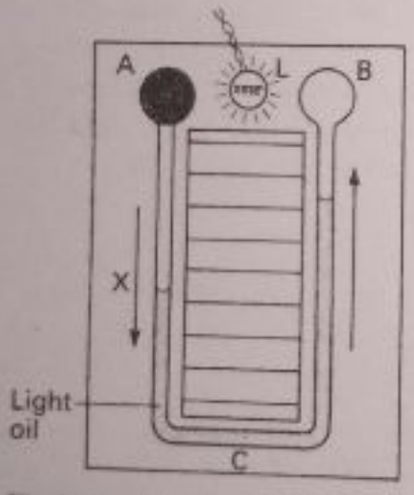


Fig. 14.14 Experiment on absorption

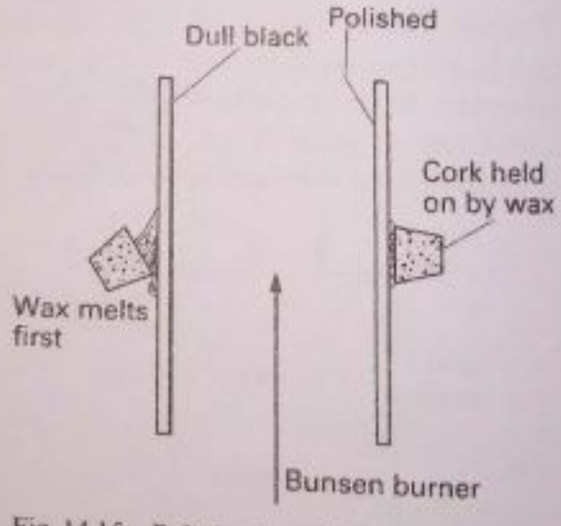


Fig. 14.15 Relative absorbing powers

Applications of radiation

Our experiments have shown that *a dull, matt surface is a good radiator and a good absorber*, especially when black; whereas *a light, polished or silvery surface is a poor radiator and poor absorber*.

For this reason a black car can be uncomfortably hot in summer on a warm day, a teapot with a silvery surface keeps the water hot for some time, and the base of an electric iron has a bright polished surface.

To keep suitable temperature conditions inside their workrooms throughout the year, the roofs of low industrial buildings are often painted with aluminium paint, which is silvery. This reflects the sun's rays during the day and reduces the

Scanned with CamScanner

heat radiated at night. For a similar reason, white robes or other clothes are worn in very hot climates.

The earth's surface absorbs heat radiated from the sun during the day and radiates heat at night to the air outside. So the ground becomes cooler at night. Clouds reflect radiation back to the earth. Thus the temperature of the earth's surface is higher on cloudy nights and frost is less likely.

In a greenhouse, the radiation from the sun passes through the glass and keeps the plants and air inside warm. Everything inside the greenhouse also radiates heat but as their temperatures are very much lower than the sun's, the longer wavelengths in the radiation (see page 345) are unable to penetrate the glass. Since more heat enters than leaves, the temperature of the greenhouse rises.

Infra-red radiation

Radiant heat, like visible light, consists of *electromagnetic waves*. They have much longer wavelengths than the longest wavelength in visible light, which is red. They are therefore called *infra-red rays* (page 345). Infra-red rays are thus not visible to the eye.

When infra-red rays fall on an opaque object, part is absorbed and converted into heat, and part is reflected. A highly polished silvered surface, as we have seen, is an excellent reflector of infra-red rays.

The reflection of infra-red rays can be demonstrated as follows.

Experiment

A small electric heater S, which emits heat rays, is placed at the focus of a large concave reflector M_1 (Fig. 14.16). Some distance away, an infra-red detector such as a thermopile (or suitable phototransistor), connected to a galvanometer, is placed at the focus X of another large concave reflector M_2. The reflecting surface of M_2 faces that of M_1 and is shielded from direct heat from the radiator S.

When S is switched on, a current flows in the thermopile. So the heat or infra-red rays are reflected by M_1 and M_2 to a focus at X, like light rays.

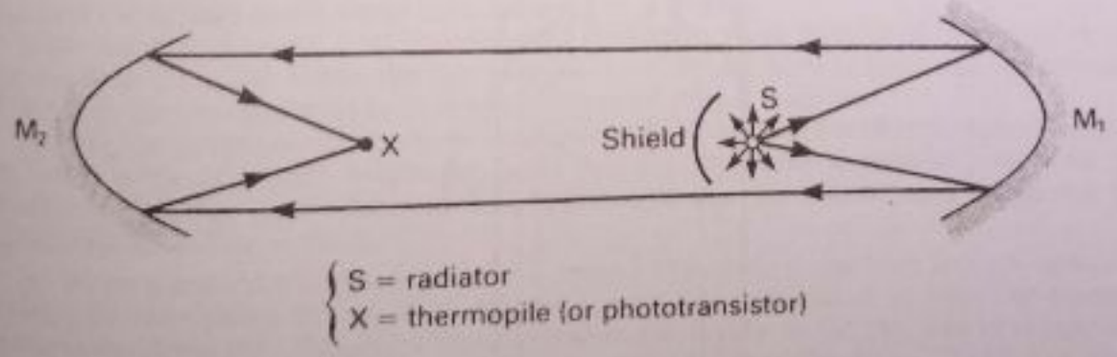


Fig. 14.16 Reflection of infra-red rays by concave mirrors

Infra-red electric heaters, used in bathrooms for example, have a filament embedded in a rod of silica. When it is hot the silica emits infra-red rays. These are reflected into the room by a concave reflector behind the rod. Solar furnaces collect infra-red rays from the sun by reflection at large mirrors.

Scanned with CamScanner

Radiation and convection in heating rooms

Metal radiators, used in a central-heating system for heating rooms, or electrically heated panels on the walls, warm the air in contact with them. The warm air rises and its place is taken by cooler air, and the process is repeated. In this way convection currents circulate in the room. Some heat is radiated directly into the room, of course, from the radiators, but since they warm the room mainly by convection, they would be more correctly termed 'convectors'. So radiators may be painted light colours without seriously affecting their heating efficiency.

A coal or log fire warms a room partly by radiation. Some of the heat, however, warms the air above it, which partly rises up the chimney. Cooler air from open windows, or through openings in doors, then flows into the room and takes its place. So fires with an open chimney not only help to heat a room but also to circulate fresh air in it. If a chimney fireplace is sealed, air heated by radiators or electric fires in the room does not escape and the air is then not as fresh as before.

Vacuum or thermos flask

About 1890 Sir James Dewar investigated the properties of liquid air. He designed a vessel to keep the very cold liquid air at its low temperature once it had been made. As a *thermos flask*, it can also keep a liquid hot.

Fig. 14.17 shows the principal features of a thermos flask. A double-walled glass vessel inside has (a) a vacuum between the walls, (b) the two interior facing walls X and Y which are silvered, (c) insulating cork supports P at the bottom, (d) an insulating plastic or cork stopper at the top. If hot tea or coffee is poured into the flask, the vacuum prevents heat losses by conduction and convection, and the silvery walls reduce losses due to radiation. The insulating stopper reduces loss of heat from the liquid by convection and evaporation. The cork P at the bottom prevents conduction from the glass to the surrounding flask C.

The design of the flask keeps liquid inside hot. For the reasons already given the flask also keeps cold liquids cold because heat is prevented from passing into

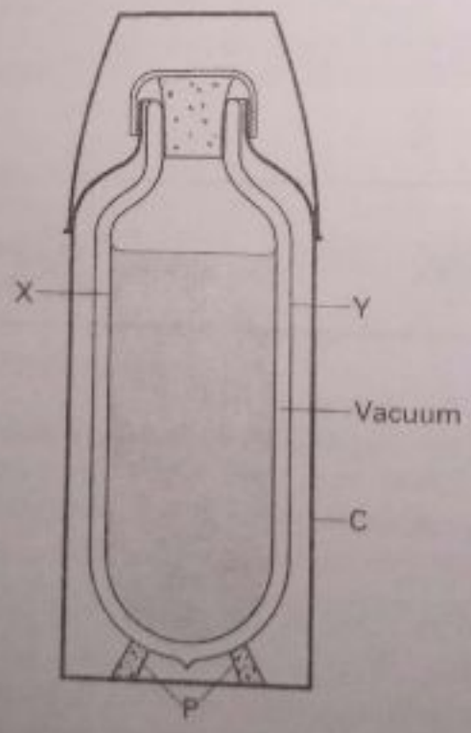


Fig. 14.17 Thermos flask

Scanned with CamScanner

the flask from the outside. A thermos flask should always be handled carefully. If the glass seal is broken by accident the vacuum is replaced by air and the flask is ruined.

Example on conduction, convection, radiation

(a) Describe an experiment which you would carry out to show how the nature of a surface affects the heat radiated from that surface in a given time.
State any precautions you would take and state your findings for two named surfaces. How would you then show that the surface which was the better radiator was also the better absorber of radiation?
(b) As the surface of a pond freezes it is found that each equal increase in the thickness of the ice takes longer to form, even when the air above the ice remains at the same low temperature. Explain why this is so.
(c) In the experiment shown (Fig. 14.18) the ice remains intact for several minutes as heating progresses. Explain how this can be so. (L)

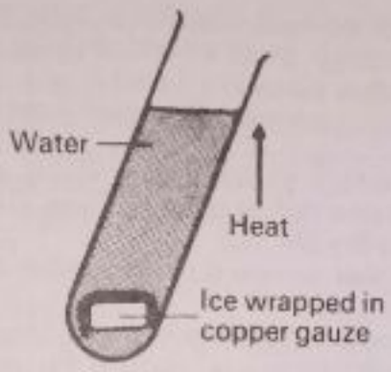


Fig. 14.18 Example

(a) A Leslie cube can be used to show how the nature of a surface affects the rate at which heat is radiated from it. This is described on page 243.
Precautions The hot water in the cube is stirred thoroughly so that the temperatures of all the surfaces are the same. Also, the thermopile must be placed near the cube, at the same distance from each surface each time.
Result A rough blackened surface is a much better radiator than a shiny white or silvery surface at the same temperature.
To show that the blackened surface is a much better absorber of radiation than the silvery surface, either experiment on page 244 can be used.

(b) The air above the ice is at a temperature well below the freezing point. After a while, the water in the pond below the ice remains at 0°C. So heat is conducted from the water through the ice and as the water loses latent heat, it freezes.
Now the rate at which heat is conducted through a solid such as ice depends on the thickness of the solid. When the two sides of the solid are kept at the same temperature difference, the heat flows faster through a thin solid than a thick one.
Each equal increase in thickness of ice requires the same amount of heat (latent heat) to be taken from the water. When the ice is thin the heat flow is relatively fast. As the thickness of the ice grows the heat flow becomes slower. So it now takes longer for the same thickness of ice to form.

(c) When a test-tube filled with water is heated from below, the heat spreads upwards quickly by convection. In Fig. 14.18, however, the water is heated at the top. The warmed water is less dense than the colder water below and so it stays at the top. The heat can only reach the ice by conduction through the water and along the glass. Water is a poor conductor of heat and so is the glass. So it takes a long time for the heat to reach the ice, which therefore remains unmelted for many minutes.

Summary

1. In *conduction* through non-metallic solids, the heat (energy) is transferred by vibrating molecules whose average position remains the same. In solid metals,

Scanned with CamScanner

the heat is transferred by free electrons. Conduction may occur in solids, liquids or gases.

2. *Convection* is the transfer of heat (energy) through a fluid by movement of the fluid itself. Convection may occur in liquids or gases but not in solids.

3. *Radiation* is the transfer of energy by electromagnetic waves. A material medium is not necessary. So radiation occurs in a vacuum. Conduction and convection cannot occur in a vacuum.

4. Dull black surfaces are good emitters of radiation and good absorbers. Highly polished silvery surfaces are poor emitters and poor absorbers of radiation.

5. A vacuum (thermos) flask prevents loss of heat due to conduction, convection and radiation. The inner glass vessel has a vacuum between the walls, silvered interior facing walls, an insulating support and an insulating stopper.

Exercise 14 (Answers, page 641)

1. Name the two processes of heat transfer in which the actual particles of the medium transmit the heat energy. State which of these cannot take place in a solid. Describe how this process takes place in a liquid or gas. (L)

2. Describe the differences in the manner in which heat is transmitted by *conduction* and *radiation*.

Describe an experiment which shows that a shiny or white surface is a poorer absorber of heat radiation than a dull or black surface. Give a brief explanation of one use made of this fact in everyday life. (L)

3. Explain the process of heat convection. Describe an experiment to show that convection takes place in air. (N)

4. It is possible to light a flame on top of a wire gauze held a few centimetres above the barrel of a bunsen burner without the gas beneath the gauze igniting. Explain. (L)

5. State the three ways by which a hot solid may lose heat.

Two copper calorimeters of identical dimensions are fitted with wooden lids. The outside of one calorimeter is coated with a thin layer of soot and the outside of the other is highly polished. Equal volumes of boiling water are now added to each, and they are allowed to cool while standing on a thick cork mat. State and explain what would be observed if the temperature of each were taken several minutes later.

Why, and in what way, would you expect the result to be different (a) if both calorimeters had been placed on a large metal bench, (b) if the coated calorimeter was near an open window? (L)

6. A hot liquid is contained in a closed vacuum flask. Describe the features in the design of such a flask which enable it to keep its contents hot for a considerable time. Illustrate your answer with a labelled diagram and explain how such features reduce the loss of heat.

A vacuum flask of heat capacity 126 J/K contains 0·02 kg of ice floating in 0·2 kg of water at 0°C. Heat is supplied at a constant rate by a small immersion heater of negligible thermal capacity and it is found that it takes 5 minutes for the ice to melt and a further 32 minutes to raise the temperature of the flask and its contents to 40°C. Determine (a) the rate at which the heat is supplied by the heater and (b) the specific latent heat of ice.

Assume that the specific heat capacity of water is 4200 J/kg K. (L)

7. Give a short account of the transfer of heat by conduction, convection and radiation, bringing out the differences between them.

Which of these processes cannot take place (a) in a vacuum, (b) inside a space ship falling freely towards the earth, (c) through solid material?

Explain briefly how you would demonstrate experimentally that water is a poor conductor of heat. (O and C)

8. Is the specific heat capacity of the land higher or lower than that of water? Describe and explain how sea and land breezes are produced.

9. How could it be shown that water is a bad conductor of heat? Show by means of a diagram the essentials of a domestic hot-water system and explain how it works. (L)

10. (a) Explain why heat losses are reduced but not completely eliminated when a warm object is: (i) wrapped in cotton-wool; (ii) separated from the surrounding air by an evacuated space.

Scanned with CamScanner

(b) Explain and compare the various means by which heat may be transferred from the white-hot filament of a gas-filled electric lamp to a nearby black surface vertically above it. (N)

11. Explain: (a) how some of the heat is wasted when a room is heated by means of a coal fire burning in an open grate; (b) how an occupant of the room sitting well away from the fire is warmed by it; (c) why an eiderdown is very effective in keeping a person warm in bed; (d) why it feels warmer to the feet to step out of bed on to a rug rather than on to linoleum. (N)

12. What are the methods by which heat is transferred from one place to another? Describe a laboratory experiment to illustrate each of these modes of transference. Describe how, in the hot-water method of heating a house, heat is transferred from the boiler to a person in a room on an upper floor. (O and C)

Multiple-Choice Questions

13. A white shawl wrapped round a baby keeps the baby warm because the shawl
- A is a poor conductor
- B is a poor radiator
- C has pockets of air trapped in it
- D conducts heat to the baby
- E stops convection currents

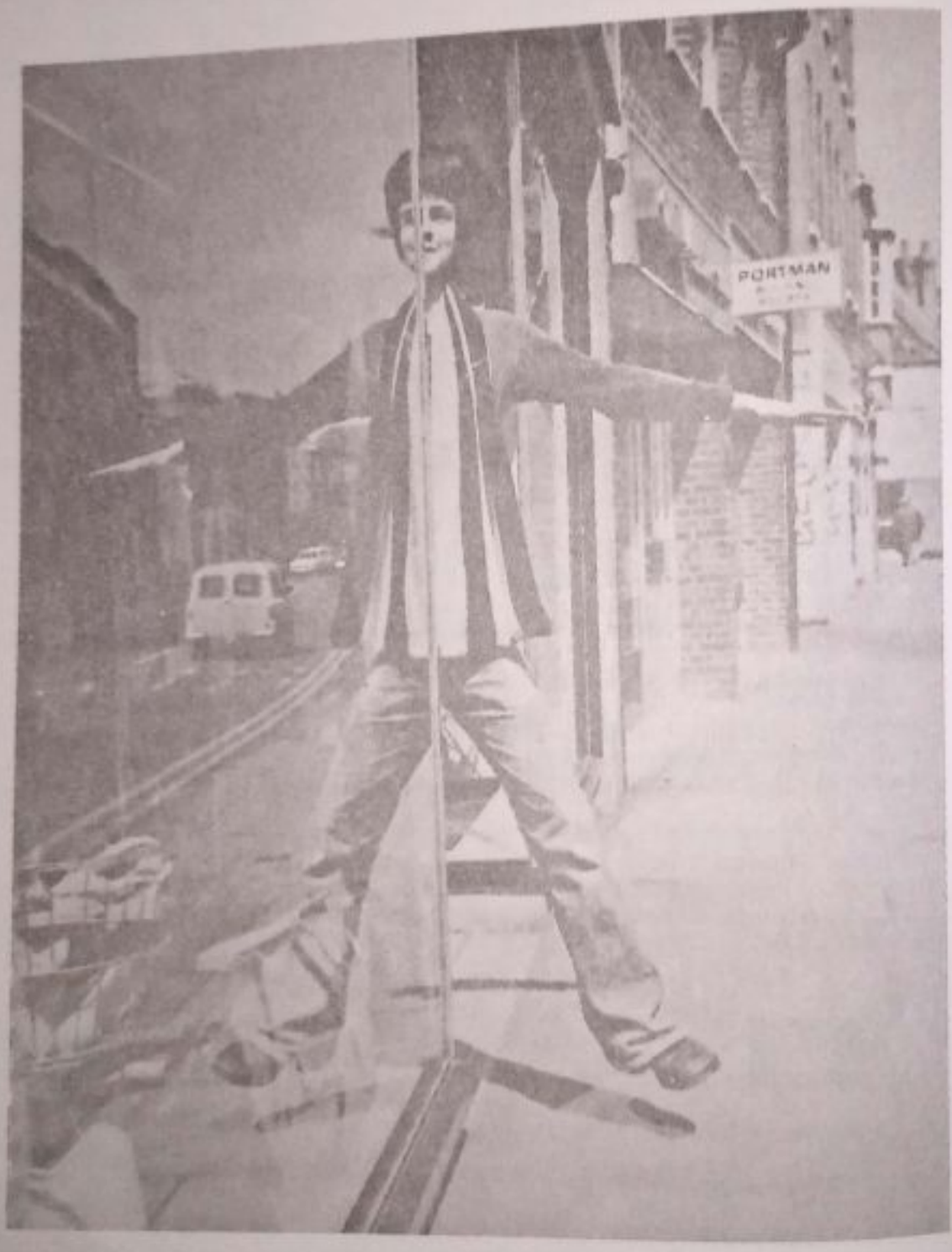
14. A vacuum (Thermos) flask is used to keep some liquid nitrogen at −160°C. The inside walls of the vacuum jacket are silvered so that heat is NOT
- A conducted towards the liquid
- B radiated towards the liquid
- C gained by convection
- D lost by convection
- E radiated away from the liquid.

15. When some ice cubes are added to a glass of warm water, the glass is cooled by a process of
- A conduction only
- B convection only
- C conduction and convection
- D conduction and radiation
- E radiation only

16. The dark driving wheel of a car feels warm when the sun shines on it because the sun
- A warms the car by conduction
- B gives energy to the wheel by conduction
- C radiates energy to the roof of the car
- D radiates energy to the wheel
- E radiates energy to the glass windows which become hot

Scanned with CamScanner

OPTICS



Reflection in a plane glass window. The image appears similar to the object but is laterally inverted. (*J. Allan Cash Ltd*)

Scanned with CamScanner

15. Straight-line Propagation of Light

Luminous and non-luminous objects

The subject of *optics* or light deals with luminous energy, that is, energy which causes the sensation of vision. The sun and the stars are *self-luminous* objects. Some living creatures such as the glow-worm or fire-fly are also self-luminous; they give out light themselves. Examples of artificial or man-made luminous sources are the electric lamp and the candle.

By contrast, the pages of this book are *non-luminous*. If you are reading it in daylight, light from the sun is falling on the page and some of it is scattered or reflected back to your eyes by the white page. The black print absorbs the light and stands out on the page. Similarly, light from an electric lamp is reflected back to your eyes if you are reading at night. A car driver at night can see a 'Stop' sign, or the rear reflector of a bicycle, by light reflected back from the car headlamps. Most objects are non-luminous. A person's face, or the pattern on a tie, for example, is seen by the sun or artificial light falling on it which is reflected back.

Rays of light · Ray-box

The direction or path along which light energy travels is called a light ray. The lines shown in Fig. 15.1a are rays which spread outwards from a very small lamp or 'point source' of light.

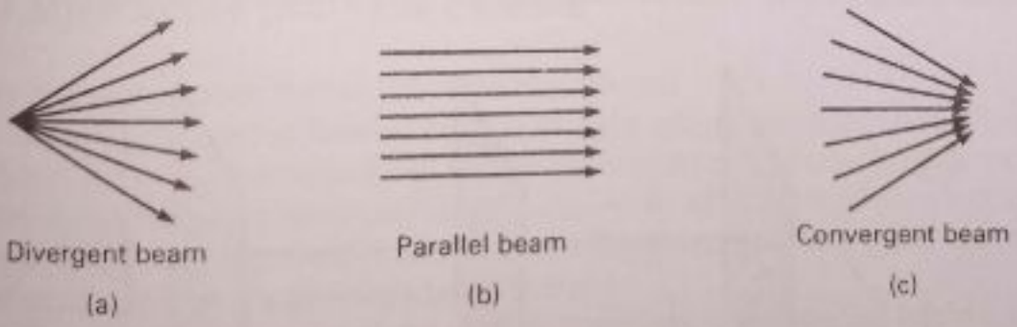


Fig. 15.1 Divergent, parallel and convergent beams and rays

A collection or beam of rays which spreads outwards as shown in Fig. 15.1a is called a *divergent* beam. A *parallel* beam is shown in Fig. 15.1b. A *convergent* beam is shown in Fig. 15.1c. For demonstration purposes, narrow beams or 'rays' of light are conveniently obtained from a ray-box B (Fig. 15.2). This has a small filament lamp L in an enclosed box, a cylindrical converging lens C and a 'comb' S containing parallel slits in front of the lens. By moving the lamp L, a diverging, parallel or converging beam of rays can easily be obtained.

Scanned with CamScanner

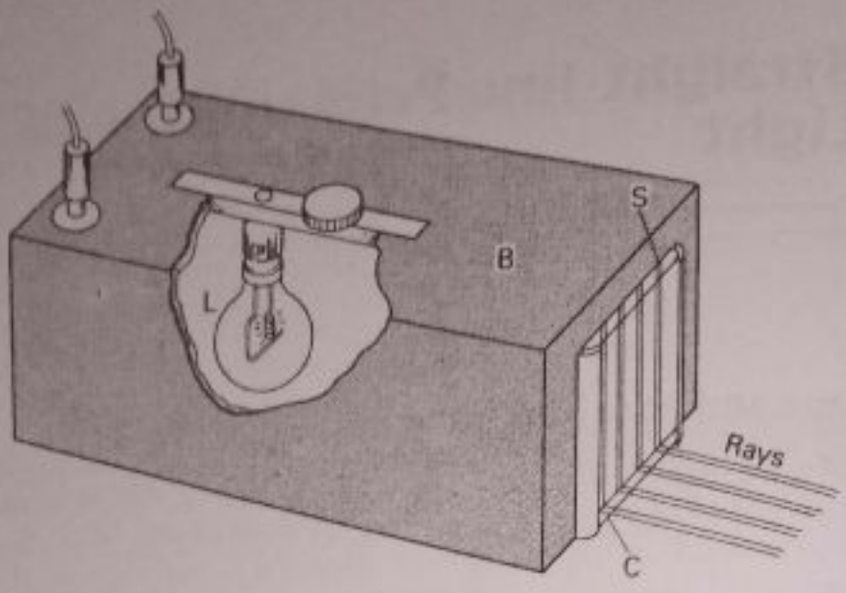


Fig. 15.2 Ray-box

Light and large apertures

If a ray-box is placed on a sheet of white paper and switched on, a thick ray is obtained through each of the rectangular slits of the 'comb' (see Fig. 15.2). This so-called ray is actually a very narrow beam of light. The edges of the ray are well defined, and so the *light can be seen to be travelling in straight lines*. We call this the 'rectilinear propagation' of light. With a much-wider opening, such as an open letter-box or a window, sunlight can be seen streaming through by light scattered from dust particles. The edges of the wide beam are again straight, showing that light travels in straight lines (see also page 355).

The rectilinear propagation of light can be demonstrated by placing an illuminated lamp L in front of a hole A in a piece of cardboard, and then moving two other pieces of cardboard, with holes B and C, until the light through A can be seen (Fig. 15.3). A thread passing through A, B and C and pulled tightly is perfectly straight along ABC, showing that the three holes are in a straight line.

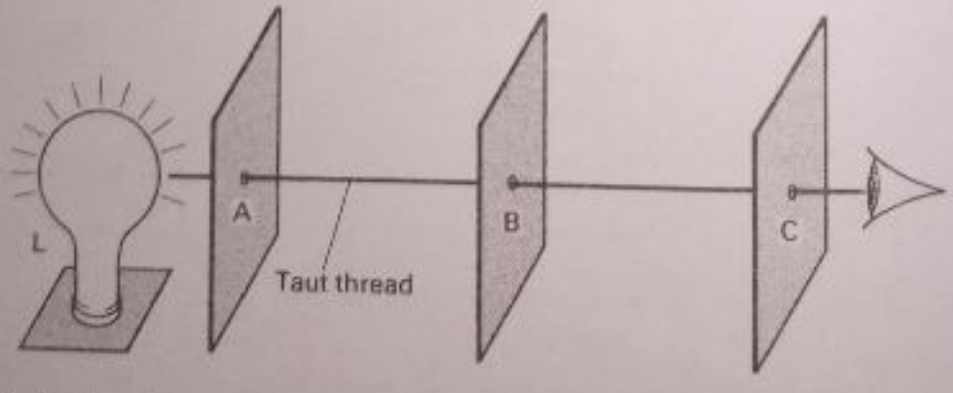


Fig. 15.3 To show the rectilinear propagation of light

Pin-hole camera

The pin-hole camera was the earliest camera. It was invented about 1550, well before lenses were made. It makes use of the fact that light travels in straight lines.

Scanned with CamScanner

Experiment

A pin-hole camera can be made simply by removing the cover from a closed box or tin, and replacing it by semi-transparent paper P (Fig. 15.4a) so that the box is sealed again from outside light. A small hole H is then made in the middle of the box opposite the paper P.

Hold up the box to a bright object such as a lamp in a darkened room (or a window). See (a) if the image on the paper screen is upside down, (b) the effect on the size of the image when the box is moved towards the bright object and then back.

Now make more pinholes near the first one and observe if the image is still clear. Finally, make a wide hole and observe the image. Now place a converging lens, whose focal length (see page 303) is about the length from the pinhole to the screen in front of the hole and observe if the image is clear.

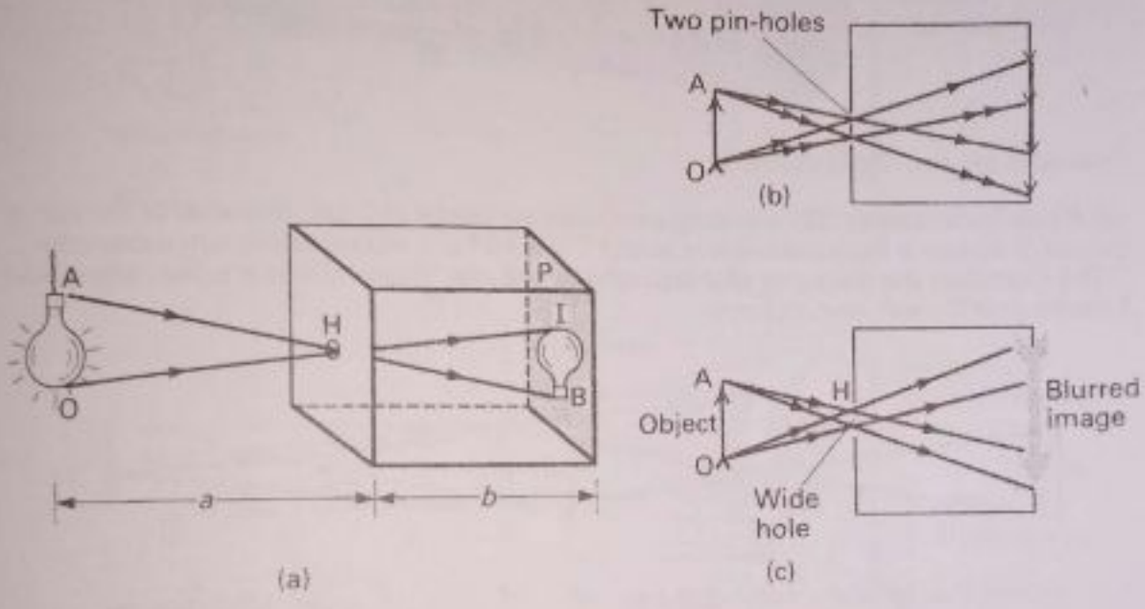


Fig. 15.4 Pin-hole camera and blurring

Images in the pin-hole camera

We can now explain the results of the experiment.

An image is always formed by rays of light which originally come from the object. A ray from a point O on the lamp, for example, passes through the hole and strikes the paper screen at I (Fig. 15.4a). A ray AH from A produces an image at B. If the hole H is small every point on the object produces a corresponding point image. The image is then fairly sharp.

The pin-hole camera has been used by surveyors. It is preferred by them to the lens camera, as the lens produces distortion. Its disadvantage is the long time required for an image to be developed on a photographic plate, as the amount of light passing through the hole is small.

With many pin-holes, or with a wide hole, the image becomes blurred. In this case each pin-hole produces its own image on the screen and the images overlap (Fig. 15.4b). A wide hole H in Fig. 15.4c is equivalent to a very large number of small pin-holes, so that appreciable overlap or blurring occurs.

A converging lens, however, brings *all* rays from a point on an object to a single point image on the screen, even though the rays are incident (fall) on different parts of the lens. So a sharp image is obtained (see the lens camera, page 316).

Scanned with CamScanner

Length of image

From the similar triangles AHO, BHI in Fig. 15.4a, it can be seen that

$$\frac{\text{BI}}{\text{AO}} = \frac{b}{a}$$

or

$$\frac{\text{image height}}{\text{object height}} = \frac{\text{image distance from H}}{\text{object distance from H}}$$

Suppose the length of the camera box is 10 cm and an object of height 2 m is photographed when it is 400 cm from the hole. Then

$$\frac{\text{image height}}{2\,\text{m}} = \frac{10\,\text{cm}}{400\,\text{cm}} = \frac{1}{40}$$

$$\therefore \text{ image height} = \frac{1}{40} \times 2\,\text{m} = \frac{1}{20}\,\text{m} = 5\,\text{cm}$$

Example on pin-hole camera

(a) A pin-hole camera 200 mm long produces an image of 2 mm diameter of the sun. If the sun's distance from the earth is about $1{\cdot}5 \times 10^8$ km, estimate the sun's diameter.
(b) Calculate the diameter of a disc which will just 'cover' the sun's disc when held 1 metre (1000 mm) from the eye.

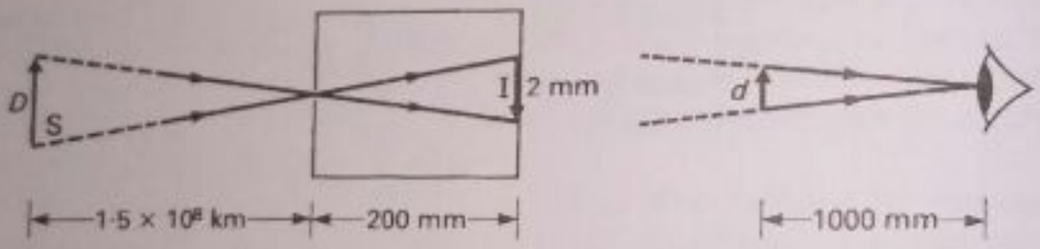


Fig. 15.5 Example

(a) Fig. 15.5a. In the exaggerated diagram, S is the sun of diameter D. By ratio

$$\frac{D}{1{\cdot}5 \times 10^8\,\text{km}} = \frac{2\,\text{mm}}{200\,\text{mm}} = \frac{1}{100}$$

So

$$D = \frac{1}{100} \times 1{\cdot}5 \times 10^8\,\text{km} = 1{\cdot}5 \times 10^6\,\text{km}$$

(b) Fig. 15.5b. Suppose d is the diameter of the disc. Then, by ratio

$$\frac{d}{1000\,\text{mm}} = \frac{\text{diameter of sun}}{\text{distance of sun}} = \frac{1{\cdot}5 \times 10^6}{1{\cdot}5 \times 10^8} = \frac{1}{100}$$

So

$$d = \frac{1}{100} \times 1000\,\text{mm} = 10\,\text{mm}$$

Shadows

Shadows occur because light travels in straight lines. When light rays arrive at an opaque obstacle, one which absorbs light, the rays just grazing the edges of the obstacle produce the outline of a shadow.

Scanned with CamScanner

The kind of shadow obtained depends on the size of the luminous object sending out the rays. In Fig. 15.6a a ray-box with a small filament, or *point source* of light, produces a sharp shadow of an opaque solid ball B on a screen placed behind B. The shadow is uniformly dark.

On the other hand, when the lamp is replaced by a *large* source of light such as a pearl lamp P the shadow is no longer uniformly dark (Fig. 15.6b). No light reaches the innermost shadow or *umbra*, the region of full darkness. The edge of the umbra corresponds to the rays PL and QM, which just graze B.

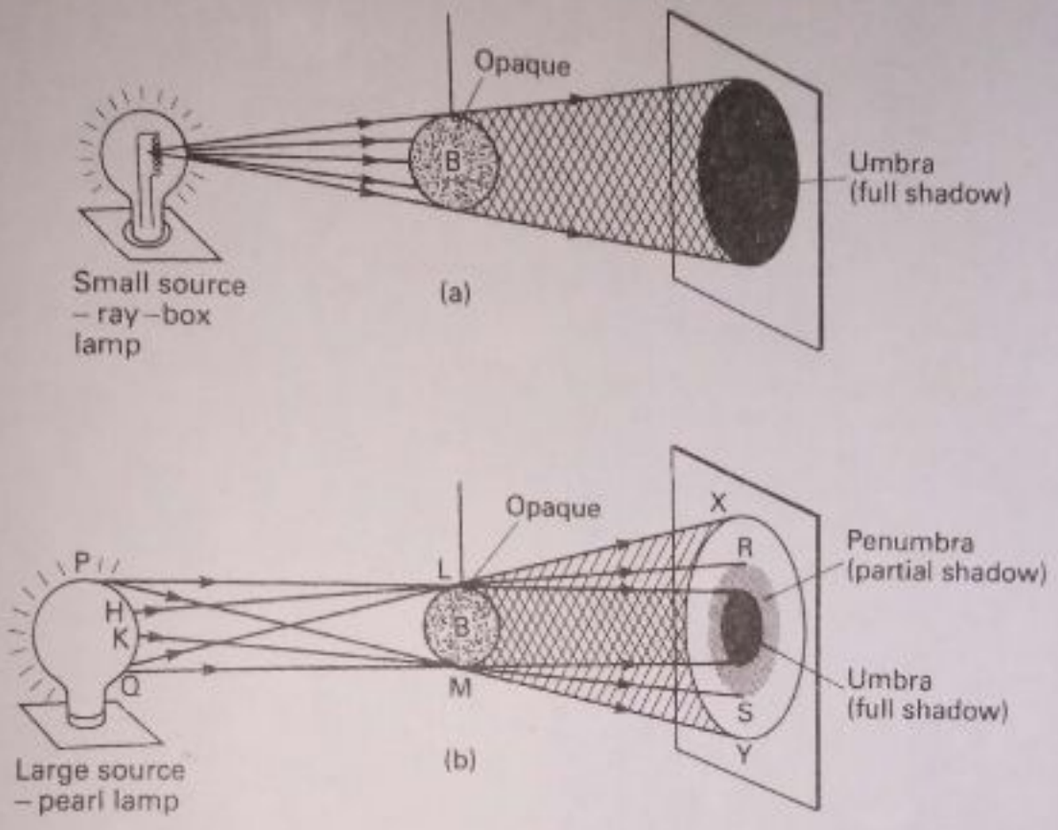


Fig. 15.6 Shadows (a) small source (b) large source

As we go away from the edge the darkness diminishes in intensity. At a point R on the screen, for example, the ray HLR shows that R receives light from that small part of the lamp corresponding to PH. The ray KM shows that the point S receives light from the small part QK. So R and S are in partial shadow, called the *penumbra*. The grazing rays QLX and PMY show that X, Y are on the edge of the penumbra. Above X or Y, points on the screen are illuminated by the whole of the lamp, and so no shadow is obtained here.

Eclipses

The moon and earth are both non-luminous. In a clear sky the moon can be seen by light from the sun, which is scattered by the moon's surface. An *eclipse of the sun* occurs when the moon, an opaque object, comes between the sun and the earth, as shown in Fig. 15.7a. Parts of the earth lie in the umbra and penumbra of the moon's shadow.

The appearance of the sun from various parts of the earth is illustrated in Fig. 15.7b. A person at *a*, as shown in Fig. 15.7a, sees a *total eclipse*, another at *b* or *d* sees a *partial eclipse*, and another at *c* or *e* sees no eclipse. On another occasion,

Scanned with CamScanner

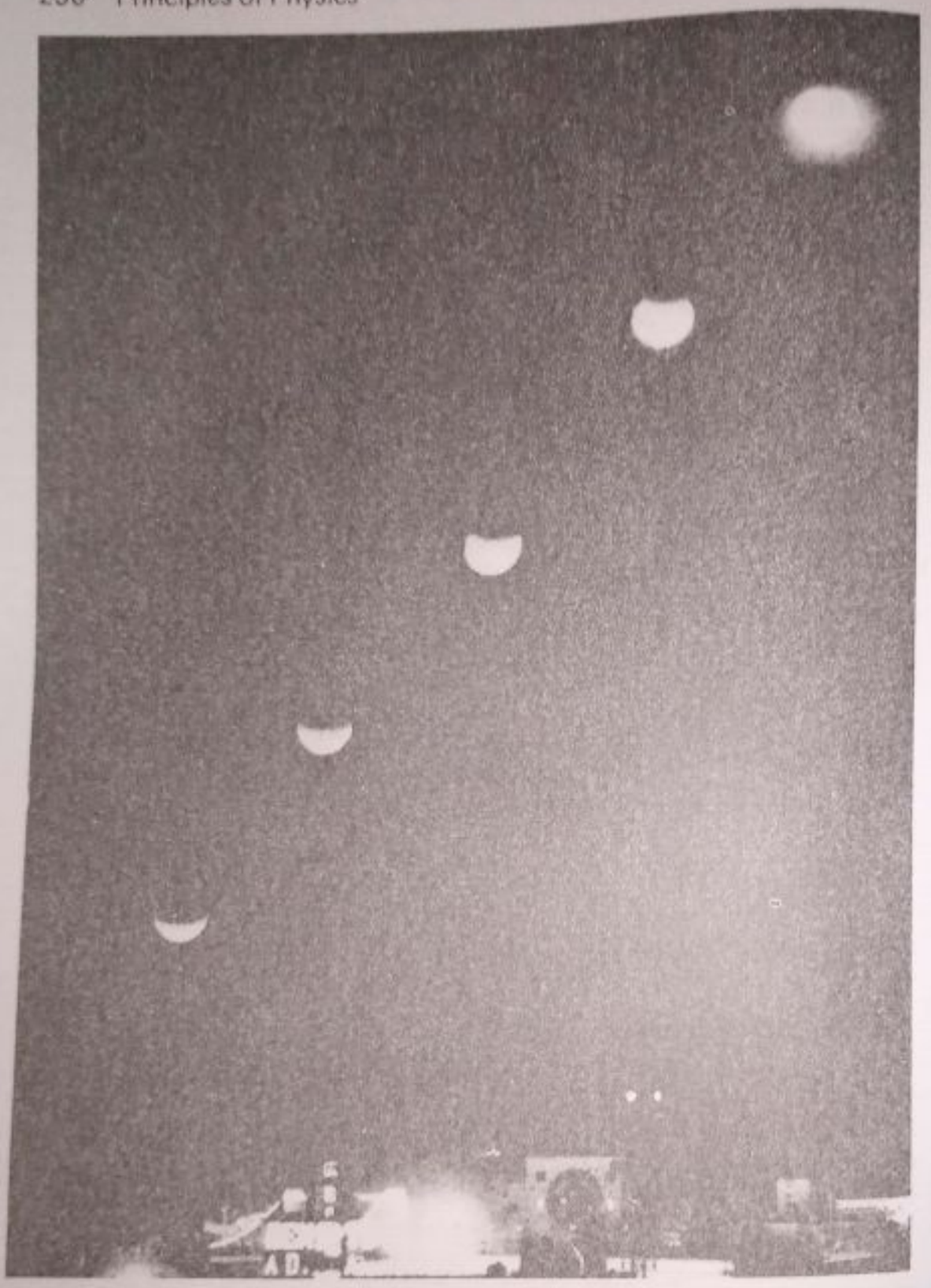


Plate 15A Stages in an eclipse of the moon over Tokyo, taken on the same film at intervals of 10 minutes from left to right (*Keystone Press Agency*)

different from that shown in Fig. 15.7a, the earth and moon may be in positions when the rays at the moon's edge intersect before reaching the earth. An observer in a position corresponding to *a'* in Fig. 15.7b then sees an *annular eclipse*, a ring of light round the shadow of the moon.

An *eclipse of the moon* occurs when the earth comes directly between the sun and the moon, when the moon is on the opposite side of the earth to the position

Scanned with CamScanner

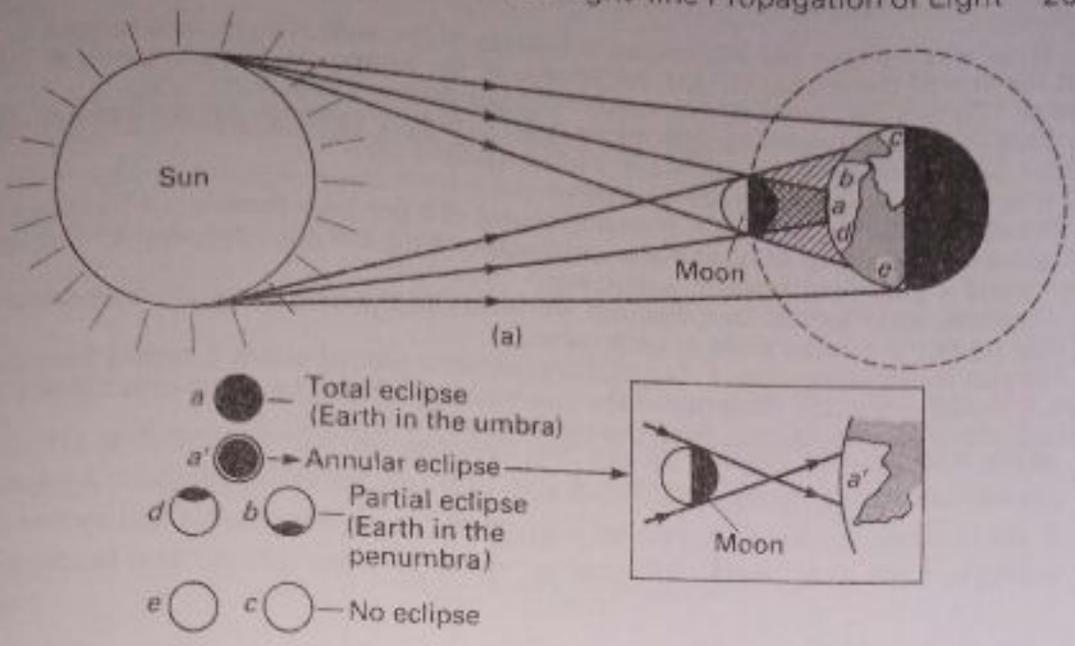


Fig. 15.7 Eclipses of the sun

shown in Fig. 15.7a. The earth is an opaque body, so the moon is then in the earth's shadow and an eclipse of the moon is seen.

Since the moon's orbit round the earth does not lie in the same plane as that of the earth's orbit round the sun, partial eclipses of the moon do not occur every time the moon is on the opposite side of the earth from the sun.

Summary

1. Luminous objects radiate light; non-luminous objects are seen by diffusely reflected light.
2. Beams of light may be parallel, convergent or divergent.
3. Shadows and eclipses are due to the rectilinear propagation of light. The umbra is the region of full shadow (total eclipse); the penumbra is the region of partial shadow (partial eclipse).
4. In the pin-hole camera the image is inverted. The size of the hole must not be too large, otherwise the image becomes blurred–a point on the image then being illuminated by light from several points on the object.

Exercise 15 (Answers, page 642)

1. In Fig. 15.8, A represents a spherical source of light and B an opaque spherical object, both of which are placed as shown. Draw a diagram showing the shape of A as seen from P. (N)

2. (a) Eclipses of the sun can be described as total, partial or annular. Explain with the aid of diagrams how the three types of eclipse are formed.

Show clearly where observers must be to see each of these eclipses. (N)

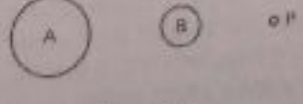


Fig. 15.8

Scanned with CamScanner

3. What effect, if any, has an increase in the size of the hole of a pin-hole camera on: (a) the size of the image; (b) the brightness of the image; (c) the sharpness of the image? What is the chief disadvantage of the pin-hole camera in use? (N)

4. Explain with a diagram the conditions necessary for a partial eclipse of the moon to occur. Why do eclipses not occur every lunar month? (N)

5. Describe how a pin-hole camera works.

The distance between the pin-hole and screen of a pin-hole camera is 12·5 cm, and the plate is 20 cm long. At what minimum distance from the pin-hole must a 1·8 m tall man stand if a full-length photo is required?

6. Describe, and illustrate by a diagram, the effect on the image in a pin-hole camera of making *two* pin-holes close to each other.

The sun is just covered by a disc of 2 cm diameter placed about 2 metres from the eye. If the length of the diameter of the sun's image formed by a pin-hole camera is 0·5 cm, calculate the distance from the pin-hole to the screen.

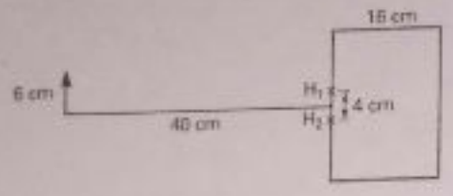


Fig. 15.9

7. Fig. 15.9 shows an illuminated arrow of length 6 cm placed 40 cm in front of a pin-hole camera 16 cm in length having two small holes, H_1 and H_2, 4 cm apart.

(a) Make a scale diagram and draw suitable rays from the luminous arrow to show how two images are formed on the back of the box.

(b) Show why one large blurred image is obtained if the cardboard between the small holes were cut away to form one large circular hole of diameter 4 cm.

(c) Draw, separately, a ray diagram to show how a suitable converging lens, placed at 40 cm from the luminous arrow, may produce a sharp real image at a distance of 16 cm from the lens and determine the focal length required. (L) (See also *Lenses*.)

8. Show by means of a diagram how a pinhole camera can produce an image larger than the object which is viewed. State and explain briefly *one advantage* and *one disadvantage* of this device compared with a lens camera. (L)

9. Describe *one* way you would demonstrate experimentally that light travels in straight lines.

A pinhole of 1 mm diameter is in the middle of a piece of black paper covering one end of a tube 1 m long. The other end of the tube is covered by a screen of tracing paper. When this 'pinhole camera' is pointed towards the sun, the diameter of the image on the screen is found to be 10 mm. Draw a ray diagram (not to scale) showing how the image is formed.

What would you expect the image diameter to be if the length of the tube were increased to 2 m? What would be the diameter of the sun's image formed by a converging lens of focal length 2 m? Mention *two* ways in which the image formed by the lens differs from that in the pinhole camera. (O and C) (See also *Lenses*.)

Multiple-Choice Questions

10. A pin-hole camera gives a sharp image
A because the image is small B if the hole is extremely small
C because the object is a long way from the screen D if the hole is small
E because the screen is a short distance from the hole

11. When illuminated by a lamp, the shadow of a table-tennis ball on a white screen is uniformly dark. This is because the lamp is
A very bright B fluorescent C very small D very weak E very large

12. A total eclipse of the sun is due to
A the moon coming between the earth and sun
B the earth coming between the moon and sun
C the moon reflecting light away from the earth
D the earth reflecting light away from itself
E the earth being transparent

Scanned with CamScanner

16. Reflection at Plane Mirrors

All highly polished or shining surfaces can act as mirrors but the best plane mirrors are made by silvering one side of a flat piece of glass. Plane mirrors are used as looking-glasses and as car mirrors. They are also used in the sextant, a navigational instrument for measuring the angle of elevation of the sun (Plate 16A).



Plate 16A A sextant used in navigation. It measures the angle of elevation of the sun. A plane mirror is used at the top of the instrument and a half-silvered mirror below. (*P and O Lines*)

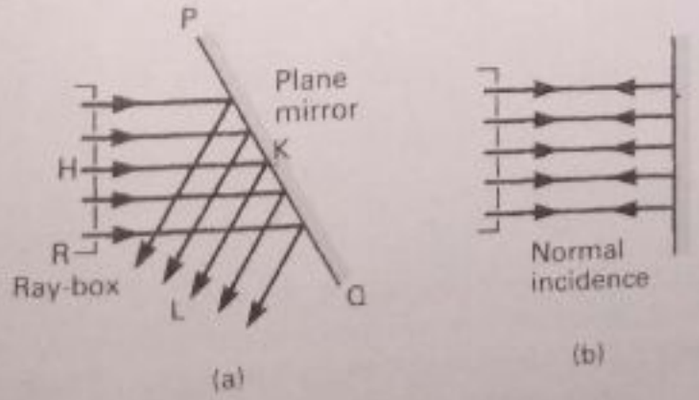


Fig. 16.1 Reflection by a plane mirror

Scanned with CamScanner

The reflection of light rays can be investigated as in Fig. 16.1a with a ray-box R and a plane mirror PQ on a sheet of white paper. A parallel beam of light HK is seen reflected by the mirror in the direction KL. When the mirror is turned, the direction of the reflected rays changes. When the light rays are incident (or fall) *normally* on the mirror, that is at right angles to the mirror, they are reflected straight back along the same path (Fig. 16.1b).

Diffuse light

A plane mirror is said to produce *regular* reflection because a parallel beam of light is reflected in one direction and not scattered. This is due to the smoothness and high polish of the reflecting surface. But most objects do not reflect light with such regularity. Paper and clothing materials are examples. This is because of the lack of smoothness, or grain, of the surface, which can be seen with a high-power microscope. Rays of light incident on such surfaces are reflected in different directions, R_1, R_2 and R_3 in Fig. 16.2. They are said to be *scattered* or *diffusely reflected*. The pages of a book, or a person's face are seen by diffusely reflected light.

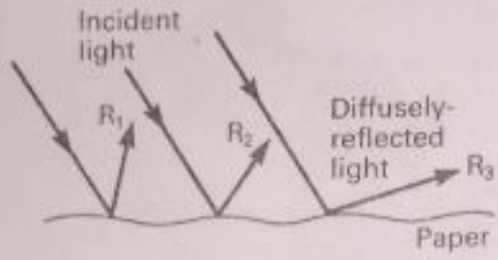


Fig. 16.2 Diffuse reflection

Investigation of reflection

Reflection of light can be investigated in more detail as follows.

Experiment

Using plasticene or blocks, support a plane mirror vertically on a sheet of white paper and draw a line ON normal to the mirror from a point O near the middle of the mirror (Fig. 16.3a). Draw lines on the paper making angles such as 20°, 45°, 60° and 80° to ON.

1. Place a ray-box R so that a *single ray* follows one of the drawn lines, say AO. Mark the reflected ray OB with two dots. (With a thick ray, use the edge of the incident ray and the corresponding edge of the reflected ray.) Now move the ray-box so that the ray is incident along each of the other drawn lines in turn and mark each reflected ray direction as before. Measure and record the angles of incidence i and the corresponding angles of reflection r in a table.

Result: Experiment shows that, in all cases, the angle of incidence i = the angle of reflection r.

Note: The angles of incidence and reflection are always measured from the *normal* (perpendicular) to the mirror.

2. Fig. 16.3b illustrates how a reflected ray can be located by using pins. Two pins, P and Q, are placed on an incident ray as shown, and then, looking into the mirror, two other pins, R and S, are positioned in line with the images A, B of P, Q. The reflected ray is the line joining R and S. Values of i and r can then be measured as in 1. above.

Scanned with CamScanner

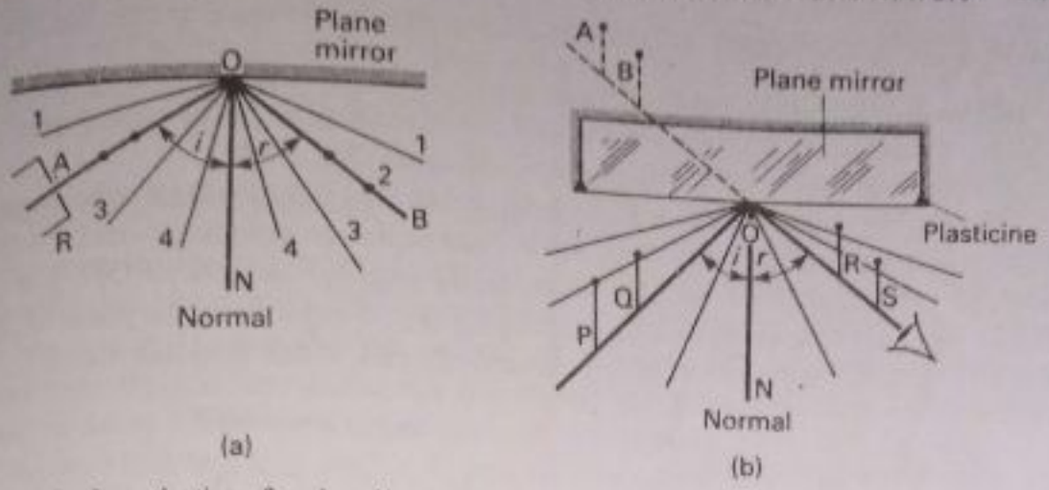


Fig. 16.3 Investigation of angles of incidence and reflection

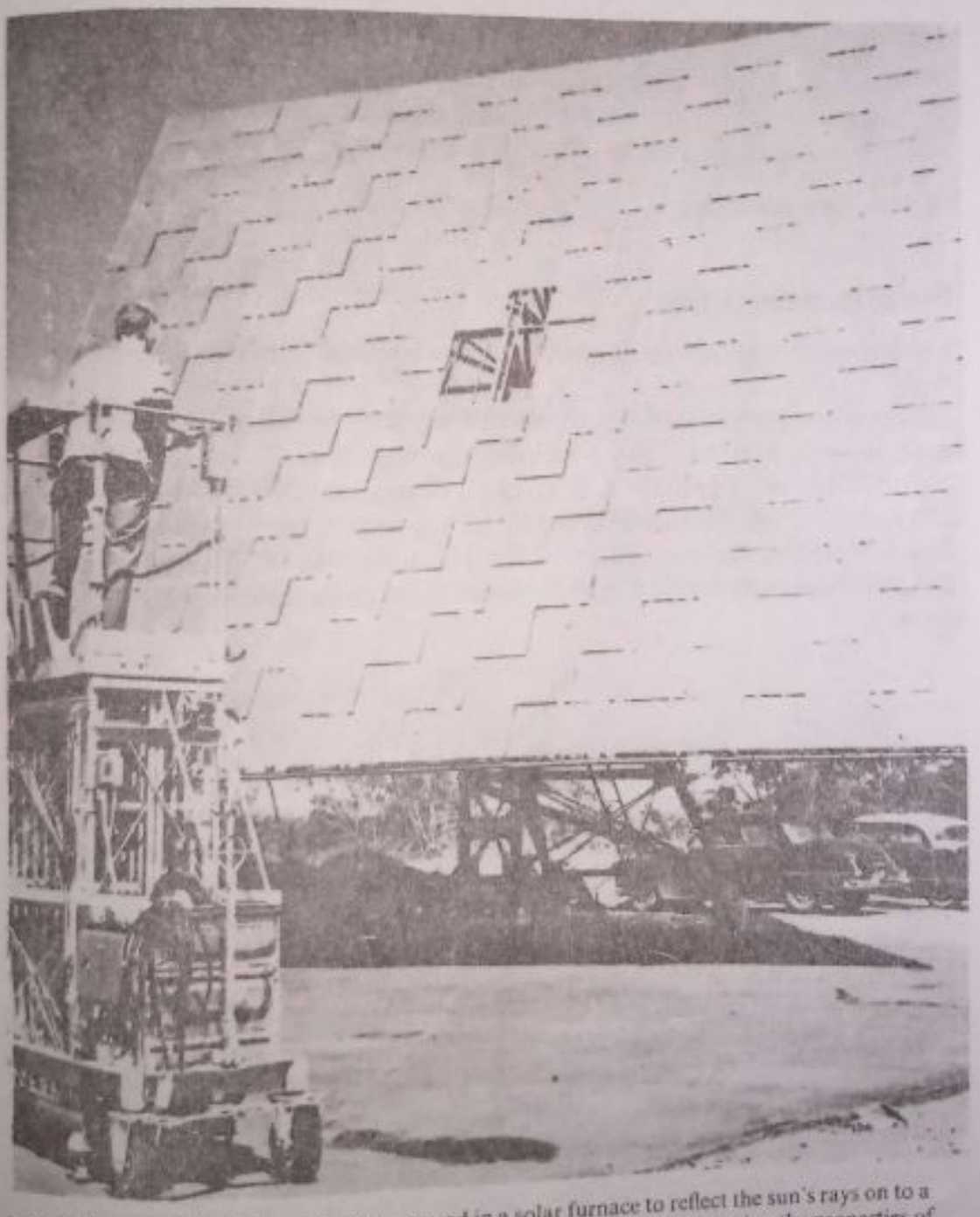


Plate 16B Hundreds of plane mirrors are used in a solar furnace to reflect the sun's rays on to a small area, thus producing very high temperatures which can be used for testing the properties of materials. (*U.S. Information Service*)

Scanned with CamScanner

Laws of reflection

The laws of reflection at plane surfaces are:

1. The incident ray, the reflected ray and the normal all lie in the same plane.
2. The angle of incidence = the angle of reflection (Fig. 16.4a).

The first law is illustrated in Fig. 16.4b. A ray AO is incident on a horizontal plane mirror at O. Many lines, such as OC, can be drawn radiating from O so that they are inclined to ON at an angle equal to the angle of incidence AON. But the first law restricts the position of the reflected ray; it must lie in the plane of AO and ON. Therefore the reflected ray can only be OB, which is in this plane.

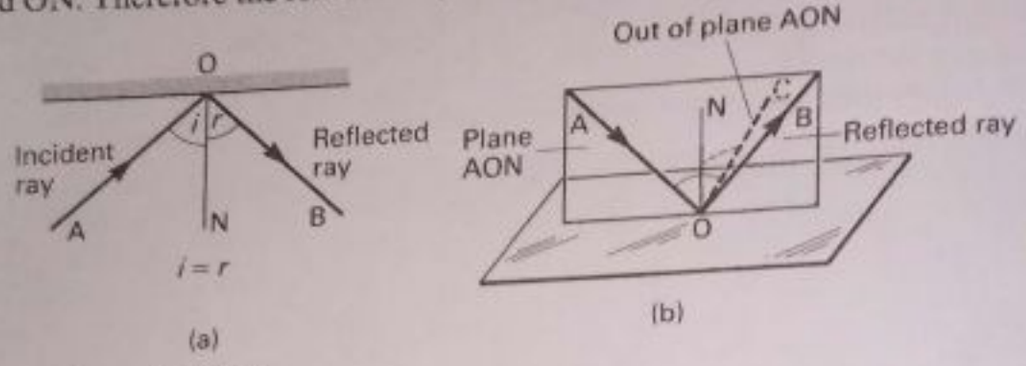


Fig. 16.4 Laws of reflection

Simple periscope

A simple periscope uses plane mirrors to see objects hidden from direct view (Fig. 16.5).

Two plane parallel mirrors, A and B, are fixed at 45° to the horizontal at opposite ends of a long tube, which has openings X and Y. A ray OL from an object O falls on the mirror A at an angle of incidence 45°. So the reflected ray LM is at 45° to the normal at L, that is, at right angles to the incident ray. LM is then reflected by the mirror B, on which it is incident at 45°. The reflected ray BS is therefore at right angles to LM. It passes through the hole Y and is seen by the eye at S.

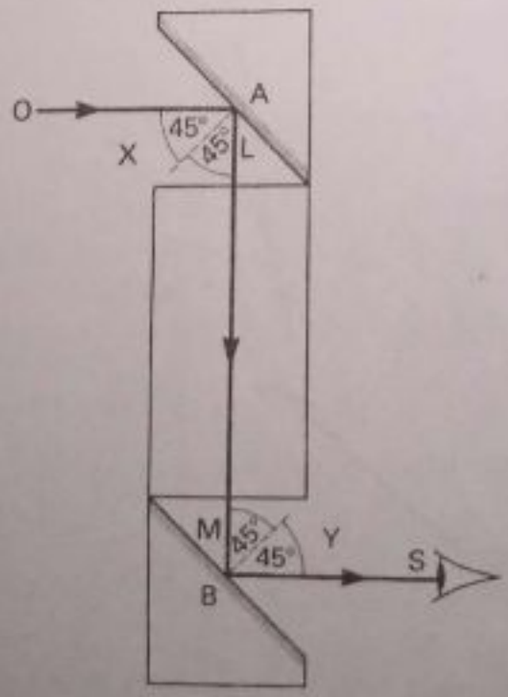


Fig. 16.5 Simple periscope

Scanned with CamScanner

In this way it is possible to see objects hidden, for example, by the heads of a crowd. The periscope will not work efficiently if the mirrors are not in their correct positions, that is they should be parallel, and at 45° to the horizontal.

Uses of plane mirrors in meters

In commercial meters, a plane mirror, in the same plane as the scale, is sometimes placed below the pointer. The scale reading of the pointer is taken when its image in the mirror is seen *directly behind* the pointer. The plane mirror helps to overcome error in the reading due to parallax (see page 469) between the pointer and the scale. This occurs when the eye is not vertically above the pointer.

A plane mirror is also used to measure the small angle of rotation of the coil in a *mirror galvanometer*. This meter measures very small electric currents.

Suppose a beam of light OM is incident normally on the mirror M (Fig. 16.6). The beam is then reflected straight back to a point O on a millimetre scale S. When the current flows, the coil rotates through a small angle θ. So the mirror attached to it turns through an angle θ, from MA to MB. The incident beam OM is now reflected along MP. The shift d of the light beam along the scale OP can be measured.

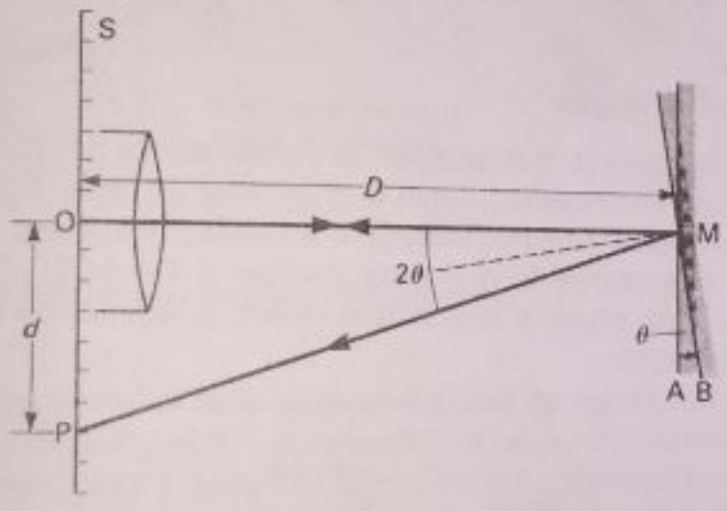


Fig. 16.6 Mirror galvanometer deflection

The angle OMP is 2θ, *twice* the angle of deflection θ. This can be seen by drawing a normal to the mirror MB at M. So $\tan 2\theta = d/D$, where OP = d and OM = D. From measurements of OP and OM, 2θ can be calculated and so θ is found. The light beam used to measure the deflection acts as a very long 'pointer' and is of negligible weight, which is another advantage.

Image position · Virtual and real images

When a small object O is seen by reflection in a plane mirror, the image I appears to be *behind the mirror*.

Fig. 16.7 explains why this happens. An observer at E, for example, looking into the mirror, sees the rays OA and OB reflected along AX and BY. AX and BY appear to come from I, the point of intersection if they are extended backwards in a straight line. Also, any other ray from O reflected from the mirror to E appears to come from I. So the *image* of O is at I. Note that a ray ON normal

Scanned with CamScanner

(perpendicular) to the mirror is reflected back along NO, so that I lies on the normal to the mirror from O.

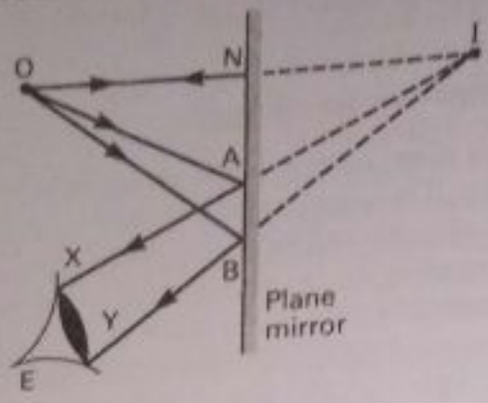


Fig. 16.7 Virtual image in a plane mirror

In Fig. 16.7, no rays actually pass through I. The image I is therefore called a virtual image; if a screen were placed in front of the mirror you would not see an image of I on it because the rays AX and BY are diverging from each other.

In contrast, a *real* image is one which could be focused on a screen and is formed by a convergent beam of rays which actually pass through it, as we see later.

Location of images

The image of an object O, such as a pin, in a plane mirror M (Fig. 16.8) can be found by placing two other pins, A and B, in line with the image I observed in the mirror. This is repeated with two more pins in a different position, such as C and D or E and F. After taking the pins away, the lines AB and CD (or AB and EF) are produced. Their point of intersection, which is behind the mirror, is the position of the image I.

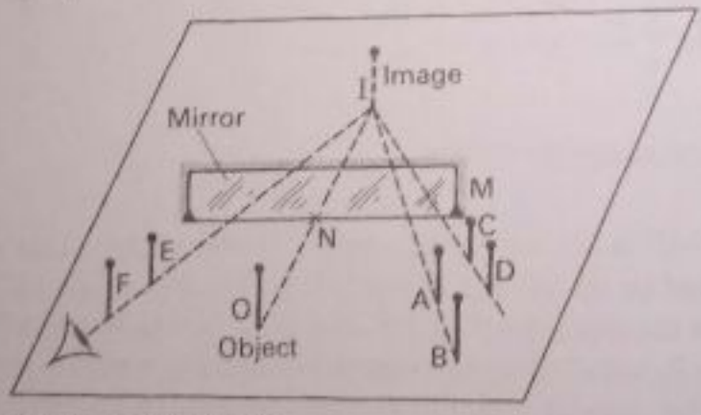


Fig. 16.8 Locating images in plane mirrors

Measurements from experiments show that the perpendicular distance IN of the image I from the plane mirror is always equal to the perpendicular distance ON of the object O from the mirror. So the image is as far behind the mirror as the object is in front.

Image of large object · Lateral inversion

The image of a large object can be investigated with the help of a sheet of plane glass.

Scanned with CamScanner

Experiment

Support the sheet of glass G vertically by clamps. Place your left hand L flat on the table and spread out your fingers on one side near G (Fig. 16.9a). Now place the *right* hand, behind G, and looking through G from the left at the same time, move this hand until all the fingers cover the image of the left hand, as shown.

Ask a friend to measure the perpendicular distance ON of your thumb O from the glass G and the perpendicular distance IN of its image I, and the corresponding distances of two other fingers and their images. Record the measurements.

Result: IN = ON for all objects and their images. So the image is as far behind G as the object is in front.

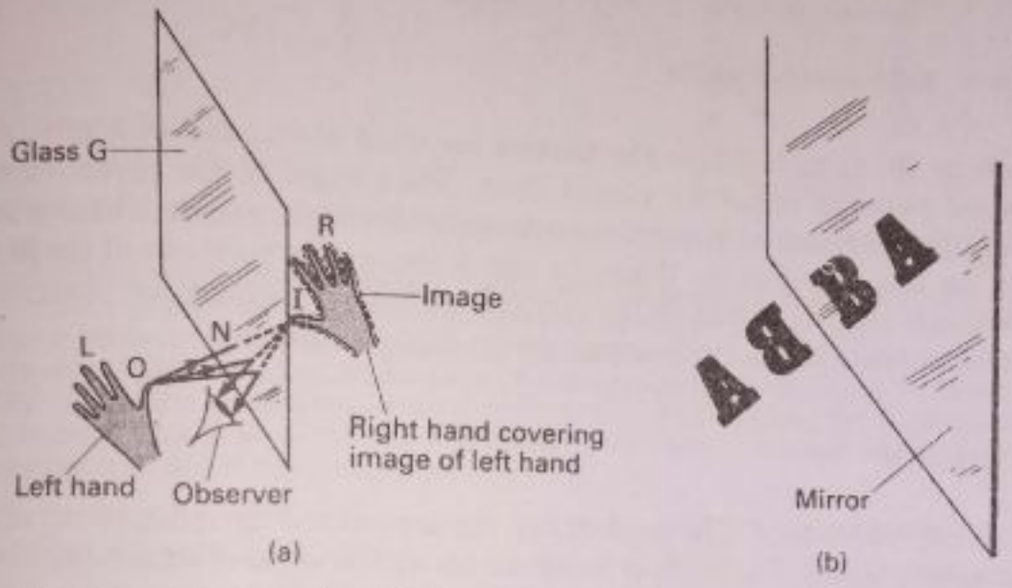


Fig. 16.9 (a) Locating image of hand (b) Lateral inversion

Since the image of the left hand looks exactly like the right hand to an observer, the image of a large object in a plane mirror is said to be *laterally inverted*. The inversion is also seen by placing lettering in front of a plane mirror (Fig. 16.9b). A right-handed tennis player will therefore appear left-handed if observed in a mirror. Words inverted on carbon paper by typing can be seen the right way round by holding the paper in front of a mirror. The famous artist and inventor, Leonardo da Vinci, used to write from right to left to avoid ink-smudges, for he was left-handed. His writing can be read by holding it up to a mirror.

Summarising, the image in a plane mirror is: 1. laterally inverted; 2. the same size as the object; 3. virtual; and 4. as far behind the mirror as the object is in front.

Inclined mirrors · The kaleidoscope

When an object is placed between two inclined mirrors, an observer can see several images, the number of which depends on the angle between the mirrors. Fig. 16.10 shows an object O placed between two mirrors M_1 and M_2 at 90° to each other. The image I_1 is due to reflection at M_1, and the image I_2 to reflection at M_2. The image $I_{1,2}$ is due to reflection at M_1 and then at M_2. The figure shows how rays starting from O reach the observer E by repeated reflection.

The kaleidoscope is a toy in which multiple images are formed by two mirrors,

Scanned with CamScanner

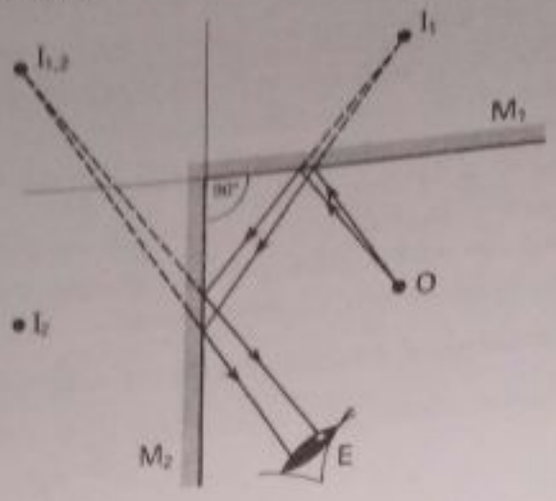


Fig. 16.10 Reflection at two mirrors

usually at 60° to each other. The mirrors are fixed at one end of a tube, and coloured pieces of paper are placed there. The images of the paper, viewed through the other end of the tube, are seen symmetrically in a circle. The coloured pieces are also seen directly. When the tube is shaken the positions of the paper change and other coloured image patterns are seen. Designers are sometimes helped in looking for new colour patterns by the kaleidoscope.

Summary

1. Laws of reflection: 1. The incident ray, the normal and the reflected ray all lie in the same plane. 2. The angle of incidence equals the angle of reflection.
2. The image in a plane mirror is laterally inverted, the same size as the object, virtual, and as far behind the mirror as the object is in front.
3. A simple periscope contains two parallel plane mirrors inclined at 45° to the horizontal.

Exercise 16 (Answers, page 642)

1. Show by a ray diagram how a suitably placed eye sees an image of a point object which is placed 10 cm in front of a *plane mirror*. Show clearly the position of the image and give two reasons why it is described as virtual. (L)
2. Explain with the aid of a sketch, why the letter F on the number plate of a following car appears as ꟻ to a driver when he sees it in his driving mirror, assumed to be plane. (O and C)
3. What are the characteristics of the image produced by a plane mirror when an object is placed in front of it?

A horizontal beam of light from a lamp falls on a plane mirror and is reflected horizontally through an angle of 90° so as to fall normally on a screen 1 metre distant from the mirror and form a spot of light. Draw a diagram to show how the mirror must be placed in relation to the incident beam and determine through what angle the mirror must be turned in order to make the spot of light move horizontally a distance of 10 cm along the screen. Suggest one useful application of such an arrangement. (L)

4. Fig. 16.11 shows a letter F in a horizontal plane in front of a vertical plane mirror AB. Draw the image of the letter seen by reflection and the path of *one* ray of light from the bottom of the letter to the eye E. (N)
5. Describe how you would find experimentally the position of the image of a pin formed by a plane mirror. State the nature and position of the image. (L)
6. State the laws of regular reflection of light. Describe how you would verify them experimentally. (O)

Scanned with CamScanner

7. Explain (a) why the image of an object formed by a plane mirror is called a virtual image; (b) why the image of each point on the object is as far behind the mirror as the object point is in front; (c) why the image appears laterally inverted.
Describe, with the help of a ray diagram, the action of a simple form of periscope. (O)

8. State the laws of reflection of light. Show that the image of an object in a plane mirror is as far behind the mirror as the object is in front. What experiment would you perform to locate the image formed by a plane mirror?
A simple periscope consists of two plane mirrors M_1 and M_2, which are 90 cm apart vertically and each inclined at 45° to the vertical. A point object O, 60 cm away from the upper mirror M_1, is viewed by an eye E, 30 cm away from M_2, both these distances being taken horizontally. Draw a ray diagram tracing the paths of two rays from O to E, and find the distance from E of the final image seen. (O)

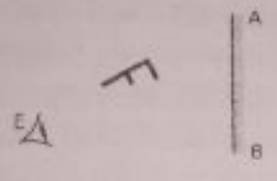


Fig. 16.11

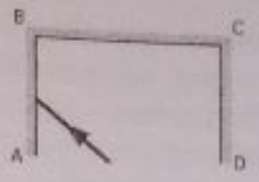


Fig. 16.12

9. Describe an experiment to determine the position of the image formed by a plane mirror.
Explain the meaning of the term 'parallax error' as applied to the reading of a pointer on a scale and describe how this error may be reduced by using a plane mirror.
Three plane mirrors are arranged along three sides of a square as shown in Fig. 16.12 and a ray of light is incident on the mirror AB at its midpoint with an angle of incidence of 40° so that the ray is afterwards reflected by BC and CD. Copy the diagram, sketch the approximate path of the ray and calculate the angle through which the ray is turned at each of the three reflections. (N)

Multiple-Choice Questions

10. A typist uses new carbon-paper for making a copy of a letter. When she holds the carbon-paper close to a plane mirror, she can read the letter. This is because the mirror
A forms an image the same size B produces an inverted image
C produces an image laterally inverted D has a virtual image
E has an image as far behind the mirror as the letters seen

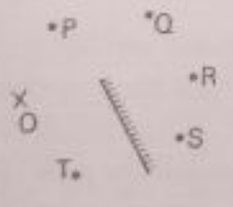


Fig. 16.13

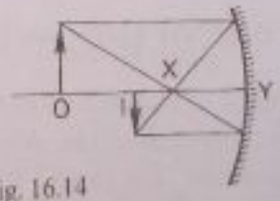


Fig. 16.14

11. In Fig. 16.13, the image of an object O is
A at P B at Q C at R D at S E at T

Curved Mirrors (to be done after Exercise 17, page 279)

12. In Fig. 16.14, the concave mirror has a focal length of 20 cm. Which statement from A to E is correct?
A The image I is virtual B the object O is at the centre of curvature
C the distance IY may be about 40 cm D the distance IY may be about 30 cm
E I is nearer Y than the focus of the mirror

A real object O is placed at a distance x cm in front of a spherical mirror of focal length a cm.

13. The image is upright and smaller than the object O.

14. The image is upright and taller than the object O.

Choose one of the statements A to E for (13) and (14):
A convex mirror and $x = 2a$ B concave mirror and $x = 2a$
C concave mirror and $x = 4a$ D concave mirror and $x = a/2$
E concave mirror and $x = 3a$

Scanned with CamScanner

17. Curved Spherical Mirrors

Curved mirrors have a large number of scientific and practical uses. The Mount Palomar or Hale telescope in America, for example, works by reflection of light from a giant concave mirror. Mirrors with a parabolic shape are used in searchlights and in car headlamps to produce powerful parallel beams of light. Convex mirrors are often used on cars for observing traffic behind; shaving mirrors are concave (Fig. 17.1).

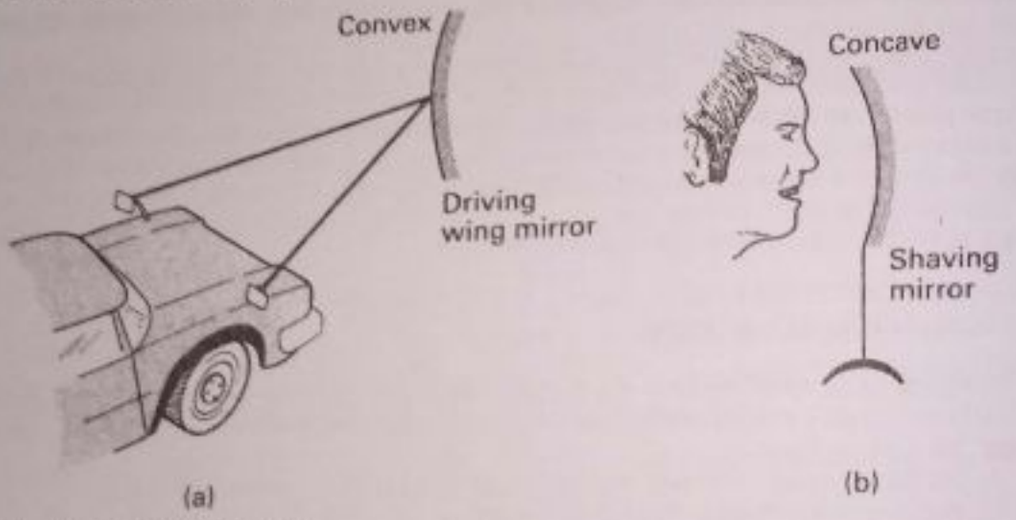


Fig. 17.1 Uses of curved mirrors

Curved mirror properties

Concave and convex mirrors are made by silvering a glass surface which is part of a sphere, like a watch-glass. As shown in Fig. 17.2, the width AB of the mirror is called the *aperture*; the centre P of the mirror is called the *pole*; the centre C of the

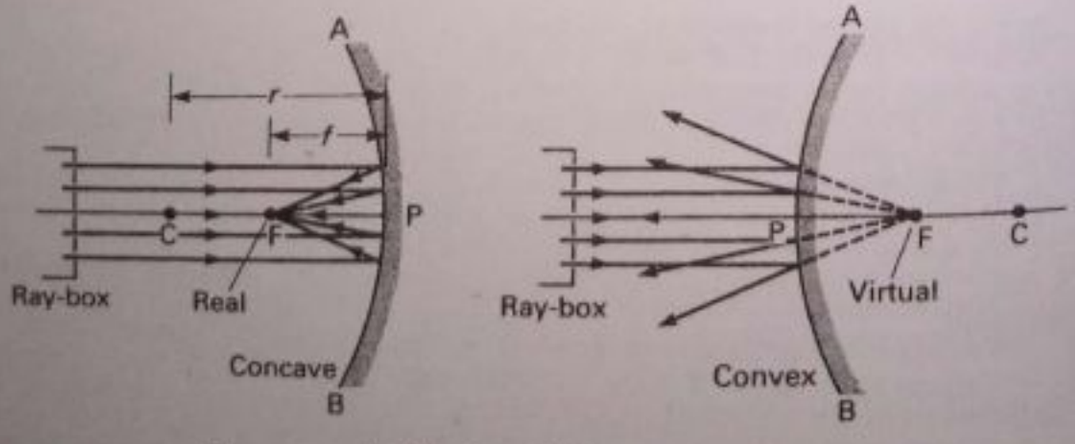


Fig. 17.2 Concave and convex mirrors: reflection of parallel beams

Scanned with CamScanner

sphere of which the mirror is part is called the *centre of curvature*; the line PC is called the *principal* (chief) *axis* of the mirror.

The *radius of curvature r* is the distance PC; the *focal length f* is the distance PF. As we see later, the position and size of an image in a mirror depends considerably on the value of its radius of curvature or focal length.

Reflection of rays from curved mirrors

When a beam of *parallel rays*, close to the principal axis CP and parallel to it, are incident on a concave mirror, the reflected rays converge to a point F on the principal axis; this point is therefore called the *principal focus* of the mirror (Fig. 17.2a).

By contrast, rays parallel and close to the principal axis *diverge* from the reflecting surface of a convex mirror (Fig. 17.2b). They appear to diverge from a point F on the principal axis behind the mirror. The principal focus of a concave mirror is a *real* focus, that is, reflected rays actually pass through it. The principal focus of a convex mirror is a *virtual* focus, which means that reflected rays do not actually pass through it.

Fig. 17.3 shows an incident beam converging to F, the principal focuses of a concave and of a convex mirror. In both cases the rays in the beam of light are reflected back parallel. Figs. 17.2 and 17.3 should be compared with each other. They show the paths of the light rays are the same, so that light rays are reversible. This illustrates a general law of optics called the *Principle of Reversibility of Light*.

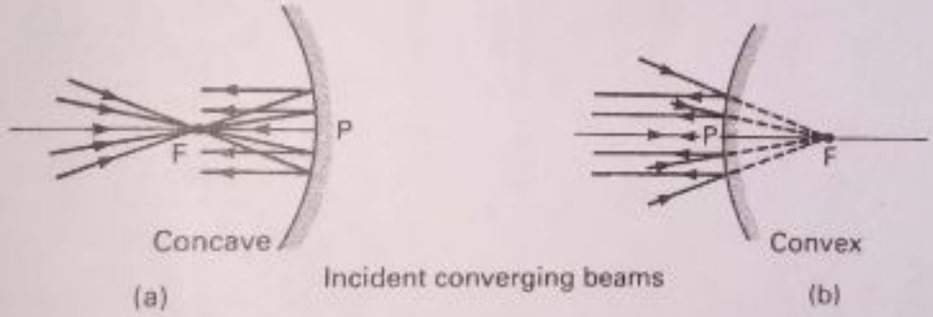


Fig. 17.3 Reflection of converging beams

Producing parallel beams · Parabolic mirrors

As we have seen, a *narrow* parallel beam of light close to the principal axis is reflected from a concave mirror to one definite point, the principal focus F (Fig. 17.4a). If, however, a *wide* parallel beam covering the whole of the aperture is

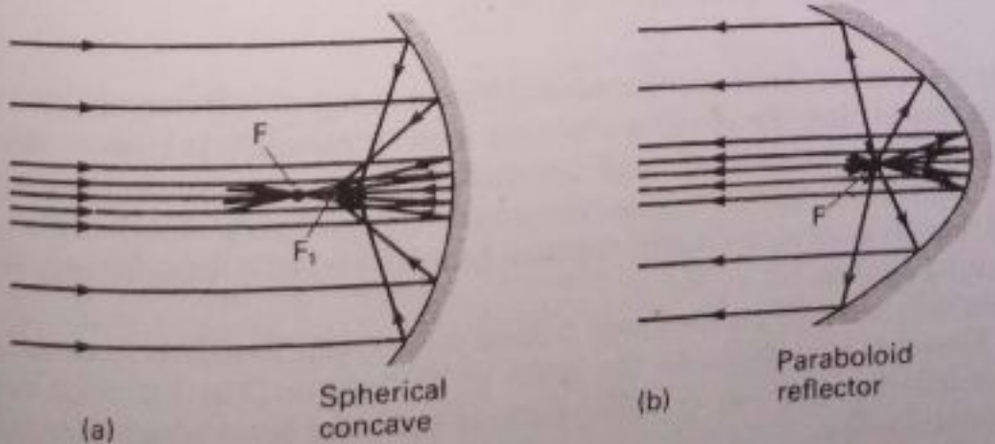


Fig. 17.4 Spherical and paraboloid reflectors

Scanned with CamScanner

incident on the mirror, experiment shows that the reflected rays well away from the axis are brought to a focus at different points, such as F_1. So the parallel beam produces a blurred focus, and this is called *spherical aberration*.

From the Principle of the Reversibility of Light, it follows that part of the light from a small lamp placed at the principal focus F will be reflected not as a parallel beam from the *outer* parts of the mirror, but as a divergent beam. The reflected light becomes weaker as its distance from the mirror increases, because the area over which it spreads becomes larger and larger.

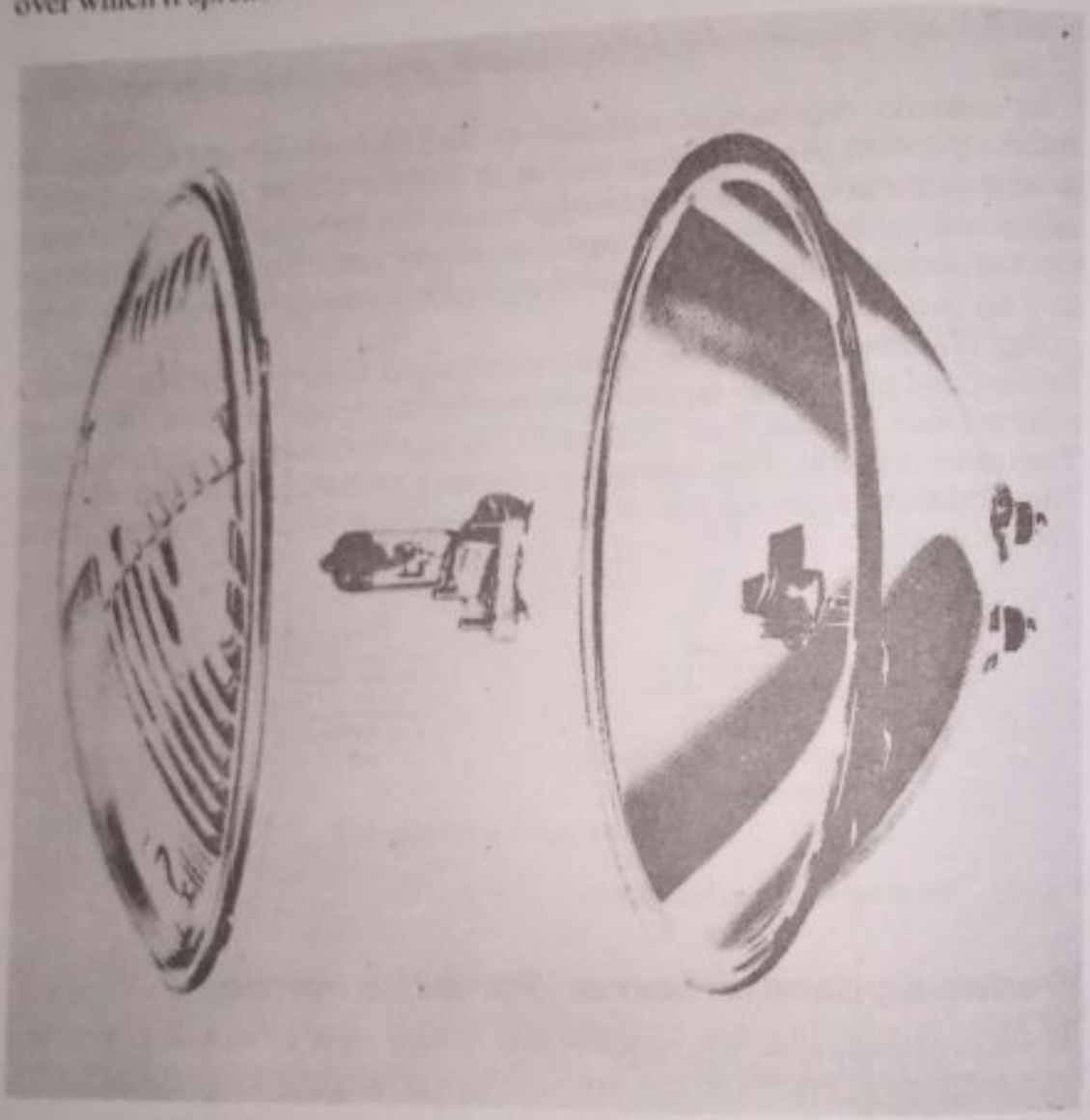


Plate 17A Car headlamp reflector. This has a parabolic shape which reflects a wide parallel beam of light from the lamp at its focus. (*Thorn Lighting Ltd*)

For this reason the concave spherical mirror is not used in searchlights or the headlamps of cars. *Parabolic mirrors* are used; a parabola is a curve similar in shape to the curved path of a ball thrown forward into the air. As shown in Fig. 17.4b, a parabolic mirror reflects rays parallel from *all* parts of its surface when a small lamp is at its focus. So the reflected beam can travel a long distance without becoming weaker.

Parabolic mirrors are also used behind the straight filaments in electric fires. The filaments are placed at the focus of the mirror. The infra-red rays are reflected in the same way as visible light, as a parallel beam which travels a long way from the fire.

Scanned with CamScanner

Measuring radius of curvature

Fig. 17.5a shows two rays CS and CR incident on a concave mirror from its *centre of curvature* C. Since CS and CR are radii of the curved surface, CS and CR are normals (perpendiculars) to the surface at S and R respectively. So the two rays are reflected back to C along their incident paths, as shown. In practice, this means that an object placed at C will form an image also at C, that is, the object and image coincide in position. This can be used to measure accurately the radius of curvature r of the concave mirror.

Experiment

Set up directly in front of a concave mirror M an illuminated object O, such as two crosswires fixed across a hole in a card or screen and with a lamp behind the wires (Fig. 17.5b). Move the mirror towards or away from O until a sharp image I of the crosswires is obtained alongside O, as shown. Measure the distance OP between O and the mirror. This is the radius of curvature r, as explained. Repeat the experiment twice more and take the average of your results for the value of r.

As explained shortly, the focal length f of the mirror $= r/2$.

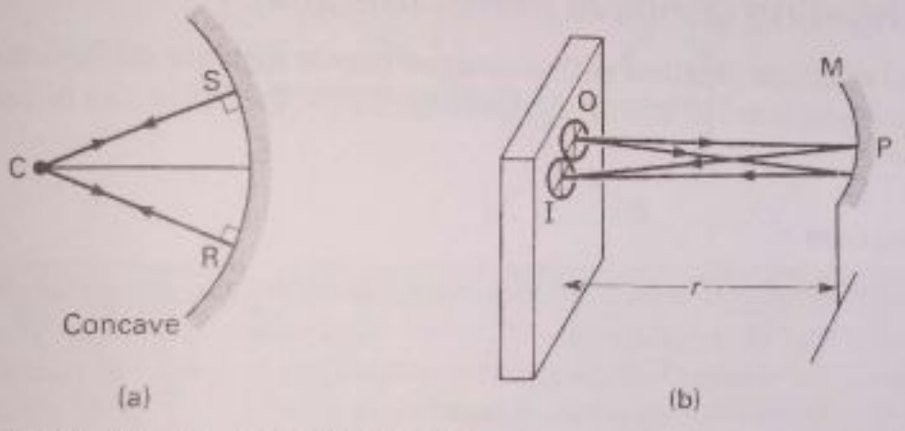


Fig. 17.5 Centre of curvature: measurement of r

Focal length · Relation between f and r

The *focal length* of a concave mirror can be found quickly, but not accurately, by holding up the mirror M at the back of a room so that the reflecting surface faces

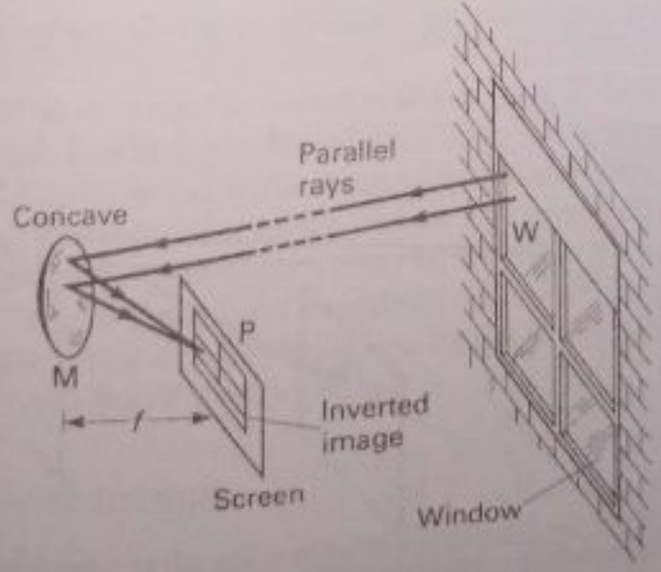


Fig. 17.6 Quick method for finding f

Scanned with CamScanner

the window W (Fig. 17.6). A sheet of paper P is then moved in front of the mirror until the *clearest* image of the window-frame is seen or, better, the image of any object such as a tree outside the window. The images are upside down. The distance from M to P is then measured. Now the mirror M receives *parallel rays* from objects a long way from it (see page 304). Since parallel rays are reflected by M towards its focus, the distance from M to P is practically equal to the focal length f of the mirror.

From the two experiments already described, we find that the *focal length is half the radius of curvature*, or

$$f = \frac{r}{2}$$

This simple relationship always holds for all curved spherical mirrors and can be explained theoretically by geometry.

An accurate value of the focal length f of a concave mirror can be found by measuring the radius of curvature r as described before, and then using the relation $f = r/2$ to calculate f. For example, if the radius r is found to be 30·2 cm, the focal length $f = 15{\cdot}1$ cm.

Investigating concave mirror images

The kind of image obtained with a concave mirror depends on how far or how close the object is to the mirror. An investigation of the image can be carried out as follows.

Experiment

Place illuminated cross-wires O in front of a concave mirror M at the same height, use a white screen S for finding the image of O, and a ruler for measuring distances. (Fig. 17.7)

As explained on page 271, first measure the radius of curvature r, so that r and the focal length $f(=r/2)$ are known. Record your results for r and f.

Starting with the object O a long way from the mirror M, write down your observations of the image I formed clearly on the screen at some position of S. Enter your results in a table similar to that below, which you can copy into your note-book. You should state whether the image is (a) inverted (upside down) or upright, (b) smaller or bigger than the object, (c) real or virtual. Repeat these observations for other distances as listed in the table.

Finally, move the mirror close to your face (that is, your face is at a distance *less* than f from the mirror). Write down how the image differs from the image formed on the screen.

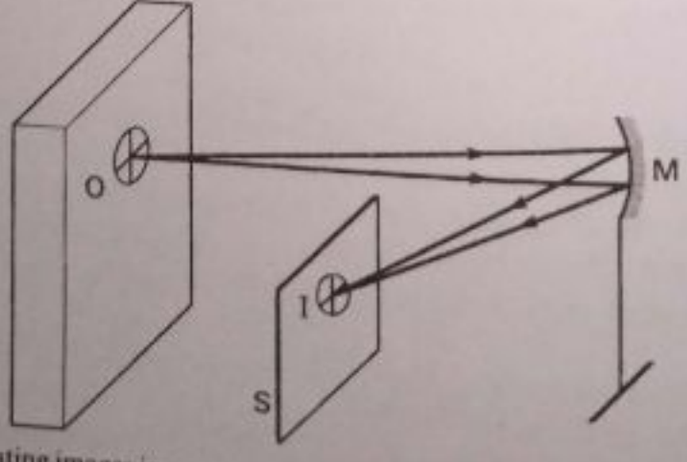


Fig. 17.7 Investigating images in a concave mirror

Scanned with CamScanner

Position of O	Long way from M	Nearer, but further than C ($2f$)	At C, distance $2f$	Between C and F	At F	Nearer than F
Image I Observation	(a) (b) (c)					

Results: As the object O moves towards the mirror, the inverted image grows in size. At C, the centre of curvature, the image has the *same* size as the object. The images are real since they are received on a screen.

When the object is nearer the mirror than the focal length, however, the image is *upright and magnified*. As we see later, this image is virtual (page 274).

Ray diagrams of images

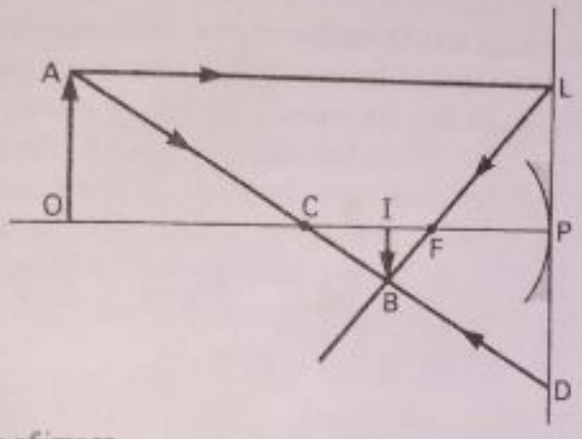


Fig. 17.8 Ray diagram of image

The images obtained in mirrors can always be drawn to scale by means of a *ray diagram*. As a small central part of the mirror is used, the reflecting surface is represented by a straight line. The object OA is drawn as a straight line perpendicular to the principal axis, with an arrow to represent its head A and its foot O on the axis (Fig. 17.8). The image of A, the top of the object, is found by drawing two rays from it. They may be:

1. a ray AL parallel to the principal axis, which passes through the principal focus F after reflection;
2. a ray AC through the centre of curvature C, which is reflected back along the same path ACD after striking the mirror at D (see page 271); or
3. a ray AF through F, which is reflected parallel to the principal axis.

The rays 1. and 2. intersect at B below the principal axis after reflection. So B is the image of A, or the top point of the image. The image of the foot O of the object lies on the principal axis, since the ray OCP strikes the mirror normally at P and is reflected straight back. Now the mirror does not change the shape of the object. So when the point B is obtained, the image is completed by drawing a perpendicular BI from B to the principal axis.

Concave mirror images

The real images formed in a concave mirror are illustrated by the ray diagrams in Fig. 17.9. These were obtained in the experiment on page 272.

Scanned with CamScanner

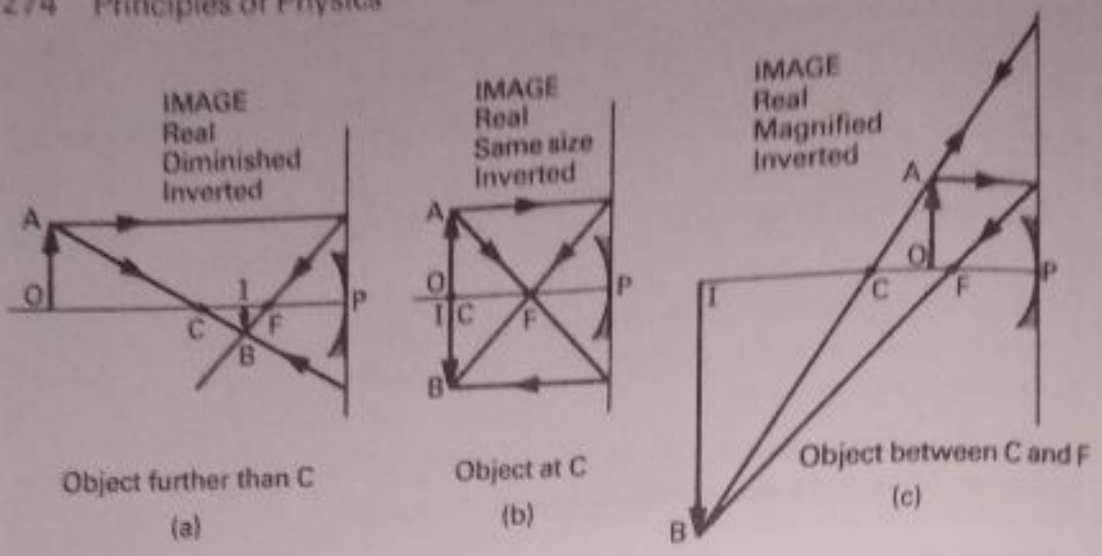


Fig. 17.9 Ray diagrams of concave mirror images

When the object is a long way from the mirror, the image is small, inverted and formed at the focus, since the rays from a distant object are parallel. See Fig. 17.6.

As the object OA approaches the mirror, its image grows in size but remains smaller than OA (Fig. 17.9a). When the object reaches C, the centre of curvature, the object and image have the same size–they are also formed at the same place, C (Fig. 17.9b). Beyond C, until F is reached, the image becomes larger than the object, that is, magnified by the mirror, and moves further back from the mirror (Fig. 17.9c). Note that all these images are inverted and real.

Magnifying mirror

When the object is between F and the mirror, a magnified upright image is seen. In this case a ray AL parallel to the principal axis is reflected along LF to pass through F (Fig. 17.10). A ray AD, drawn in the direction CAD from the centre of

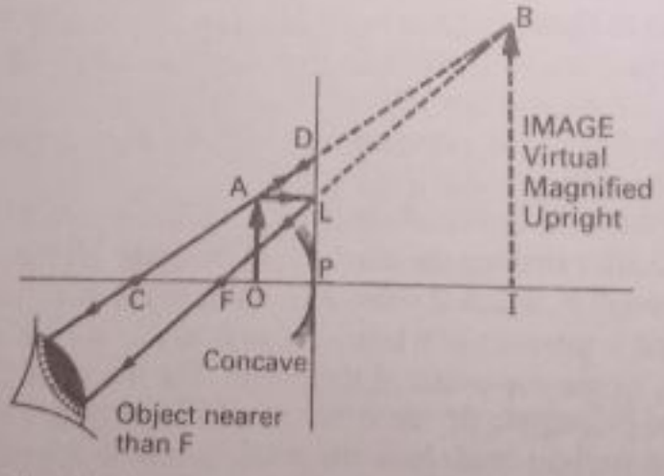


Fig. 17.10 Magnified upright image

curvature C, is reflected back along the same path DA. Since DA and LF intersect at B *behind* the mirror, the image is virtual. It is also magnified and upright. This is why the concave mirror can be used as a shaving mirror or as a dentist's mirror; in either case the mirror is placed close to the object so that the object distance is less than the focal length.

Scanned with CamScanner

Convex mirror images

Experiments with an illuminated object such as cross-wires show that the light reflected from a convex mirror can never be focused on to a screen. The image seen in this mirror is therefore *virtual*. As we shall see, the image is also always upright.

Fig. 17.11a shows a ray diagram of a typical image produced by a convex



Plate 17B Convex mirrors are often used at difficult road corners to help motorists see oncoming traffic. Note the wide field of vision. (*J. Allan Cash Ltd*)

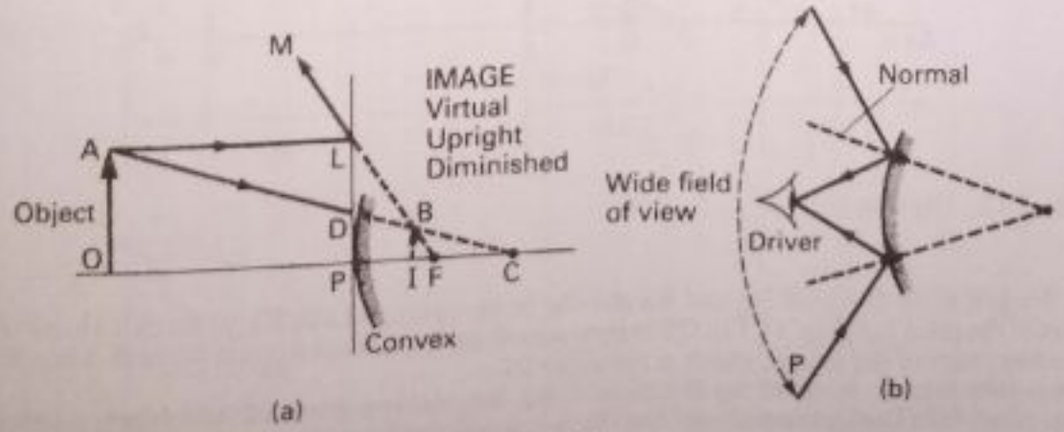


Fig. 17.11 Convex mirror (a) image (b) wide field of view

Scanned with CamScanner

mirror. A ray AL parallel to the principal axis is reflected along LM as if it diverged from F, the principal focus, which is behind the mirror (see page 268). A ray AD, which would pass through the centre of curvature C if produced, is reflected back along the same path DA. Since ML and AD intersect at B behind the mirror, the image IB is virtual. If the object OA is moved nearer the mirror and the image is redrawn, it will still be virtual, upright and smaller than the object.

As the image is always upright, the convex mirror is used as a driving mirror. Objects such as P within a wide angle round the mirror can be seen (Fig. 17.11b). So the convex mirror has the advantage of a wide field of view and overtaking traffic can be seen. If a plane mirror were used in place of the convex mirror, only traffic mainly behind the mirror would be seen and the field of view would be narrower.

Magnification by curved mirrors

The *linear* or *transverse magnification* m produced by a mirror is defined as

$$m = \frac{\text{height of image}}{\text{height of object}} = \frac{IB}{OA}$$

So by representing the height of the object OA by some convenient length, and measuring the height IB of the image obtained from a ray diagram, the magnification can be found.

Example on magnification by drawing

A concave mirror of focal length 20·0 cm produces an upright image of magnification 3. By drawing, find the distance of the object from the mirror.

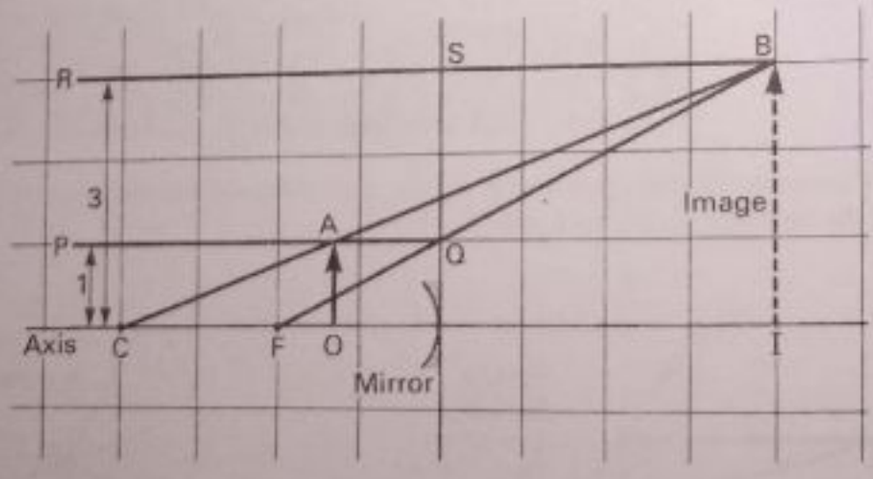


Fig. 17.12 Drawing of image

We first draw two lines PQ and RS parallel to the principal axis CF at heights above the axis in the ratio 1:3 (Fig. 17.12). QF is then joined and produced to meet RS at B. Then B is the top point of the image, which is therefore BI.
Now join B to C, intersecting PQ at A. Then A is the top point of the object, which is therefore AO. The distance from O to the mirror can be found from the scale used for the focal length and is 13·3 cm.

Scanned with CamScanner

Mirror formulae · Sign rule

If an object is at a distance u from a curved mirror, concave or convex, of focal length f and radius r, and the image is formed at a distance v from the mirror, experiment and theory show that the distances are related by a general formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{r}$$

When this formula is used, it is necessary to add positive (+) or negative (−) signs to each of the distances, according to a sign rule or convention.

Real is Positive (RP)

On this convention, real objects and real images are considered to be at a *positive* distance from the mirror; virtual images are considered to be at a *negative* distance from the mirror.

A concave mirror has a real focus. The focal length is therefore *positive*. A convex mirror has a virtual focus; its focal length is therefore *negative*.

New Cartesian (NC)

On this convention distances measured to the right of the mirror are *positive*; distances to the left are *negative*. The object is always placed on the left of the mirror.

A concave mirror has a real focus, which is left of the mirror; its focal length is hence *negative*. A convex mirror has a virtual focus, which is right of the mirror; its focal length is therefore *positive*.

The magnification m produced by a mirror is given by

$$m=\frac{v}{u}$$

Example on mirror formulae

An object is placed (a) 15 cm, (b) 5 cm in front of a concave mirror of radius of curvature 20 cm. Calculate the position, nature and magnification of the image in each case.

RP

The focal length $f=+\frac{20}{2}=+10\,\text{cm}$

(a) The object distance $u=+15\,\text{cm}$

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{v}+\frac{1}{(+15)}=\frac{1}{(+10)}$$

$$\therefore \frac{1}{v}+\frac{1}{15}=\frac{1}{10}$$

$$\therefore \frac{1}{v}=\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$$

$$\therefore v=30\,\text{cm}$$

So the image is 30 cm from the mirror and since v is +ve the image is real (see Fig. 17.9c)

NC

The focal length $f=-\frac{20}{2}=-10\,\text{cm}$

(a) The object distance $u=-15\,\text{cm}$

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{v}+\frac{1}{(-15)}=\frac{1}{(-10)}$$

$$\therefore \frac{1}{v}-\frac{1}{15}=-\frac{1}{10}$$

$$\therefore \frac{1}{v}=-\frac{1}{10}+\frac{1}{15}=-\frac{1}{30}$$

$$\therefore v=-30\,\text{cm}$$

So the image is 30 cm on the left of the mirror, on the same side as the object, and is therefore real (see Fig. 17.9c)

Scanned with CamScanner

RP

Also $m = \frac{v}{u} = \frac{30}{15} = 2$

(b) The object distance $u = +5\,\text{cm}$

$$\therefore \frac{1}{v} + \frac{1}{(+5)} = \frac{1}{(+10)}$$

$$\therefore \frac{1}{v} + \frac{1}{5} = \frac{1}{10}$$

$$\therefore \frac{1}{v} = \frac{1}{10} - \frac{1}{5} = -\frac{1}{10}$$

$$\therefore v = -10\,\text{cm}$$

So the image is 10 cm from the mirror, and since the sign of v is negative the image is a virtual one behind the mirror (see Fig. 17.10).

NC

Also $m = \frac{v}{u} = \frac{-30}{-15} = 2$

(b) The object distance $u = -5\,\text{cm}$

$$\therefore \frac{1}{v} + \frac{1}{(-5)} = \frac{1}{(-10)}$$

$$\therefore \frac{1}{v} - \frac{1}{5} = -\frac{1}{10}$$

$$\therefore \frac{1}{v} = -\frac{1}{10} + \frac{1}{5} = \frac{1}{10}$$

$$\therefore v = 10\,\text{cm}$$

Since v is +ve, the image is 10 cm on the right of the mirror, or virtual (see Fig. 17.10).

$$\text{Magnification } m = \frac{v}{u} = \frac{10}{5} = -2 \text{ (RP and NC)}$$

The minus indicates that the image is the same way up as the object.

Summary

1. A concave mirror converges incident parallel rays. A convex mirror diverges incident parallel rays.
2. A concave mirror has a real focus; a convex mirror has a virtual focus.
3. The images in a concave mirror are real and inverted unless the object is nearer the mirror than its focal length, when the image is upright, magnified and virtual. At C, the centre of curvature, which is $2f$ from the mirror, the image is the same size as the object. The image in a convex mirror is always virtual and diminished.
4. Mirror formulae (a) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ (b) $m = \frac{v}{u}$.

When using the formulae, the sign conventions must be used.

RP

Real is +ve,
Virtual is −ve
Concave mirror, f is +ve.
Convex mirror, f is −ve.

NC

Distances to the right of the mirror are +ve, to the left −ve.
Concave mirror, f is −ve.
Convex mirror, f is +ve.

Exercise 17 (Answers, page 642)

1. In Fig. 17.13, the mirror on the left is concave and that on the right is a convex spherical mirror. In each diagram two rays of light are shown incident on the mirror. Draw the approximate path of each ray after reflection. (N)

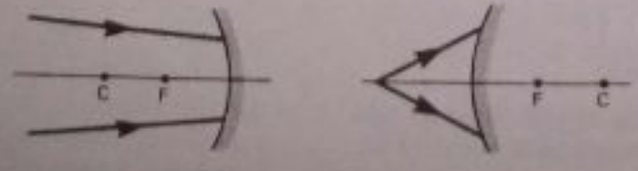


Fig. 17.13

Scanned with CamScanner

2. Draw ray diagrams to show the formation of an image by means of (a) a plane mirror, (b) a convex mirror.

State *one* advantage to be gained if the driving mirror of a car is convex. (N)

3. Explain, with the aid of a diagram, the action of a concave spherical mirror on a narrow incident beam of light travelling parallel to its axis.

A concave mirror of radius of curvature 40 cm forms a sharply focused image of a small object on a screen placed at a distance of 80 cm from the mirror. Determine, graphically or otherwise, (a) the position of the object and (b) the linear magnification.

Give a detailed account of a reliable experimental method for determining the focal length of a concave mirror. (L)

4. Define principal focus of a convex mirror. A concave mirror is used to form an image of an object pin. Where must the object be placed to obtain: (a) an upright, enlarged image; (b) an image the same size as the object? (N)

5. What is meant by the *radius of curvature* of a *concave* mirror and how is it related to the *focal length*?

Show clearly by means of a ray diagram how such a mirror produces an upright image of an object placed before it. State one practical application of the concave mirror when used this way.

A concave mirror of radius of curvature 30 cm produces an inverted image 4 times the size of an object placed on and perpendicular to the axis. Determine by an accurate scale drawing or otherwise the positions of the object and image. (L)

6. Explain the term *virtual image* as used in connection with a concave mirror, and illustrate your answer with a diagram.

Make a diagram showing how an image of the sun can be formed by a concave mirror.

What is the diameter of the image of the sun formed by a mirror of radius of curvature 4·0 m if the sun itself is just obscured by a disc 1·0 cm in diameter when held at a distance of 1·0 m from the eye? (O and C)

7. Define *principal focus* of a concave mirror. An object 2 cm high is situated on and perpendicular to the axis of a concave mirror of radius of curvature 30 cm and is 10 cm from the mirror. Find, graphically or by calculation, the position and size of the image and indicate on a diagram the paths of three rays from a point on the object to an eye viewing the image. (L)

8. A pin object is placed 10 cm in front of a *concave mirror* of radius of curvature 12 cm. Draw a ray diagram to show how an image is formed, and determine its position. State two ways in which this image differs from the image of the same pin placed at the same distance from the plane mirror.

State one example of the use of a *convex mirror* and indicate why it is preferred to a plane mirror. (L)

9. Describe fully the apparatus used, how it is set up, and the precautions which are taken, in order to obtain an accurate value for the focal length of a concave spherical mirror. Show how the observations made are used to give the final result.

A concave spherical mirror has a focal length of 10 cm. Where must an object be placed in order to produce a *real* magnified image three times as tall as the object?

How far, and in what direction, does the object need to be moved in order to produce a *virtual* image three times as tall as the object? (L)

10. What is meant by the *focal length of a concave mirror*?

Draw ray diagrams to show how (a) a real image, (b) a virtual image, can be formed by a concave mirror. The diagrams must show at least *two* rays from a point on the object not on the axis.

Give, with brief explanations, *one* practical application of a concave mirror in which a real image is used and *one* in which a virtual image is used. (O and C)

11. An object is placed on the axis of a converging (concave) mirror of focal length 200 mm. The image produced is inverted and has a magnification of 1·5. By calculation or by scale drawing on graph paper determine the position of the object. (AEB, 1982 *part qn.*)

Scanned with CamScanner

18. Refraction at Plane Surfaces

It is well known that the bottom of a swimming pool appears to be nearer the surface than it really is (Fig. 18.1a). Also, letters in print seem to be nearer when a thick block of glass is placed over them (Fig. 18.1b). As we shall see, this is due to a change in direction of light when it travels from water or glass into air. The phenomenon is called *refraction*. Refraction occurs when light travels from air into a glass lens. A scientific study of refraction has helped to make very efficient microscopes and telescopes.

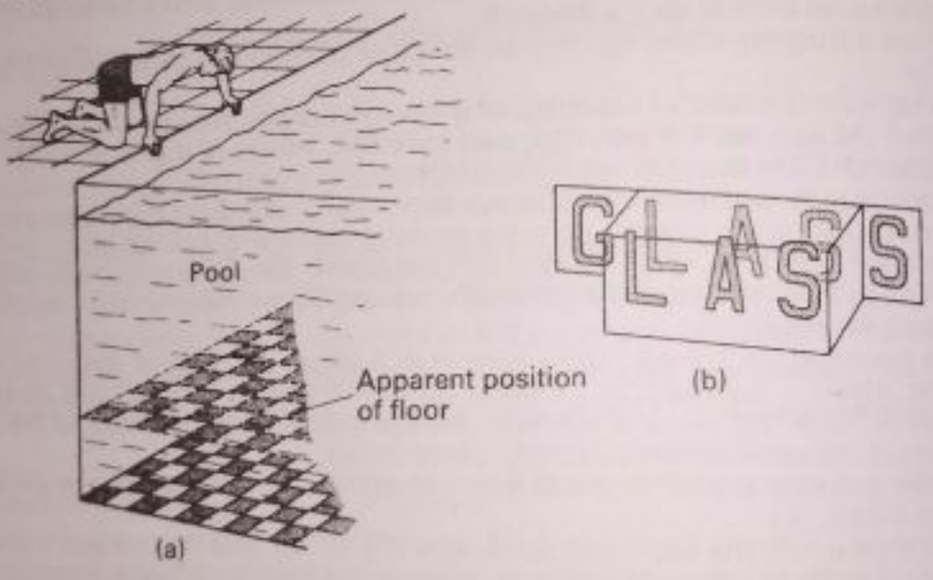


Fig. 18.1 Refraction effects in (a) water, (b) glass

Refraction through rectangular block and prism

The refraction of rays of light from air to glass or from glass to air can be investigated with a ray-box. A parallel beam of light from the box R is incident on a rectangular glass block ABCD (Fig. 18.2a). When the rays are normal, i.e. at 90°, to the side AB, they pass straight through the glass and come out without change of direction. In this case we can see no refraction.

But when the ray-box is turned so that the light rays meet the glass surface at an acute angle, say 60°, there is a noticeable change. The beam inside the glass now travels in a different direction XZ (Fig. 18.2b). Refraction has taken place. The light beam comes out into the air at Z in the same direction as the incident beam at X, but is displaced sideways.

Glass *prisms* are used in optics. Fig. 18.2c shows that the beam refracted by a triangular glass prism PQS comes out or emerges in a completely different direction to the incident beam. So the incident light is *deviated* by the prism.

Refraction through water can be seen by placing a little fluorescein in a

Scanned with CamScanner

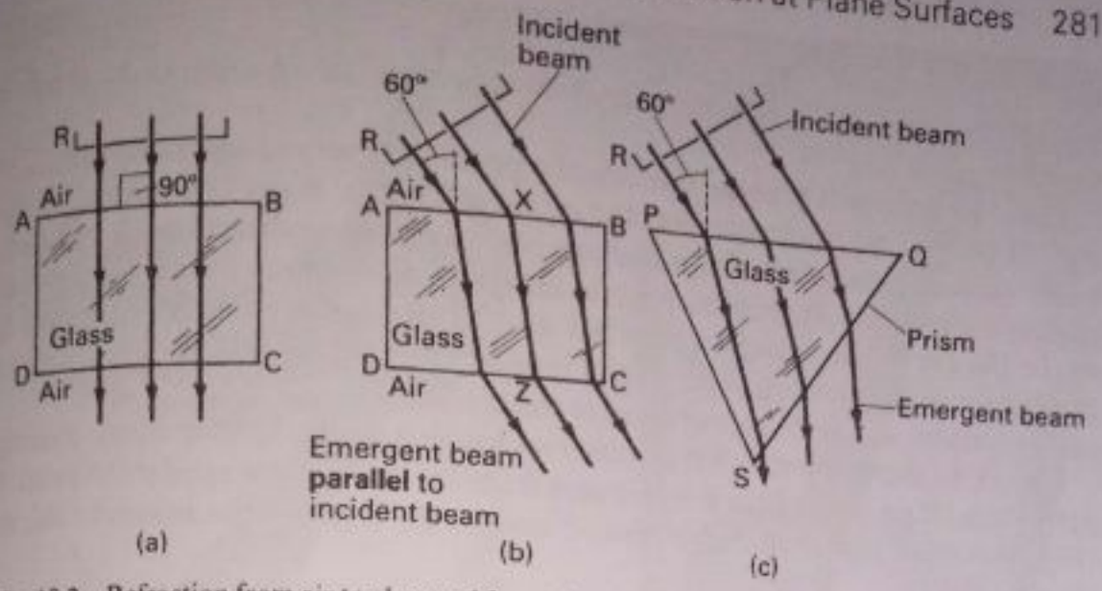


Fig. 18.2 Refraction from air to glass and from glass to air

rectangular tank of water. The change of direction of the ray when a beam enters the back face of the tank is then clearly visible.

Rules for refraction at boundaries

When a ray PA is incident in air on the air–glass boundary, it is refracted *towards* the normal NAM when it enters the glass (Fig. 18.3). Some of the light is also reflected at A along AS but AS is a weak ray compared with the refracted ray.

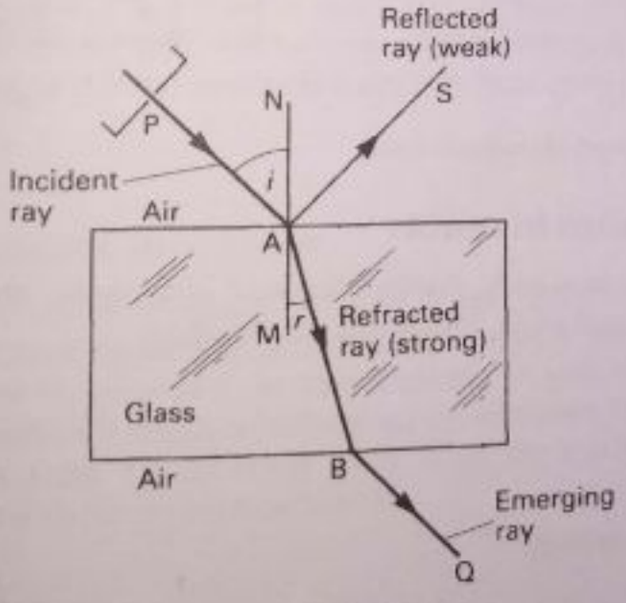


Fig. 18.3 Refraction through a glass rectangular block

We also see that the ray AB in glass is refracted at the glass–air boundary along BQ in air, *away from* the normal at B. Similar results are obtained at an air–water boundary and a water–air boundary.

Generally, you should remember that *Light travelling from one medium such as air to an optically denser medium such as glass or water is refracted towards the normal. When travelling from glass or water to a less dense medium such as air, the light is refracted away from the normal.*

Scanned with CamScanner

Apparent depth

We can now explain why a swimming pool appears less deep when looking down into it.

Consider an object O at the bottom of a pool in the very exaggerated sketch of Fig. 18.4a. To find the image, we take *two* rays from O. A ray ON normal to the water surface passes into the air along NR in the same direction. A ray OA, slightly inclined to ON, is refracted into the air along AP *away from* the normal at A. To the eye E, the emerging rays NR and AP appear to come from a point I, their point of intersection. The point I is *nearer* the surface than O. So an object on the bottom of a pool appears nearer the surface.

Fig. 18.4a shows the case for an observer looking at an object in water directly below him. If an object O in water is seen from the side, it now appears to be at A (Fig. 18.4b). A is a little higher than I and to the right of the normal ON, as shown.

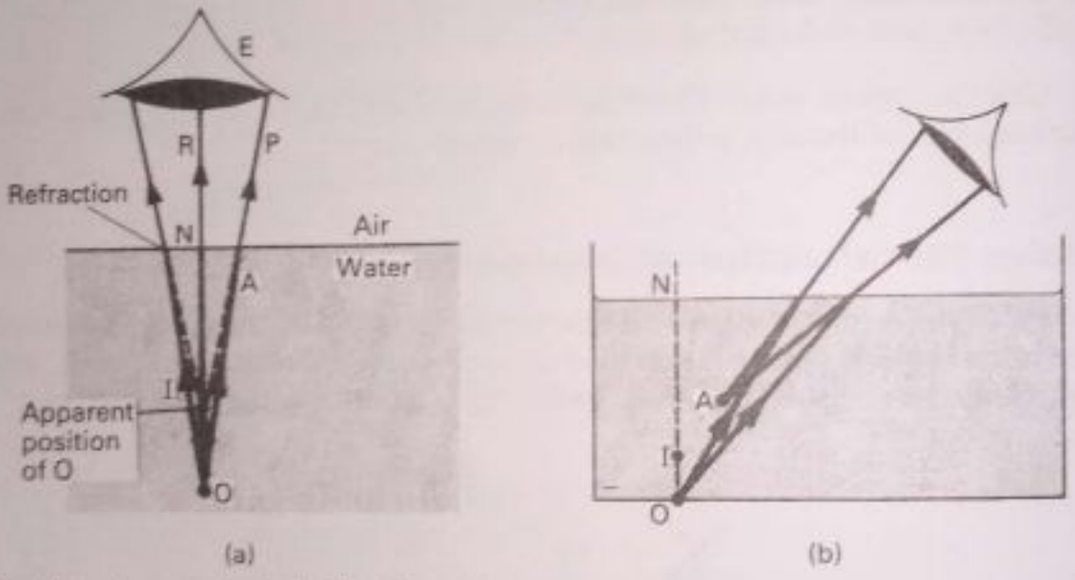


Fig. 18.4 Apparent depth of objects in water

Part-immersion in water

A drinking straw or a stick, partly immersed in a liquid, appears bent at the surface (Fig. 18.5 and 18.6). This is due to refraction.

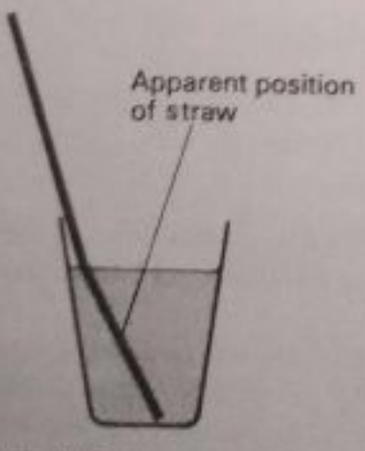


Fig. 18.5 Appearance of straw in liquid

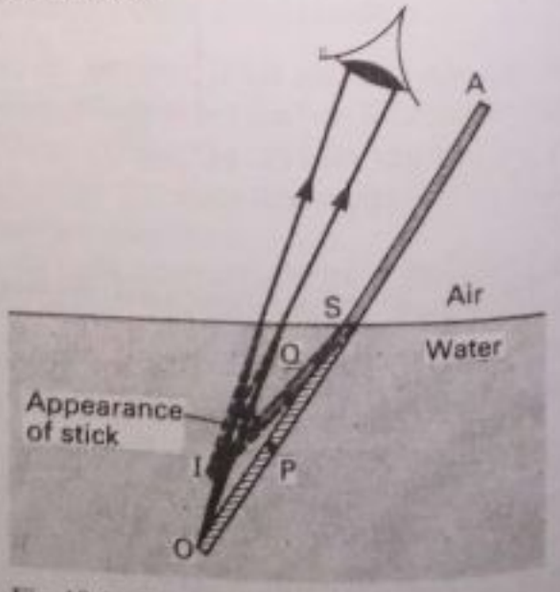


Fig. 18.6 Bending effect explained

Scanned with CamScanner

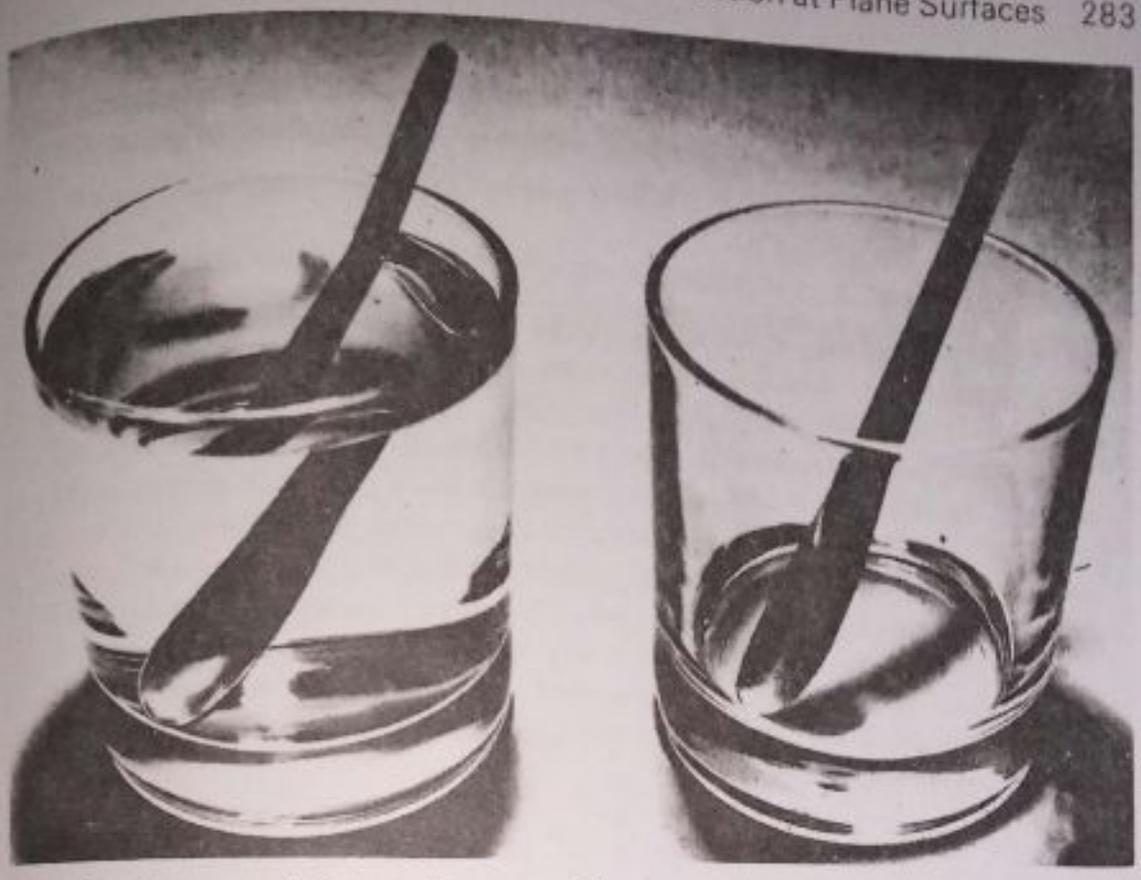


Plate 18A A spoon partly immersed in water and in an empty glass, showing the effect of refraction. Note also the different shadows cast by the full and empty glasses. (*Paul Brierley*)

Suppose OA is the actual position of the stick. If an observer looks at O from above, O will appear nearer the surface as explained under 'apparent depth'. In the same way, a point P on the stick will appear higher, at Q. So all the points between O and S appear higher. The image of OS is therefore IS. It is not in the same straight line as AS and so the stick appears bent upwards as shown in Fig. 18.6.

Bringing objects into view

If an object O is placed in a vessel C so that it is just hidden from an observer at E it can be seen by pouring water into the vessel (Fig. 18.7). In Fig. 18.7a rays from O just pass below the edge of C, so that it cannot be seen by E. When water is poured in to a sufficient depth refraction occurs as shown (exaggerated) in Fig. 18.7b. The rays from O, refracted away from the normal, now enter the observer's eye and so the object is seen at I.

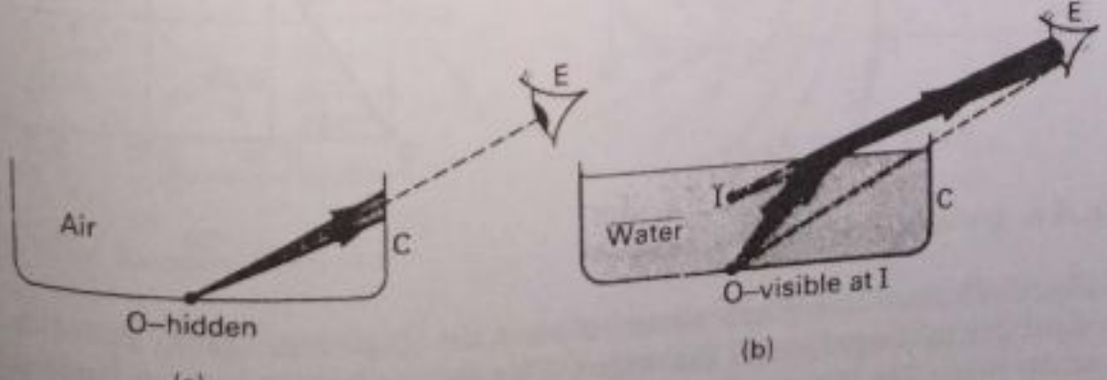


Fig. 18.7

Scanned with CamScanner

Laws of refraction · Snell's law

More exact investigations on the refraction of light from air to glass can be made by using a single ray from a ray-box R and a rectangular block of glass XYML (Fig. 18.8a).

Experiment

Draw a line XY on a sheet of white paper. Using a protractor, draw (a) four lines such as AO radiating from O at convenient angles such as 30°, 40°, 50° and 70°, and (b) the normal (perpendicular) ON to XY at O (Fig. 18.8a).

Now place the glass block so that its longer boundary edge lies along XY. Direct a single ray from R along the first line AO and mark with a pencil two points C, D on the ray passing out from the block, as shown. Join CD to cut the lower glass edge at P and join OP after taking the block away. Then OP is the refracted ray in the glass when AO is the incident ray in air. Measure the angle of refraction r, which is angle POQ, the angle with the normal. Enter the value of i, the angle of incidence, and r in a table, as below.

Move the ray-box so that the ray is now incident on the block along the next line from O. Repeat the measurement of r and enter the results for i and r in the table. Repeat for two more values of i and r.

Measurements

i	r	$\sin i$	$\sin r$	$\sin i/\sin r$

Results: Some typical results are shown in the table below. The sines are given to two decimal places.

i	r	$\sin i$	$\sin r$	$n = \sin i/\sin r$
30°	19°	0·50	0·33	1·54
40°	25°	0·64	0·42	1·52
50°	30°	0·77	0·50	1·53
70°	38°	0·94	0·62	1·53

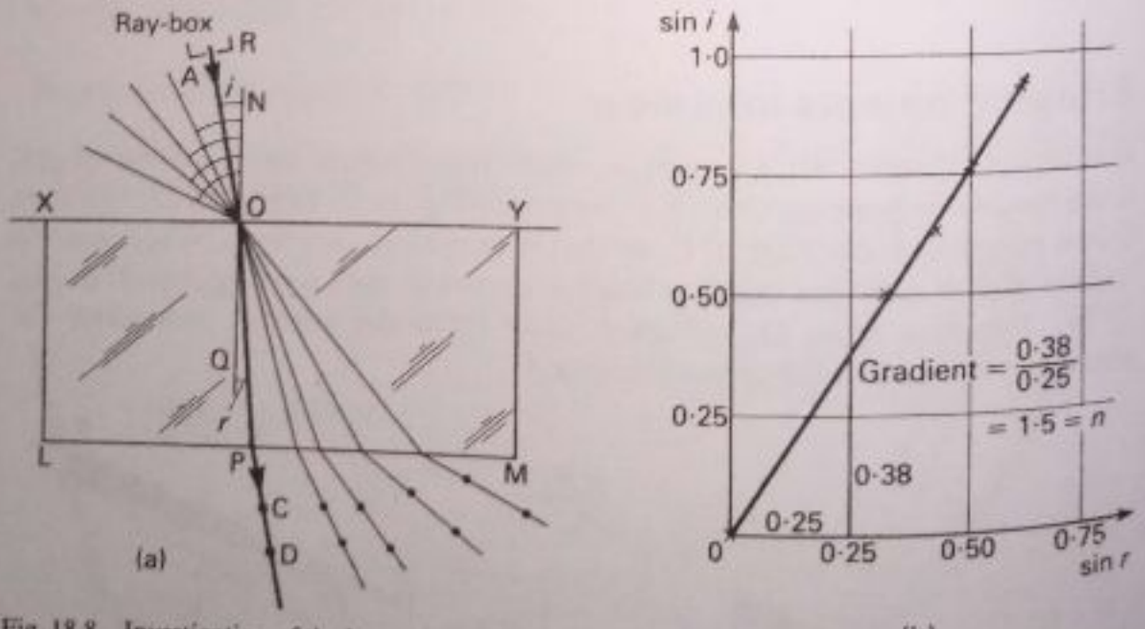


Fig. 18.8 Investigation of the laws of refraction

Fig. 18.8b shows the graph obtained when sin i is plotted against sin r. It is a straight line passing through the origin. This shows that *the ratio sin i/sin r is a constant value*. The law was found by Snell, a Dutch scientist, about 1620 and is

Scanned with CamScanner

known as *Snell's law of refraction*. (As we see shortly, the ratio is called the 'refractive index' of the glass, symbol n; from the gradient of the graph, $n = 1{\cdot}5$ approximately.)

A second law of refraction fixes the *plane* in which the refracted ray travels. This states *The incident ray, the normal, and the refracted ray all lie in the same plane*.

So, in Fig. 18.9, the incident ray AO, the normal ON and the refracted ray OB in the glass all lie in the plane AON shown in faint outline (compare the *law of reflection*, page 262).

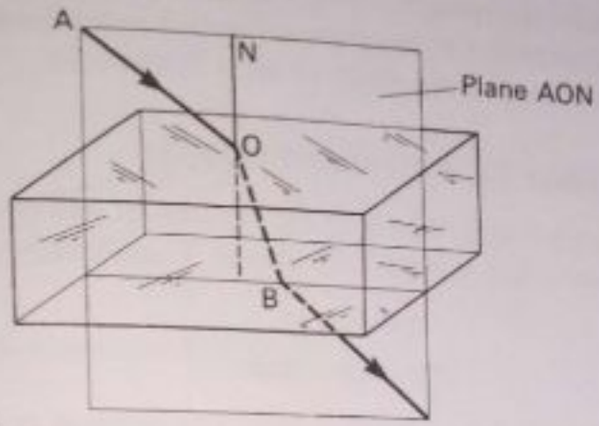


Fig. 18.9 Plane of refraction

Refractive index

The ratio $\sin i/\sin r$ is called the *refractive index* from air to glass. It is a number which gives a measure of the refraction or 'bending' of light when it travels from one medium to another. We shall use the symbol n for the refractive index, so that $n = \sin i/\sin r$. In the section on waves, it is shown that the refractive index n is the ratio of the velocities of light in the two media (page 352).

Fig. 18.10 shows refraction from air to crown glass and flint glass, and from water and glass to air. In each case the angle of incidence i is the angle in the first medium, and the angle of refraction r is that in the second medium. Suppose the respective angles of refraction are 35°, 32°, 42°, 75°. Then

(a) n, air to crown glass = $\sin 60°/\sin 35° = 1{\cdot}51$
(b) n, air to flint glass = $\sin 60°/\sin 32° = 1{\cdot}63$
(c) n, water to air = $\sin 30°/\sin 42° = 0{\cdot}75$
(d) n, glass to air = $\sin 40°/\sin 75° = 0{\cdot}67$

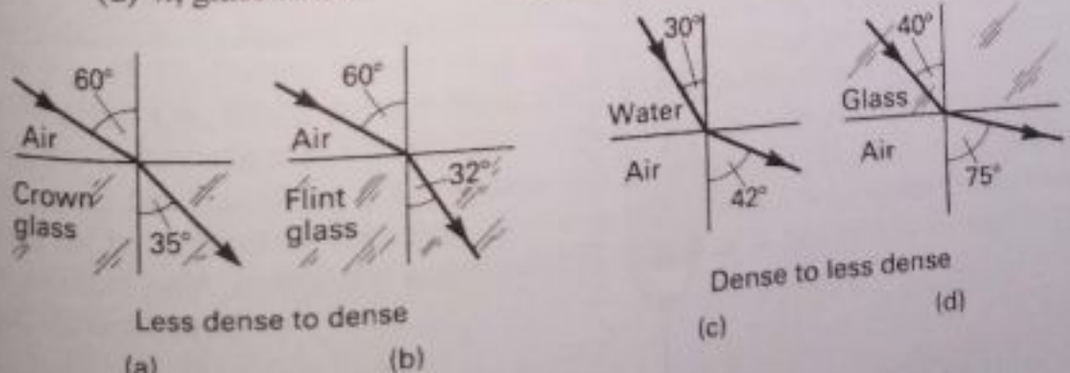


Fig. 18.10 Examples of refraction

In tables, refractive indices are usually given with a vacuum as the incident medium. For most purposes, where great accuracy is not required, the refractive index can be measured using light incident from air.

Scanned with CamScanner

Glass–air and water–air refractive indices

From the result in (a), the refractive index from air to glass is $\sin 60°/\sin 35°$, which is about 1·5 or 3/2. When light travels from *glass to air* its path reverses exactly, so the angle of incidence is now 35° and the angle of refraction is 60°. Hence

$$\text{refractive index for glass to air} = \frac{\sin 35°}{\sin 60°} = \frac{1}{1{\cdot}5} = \frac{2}{3}$$

We therefore *invert* the refractive index value from air to glass to find the value from glass to air. For example, since the refractive index from air to water is 1·33 or 4/3, the refractive index from water to air is 3/4 or 0·75.

Refractive index by apparent depth

We have already seen that an object at the bottom of a pool of water appears to be nearer the surface when viewed from above. Theory shows that

$$\frac{\text{true depth}}{\text{apparent depth}} = n$$

where n is the refractive index of the water. The relation is true only when the object at the bottom is viewed directly above it. Since $n = 4/3$ for water, a pool of water 4 m deep appears to be only 3 m deep when viewed directly from above.

Experiment

The refractive index of a *liquid* such as water can be found from this relation. A 'travelling microscope' is needed; this is a microscope whose tube M can be moved up or down by turning a screw and whose position is obtained from a vertical scale (Fig. 18.11). Three readings are taken on this scale; when the microscope is focused on (a) a pin O at the bottom of an empty beaker, (b) the pin again, seen at I, after a small quantity of water is added to the beaker, (c) lycopodium powder sprinkled on the water surface at P. The true depth OP is the difference in the readings (c) and (a); the apparent depth IP is the difference in the readings (c) and (b).

Calculation: $n = \dfrac{\text{OP}}{\text{IP}}$

n is calculated from the ratio true depth/apparent depth. The use of the microscope ensures that the object is viewed directly above it.

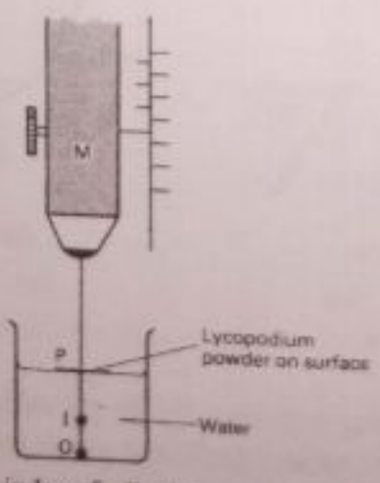


Fig. 18.11 To find the refractive index of a liquid

Scanned with CamScanner

Refraction by a glass prism

Prisms are used in optical instruments. They consist of a glass block with inclined faces, so that a section such as ABC is a triangle (Fig. 18.12). Prisms with angles of 45°, 45° and 90°, right-angled prisms, are used in prism binoculars. Prisms with angles of 60° are also used in optical experiments. The refraction of light by a prism, and the measurement of the refractive index of its material, can be shown using a ray-box.

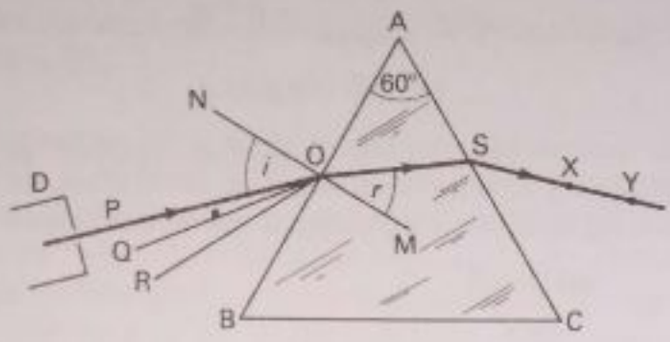


Fig. 18.12 Refraction through a glass prism

Experiment

With a pencil, draw the outline ABC of a 60° prism placed on a white sheet of paper (Fig. 18.12). Remove the prism and using a protractor, draw (a) the normal ON to AB at a point O between A and B, (b) three lines OP, OQ, OR making angles such as 35°, 45°, 55° respectively with ON.

Replace the prism ABC on the paper. Now direct a ray from a ray-box D along PO. Mark two points X, Y on the ray passing out of the prism, as shown. Remove the prism, join X, Y and produce XY to meet AC at S. Join OS. Then OS is the refracted ray in the glass when OP is the incident ray.

Measure the angle of refraction r in the glass and record in a table the values of i and r. Repeat the whole experiment using the larger angles of incidence made by lines QO and then RO.

Measurements

i	r	$\sin i/\sin r = n$

Results: Work out the ratio $\sin i/\sin r$ from your three results and find the average value. This is the refractive index n of the prism material.

In Fig. 18.12, the ray PO entering the glass is refracted towards the normal NOM along OS. When the ray SO is refracted at O into the air, it is refracted away from the normal at S along SXY. From the principle of the reversibility of light, a ray incident along YXS would travel along SO in the glass and pass out along OP into the air. We see that, in general, the prism changes the direction of the incident light, unlike the rectangular block; that is, the prism 'deviates' the light incident on it.

Direction of refracted ray

The direction of a refracted ray can be found by calculation or by drawing.

1. *Calculation* Suppose light is incident in air at 50° on a plane water surface and

Scanned with CamScanner

the refractive index n for water is 1·33 (Fig. 18.13a). The angle of refraction r in the water is then given by

$$\frac{\sin 50^\circ\ (i)}{\sin r} = n = 1{\cdot}33$$

$$\therefore \frac{0{\cdot}766}{\sin r} = 1{\cdot}33$$

or

$$\sin r = \frac{0{\cdot}766}{1{\cdot}33} = 0{\cdot}576$$

$$\therefore\ r = 35^\circ \text{ (approx.)}$$

Again, consider a ray of light in glass incident at 30° on the boundary with air (Fig. 18.13b). Since $n = 1{\cdot}5$ for light passing from air to glass, the refractive index from *glass to air* $= 1/1{\cdot}5$. So if r is the angle of refraction in the air

$$\frac{\sin 30^\circ\ (i)}{\sin r} = \frac{1}{1{\cdot}5}$$

$$\therefore\ \sin r = 1{\cdot}5 \times \sin 30^\circ = 0{\cdot}75$$

So

$$r = 49^\circ$$

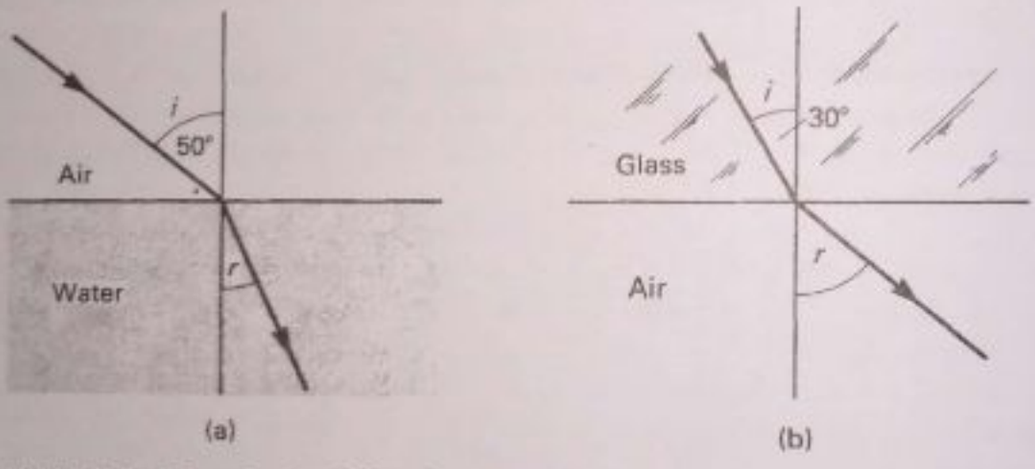


Fig. 18.13 Calculation of angle of refraction

2 *Drawing* A geometrical method can also be used to find the refracted ray. Consider a ray QO incident at an angle i in air on the surface of a medium such as glass with a refractive index $n = 1{\cdot}5$ (Fig. 18.14a). Draw two circles with a

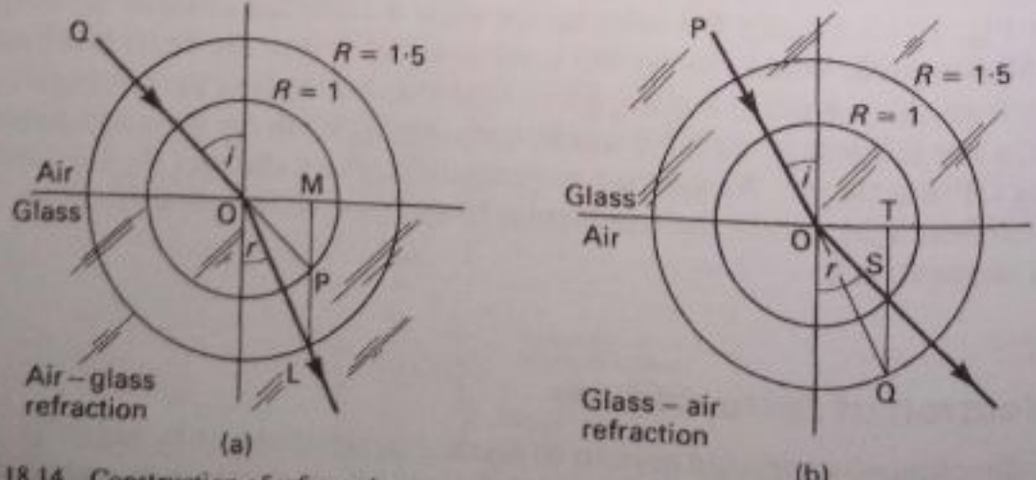


Fig. 18.14 Construction of refracted rays

Scanned with CamScanner

common centre at O, the point of incidence, and with radii R in the ratio $1:n$ or 1:1·5. Produce QO to cut the inner circle at P. Draw a line through P parallel to the direction of the normal and cutting the outer circle at L. OL is then the refracted ray.

Proof i = angle OPM, r = angle OLM.

So
$$\frac{\sin i}{\sin r} = \frac{\text{OM/OP}}{\text{OM/OL}} = \frac{\text{OL}}{\text{OP}} = 1{\cdot}5$$

Fig. 18.14b shows the construction for a light ray PO refracted from *glass to air*. The refracted ray is OS.

Field of view under water

A fish under water, or people swimming under water on their backs, will see rays of light such as BC which are refracted from air along CE in the water (Fig. 18.15). The edge or boundary of the light beam from the outside reaching E corresponds to rays such as LO which just graze the air–water boundary. The angle of incidence in air is then 90°. If the angle of refraction in the water is r, it follows that all the light from the outside entering the water is concentrated into a cone of half-angle r.

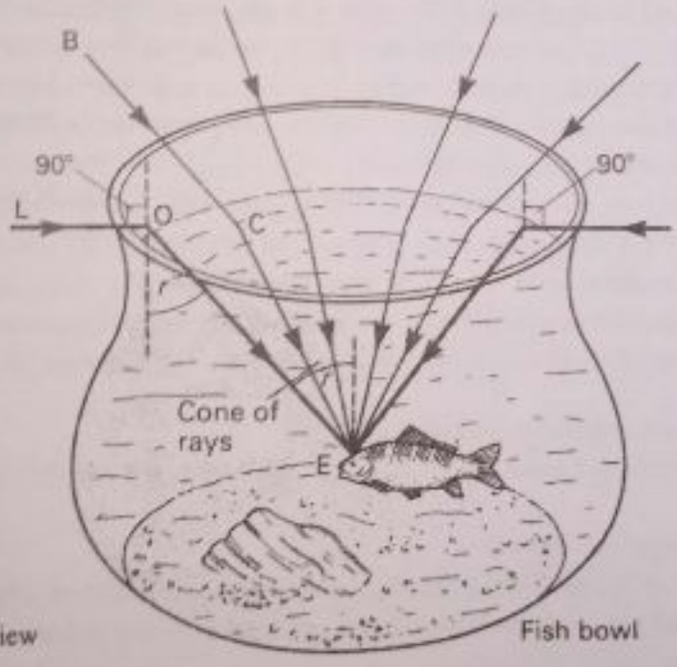


Fig. 18.15 Fish's view

The angle of the cone can easily be calculated, using $n = 4/3$ for water. Then
$$\frac{\sin 90^\circ\,(i)}{\sin r} = \frac{1}{\sin r} = \frac{4}{3}, \text{ or } \sin r = \frac{3}{4} = 0{\cdot}75$$
$$\therefore\ r = 49^\circ \text{ (approx.)}$$

So a fish can see everything in the air above the water by rays in a cone of half-angle about 49°

Total internal reflection

We have already seen that rays incident in air on an air–glass boundary are always refracted into the glass. Some of the incident light is also weakly reflected

Scanned with CamScanner

(Fig. 18.16). Refraction always occurs from air to glass. With the ray XK, for example, which is incident on the glass at nearly 90°, a strong refracted ray KY and a weak reflected ray KZ is produced.

A striking change occurs when the situation is reversed optically, so that light passes from glass to air.

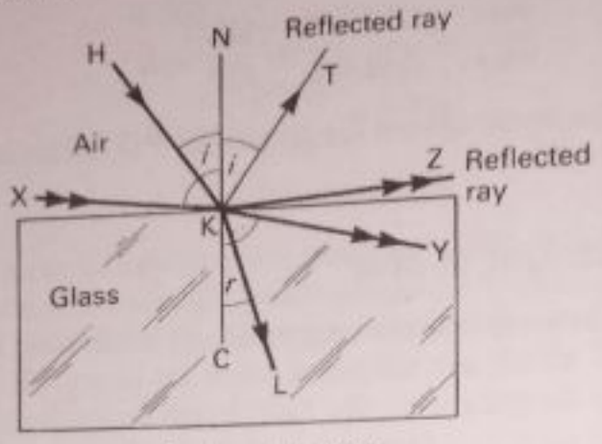


Fig. 18.16 Refraction and partial reflection–air to glass

Experiment

Place a semi-circular glass block G on a sheet of white paper and draw its outline with a pencil (Fig. 18.17a). Mark the middle O of its plane side. Using a ray-box R, direct a ray AB *towards O*. Since BO is a radius of the semicircle and therefore a normal to the glass at B, the ray AB passes straight through to O without any change in direction. BO is now a ray *in glass* incident on a glass–air boundary at O.

Increase slowly the angle of incidence *i* in the glass, that is, angle BON, by moving the ray-box round from the direction NO. Observe the reflected ray OP and the refracted ray OCD as *i* is increased from zero to a large value. When the refracted ray OCD *just* disappears, measure the angle of incidence *c* in the glass by marking the point Q where the incident ray enters the block, and joining OQ after removing the glass (Fig. 18.17b).

Results: (a) Small angle of incidence *i*, e.g. 20 .

There is a strong refracted ray OCD and a weak reflected ray OP in the glass (Fig. 18.17a).

(b) Critical angle of incidence *c*.

As the angle of incidence in the glass is increased, the angle of refraction in the air increases. At one special angle of incidence, called the *critical angle c*, the refracted ray

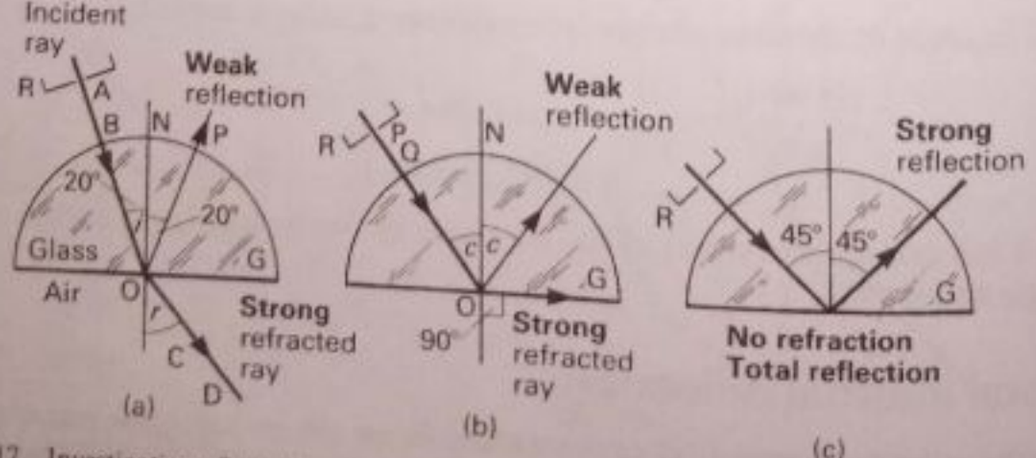


Fig. 18.17 Investigation of total internal reflection–glass to air

Scanned with CamScanner

OG travels along the glass–air boundary. The angle of refraction in the air is then 90° (Fig. 18.17b). A weaker reflected ray is again obtained, making an angle c with the normal ON.

(c) Angle of incidence greater than c.

When the angle of incidence is greater than c no refracted ray is obtained. The reflected ray in the glass is now almost as bright as the incident ray (Fig. 18.17c). Nearly all the incident light energy appears in the reflected light. So the glass–air boundary acts like a mirror—it reflects the light strongly.

The strong reflection takes place sharply as the critical angle is exceeded. For example, if the critical angle is 42° a ray incident in the glass at 41° will produce a strong refracted ray and a weak reflected ray. But if the angle of incidence is 43° the reflected ray is strong and there is no refracted ray. Since almost all the light incident is reflected back into the glass for angles greater than the critical angle, the effect is known as the *total internal reflection of light*.

Critical angle values

Can total internal reflection ever occur when light is incident in air on a glass block? The answer is *No*. As shown in Fig. 18.16, the angle of refraction in the glass is always less than the angle of incidence in air. So even when the angle of incidence in air is nearly 90°, a refracted ray is obtained.

Reflection always occurs at an air–glass boundary, as shown in Fig. 18.16, but this is only a partial reflection, since most of the light passes into the glass. It is therefore important to understand that *total* internal reflection can only occur when light passes from an optically dense medium to an optically less dense medium, for example from glass to air or from water to air.

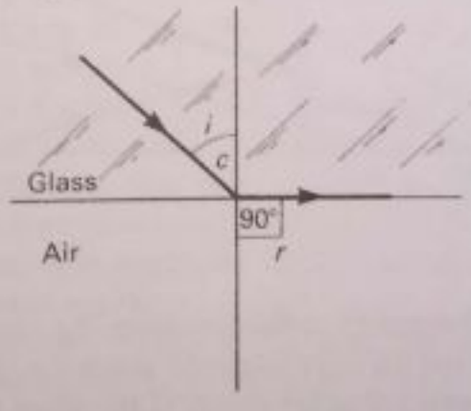


Fig. 18.18 Value of c

Suppose the angle of incidence in glass is c, the critical angle, so that the angle of refraction in air is 90° (Fig. 18.18). If the glass has a refractive index n of 1·5 or 3/2 for light passing from air to glass, then for light travelling from glass to air the refractive index is 2/3 (see page 286).

$$\therefore \frac{\sin i}{\sin r} = \frac{\sin c}{\sin 90^\circ} = \frac{2}{3}$$

$$\therefore \sin c = \frac{2}{3} \times \sin 90^\circ = \frac{2}{3} \times 1 = 0{\cdot}6667$$

$$\therefore c = 42^\circ \text{ (approx.)}$$

Total internal reflection occurs for any angle greater than c. So an angle of incidence of 45° in glass produces total internal reflection with an air medium on the other side of the glass.

Scanned with CamScanner

The glass with a refractive index 1·5 has a critical angle of about 42°, as we have just discussed. The refractive index of a glass with a critical angle of 45° is given by $n = \sin 90°/\sin 45° = 1{\cdot}41$ from tables. It follows that a glass with a refractive index lower than 1·41 will *not* produce total internal reflection when the angle of incidence in glass is 45°.

The refractive index for air to water is 4/3. So the refractive index for light passing from water to air is 3/4. The critical angle c for water–air is therefore given by

$$\frac{\sin c}{\sin 90°} = \frac{3}{4}$$

$$\therefore \sin c = \tfrac{3}{4} \times \sin 90° = 0{\cdot}75 \times 1 = 0{\cdot}75$$

$$\therefore \; c = 49° \text{ (approx.)}$$

Multiple images

When the image of an object in plane mirrors is observed closely, several faint images can be seen besides a prominent one. The presence of multiple images is due to partial reflection and refraction at the non-silvered glass surface of the mirror, as illustrated in Fig. 18.19.

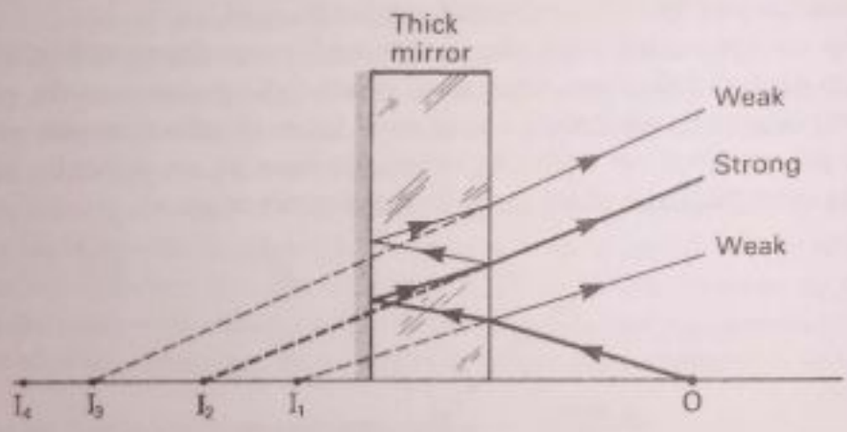


Fig. 18.19 Multiple images

A weak image I_1 is formed by reflection from the glass surface of part of the incident light at O. Most of the light passes through the glass, and is then reflected at the silvered surface and refracted again at the glass to form the main image seen, I_2, which is bright. A small part of the light reflected from the silvered surface is reflected at the glass, and further weak images I_3 and I_4, due to reflection and refraction, are thus obtained. The images are more widely separated if the glass is thicker.

Totally reflecting prisms

Since they produce multiple images, plane mirrors are not used as reflectors in high-quality periscopes, such as those needed in submarines for accurate sighting of ships. Instead, glass prisms with angles of 90°, 45°, 45° are used as reflectors. Using total internal reflection, these right-angle prisms have the advantage that only one image is produced.

Fig. 18.20a illustrates the optical action of a totally reflecting prism.

Scanned with CamScanner

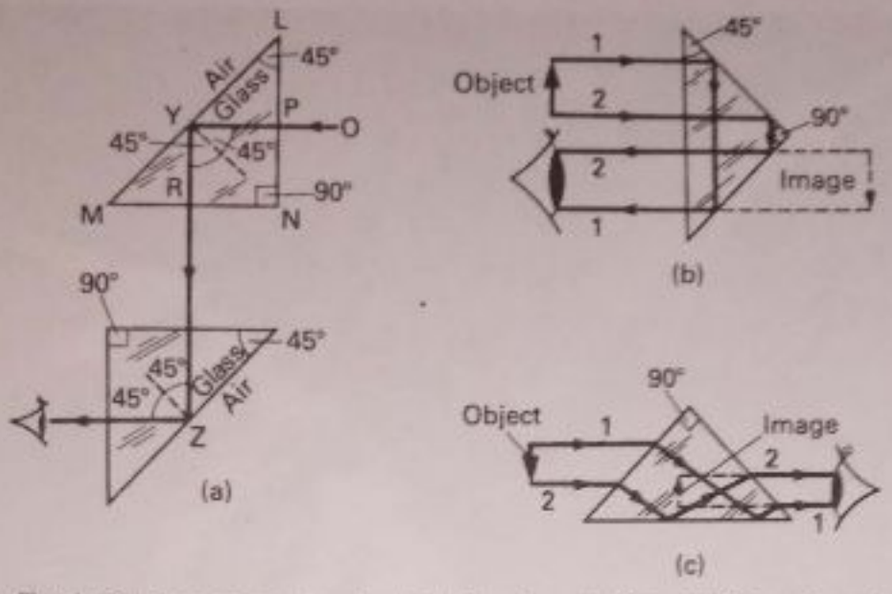


Fig. 18.20 Total reflecting prisms: changing light directions by 90° and 180°, and inverting images.

Suppose that light from an object O is incident normally on the side LN. When it strikes the hypotenuse at Y, the angle of incidence in the glass is 45°. This is greater than the critical angle between glass and air, which is 42° for glass of refractive index 1·5. So total internal reflection takes place at Y. All the light is reflected at Y in a direction YR at 45° to the normal, which is perpendicular to OY.

The ray YR is incident normally on MN and so it comes out into the air perpendicular to its original direction. The emergent light is almost as bright as the incident light from O. Also, only one reflected image is produced; there are no multiple images as with a plane mirror. Another advantage of the prism is that it acts as a mirror without any silvering, whereas the silvering on a plane mirror may disappear or tarnish with time.

A periscope is made by placing a second right-angled isosceles prism below the first. Light falling on this prism behaves in exactly the same way as described above. Total internal reflection thus takes place in the glass at Z (Fig. 18.20a). The emergent ray enters the eye in a direction parallel to OY, and so the object O is seen. Right-angle prisms with angles 90°, 30°, 60° do not act as totally reflecting prisms, as the reader should verify.

Fig. 18.20b, c illustrates two other cases of total reflection by right-angled 45° prisms. Fig. 18.20b shows how an object is inverted, as in prism binoculars (page 324). Here the light beam is turned through 180°, so that the light returns along a path parallel to its incident direction as shown.

Total internal reflection explains the brilliance of cut diamonds. The refractive index n of a diamond is high, so its critical angle, given by $\sin c = 1/n$, is low. Much of the light entering the diamond is therefore totally reflected to the observer.

Optical fibres

Total internal reflection is used in optical fibres, following an idea due originally to Baird, the founder of television, in 1927. Optical fibres consist of many tens of thousands of long fine strands of high-quality glass coated with glass of lower refractive index. The strands may be 1/2000 cm in diameter and the refractive indices of the respective glasses about 1·7 and 1·5.

Scanned with CamScanner



Plate 18B Optical fibres, showing the light rays curving and emerging at their ends after total internal reflection inside the fibres. British Telecom are now using optical fibres in telephone and transmitting cables; each fibre can carry as many as 2000 telephone conversations (*Paul Brierley*)

When light is incident on one end of the fibre at an angle less than about 60°, it passes inside. (Fig. 18.21). It then undergoes total internal reflection repeatedly at the walls because the angle of incidence is greater than the critical angle between the high and low refractive index glass. The trapped light travels along the fibre, no matter how it may be curved, and comes out with a high intensity at the other end. So a bundle of flexible fibres enables an image of an object at one end to be seen at the other end. If the fibres are tapered a magnified image can be produced.

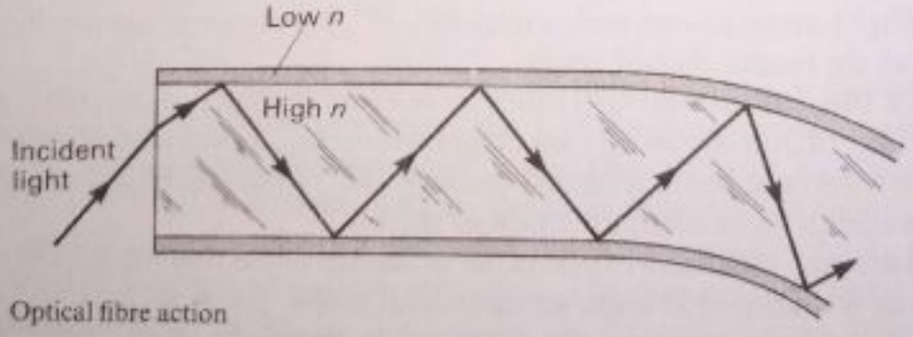


Fig. 18.21 Optical fibre action

Optical fibres are flexible guides which can 'pipe' light to the other end. They are used medically, for example to examine the inside of the throat, and they have many engineering uses.

Mirages

The mirage is another example of total internal reflection, but this time it occurs at the boundary of a hot and a cold gas.

Scanned with CamScanner

The air in a desert is hot near the ground and cools rapidly with height. The hotter air is optically less dense. So a ray from part of the sky, passing from a colder to a warmer air layer (i.e. from a denser to a less dense medium) will bend gradually from the incident direction until it enters a layer of air near the ground, at h, where total internal reflection occurs (Fig. 18.22). The ray is then reflected upwards into the denser air.

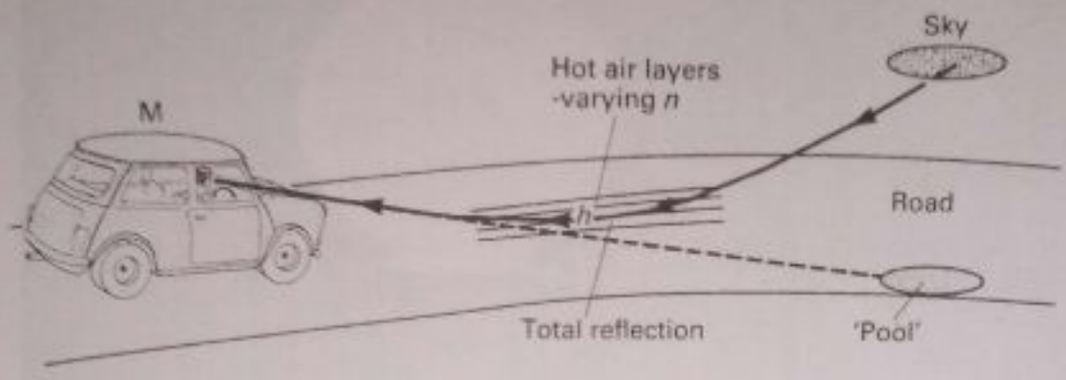


Fig. 18.22 Mirage

After undergoing refraction in a gradually increasing upward direction the ray may enter the eye of an observer in M. To him, the ray seems to come from a place X. This gives the illusion of a reflecting pool of water in the road some distance away. In hot deserts, people may see an illusion of an inverted image of a tree in a pool of water below the actual tree.

Atmospheric refraction

The sun is visible even after it has set below the horizon. The reason for this is shown in Fig. 18.23a. When light rays from the sun enter the earth's atmosphere, a denser medium, the rays are refracted towards the earth. An observer at A will therefore see an image of the sun, although it is actually below A's horizon.

For a similar reason, atmospheric refraction also produces changes in the position of stars. Astronomers make allowance for this source of error when measuring positions of stars through the earth's atmosphere (Fig. 18.23b). Corrections are also needed in measurements of the sun's altitude which are used for navigation.

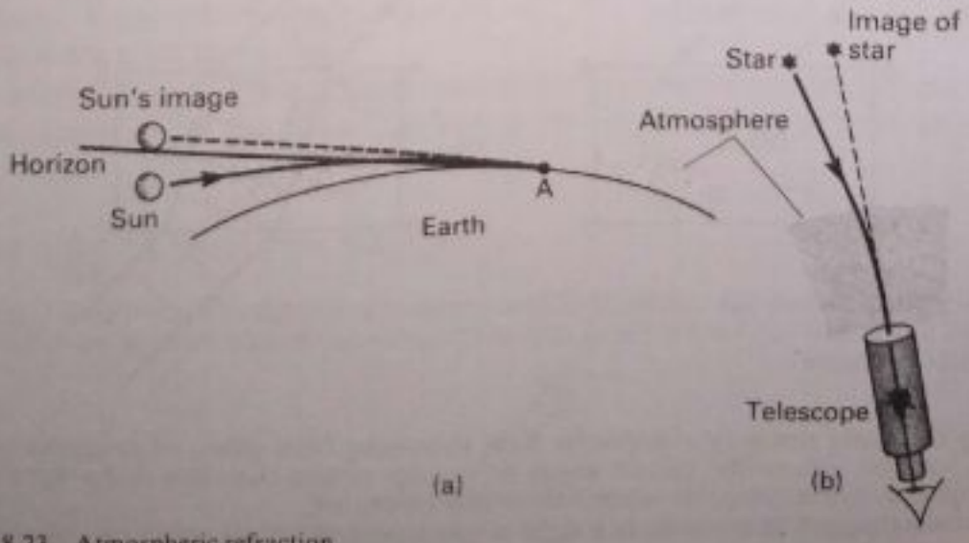


Fig. 18.23 Atmospheric refraction

Scanned with CamScanner

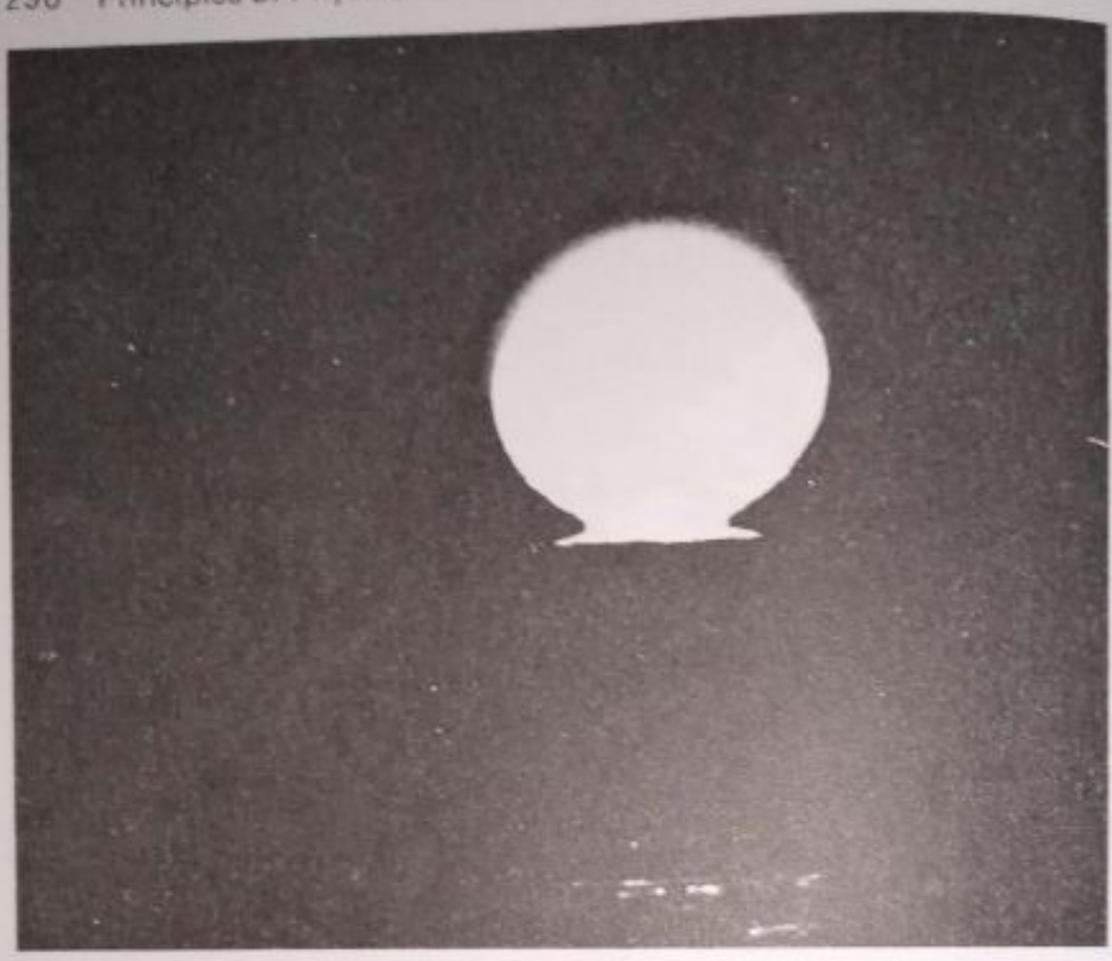


Plate 18C Sun about to set below the horizon. Atmospheric refraction of the sun's rays produces the distortion. (*Camera Press*)

Example on refraction and critical angle

(a) Fig. 18.24a shows a parallel beam of monochromatic light incident at 45° on the surface AB of the glass block ABCD. Copy the diagram, and complete it to show some of the *wave-fronts* of the light (i) approaching AB, (ii) travelling in the glass, (iii) after emergence from CD.

Explain why the direction of travel of a wave-front changes on entering the glass. If the refractive index of the glass is 1·55, what angle do the wave-fronts in the glass make with AB?

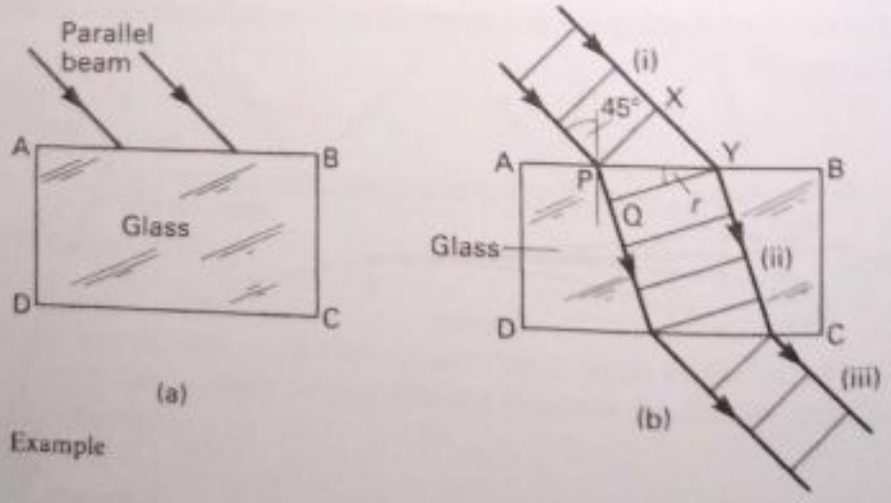


Fig. 18.24 Example

(b) Calculate the *critical angle* for light emerging from glass, of refractive index 1·55, into air. Would the critical angle be greater or less than this if the light were emerging from the glass into water? Give your reasoning.

Draw a diagram to show how a right-angled isosceles glass prism can be used to

Scanned with CamScanner

turn a ray of light through 90°. What are the advantages of using a prism, rather than a silvered mirror, for this purpose? (C)

(a) Some of the wave-fronts are shown in Fig. 18.24b for (i), (ii) and (iii).

The direction of travel of a wave-front changes in glass because the speed of light in glass is less than in air. So while the light travels from X to Y in the air, the light in the glass travels a smaller distance P to Q. The wave-front in the glass thus becomes YQ. (See also page 352).

The wave-front YQ in the glass makes an angle r with AB which is the angle of refraction, since angle PQY is 90°. From $\sin i/\sin r = n = 1{\cdot}55$

$$\frac{\sin 45^\circ}{\sin r} = 1{\cdot}55$$

$$\therefore \sin r = \frac{\sin 45^\circ}{1{\cdot}55} = \frac{0{\cdot}707}{1{\cdot}55} = 0{\cdot}456$$

$$\therefore r = 27^\circ \text{ (approx.)}$$

(b) The critical angle c for $n = 1{\cdot}55$ is given by

$$\sin c = \frac{1}{n} = \frac{1}{1{\cdot}55} = \frac{100}{155} = 0{\cdot}645$$

$$\therefore c = 40^\circ \text{ (approx.)}$$

When light travels from glass to water it is refracted away from the normal by a *smaller* angle than when the light travels from glass to air. So the angle of refraction in the water will be 90° when the angle of incidence in the glass is *greater* than in the case of glass to air. This angle of incidence is c, the critical angle.

The 90° deviation of light by a 45°, 45°, 90° prism is discussed on page 293, and the advantages on page 292.

Summary

1. When light passes from one medium to an optically denser medium it is refracted towards the normal. When it passes to an optically less dense medium it is refracted away from the normal.

2. *Laws of refraction* 1. The incident ray, the normal and the refracted ray all lie in the same plane.

2. The ratio $\sin i/\sin r$ is a constant (Snell's law).

3. Refractive index $n = \sin i/\sin r$. Real depth/apparent depth $= n$, at near-normal incidence. The direction of a refracted ray can be calculated by using $\sin i/\sin r = n$ or by drawing two circles whose radii are in the ratio n: 1.

4. Total internal reflection occurs when (a) light passes from a medium (such as glass) to a medium optically less dense (such as air); and (b) the critical angle is exceeded. Total reflecting prisms are 90°, 45°, 45°; unlike plane mirrors, they do not produce multiple images.

5. The critical angle c is the incident angle in the *denser* medium when the angle of refraction in the less dense medium is 90°. If the less dense medium is air, $\sin c = 1/n$.

Exercise 18 (Answers, page 642)

1. Fig. 18.25 shows the paths of two rays which enter the extremities of the pupil of the eye from a small object immersed in water. Draw lines in a diagram to show a

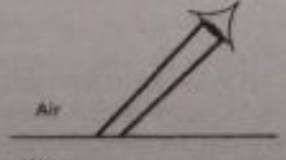


Fig. 18.25

Scanned with CamScanner

possible true position of the object and its apparent position in the eye. Mark the directions of rays with arrows. (L)

2. Fig. 18.26 shows a ray of light incident on a face of a rectangular glass block. Complete the diagram indicating the approximate path of the ray through the glass and mark the angle of incidence *I* and the angle of refraction *R* at the first surface. (N)

3. A ray of white light is to be reflected, Fig. 18.27, so that it is turned through 90° in each case using different optical devices. Complete the diagrams (a) and (b), without altering the paths of the rays, to show how this can be done. Label the parts you add and mark the values of important angles. (N)

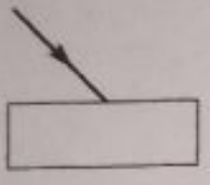


Fig. 18.26

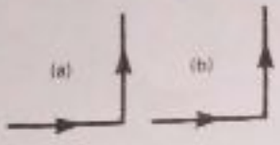


Fig. 18.27

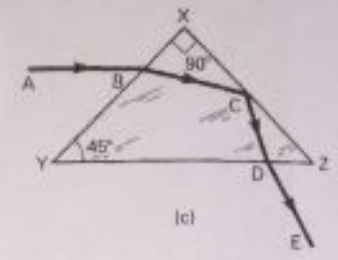


Fig. 18.28

4. Fig. 18.28 shows the path of a ray AB incident on a glass prism XYZ with angles of 90°, 45°, 45° and finally emerging along DE.

(a) Explain the statement 'the refractive index of the glass is 1·5'.
(b) Why does the ray AC not emerge into the air at C?
(c) Why does the ray DE refract away from the normal at D to the side YZ?

5. Define refractive index.

Explain with the help of a ray diagram, why water in a swimming pool appears to be shallower than it really is. What is the apparent depth of a swimming-pool if its real depth is 2·4 m and it is viewed from a springboard vertically above? (Refractive index of water is 4/3.) (L)

6. In each of the diagrams shown in Fig. 18.29, AB shows the path of a ray of light which enters one face of a triangular glass prism at B, and CD the path of the same ray emerging from another face of the prism at C. Draw a possible outline of the prism in each instance and complete the path of the ray through it. (L)

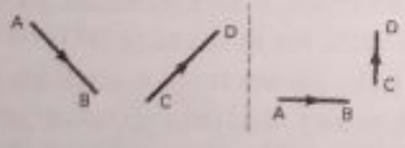


Fig. 18.29



Fig. 18.30

7. State the *laws of reflection* of light.

If a small illuminated object is placed in front of a thick plate glass mirror, silvered on the back, several images are seen. Explain the formation of these images and indicate which one is usually brightest. (L)

8. 'Paraffin oil has a greater refractive index than water.'

What information does the above statement give with regard to (a) the relative velocities of light in paraffin oil and in water and (b) the path of a ray of light when passing from water into a layer of paraffin oil floating on top of it?

A small but intense source of light is situated at the bottom of a large tank of clear water. Explain and illustrate by means of a diagram why the source does not appear to be as far below the surface as it actually is. (L)

9. ABC is the triangular section of a glass prism whose angles are as shown in Fig. 18.30. PQ shows the path of a ray of light which meets the side AB normally.

Determine, graphically or otherwise, the angle that the ray which emerges from the side AC makes with the normal at the point of emergence.

Scanned with CamScanner

Explain why the ray PQ would not emerge from the side AC, as above, if the angles of the prism at A and C were both 45°.

(Assume that the refractive index of glass relative to air is 1·5.) (L)

10. Give the meaning of the terms *refraction, refractive index*. In what circumstances may *total (internal) reflection* occur? Illustrate your answer by sketches.

Describe a method you have used to measure the refractive index of water.

A small electric light bulb is switched on 210 cm below the surface of a pool of water. Through what area of the surface will light emerge? What happens to the rest of the light from the bulb?

Take the refractive index of water as 4/3 and π as 22/7.

11. Explain why a pool of water appears to be less deep than it really is.

Describe an experiment to determine the refractive index of water by measuring the real depth and the apparent depth. (L)

12. State the laws of refraction and illustrate your statement by a ray diagram, marking clearly thereon the angles of incidence and refraction.

Describe *two* methods of measuring the refractive index of glass, supplied in the form of a rectangular glass block.

Water is poured into a beaker to a depth of 12 cm. To an eye looking vertically down through the water surface, the bottom of the beaker appears to be raised 3 cm. Calculate the refractive index of water. (O and C)

13. What is meant by *total internal reflection* of light? State the conditions under which this occurs. Define *critical angle* and explain how the critical angle for a material in air depends on its refractive index.

14. Give the meaning of the terms *refraction, refractive index* and *critical angle*. In what circumstances may *total internal reflection* occur?

Describe briefly *one* example of the occurrence of total internal reflection in everyday life or of the use of total internal reflection in an optical instrument. Illustrate your answer with a diagram.

Calculate the critical angle for a water–air surface, taking the refractive index of water as 4/3. (O and C)

Multiple-Choice Questions

15. In Fig. 18.31, a ray XO is incident on a rectangular glass block. Which ray, from A to E, is the correct refracted ray?

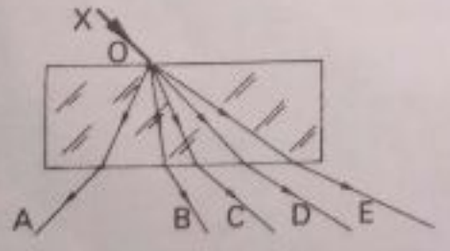


Fig. 18.31

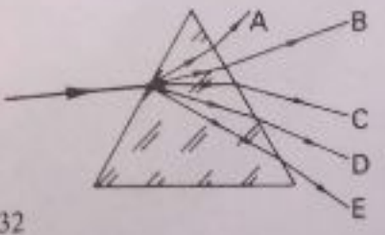


Fig. 18.32

16. In Fig. 18.32, a ray is incident on a 60° glass prism. Which of the rays A to E is the best answer for the ray refracted through the prism into the air?

17. If n is the refractive index of water in air, the critical angle for an air–water boundary is the angle of incidence i

A in air when $\sin i = 1/n$
B in water when the angle of refraction in air is 90°
C in X when $\sin i = n$
D in X which gives total reflection
E in air when i is 90°

18. The letters X and Y are written on paper and appear nearer when a thick glass block is placed on them. This is because light from X and Y

A passes straight through the glass
B is refracted towards the normal after passing into air
C is partly reflected at the glass surface
D is refracted away from the normal after passing into air
E is totally reflected at the top glass surface

Scanned with CamScanner

19. Refraction through Lenses

Lenses consist of pieces of glass of thickness varying from the middle to the edges, with spherical surfaces on one or both sides. They are used in spectacles to correct defects of vision, and in optical microscopes for looking at objects too small to be



Plate 19A A sunshine recorder. The glass sphere, diameter about 10 cm, brings the sun's rays to a focus on a card graduated in hours in a metal bowl. This chars the card so that the hours of sunshine can be read. (*Crown Copyright reserved/Daily Mail*)

Scanned with CamScanner

seen by the naked eye. They are also used in telescopes and prism binoculars for seeing distant objects, and in cameras and film projectors.

Lenses have been used to concentrate light since ancient times, and they were then called burning glasses. A sunshine recorder uses a glass sphere as a lens to burn a line on a moving strip of graph paper, and this is still used at the Air Ministry and seaside resorts to record the hours of sunshine.

Types of lenses

Some types of lenses are shown in Fig. 19.1. The first three lenses in Fig. 19.1 are *converging* (or *convex*) lenses; these lenses are thicker at the centre than at the edges. The last three lenses in Fig. 19.1 are *diverging* (or *concave*) lenses; these lenses are thinner at the centre than at the edges. Lenses which are biconvex or biconcave are commonly used in the laboratory. Plano-convex and plano-concave lenses have only one curved surface and are used in optical instruments. Converging and diverging meniscus lenses are used as contact lenses to fit the curvature of the eyeball to correct people's sight.

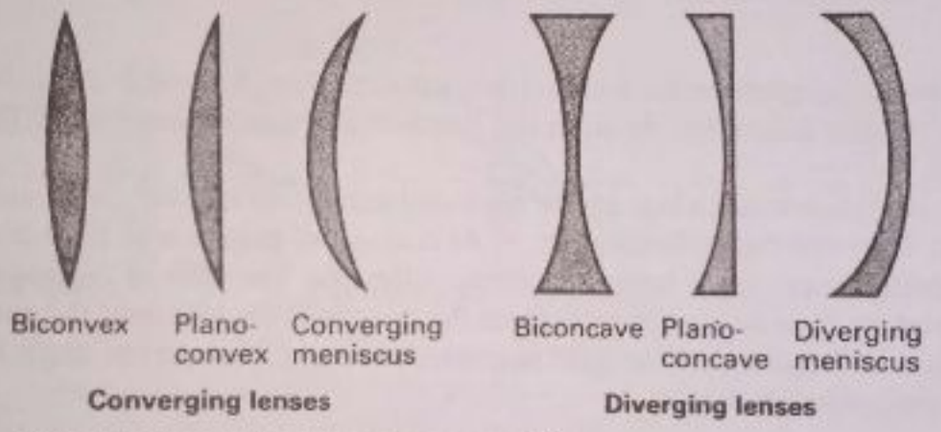


Fig. 19.1 Converging (convex) and diverging (concave) lenses

The eye has a natural converging lens, and a camera has a glass converging lens. Some spectacles may contain diverging lenses, others may contain converging lenses. One type of telescope has both a converging and a diverging lens inside it.

How lenses refract parallel light beams

Fig. 19.2a shows how a 'model' flat glass converging lens refracts a parallel beam of light from a ray-box R. In Fig. 19.2b, an actual converging lens, pushed half-

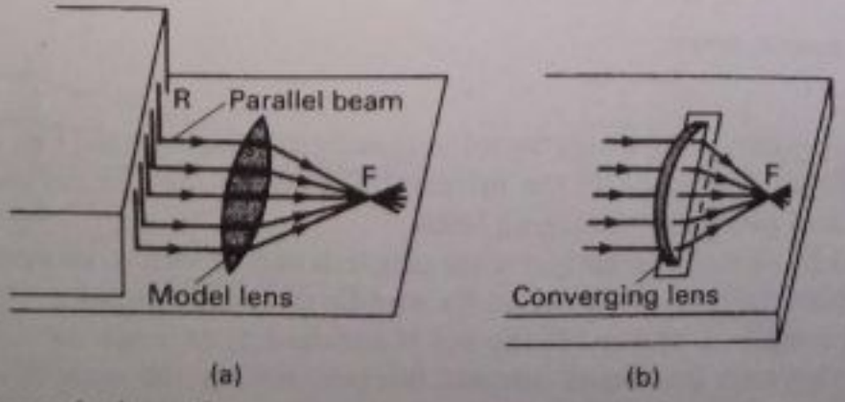


Fig. 19.2 Converging lens action

Scanned with CamScanner

way through a board, is used. The lens brings the rays to a focus F in front of the lens, as shown.

Fig. 19.3 shows the action of a diverging lens on a parallel beam. After refraction through the lens a diverging beam of light is obtained and this beam would pass through a point F behind the lens if produced back. The effect of this type of lens is therefore opposite to that of the converging lens.

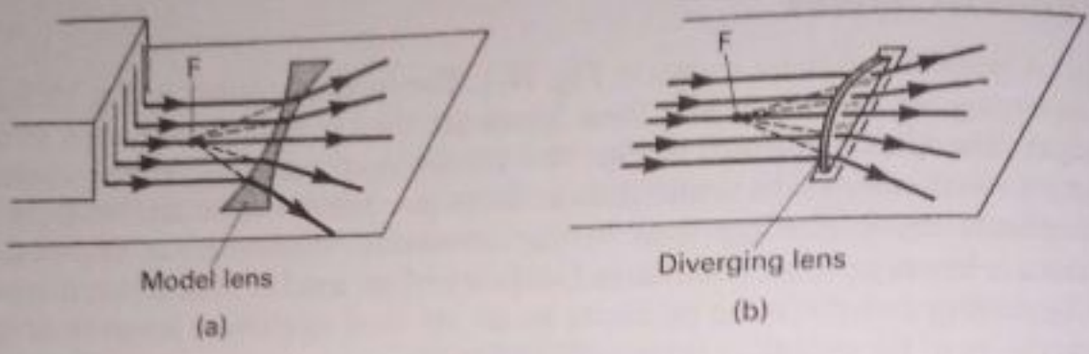


Fig. 19.3 Diverging lens action

As the names given to the lenses show, a converging lens makes a parallel beam converge after refraction through the lens and a diverging lens makes the beam diverge.

Fig. 19.4 shows that a lens can be regarded as built up of a very large number of prisms. The converging lens in Fig. 19.4a consists of prisms with their triangular top missing except at the top and bottom of the lens. The sides of the prisms slope more and more as we go upwards from the middle of the lens, and the amount of deviation or 'bending' of the light increases as shown because the angle between the sloping sides increases.

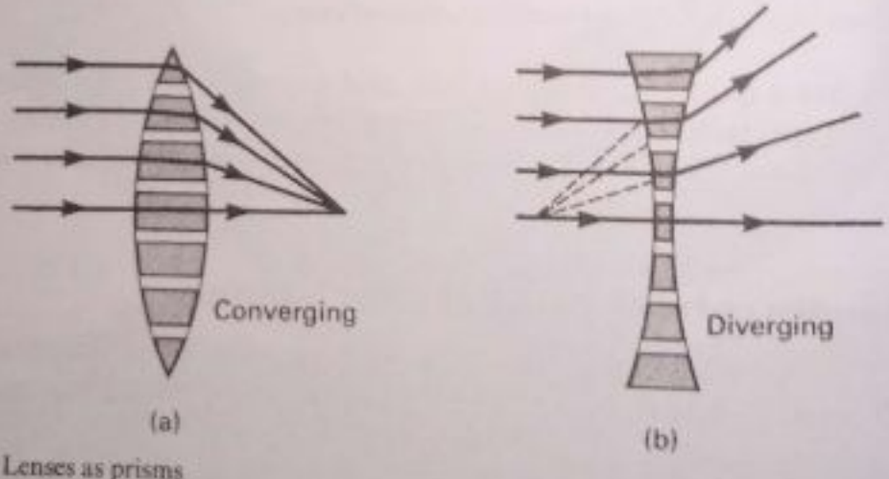


Fig. 19.4 Lenses as prisms

A similar explanation holds for refraction by a diverging lens (Fig. 19.4b). This time, however, the bases of the prisms are upwards. So the rays are deviated upwards, and produce a diverging beam.

The middle of a converging or diverging lens can be seen to be a parallel-sided piece of glass. So a ray incident on the middle of a lens normal to the glass goes straight through, as shown. If the ray is incident at an angle to the glass, it is displaced sideways by a small amount but continues in the *same direction* (page 281).

Scanned with CamScanner

Principal focus · Focal length

Fig. 19.5a shows three typical rays in a parallel beam of light incident on a converging lens. Ray 1 lies along the *principal axis* of the lens; this is the line joining the mid-points of the opposite faces of the lens. Rays 2 and 3 converge to the point F on the principal axis. F is called the *principal focus* of the lens. It is defined as *the point on the principal axis to which rays parallel and close to the principal axis converge after passing through the lens.*

Fig. 19.5b shows three parallel rays incident on the lens at an angle to the principal axis. The ray incident on the middle C goes straight through because the glass is thin and has parallel sides here as explained before. The beam now converges to a point F_1 *below F* called a secondary focus.

Fig. 19.5c shows the principal focus F of a diverging lens. It is the point on the principal axis from which rays parallel and close to the principal axis appear to diverge after they pass through the lens. From the ray diagrams, we see that:

1. The principal focus of a converging lens is real;
2. The principal focus of a diverging lens is virtual.

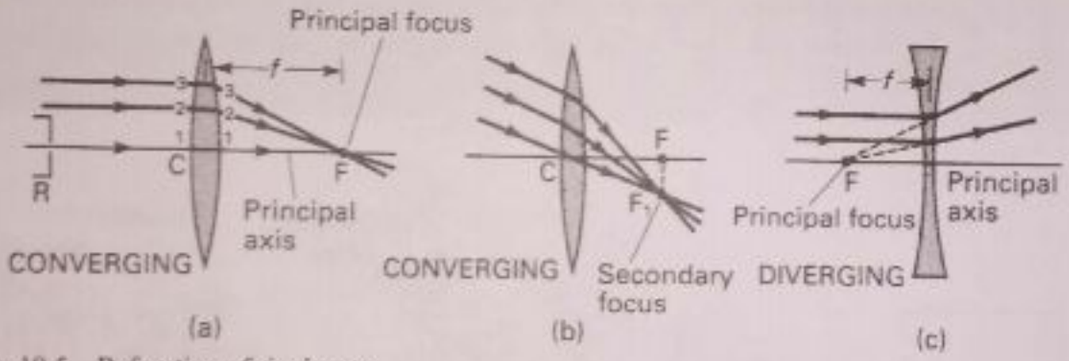


Fig. 19.5 Refraction of single rays

The *focal length f* of a lens is the distance from the principal focus to the middle of the lens. In this book we deal only with thin lenses, and we shall neglect the thickness of the lens in focal length measurement or calculation.

The *power* of a lens depends on its focal length. A converging lens with a long focal length, of 100 cm for example, will refract a parallel beam to a focus 100 cm away, whereas a converging lens of focal length 10 cm will refract a parallel beam to a focus only 10 cm away. The converging action of the 10 cm focal length lens is therefore much more powerful than that of the 100 cm focal length. The shorter the focal length *f* the greater will be the power of the lens (Fig. 19.6). The power of a lens is equal to $1/f$, and is measured in *dioptres* when *f* is in metres.

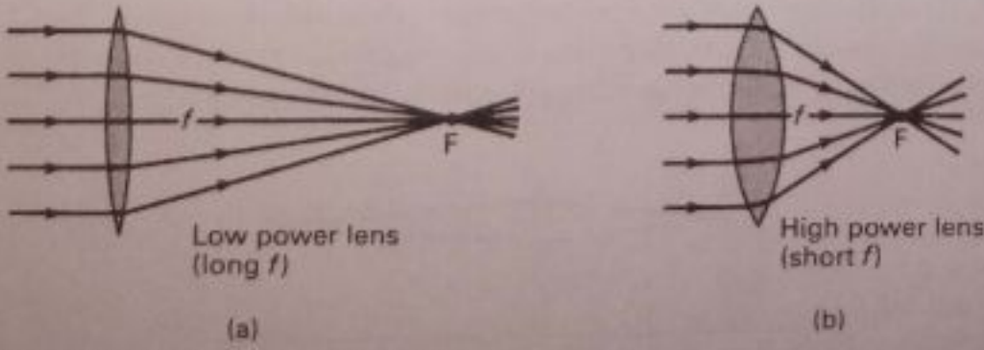


Fig. 19.6 Low power and high power lenses

Scanned with CamScanner

Experiment to measure focal length of converging lenses

1. *Plane mirror method* Support a converging lens L in a holder and place a plane strip mirror M behind it (Fig. 19.7a). Place an illuminated object O, such as crosswires with a lamp behind it, directly in front of the lens L and at the same height. Move O towards or away from L until a sharp image of the crosswires is obtained *beside O*, that is, on the surface containing the crosswires. Then measure the distance LO. Move the crosswires away and repeat the experiment. Take the average of three values. This is the focal length *f* of the lens.

Explanation: Light which passes from O through L is reflected by the mirror M back towards O. When a sharp image I is obtained back near O, the beam of light striking M must now return from M practically along its original path, as shown in Fig. 19.7a. This means that the beam is incident *normally* on M. The beam incident on the lens L from M is therefore a parallel beam, and so it is refracted to the focus at O.

2. *Rough measurement*: Hold the lens near the back of the room and move it until a sharp image of the window frame is obtained on a sheet of paper or the wall (Fig. 19.7b). Measure the distance from the lens to the paper screen or wall. Since parallel rays reach the lens when the window is a long way from it, this distance is roughly equal to the focal length *f*. This is not an accurate method but it can be used for getting a rough value of *f* very quickly.

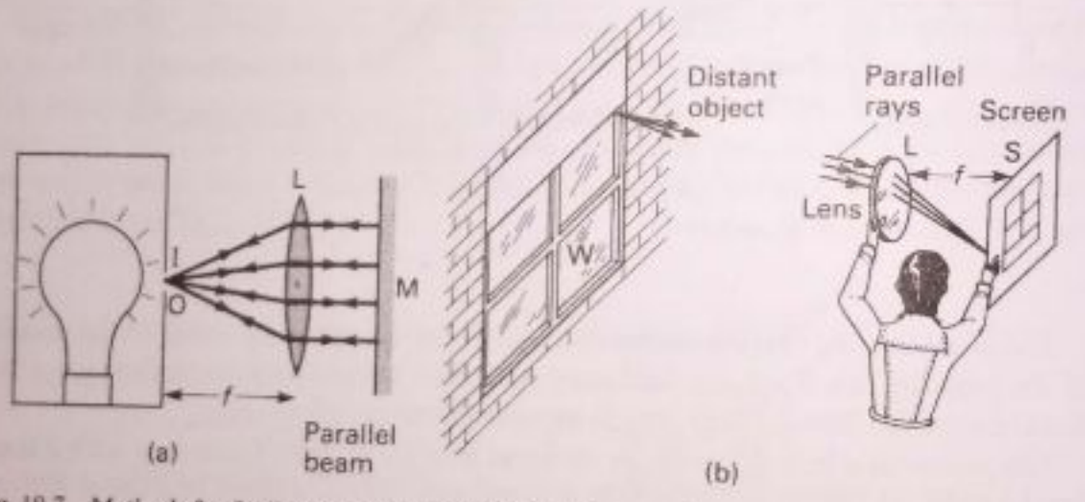


Fig. 19.7 Methods for finding *f* (a) Accurate (b) Quick but rough value

Images in converging lenses

Experiment

1. To investigate the nature and size of the images produced by a converging lens, use illuminated crosswires (or a perspex ruler with a lamp behind it) as the object O and a white screen S behind the lens L to receive the clear image (Fig. 19.8).

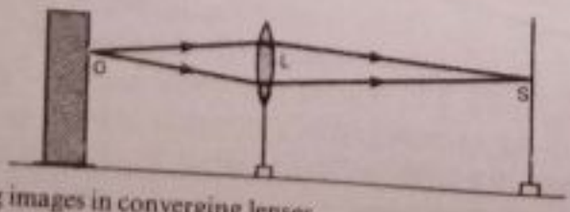


Fig. 19.8 Investigating images in converging lenses

Scanned with CamScanner

2. Use typical object distances from the lens such as (a) a long way from it, (b) further than $2f$, where f is the focal length, (c) at $2f$, (d) between $2f$ and f. In each case, note if the image is (i) upright or inverted, (ii) smaller or bigger than the object, (iii) real or virtual. Enter your results in a table similar to that copied from page 273 for the images in a concave mirror.

3. Finally, hold the lens near the printed words or diagrams in this book, that is, at a distance *less* than the focal length f. Record how the image has changed.

Results Fig. 19.9 shows the images formed by typical rays from the object. As the object moves nearer the lens the image IB increases in size and remains real and inverted. At a distance $2f$ the image is the same size as the object (compare the concave mirror, page 274). The image becomes bigger than the object as it is moved between a distance of $2f$ and the principal focus F. At the focus, however, no image is received on the screen; the rays emerging from the lens are parallel and the image is said to be formed 'at infinity'.

When the object is nearer the lens than its focal length, the image becomes upright and magnified as discussed on page 306.

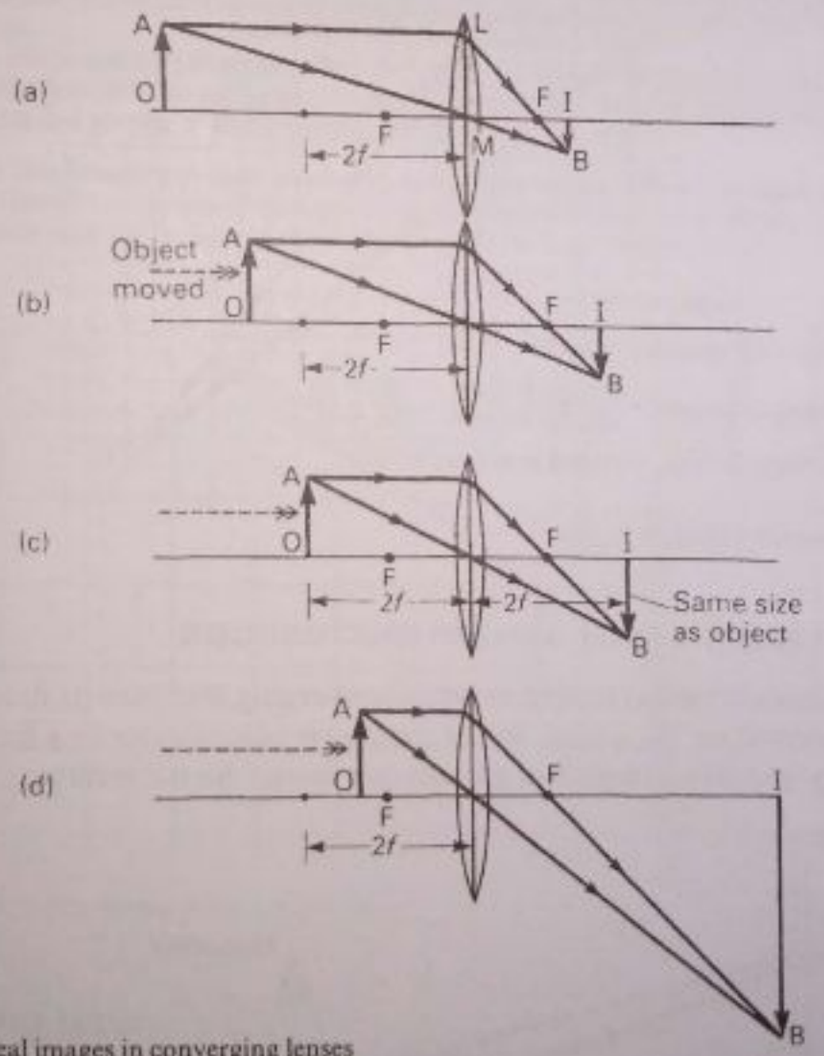


Fig. 19.9 Real images in converging lenses

Lens images by drawing

In finding lens images by drawing, we represent the lens position by a straight vertical line as the lens is thin. The object is represented by a straight line with its foot on the principal axis and usually only rays are needed from the top of the

Scanned with CamScanner

object to find the position of the top of the image. This is illustrated in the example which follows.

Example on lens image by drawing

An object (OA) is placed 30 cm in front of a converging lens of focal length 20 cm. Obtain the image by drawing. State (a) its distance from the lens; (b) its magnification; (c) its description.

(a) Represent the lens by a straight line XY and mark the position of the principal focus F from the centre C of the lens to a scale; that is, CF represents the focal length (Fig. 19.10). State the scale used.

(b) Place the object OA at the distance CO from the lens, on the same scale.

(c) From the top point A of the object, draw: (i) a ray AD parallel to the principal axis–this is refracted to pass through F; (ii) a ray through the centre C–this passes straight through without changing direction. The top point B of the image is the point of intersection of the two emerging rays CB and DB. Complete the image by drawing BI normal to the principal axis.

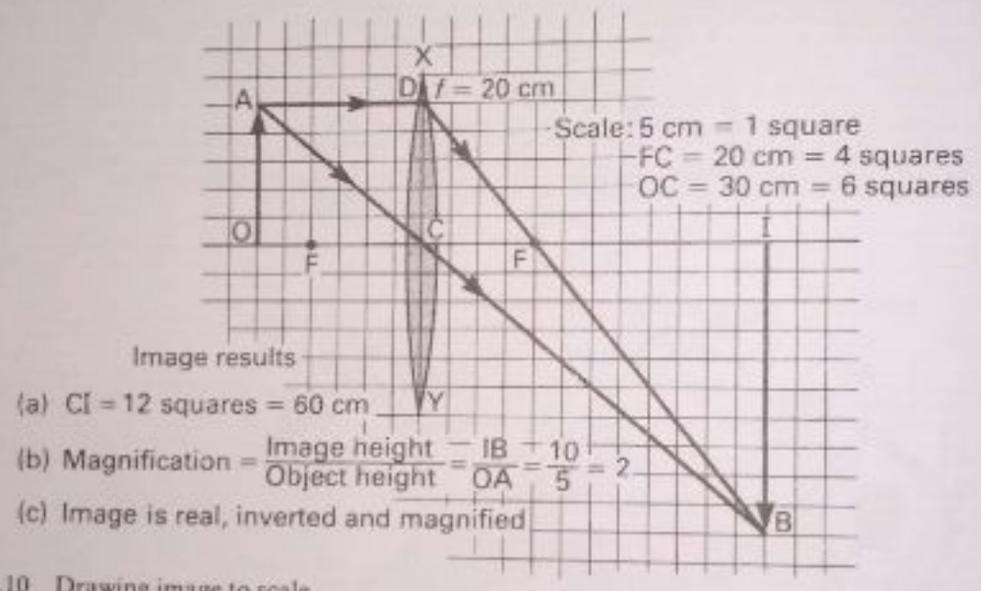


Fig. 19.10 Drawing image to scale

Magnifying glass or simple microscope

If an illuminated object is moved nearer a converging lens than its focal length no image is received on the screen. But if the lens is placed close to a fingernail, for example, the eye sees a *magnified* and *upright* image. So a converging lens can be

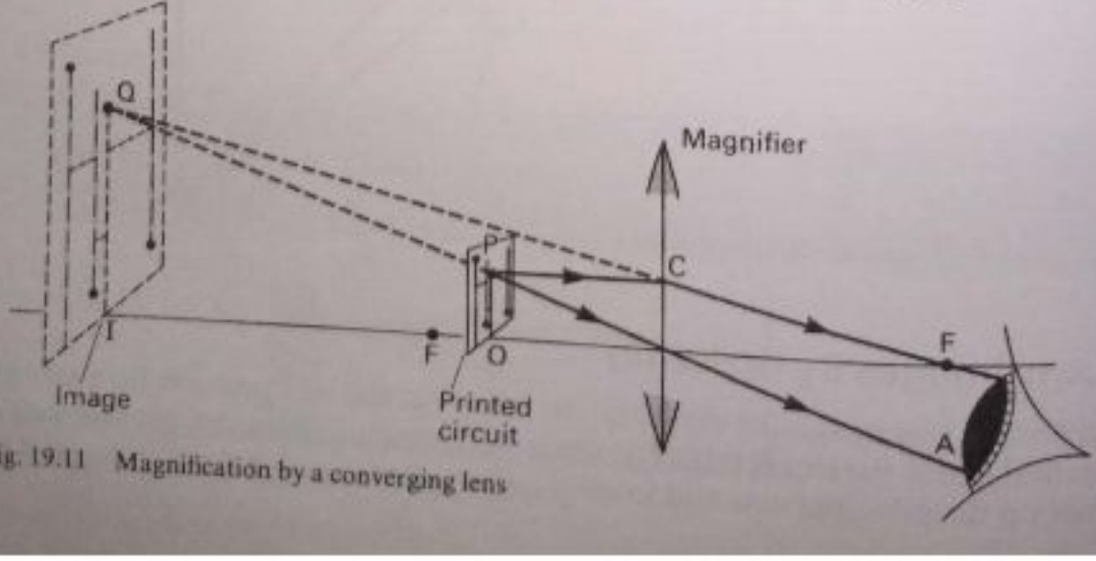


Fig. 19.11 Magnification by a converging lens

Scanned with CamScanner

used as a magnifying glass or simple microscope. To look closely at small print or a map the magnifying glass must be held nearer to the object than its focal length. Otherwise, the image will be upside down and not particularly large.

Fig. 19.11 illustrates how the lens produces a magnified upright image of part of a printed circuit used in electronic equipment. Two rays from the point P form a diverging beam after refraction along CF and PA, as shown. So the object PO is seen at QI by the eye, now looking very much bigger, and upright. The image QI is a virtual one because it is produced by a divergent beam that appears to come from the image and cannot be obtained on a screen.

Example on drawing magnified lens image

A converging lens of focal length 20 cm produces an *upright* image of an object which is magnified 4 times. By drawing, find the object distance from the lens.

Draw CQ to represent the position of the lens. Mark the focus F according to a chosen scale (Fig. 19.12); in this diagram one square represents 10 cm. Then draw two lines LP and MQ parallel to the principal axis at distances of 1 : 4 above the axis, using any suitable scale. Join FP and produce it to meet MQ at B. Then B is the top point of the virtual image. BI is the image.

Now join BC, cutting LP at A. Then A is the top point of the object, and AO is the object. The distance OC is the object distance, which can be found from the scale used to represent the focal length, CF. From Fig. 19.12, OC = 15 cm.

Note that the image position is on the same side as the object because the image is upright and therefore virtual. If the image was inverted and real, it would have to be drawn on the opposite side of the lens to the object.

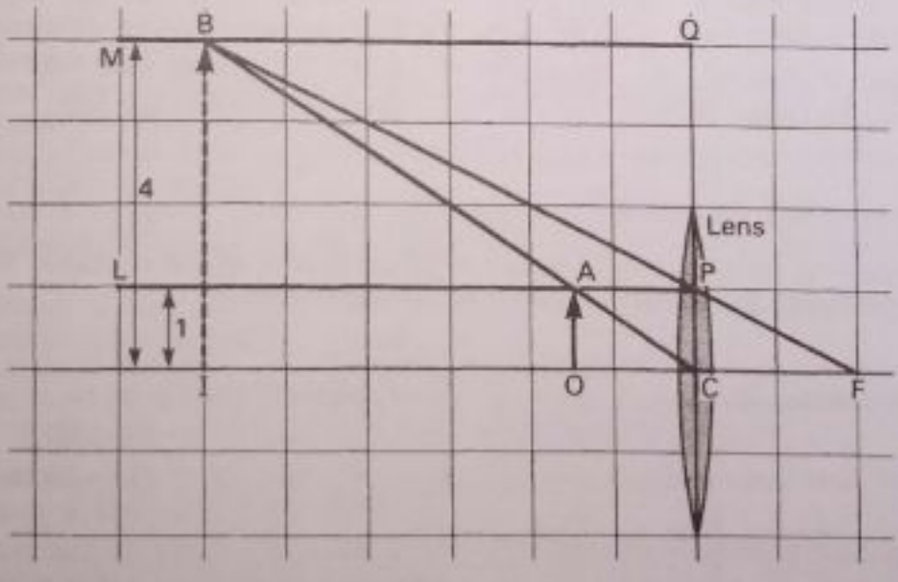


Fig. 19.12 Drawing of magnified upright image

Diverging lens

When an illuminated object such as crosswires is placed in front of a diverging lens no image is received on a screen. On looking through the lens at a finger-nail or at print, however, the virtual image is seen to be *diminished* and *upright*. This is always the case wherever the object is positioned. In contrast, the images in a converging lens are sometimes upside down and on other occasions upright, depending on the object position.

Scanned with CamScanner

Fig. 19.13 illustrates by a ray diagram why the image in a diverging lens is always diminished, upright and virtual.

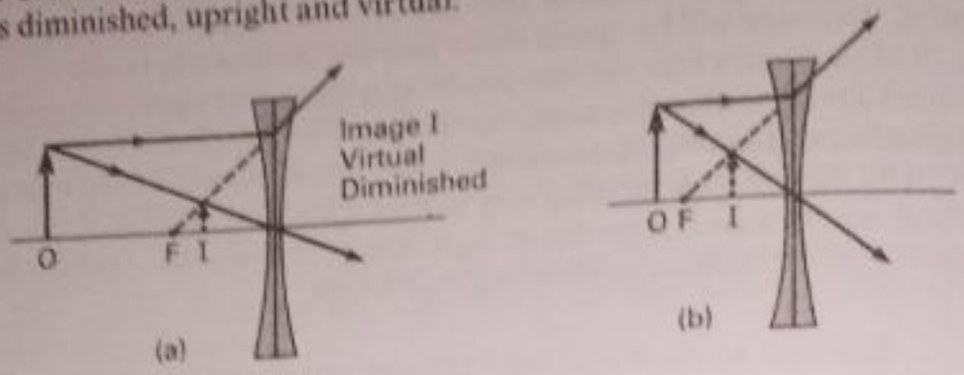


Fig 19.13 Images in a diverging lens

Lens formula

Measurements of the object and image distances, u and v, from a converging lens, and of the focal length f, show that u, v and f are related by a general formula given below. This formula, which can be proved theoretically, applies both to converging and to diverging lenses provided a sign rule or convention, given on page 277, is used with the distances u, v and f. Further, the magnification m produced by a lens is given numerically by $m = v/u$ (see page 277).

RP
On the Real is Positive convention

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Remember that f is +ve for a converging lens, and f is −ve for a diverging lens (page 303).

NC
On the New Cartesian convention

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

Remember that f is +ve for a converging lens, and f is −ve for a diverging lens (page 303).

Examples on lens formulae

1. A converging lens has a focal length of 20 cm and an object is placed 30 cm from the lens. Calculate the image position and its magnification.

RP

The lens is converging, so

$$f = +20$$
$$u = +30\,\text{cm}$$

From $\dfrac{1}{v}+\dfrac{1}{u}=\dfrac{1}{f}$

$$\frac{1}{v}+\frac{1}{(+30)}=\frac{1}{(+20)}$$
$$\therefore \frac{1}{v}+\frac{1}{30}=\frac{1}{20}$$
$$\therefore \frac{1}{v}=\frac{1}{20}-\frac{1}{30}=\frac{1}{60}$$
$$\therefore v = 60$$

Also $$m=\frac{v}{u}=\frac{60}{30}=2$$

The drawing of the image to scale is shown in Fig. 19.10, page 306.

NC

The lens is converging : so

$$f = +20$$
$$u = -30\,\text{cm}$$

From $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$

$$\frac{1}{v}-\frac{1}{(-30)}=\frac{1}{(+20)}$$
$$\therefore \frac{1}{v}+\frac{1}{30}=\frac{1}{20}$$
$$\therefore \frac{1}{v}=\frac{1}{20}-\frac{1}{30}=\frac{1}{60}$$
$$\therefore v = 60$$

Also $$m=\frac{v}{u}=\frac{60}{30}=2$$

Scanned with CamScanner

2. The image in a converging lens is upright and magnified four times. Calculate the object distance if the focal length is 20 cm.

RP

$$m = \frac{v}{u} = 4$$

$$\therefore\ v = 4u$$

Thus if x cm $= u$, $4x$ cm $= v$

Now an upright image is a virtual one (page 306).

$$\therefore\ v = -4x,\ u = +x$$

From $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

$$\frac{1}{(-4x)} + \frac{1}{(+x)} = \frac{1}{(+20)}$$

$$\therefore\ -\frac{1}{4x} + \frac{1}{x} = \frac{1}{20}$$

$$\therefore\ \frac{3}{4x} = \frac{1}{20}$$

$$\therefore\ 4x = 60, \text{ or } x = 15$$

$\therefore$ Object distance $= 15$ cm

NC

$$m = \frac{v}{u} = 4$$

$$\therefore\ v = 4u$$

Thus if x cm $= u$, $4x$ cm $= v$

Now an upright image is left of the lens (page 306).

$$\therefore\ v = -4x,\ u = -x$$

From $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$$\frac{1}{(-4x)} - \frac{1}{(-x)} = \frac{1}{(+20)}$$

$$\therefore\ -\frac{1}{4x} + \frac{1}{x} = \frac{1}{20}$$

$$\therefore\ \frac{3}{4x} = \frac{1}{20}$$

$$\therefore\ 4x = 60, \text{ or } x = 15$$

$\therefore$ Object distance $= 15$ cm

Example on lenses

The diagram, Fig. 19.14, shows a method of obtaining the focal length of a lens by placing a pearl lamp behind a circular hole in a sheet of cardboard and finding a focused image of the edge of the hole on a screen.

(a) If you had no means of cutting a clean hole in the cardboard, what would you do to make it easier to focus the image?

(b) If you could not obtain a clearly focused image by moving the screen only, what would most likely be wrong?

(c) When a focused image was obtained, the distance from the lens to the cardboard was 0·18 m and from the lens to the screen 0·36 m. Find the focal length of the lens.

(d) Without moving the cardboard or screen, where could you place the lens to obtain another focused image?

(e) What would be the ratio between the sizes of your first and second images?

(f) If the top half of the lens were covered by an opaque sheet, what effect would this have on the size of the image and on the brightness of the image? (L)

(a) To make it easier to focus the image, a piece of thin wire can be placed across the hole and the image of the wire focused on the screen.

(b) If an image can not be obtained on the screen, the hole is nearer to the lens than the focal length (a virtual image is then obtained).

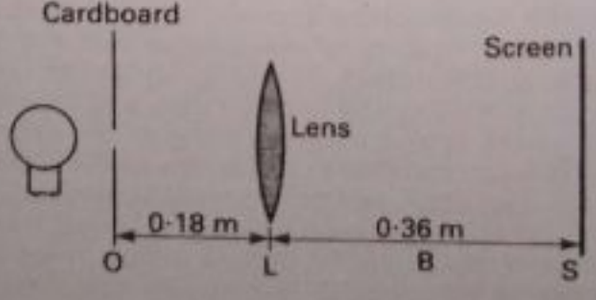


Fig. 19.14 Example

Scanned with CamScanner

(c) Using the Real is Positive sign convention, $u = +0{\cdot}18$ m, $v = +0{\cdot}36$ m. Substituting in the lens formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

then
$$\frac{1}{0{\cdot}18}+\frac{1}{0{\cdot}36}=\frac{1}{f}$$

So
$$\frac{1}{f}=\frac{2}{0{\cdot}36}+\frac{1}{0{\cdot}36}=\frac{3}{0{\cdot}36}$$
$$\therefore\ 3f = 0{\cdot}36, \text{ or } f = 0{\cdot}12\text{ m}$$

(d) To obtain another image, move the lens to B in Fig. 19.14, where BS = 0·18 m

(e) Magnification of the first image $= v/u = 0{\cdot}36/0{\cdot}18 = 2$.

Magnification of the second image $= 0{\cdot}18/0{\cdot}36$
$= 0{\cdot}5$ since v is now 0·18 m and u is now 0·36 m.

So the ratio of the size of the first to the second image $= \dfrac{2}{1/2} = 4$.

(f) If the top half of the lens is covered, the size of the image is not affected. The brightness is reduced to about one half because the image now receives through the lens only about half the original amount of light.

Summary

1. The *focal length* of a *converging lens* is the distance from the lens to the principal focus; the *principal focus* is the point on the principal axis to which rays parallel and close to the principal axis converge after refraction through the lens. The focal length is a +ve length.
2. The *focal length* of a *diverging lens* is the distance from the lens to the principal focus; the *principal focus* is the point on the principal axis from which rays parallel and close to the principal axis appear to diverge after refraction through the lens. The focal length is a −ve length.
3. RP convention | NC convention

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f},\ m=\frac{v}{u} \qquad \frac{1}{v}-\frac{1}{u}=\frac{1}{f},\ m=\frac{v}{u}$$

4. Images in a converging lens are inverted and real when the object is further from the lens than the focal length. Nearer the lens than f, the image is upright, magnified and virtual (principle of magnifying glass or simple microscope).
5. Images in a diverging lens are always virtual, diminished and upright.
6. Focal length of converging lens: for a quick method use the window as a distant object; for an accurate method use a plane mirror or the lens formula.

Exercise 19 (Answers, page 642)

1. (a) Draw ray diagrams showing how a converging lens produces (i) a *real* inverted image larger than the object, (ii) a *virtual* image larger than the object.
(b) An object is placed 15 cm from a converging lens of focal length 10 cm. By drawing, find the distance of the image from the lens, and the magnification.
(c) An illuminated object O and a screen S are 60 cm apart. A converging lens between O and S produces an inverted image on the screen which is twice the size of the object. (i) Using the magnification formula (or otherwise), find the distance of the object from the lens, (ii) draw a ray diagram showing how this image is formed.
2. A converging lens is used to form an upright image, magnified five times, of an object placed 6 cm from the lens.
(a) What type of image is formed?

Scanned with CamScanner

(b) Find, by scale drawing or calculation (stating the sign convention used), the focal length of the lens. (N)

3. (a) Describe how a reliable value of the focal length of a converging lens can be obtained by experiment.

(b) An illuminated object and a screen are fixed 80 cm apart. There are two positions where a converging lens can be placed so as to form a sharply focused image of the object on the screen. In the first position the image is three times the size of the object. How far is the lens from the object in this position? How far is the lens from the object in the second position where the image is one-third the size of the object? Show your method. (L)

4. A converging lens of focal length 12 cm is used to produce a real image of an object enlarged four times. How far must the object be placed from the lens? (N)

5. In Fig. 19.15, AOB represents the position of a thin diverging lens; F, F are its principal foci. Determine the position of the image of the point object P and complete the ray diagram. (N)

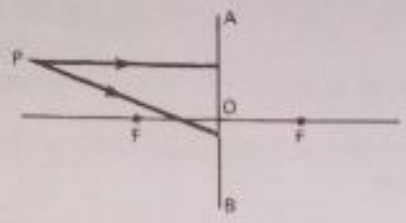


Fig. 19.15

6. Define *focal length* of a converging lens and describe a good method of determining its value for a given lens.

A converging lens of focal length 10 cm is used to produce an image three times the size of the object. Find the distance of the object from the lens when the image is (a) virtual; (b) real. (If the answers are obtained by calculation, state the sign convention used.) (L)

7. Describe briefly an approximate method, and then in detail an accurate method, of finding the focal length of a converging lens.

An object 1 cm high is placed 6 cm away from a converging lens of 12 cm focal length. Find by calculation, or by drawing, the position, height and nature of the image. (N)

8. Explain the meaning of *real image*, *virtual image*.

Draw scale diagrams to show the passage of a pencil of rays from a point on a small object, through a converging lens of focal length 10 cm for the following distances: (a) 15 cm; (b) 5 cm; (c) 10 cm. The point must not be on the axis of the lens.

In the case (b) deduce the magnification, and state the nature of the image. (O and C)

9. (a) Explain, with the aid of ray diagrams, how a converging lens may be used (i) as a magnifying glass, (ii) in a simple camera. State the properties of the image produced in each case.

(b) A small object is placed 30 cm away from a diverging lens of focal length 10 cm. Determine by scale drawing or by calculation (stating any sign convention used) the position and nature of the image. (N)

10. Define *principal focus* and *focal length*. Describe how you would determine the focal length of a thin converging lens with the help of a plane mirror.

Draw a diagram showing how a thin converging lens forms an image of a very distant extended object, such as the sun.

The image of the sun formed by a lens of focal length 1·8 m is 1·7 cm in diameter. Taking the distance of the sun to be 150 million km, calculate the sun's diameter. (O)

11. Describe an appropriate method, and an accurate method, of finding the focal length of a converging lens.

An object 4 cm high is at right angles to the principal axis of a diverging lens of focal length 20 cm, and 30 cm from it. Determine the position of the image and its size. (N)

12. An object is set up 0·20 m (20 cm) in front of a lens A and the details of the image are noted and are shown bleow. The process is repeated for a different lens B.

Lens A. Real, inverted, magnified and at a great distance.
Lens B. Real, inverted and the same size as the object.

State the type of each lens and give as definite a value as possible for each focal length, explaining how you arrive at the values you give. (N)

Scanned with CamScanner

13. A slide projector produces, from a transparent photograph 30 mm wide, a picture 4·5 m wide on a screen which is 15 m from the projection lens.

How far from the lens must the photograph be placed? What type of lens is used? Should its focal length be (a) 5 cm, (b) slightly less than 10 cm, or (c) 15 cm? (O and C)

Multiple-Choice Questions

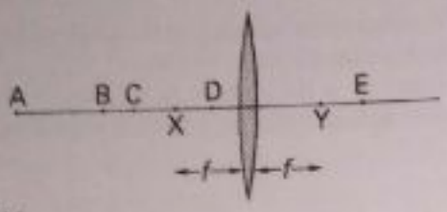


Fig. 19.16

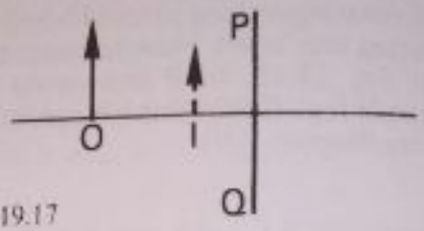


Fig. 19.17

In Fig. 19.16, a converging (convex) lens has a principal focus at X and at Y. The focal length is *f*.

Choose one answer from the points A, B, C, D and E for the best position of an object which gives an image

14. at E

15. at B

16. further than 2*f* from the lens.

17. In Fig. 19.17, PQ represents a *mirror* or a *lens*. O is an object which forms a virtual image at I. Then PQ is a

A diverging (concave) lens
B converging (convex) lens
C convex mirror
D concave mirror
E plane mirror

Scanned with CamScanner

20. Applications of Lenses

The eye

The eye is one of the most intricate and sensitive instruments developed by nature. Its chief optical features are (Fig. 20.1):

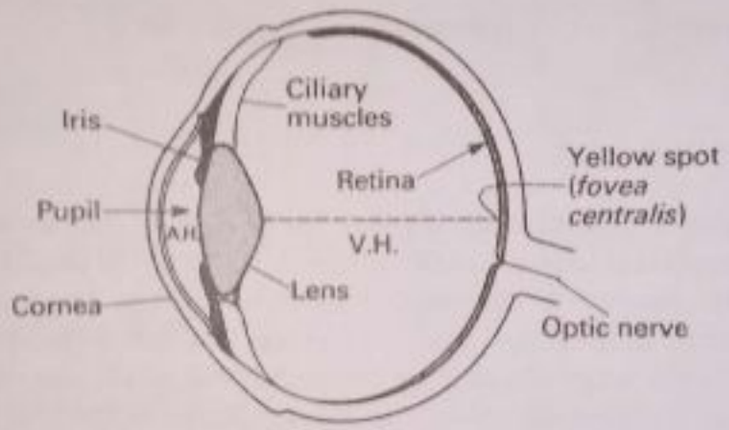


Fig. 20.1 Optical features of the eye

1. the *eye-lens*, which focuses light entering the eye;
2. ciliary muscles, which are attached to the eye-lens surfaces and alter the focal length;
3. the *retina*, the light sensitive area of cells at the back of the eye;
4. the *yellow spot* (*fovea centralis*), the most light sensitive spot on the retina;
5. the *iris*, the coloured circle round the eye-lens;
6. the *pupil*, the circular opening or diaphragm in the iris through which light passes;
7. the *cornea*, the thick transparent protective covering in front of the eye-lens which refracts the light most;
8. the *aqueous humour* (A.H.) and *vitreous humour* (V.H.), which are liquids respectively in front of and behind the eye-lens in which the eye-lens is suspended.

Binocular vision

If you close one eye and try to pick up a pen or other object at the corner of the table you will find that you have a poor sense of distance compared with using two eyes. In addition, binocular vision provides two images of the same object which are slightly different in perspective. The brain combines the two images and gives an impression of depth or solidity which is missing if only one eye is used.

Scanned with CamScanner

In Fig. 20.2a, for example, a view of a pyramid P is seen slightly differently by the left eye L and the right eye R, as indicated in (b). Two retinal images of slightly differing perspective are always necessary to provide depth or a stereoscopic effect. A photograph appears flat because only one image is formed by the camera lens.

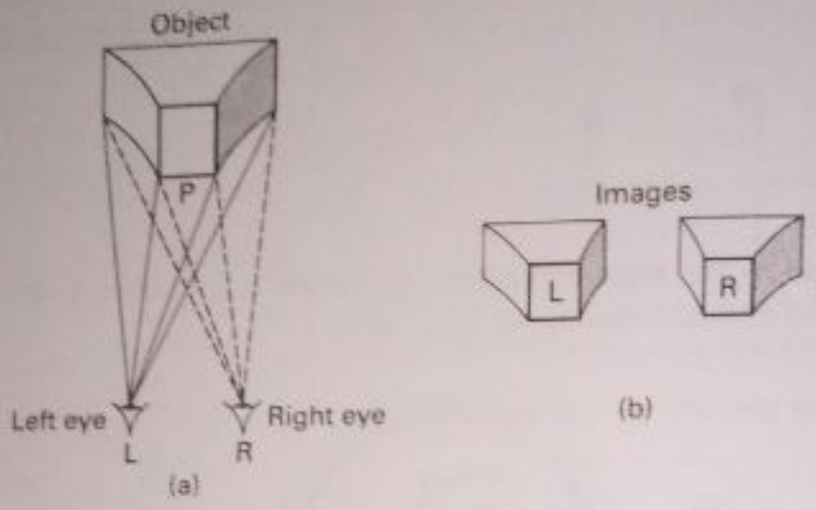


Fig. 20.2 Binocular vision

Binocular vision is possible only if the fields of view of the two eyes overlap. This clearly depends on the position of the eyes in the head. In small birds, such as the blackbird, the eyes are at the sides of the head. In this case each eye views space independently of the other; that is the visual fields are entirely separate. So the blackbird cannot judge distance by binocular vision. On the other hand, in birds of prey, such as the hawk or owl, the eyes are in the front of the head, so that this bird has stereoscopic vision. The development of binocular vision is most marked in the more advanced creatures. Man possesses this advantage to a greater degree than any other animal.

Persistence of vision · The iris

When a bright light is viewed and then switched off the sensation of vision lasts for a short but definite time afterwards. This is called *persistence of vision*. If a light flickers more than 10 times per second, then, owing to persistence of vision, the light is seen constant in intensity. In cinema films, bright images are projected rapidly one after the other on to a screen and persistence of vision makes any action look continuous. If the film slows down by accident, a flickering effect is seen.

In a dim light the iris or diaphragm round the pupil of the eye opens more, so increasing the size of the pupil and allowing more light to enter the eye. In bright light the iris automatically decreases in size. This reduces the amount of light entering the pupil, so protecting the eye from excessive brightness and possible damage to the retina.

Accommodation · Normal vision

The ability of the eye-lens to focus points at different distances on the retina is called its *power of accommodation*. Accommodation is produced by the action of the ciliary muscles. When a person with normal vision looks at a very distant

Scanned with CamScanner

object, at 'infinity', the lens brings parallel rays to a focus on the retina (Fig. 20.3a). The furthest point which the eye can see distinctly is called the 'far point' of the eye. For normal vision, then, the far point is at infinity. The ciliary muscles are then fully relaxed and the lens is slim.

When a near object is viewed, the muscles increase the curvature of the lens surfaces. A 'fatter' lens, one with a shorter focal length and greater power, is then produced and the rays are again focused on the retina (Fig. 20.3b). People with normal vision are able to focus near objects such as reading books when these are about 25 cm from the eye. This is the limit to which the ciliary muscles can operate. The closest point at which an object can be seen clearly is called the 'near point' of the eye.

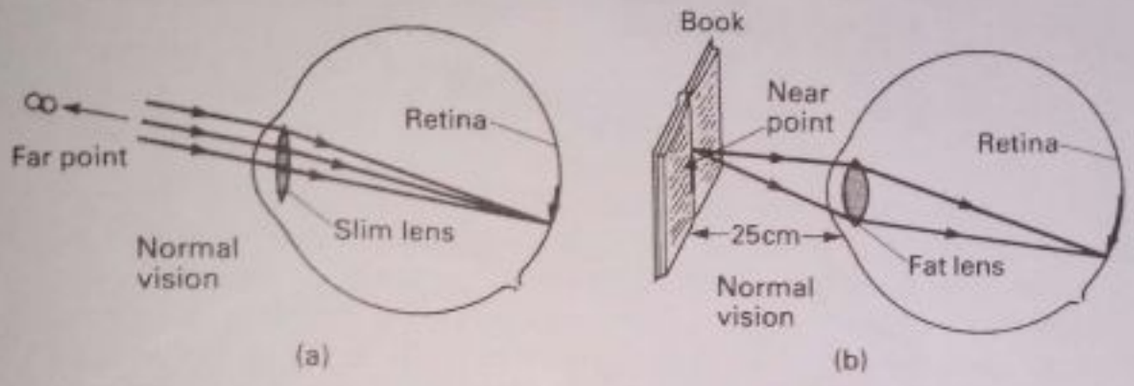


Fig. 20.3 Normal vision. (a) Far point (b) Near point

Defects of vision

Long sight and correction

If you wear spectacles for reading only, your near point N may be say 30 cm from the eye instead of 25 cm (Fig. 20.4). This defect of vision is called *long sight*. The eyeball is then too short. Rays from a point A 25 cm from the eye, the normal near point, are now brought to a focus beyond the eyeball instead of on the retina.

Fig. 20.4 shows how a *converging* lens L corrects this defect of vision. It refracts light rays from a book at A, the normal near point, so that they enter the eye *as if they come from N*. The book at A can then be seen clearly.

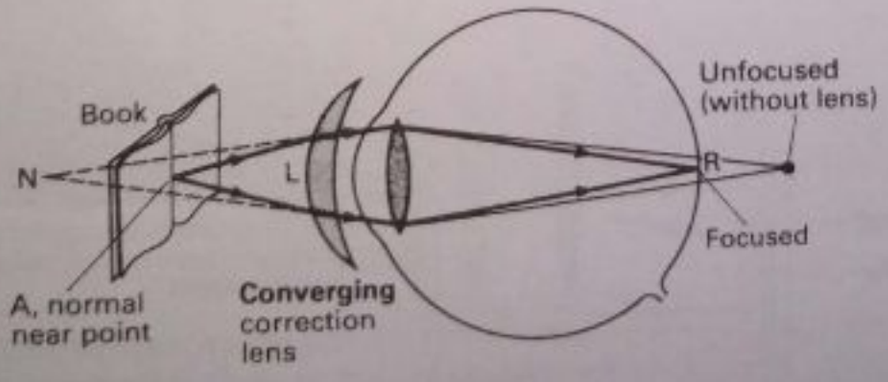


Fig. 20.4 Long sight and correction

Short sight and correction

Some people are unable to see clearly distant objects such as road signs, car number plates or footballers on a playing field. In this case the eyeball is too long.

Scanned with CamScanner

Parallel rays from these distant objects are brought to a focus in front of the retina (Fig. 20.5). The furthest point A seen clearly may only be 2 metres away. This defect is known as *short sight*.

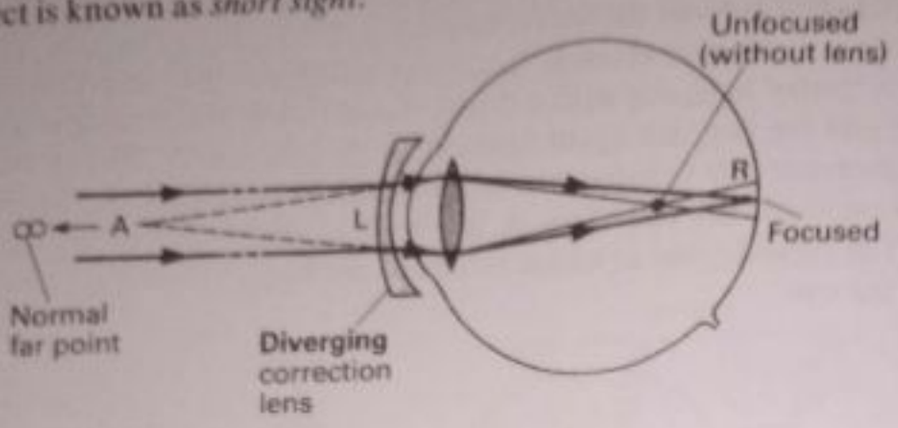


Fig. 20.5 Short sight and correction

Since parallel rays are incident on the lens, A is the focus of the lens. So AL is the focal length. A *diverging* lens of focal length 2 metres is therefore needed to see distant objects clearly if the far point A is 2 metres from the eye.

With age, when the lens of the eye becomes inelastic and unable to accommodate, two pairs of spectacles may be necessary; one for distance and one for reading. *Bifocal lenses* have an upper portion for long-distance viewing and a lower portion used for reading.

Lens camera

The lens camera is a man-made 'copy' of the eye. It has an opening or aperture similar to the pupil to allow light to enter, and a light-sensitive film at the back similar to the light-sensitive retina of the eye. There are differences, however, in the lens system. In the eye, the focal length of the lens is altered by ciliary muscles.

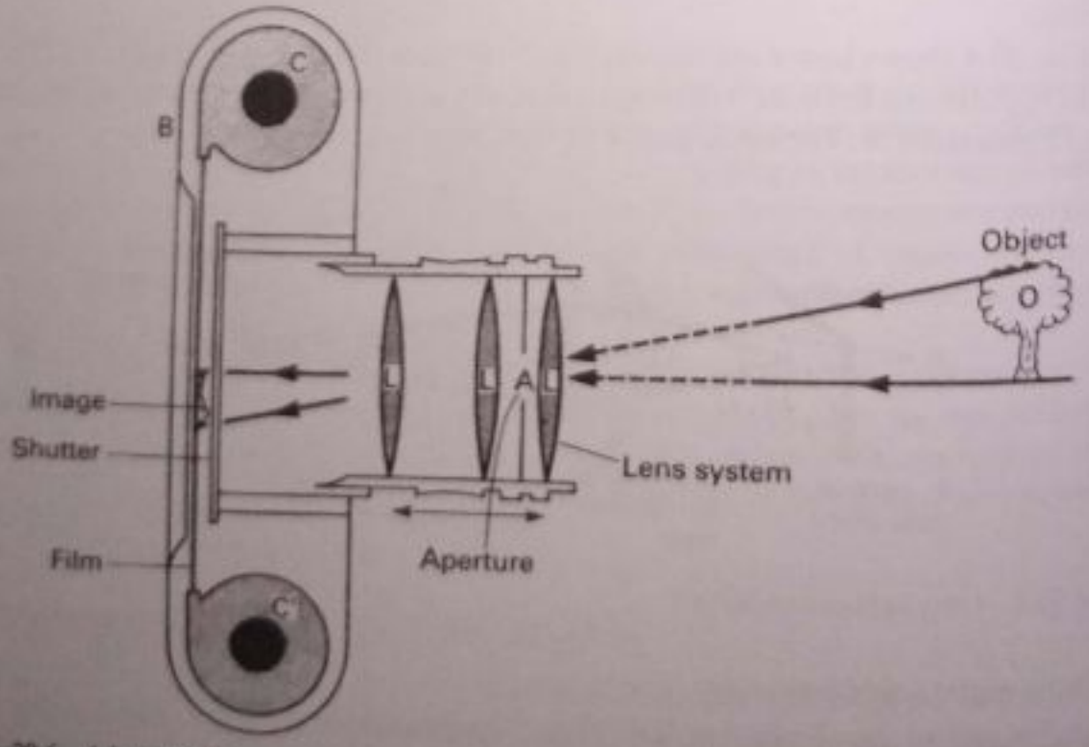


Fig. 20.6 A lens camera

Scanned with CamScanner

This helps the eye to see clearly objects at different distances. In the lens camera, however, the focal length of the lens is constant. Objects at different distances are focused on the film by moving the lens.

The modern lens camera is a complex instrument and photography is a specialist subject. Only some of the main points are now discussed briefly.

1. *Lens action* Basically, the camera consists of a light-proof box B, with a lens system L in front (Fig. 20.6). This acts effectively as a converging lens. It is used to focus light from an object O on to the film. The image is inverted and small. A shutter is placed between the lens and the film. When a photograph is taken, the shutter opens and closes rapidly, so exposing the film for a short time to light entering the camera through the lenses.



Plate 20A Photograph of the inside of a ship's hold taken by a camera using a 'fish-eye' lens, which gives a 180° field of view. (*P and O Lines*)

By coupling the lens to a rangefinder, which locates the object distance, the lens system can be moved to focus on the object. After taking a photograph the film is wound between two spools, C and C', until it is all used up.

Plate 20B shows the exterior of one type of camera; some details necessary for taking a good photograph, which we now explain, are shown in Fig. 20.7.

2. *f-numbers* The images produced by the lens system have to be bright enough to 'expose' the film sufficiently in the short time the shutter is open. The main

Scanned with CamScanner

factors controlling the brightness are (a) the diameter, d, of the aperture A or circular opening through which the light passes–this controls the amount of light entering the camera; (b) the focal length, f, of the lens–this controls the size or area of the image on the film; (c) the exposure time, discussed shortly.

Since it is linked with the brightness, the diameter d of the aperture is expressed as a fraction of f in 'f-numbers' (Fig. 20.7). An aperture of 'f-4' means one whose diameter is $f/4$. An aperture 'f-16' is one of diameter $f/16$. This is an aperture one-quarter the diameter of f-4. As shown in Fig. 20.7, some f-numbers used in cameras are f-2·8, 4, 5·6, 8, 11, 16, 22; each f-number produces about half the illumination of the previous one. The smallest aperture here is f-22; the largest is f-2·8.

The diameter of the aperture is varied by the camera iris. This consists of a number of overlapping sections which can move in or out, thus producing a circular hole or aperture of variable size.

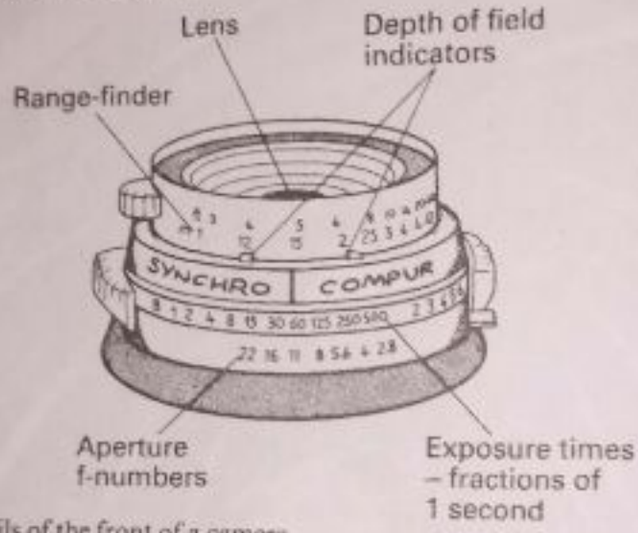


Fig. 20.7 Some details of the front of a camera

3. *Exposure time* For best results in using the camera normally, the shutter should open and close very fast so that the film is exposed for a short fixed time. When taking photographs in bright sunlight the brightness of the image will be high, and only a short exposure time, such as 1/250 s, is needed. If it is a dull day and the same aperture is used, a longer exposure time, such as 1/50 s, is needed. Thus the exposure time needed increases as the brightness decreases.

If a photograph is taken of a fast-moving car, a short exposure time, such as 1/250 s, is needed, otherwise the image will be blurred. A wide aperture is needed to collect more light because of the short exposure time, for example f-2·8.

4. *Film speed* One of the most important properties of photographic film is its 'speed'. The film speed is a measure of how quickly the film will become correctly exposed when in use, that is how long some of the light-sensitive chemicals on it will take to react to light. A 'fast' film needs a relatively short exposure time for correct exposure. A 'slow' film needs a much longer exposure time, so fast films are used in poor lighting. Here, the relatively long time needed with a slow film would be inconvenient. Slow films can be used in taking photographs of portraits or still objects, or in very good lighting conditions such as bright sunlight.

5. *Exposure meter* A professional photographer measures the brightness of the subject photographed with a separate exposure meter or light meter. Many cameras now have a built-in light meter. This has a light-sensitive surface. When the surface is illuminated an electric current whose strength is proportional to the brightness flows in the meter.

Scanned with CamScanner

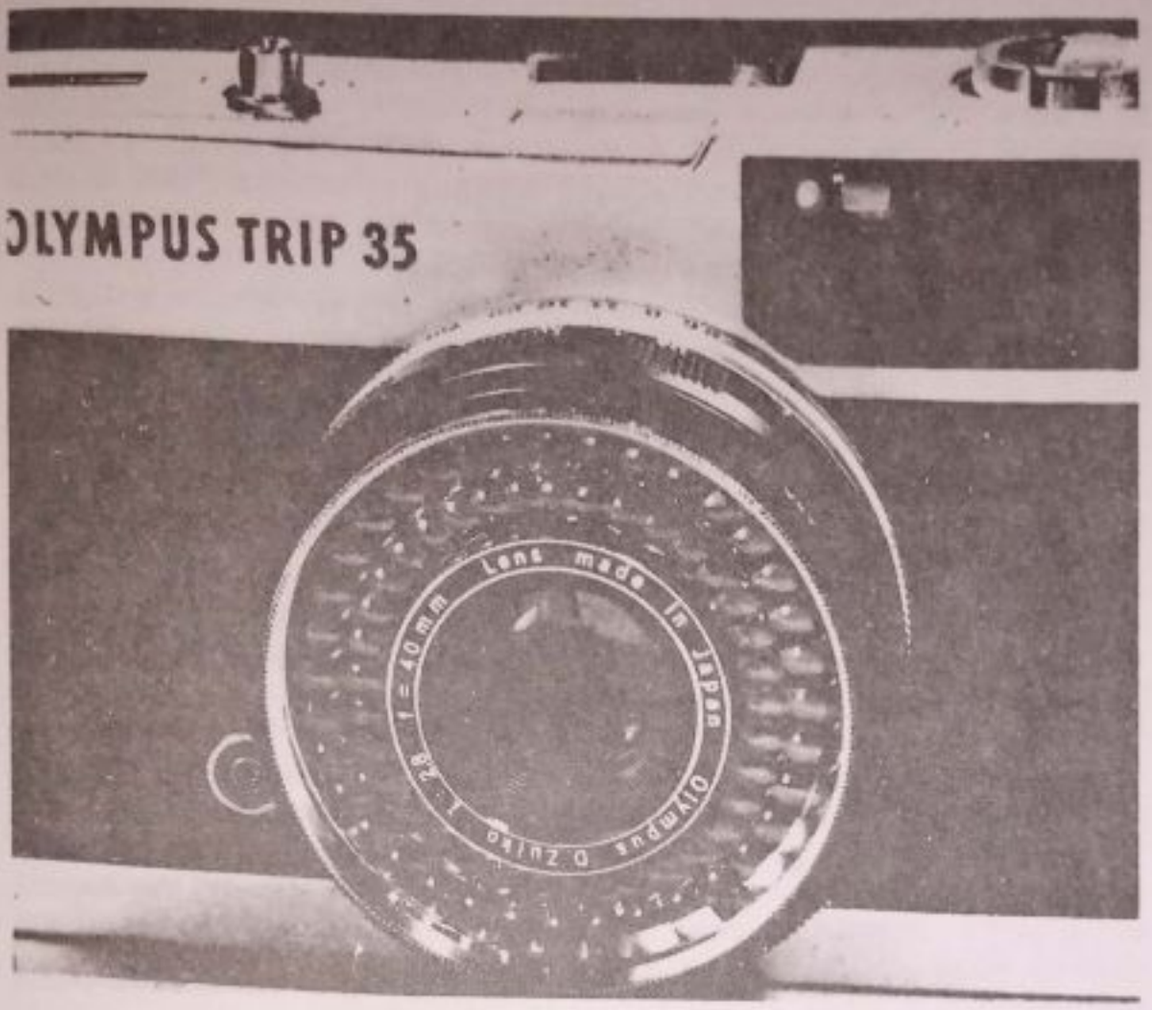


Plate 20B A modern lens camera. The light sensitive surface is round the outside of the lens. (*Olympus Optical Co*)

Slide projector

The slide projector (or projection lantern) is used for showing audiences large images of slides or other objects on a white screen. The essential features are (Fig. 20.8): (a) a very powerful small source of light S; (b) a condenser C of two converging lenses–this collects the light and beams it towards the slide O, which is

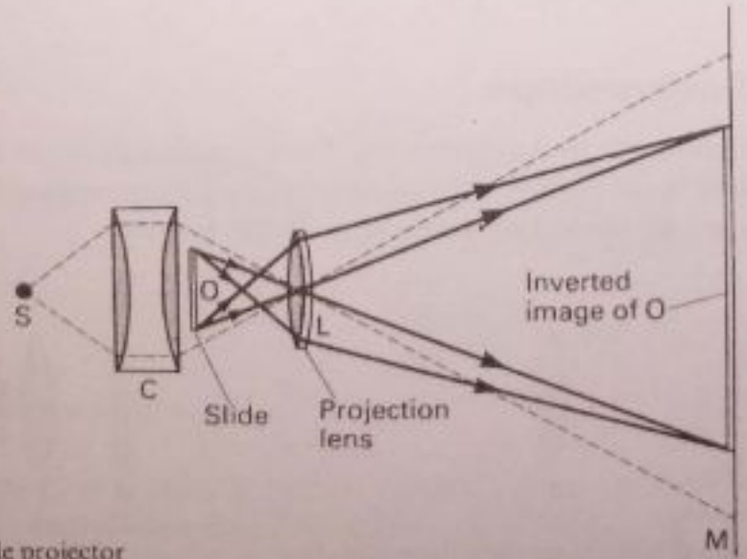


Fig. 20.8 Slide projector

Scanned with CamScanner

therefore strongly illuminated; (c) a projection lens L near the slide, which produces an enlarged inverted image of O on a distant screen M.

Without the condensing lenses C, the slide would be weakly illuminated. The condenser collects light and sends it through O. A strong source of light is essential as O is non-luminous. The lens L is slightly further from O than its focal length, so that an enlarged image is obtained on M. The image is inverted, and so the slide must be put upside down into the projector.

The image of the lamp filament S is produced at L itself, and so the projection lens does not produce an image of the filament on the screen, which would spoil the picture.

Microscopes

Simple microscope

The simple microscope, or magnifying glass, was one of the earliest optical instruments. Over 300 years ago, Hooke discovered the existence of the cell in his biological studies by using a converging lens.

If the lens is used as a simple microscope, the object O is placed nearer to the lens than its focal length or principal focus F (page 306). The image produced is then upright and magnified. The lens is moved until the image is seen distinctly 25 cm from a normal eye. As shown in Fig. 20.9b, the image subtends a greater angle y at the eye than the angle x when the object is observed with the unaided eye as in Fig. 20.9a. It therefore looks much bigger. The greatest magnifying power is obtained with lenses which have short focal lengths.

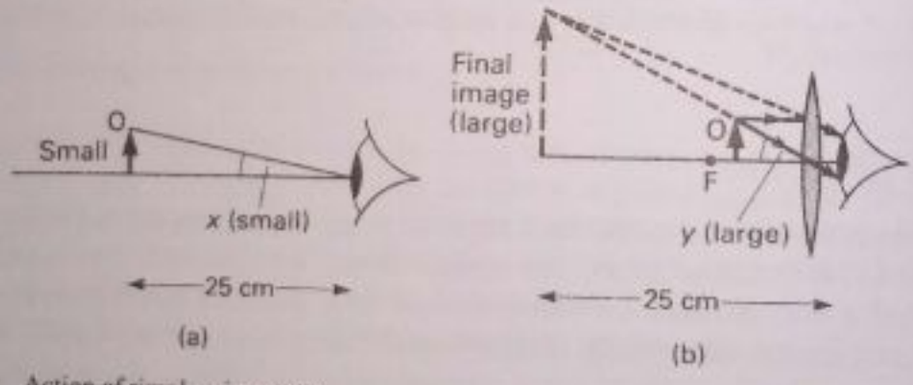


Fig. 20.9 Action of simple microscope

Compound microscope

A compound microscope produces much higher magnification than a single lens. This microscope is made by using two short focal length converging lenses. High power microscopes are used in medical research and science.

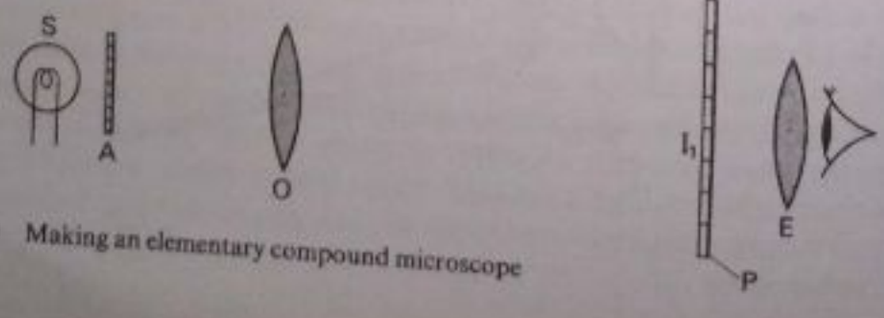


Fig. 20.10 Making an elementary compound microscope

Scanned with CamScanner

Experiment

To make an elementary form of compound microscope, a transparent graduated scale such as a perspex ruler A is illuminated by a lamp S behind it (Fig. 20.10). A converging lens O of *short* focal length is then moved away from A until a clear large inverted image I_1 is obtained on a semi-transparent paper screen P. Place your eye close to a second converging lens E of short focal length and move the lens near to I_1 until a magnified clear image of I_1 is seen. E is now acting as a magnifying glass. Now remove the paper screen.

On looking at the scale A through *both* lenses, a large inverted image of A is seen. If necessary, the position of the lens E is adjusted to see the magnified image clearly.

Using different lenses for O and E, the greatest magnifying power is obtained when both lenses have short focal lengths.

Fig. 20.11 illustrates how the final image is formed in the compound microscope. The lower lens O nearer the object X is called the *objective* lens. It is positioned so that X is just further from it than F_O, the principal focus, so that a real, inverted and magnified image I_1 is obtained. The lens E through which I_1 is viewed is called the *eyepiece* lens. This is moved until I_1 is nearer to it than its principal focus F_E. The lens then acts as a magnifying glass, and so a magnified image of I_1 is obtained at I_2. The observer automatically adjusts the eyepiece so that I_2 is at his or her near point, 25 cm from the eye for normal vision, when it is seen most distinctly.

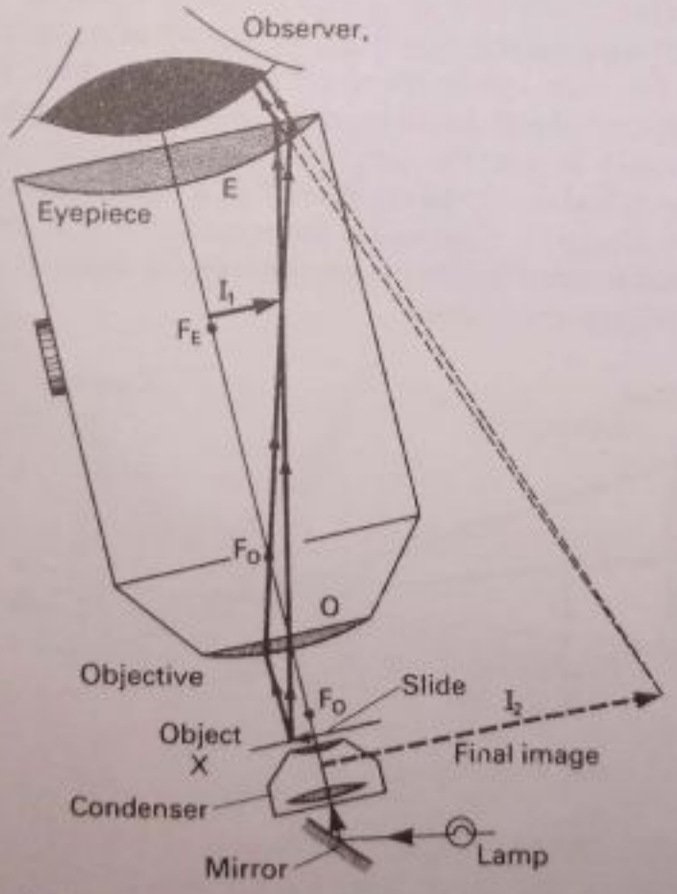


Fig. 20.11 Compound microscope

The image I_2 is much bigger than the tiny object X on the slide. By using converging lenses of short focal length for both O and E, high magnifying power such as × 500 can be obtained. The image I_2 is inverted compared with the object

Scanned with CamScanner

but this does not matter. The object must be strongly illuminated as it is non-luminous.

Astronomical refractor telescope

In contrast to the microscope, the astronomical telescope produces angular magnification of *distant* objects.

Experiment

A simple astronomical telescope can be made by setting up at the same height a converging lens O of long focal length such as 50–100 cm and a converging lens E of short focal length such as 5–10 cm as the eyepiece. First, however, measure the focal length of each lens by the method on page 304. Suppose these are 80 cm and 10 cm respectively. The lenses O and E are then set up in front of the window so that their separation is equal to the *sum* of the focal lengths, 90 cm in this case, with O nearer to the window. On looking through E towards O, and adjusting the position of E slightly, distant objects outside the window can be seen clearly. The distant object appears *upside down* in this telescope.

Experiment shows that the highest magnifying powers of the simple astronomical telescope are obtained when O has a long focal length and E has a short focal length.

A simple astronomical telescope (Fig. 20.12) uses two converging lenses as we have just seen. The front lens O, the *objective*, has a long focal length and collects light from the distant object. In the ray diagram, note (a) that the rays incident on this lens are parallel; (b) that the parallel rays come from the top point or head of the distant object, and are therefore inclined at a small angle α to the principal axis. The inverted image I_1 is formed at the principal focus F_O of the objective. To draw I_1, note that its head lies on the ray through the centre of the objective lens, which passes through undeviated.

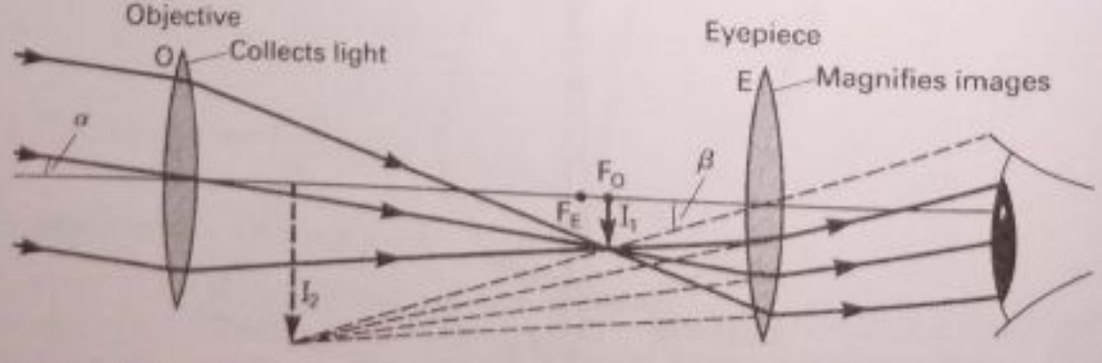


Fig. 20.12 Action of astronomical telescope

The eyepiece E is used as a magnifying glass. It is moved so that I_1 is nearer the lens than its focus F_E. The final image subtends a large visual angle β at the eye placed close to E, so the image is seen larger. High magnifying power occurs when the objective O has a *long* focal length and the eyepiece E has a *short* focal length.

Fig. 20.13 shows the positions of the lenses and inverted final image of the moon in an astronomical telescope. Only half each lens is shown.

Scanned with CamScanner

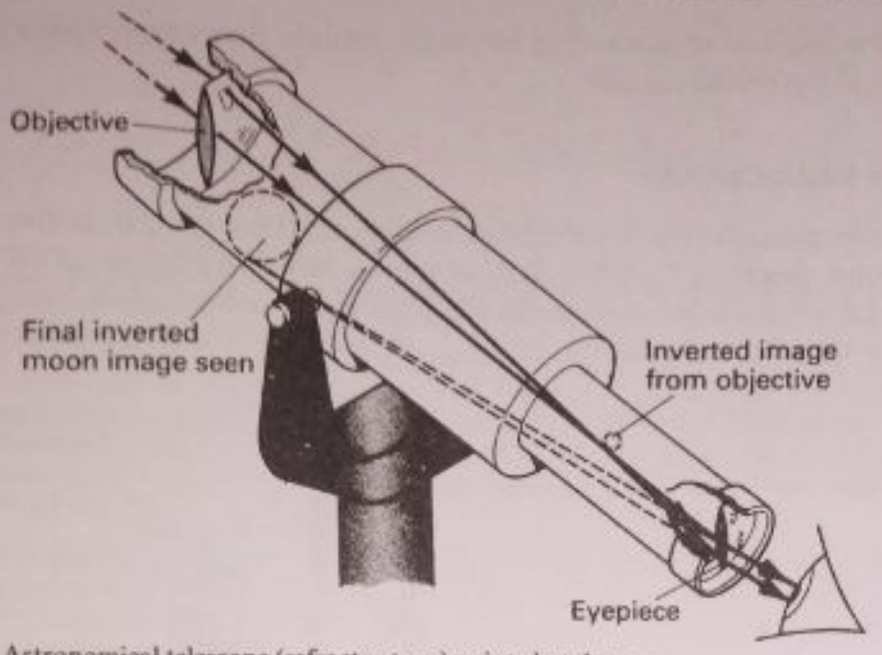


Fig. 20.13 Astronomical telescope (refractor type) pointed at the moon

Other telescopes

Telescopes using lenses are called *refractor* telescopes. Using them, some light is lost from distant stars or other sources by reflection at the lens surfaces. Newton therefore suggested using a large concave *mirror* as a telescope objective. This is called a *reflector* telescope. Today, many observatories throughout the world use reflector telescopes.



Plate 20C A radio telescope. The large parabolic reflector focuses weak radio signals from distant sources on to the central sensitive receiver. This passes the signal to equipment for automatic recording. This telescope at Jodrell Bank received the first television pictures of the moon's surface. (*Barnaby's Picture Library*)

Scanned with CamScanner

Distant galaxies of stars send out radio signals. So radio reflector telescopes are used to detect the signals.

Prism binoculars

Prism binoculars are short telescopes. Each has a converging objective lens and a converging eyepiece, together with two total reflecting prisms of 90°, 45°, 45° angles. Light which passes through the objective is reflected from one prism P to the other Q and then comes out through the eyepiece. This is shown in Fig. 20.14.

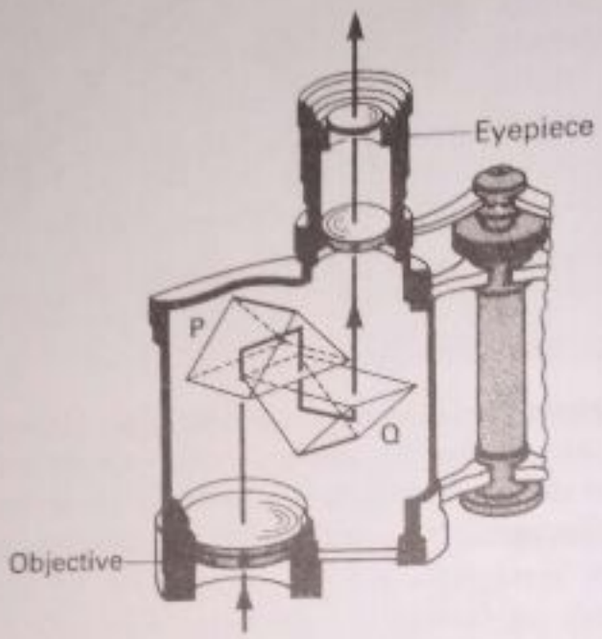


Fig. 20.14 Prism binoculars

Without the prisms the image would be upside down and laterally inverted. One prism turns the image round and corrects for lateral inversion; the second prism turns the image the right way up. So the final image is seen upright and magnified.

Examples on the camera and projector

1. How is the amount of light energy reaching the film of a camera controlled? A camera has a lens of focal length 50 mm. It is used (a) first for taking a photograph of an object a long distance away, and (b) then for taking a photograph of an object 1·00 m from the camera.

What is the distance of the film from the lens in (a)? In what direction must the lens be moved, and through what distance, to take the photograph in (b)?

The amount of light reaching the film is controlled by the diameter of the aperture or opening in front of the lens and by the time during which the shutter opens to allow the light to fall on the film.

(a) For distant objects

$$\text{image distance from lens} = \text{focal length} = 50\,\text{mm} = \text{film distance}$$

(b) Using the Real is Positive convention, we have object distance $u = +1000\,\text{mm}$ and focal length of converging lens $f = +50\,\text{mm}$. If v is the image or film distance, the lens equation is

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Scanned with CamScanner

So
$$\frac{1}{v}+\frac{1}{+1000}=\frac{1}{+50}$$
$$\therefore \frac{1}{v}=\frac{1}{50}-\frac{1}{1000}=\frac{20-1}{1000}=\frac{19}{1000}$$
$$\therefore v=\frac{1000}{19}=52{\cdot}6\,\text{mm}$$

So the distance of the lens from the film is now 52·6 mm, compared to 50 mm for a distant object. The lens is therefore moved 2·6 mm away from the film for a sharp image to be formed on the film.

2. When a slide projector is used, the slide must be put in upside down and the image is magnified laterally (sideways) as well as vertically. Explain this statement.
A projector lens is used to produce a sharp image of an object when the distance between the object and screen is 160 cm. If the linear magnification is 7, calculate the focal length of the lens.

The slide is placed upside down because the real image produced on the screen by the converging lens is inverted. The image is magnified laterally as well as vertically because the lens curvature is spherical in a horizontal direction as well as in a vertical direction and so magnifies in both directions.

Since the linear magnification is 7, the ratio image distance to lens: object distance to lens = 7:1. So

$$\text{image distance from lens, } v=\frac{7}{8}\times 160\,\text{cm}=140\,\text{cm}$$

and

$$\text{object distance from lens, } u=\frac{1}{8}\times 160\,\text{cm}=20\,\text{cm}$$

Using the Real is Positive sign convention, $v=+140$ cm and $u=+20$ cm.
Substituting in the lens equation $1/v+1/u=1/f$, we have
$$\frac{1}{+140}+\frac{1}{+20}=\frac{1}{f}$$
$$\therefore \frac{1+7}{140}=\frac{8}{140}=\frac{1}{f}$$
$$\therefore 8f=140\times 1=140$$
$$\therefore f=\frac{140}{8}=17{\cdot}5\,\text{cm}$$

Summary

1. The eye has a lens whose focal length can be varied by ciliary muscles for accommodation, a coloured iris with a pupil whose diameter varies with the brightness of the light and a retina with a very light sensitive spot (yellow spot).
2. Binocular vision gives a sense of perspective or solidity. Persistence of vision enables cinema films to be seen as if action was continuous. Accommodation is the ability of the eye-lens to focus points at different distances.
3. The near point for normal vision is about 25 cm from the eye; the far point for normal vision is at infinity. Short sight (far point nearer than infinity) is corrected by a suitable diverging lens; long sight (near point farther than 25 cm) is corrected by a suitable converging lens.
4. The length of the image on the retina is proportional to the angle subtended at the eye by the object.
5. The simple microscope (magnifying glass) has one converging lens of short focal length.

Scanned with CamScanner

6. The compound microscope has two converging lenses of short focal length.
7. The simple astronomical telescope has a converging objective of long focal length and a converging eyepiece of short focal length.
8. Prism binoculars are short astronomical telescopes, each containing two right-angle 45° prisms, which produce a final upright image.

Exercise 20 (Answers, page 642)

1. (a) State two advantages of binocular vision (two eyes).
(b) In using a slide projector, the image of the slide must be (i) the right way up, (ii) magnified. Explain briefly how this is done.
(c) How does the eye adjust itself to deal with (i) light of different intensity, (ii) objects at varying distances?
2. Give a clearly-labelled drawing of the optical system of the eye, indicating the functions of the various parts.
What is meant by the least distance of distinct vision?
Explain with a ray diagram how a converging lens of suitable focal length may be used as a simple magnifier. (O and C)
3. Fig. 20.15a shows a simplified diagram of a shortsighted eye. Show, on the diagram, what happens to the parallel rays of light on passing into the eye. On Fig. 20.15b show the effect of inserting a suitable lens in front of the eye which will correct its defect. (C)

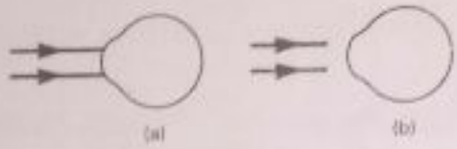


Fig. 20.15

4. A slide 7·5 cm × 7·5 cm is used in a projection lantern to produce a picture 180 cm square on a screen in a lecture hall. The slide is 30 cm from the lens. What kind of lens is used? What can you say about: (a) its focal length; (b) the length of the hall? (O and C)
5. You are supplied with converging lenses of focal length 50 cm, 5 cm and 0·5 cm. Which would you use as: (a) the objective of a compound microscope; (b) the eyepiece of a compound microscope; (c) the objective of a telescope? (N)
6. Name the type of lens used as: (a) the eyepiece of an astronomical telescope; (b) a spectacle lens to correct short sight; (c) a magnifying glass. (N)
7. Explain, with ray diagrams, the use of a lens (a) as a magnifying glass, (b) in a camera. State the characteristics of the image formed in each instance.
The distance between the film and the lens in a camera is 6·25 cm when it is focused on an object 1·5 m from the lens. How far must the lens be moved, and in what direction, in order to focus the camera on distant objects? (L)
8. (a) Briefly describe how the human eye can focus objects at different distances and how it allows objects to be seen clearly in both bright and dim light.
(b) Draw *two* labelled diagrams showing how light from a distant object passes through a shortsighted eye, one without a spectacle lens and the other with a correcting lens. (N)
9. Describe a simple photographic camera and explain how it works.
State *two* similarities and *two* differences between the optical arrangements of the camera and those of a human eye.
A person whose least distance of distinct vision is 20 cm uses a converging lens of focal length 5 cm as a magnifying glass. What is the magnification when the lens is held close to the eye and the image is formed at the least distance of distinct vision? (N)
10. Sketch *two* diagrams indicating the type of lenses used and the positions and nature of the images in an astronomical telescope. (N)
11. Make a clear diagram to show how a converging lens placed near to the eye may be used as a magnifying device. Show the angle subtended at the lens by the image. How does this compare with the angle subtended by the object? Explain how a magnifying effect is achieved. (O and C)

Scanned with CamScanner

12. Draw a diagram of the optical system of a projection lantern. Explain the purpose of each part shown and draw a separate ray diagram showing how the image is formed.

A heat filter is often added to the optical system. Why is it needed and where would it be placed?

A projection lantern is used to give an image of a slide 5 cm × 5 cm on a screen. The image is 120 cm wide on the screen which is 75 m from the projecting lens. Where must the slide be placed and what is the focal length of the lens required? (State any sign convention used.) (N)

13. With the aid of a labelled diagram describe the construction and explain the action of a compound microscope.

State one way in which a compound microscope could be modified to give greater magnification.

A magnifying glass gives a five times enlarged image at a distance of 25 cm from the lens. Find, by ray diagram or by calculation, the focal length of the lens. (N)

Multiple-Choice Questions

14. In the optical systems of the eye and lens camera, which statement is NOT true?
- A The ciliary muscles control the focal length of the eye lens
- B The focal length of the camera lens is fixed
- C The retina and camera film have similar purposes
- D The focal length of the eye lens is fixed
- E The eye pupil controls the brightness of the image

15. In the slide projector, which statement is NOT true?
- A A converging (convex) lens is used
- B A condenser is necessary
- C The image is magnified and real
- D A lamp is essential
- E The lens increases the image brightness

16. A lens camera is used to photograph a fast-moving car without blur in bright daylight.

Which of the statements A to E is true?
- A The image is the same way up as the object
- B The aperture must be opened wide
- C The shutter speed must be fast
- D A diverging lens is used
- E The shutter speed must be slow

Scanned with CamScanner

21. Colours of Light · The Spectrum

Colours in white light

The colours in white light can be investigated using a glass prism.

Experiment

A ray-box R can provide a narrow beam of white light through a single slit O (Fig. 21.1). A white image I of the slit is then seen on a white screen S opposite O. When a 60 glass prism X is placed to intercept the white beam, and the screen S is moved round, the image produced by refraction is now *coloured*. The edges of the image are red (R) and violet (V), as shown. Also, the coloured image is appreciably wider than the narrow white beam of light seen originally on S.

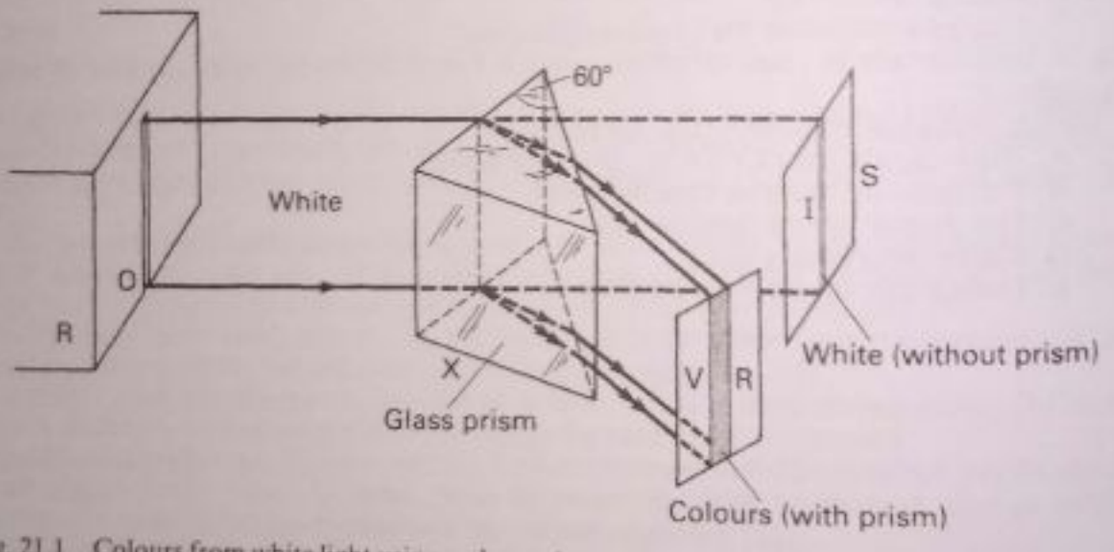


Fig. 21.1 Colours from white light using a glass prism

The prism experiment outlined here was first carried out by Sir Isaac Newton in Cambridge in 1666. He made a small circular opening in a shutter in a darkened room and placed a prism near the hole, so that the light was refracted on to the opposite wall (Fig. 21.2). The colours produced in this way were, in order from the apex side A of the prism

Red, orange, yellow, green, blue, indigo, violet (ROYGBIV)

These colours are called the *spectrum* of white light. Our natural white light comes from the sun. The beautiful colours in the rainbow are due to dispersion of sunlight by water droplets suspended in the air after rain has occurred. A similar colour effect is produced by drops of water from a garden hose when sunlight is present.

Scanned with CamScanner

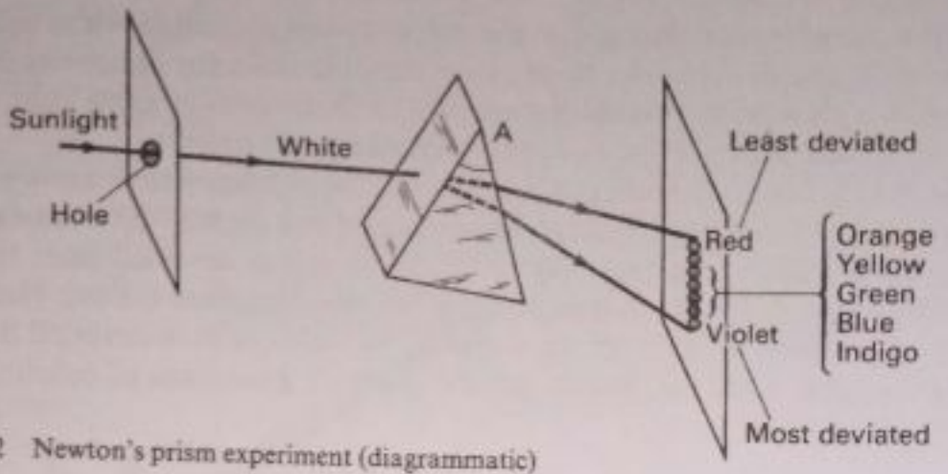


Fig. 21.2 Newton's prism experiment (diagrammatic)

Dispersion and recombination with prism

Newton began a series of experiments to find out whether colours were already in white light, or whether colour was somehow added to it by the prism. We can follow his experiment.

Experiment

As in Fig. 21.1, the coloured images of the slit illuminated by white light are obtained on a screen S using a prism X. Another 60° glass prism Y is now placed to intercept the coloured beam before it reaches the screen (Fig. 21.3a). On moving S round, the colours are now seen to be more widely separated than before, but are still red, orange, yellow, green, blue, indigo and violet. So prism Y does not create colours.

The prism Y is now turned round so that its 60° angle points the other way to that of the prism X and the sides of the prisms are parallel (Fig. 21.3b). When the screen S is moved the image on S is seen to be white. The prism Y has now neutralised the colour effect produced by X.

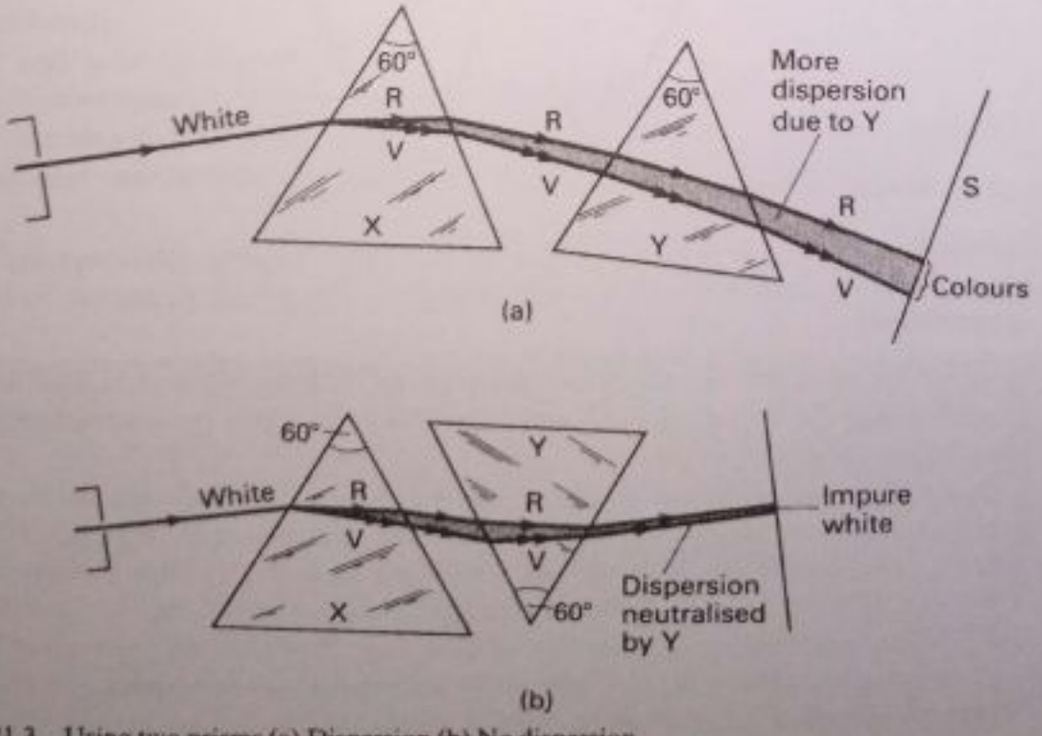


Fig. 21.3 Using two prisms (a) Dispersion (b) No dispersion

Scanned with CamScanner

Newton therefore concluded that the colours were present in white light. The glass prism X simply separates them; they travel in different directions in glass. We say that a glass prism produces *dispersion* of the colours in white light. In Fig. 21.3a, the second prism, Y, helps to produce greater dispersion.

Dispersion is due to the different speeds that the coloured rays move in glass. Each colour in a ray of white light is then refracted in a slightly different direction on entering a glass prism (see page 287). Since red is deviated least from its original direction in air, its change of speed on entering glass is least. Hence red light travels in glass with the greatest speed. As violet light is deviated most on entering the glass, it has the slowest speed in glass. In a vacuum, all colours travel with the same speed.

Newton's colour disc · Adding coloured lights

In order to see the results of adding colours, Newton made a circle and divided the circumference into seven parts. The seven colours were painted on the sectors in a similar way to Fig. 21.4. If a disc painted in this way is spun fast, its colour seems to be grey-white. As the disc slows down the individual colours are seen again. This experiment confirms that 'white' is a blend of seven main colours. The sun, for example, is yellowish-white, since more yellow than any other colour is present in sunlight.

White light can also be obtained by adding lights of three pure colours.



Fig. 21.4 Newton colour disc

Experiment

Blue and red filters are placed in gaps or wide slits on the sides of a ray-box, and plane mirrors are placed at an angle in front of the filters (Fig. 21.5). A green filter is placed in the front wide slit as shown.

By turning the two mirrors to deviate the blue and red light, the three coloured images can be made to overlap on a white screen. The three coloured lights added together are seen to produce an impure white. When one colour is masked out so that only the two other colours are added, (a) red and green lights produce yellow, (b) red and blue produce magenta and (c) green and blue produce cyan. Red, blue and green are called *primary* colours since they produce white light when these lights are added. Fig. 21.5b shows the colours obtained by adding different primary colours.

Scanned with CamScanner

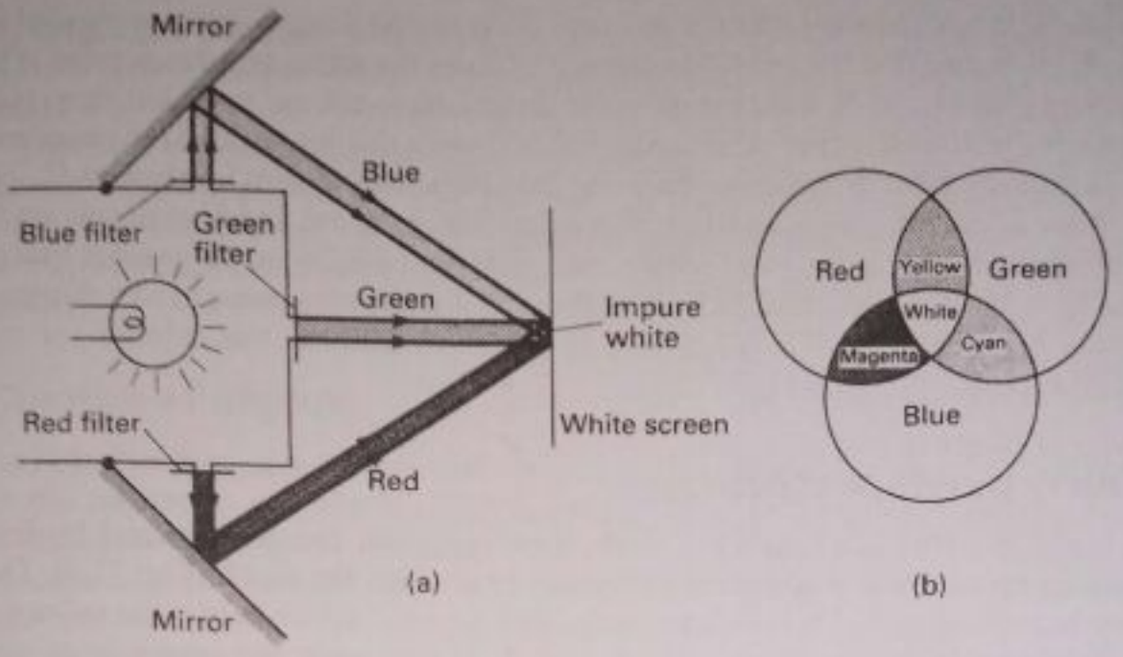


Fig. 21.5 Adding coloured lights

How to form a pure spectrum

These experiments or demonstrations with a prism produce an impure spectrum, that is, one in which the different colours overlap. In research and industry we may require a pure spectrum, that is, one in which the colours are clearly separated from each other.

Experiment

To produce a pure spectrum, the arrangement in Fig. 21.6 is used. This consists of:

1. a narrow slit O as a source of light–this produces a series of narrow coloured images, which lessens the chance of overlapping colours;
2. a converging lens A placed with O at its focus, so that a beam of parallel light is produced;
3. a 60° prism for dispersion;
4. a converging lens C for collecting the parallel beams of different colours;
5. a screen S at the focus of C on which the pure spectrum can be projected, or an eyepiece near the focus through which the pure spectrum can be observed.

If the spectrum of *white light* is required a bright filament lamp can be used in front of the slit O. In Fig. 21.6 observe that the rays XH and YK in the incident

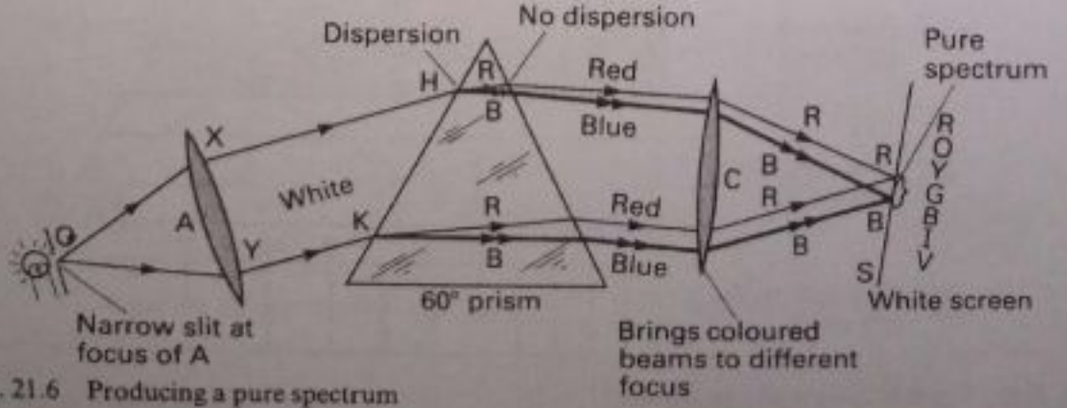


Fig. 21.6 Producing a pure spectrum

Scanned with CamScanner

white light are parallel, that the red rays R inside the prism are hence parallel to each other and that the red rays coming out from the prism are also parallel. The blue rays at H and K travel in different directions to the red rays, but they, too, are parallel to each other inside and outside the prism. So a parallel red beam and a parallel blue beam, each in a different direction, are incident on the lens C.

Now a lens brings parallel rays to a focus. Since the red and blue beams are in different directions, the lens forms a red image R at a different focus from that of the blue image B (see page 303). All other colours are also brought to a different focus between R and B. So a pure spectrum is formed; the colours are not overlapping.

Fairly pure spectrum

If only one lens is available, a fairly pure spectrum can be obtained by first projecting an image I of the slit S on a screen through the lens L (Fig. 21.7). The beam converging to I is then intercepted by a prism P, which brings the coloured beams to a different focus, as at R and B. If necessary, the screen is moved around. The best spectrum is obtained when P is moved so that the spectrum is formed as near to I as possible.

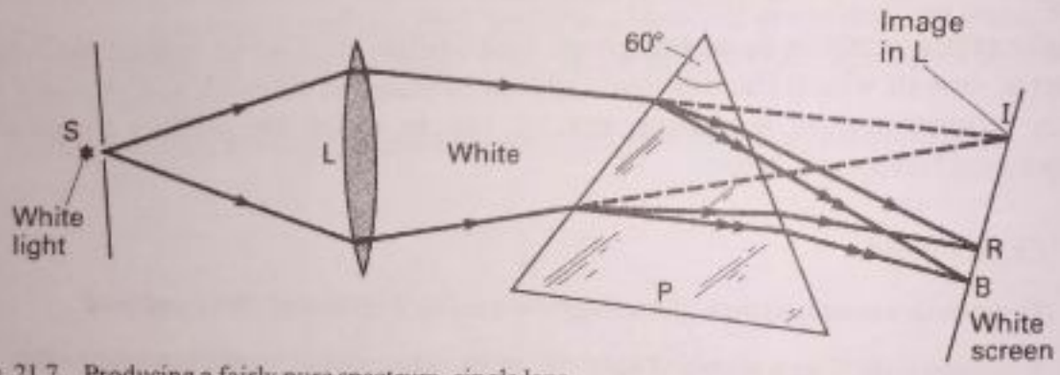


Fig. 21.7 Producing a fairly pure spectrum–single lens

Other spectra

White light produces a *continuous spectrum*, that is, all colours from red to violet are present in a continuous band (Fig. 21.8a). On the other hand, the spectrum of sodium, obtained from the hot flame of a sodium burner, has a bright yellow line (strictly, two lines extremely close together). It is an example of a *line spectrum* (Fig. 21.8b). Mercury vapour has a line spectrum of mainly green and blue lines.

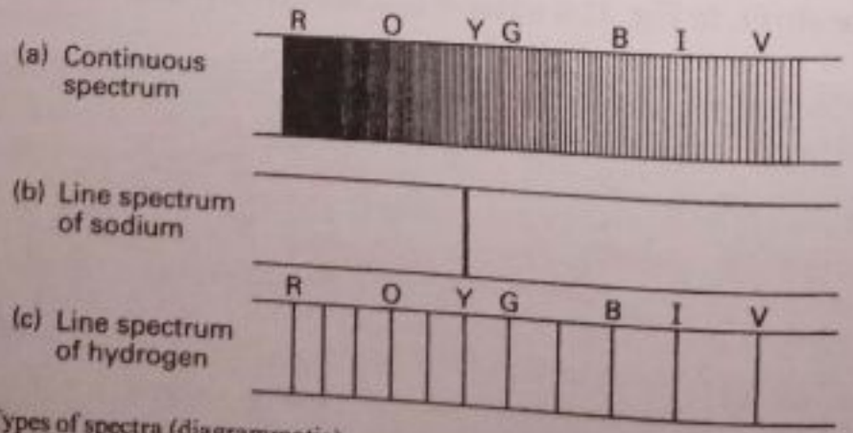


Fig. 21.8 Types of spectra (diagrammatic)

Scanned with CamScanner

Hydrogen gas which glows has a line spectrum consisting of many colours (Fig. 21.8c). Carbon dioxide gas has a *band spectrum*, consisting of many bands of fine lines.

These are *emission spectra*. Dark lines called 'Fraunhofer lines' are found in the sun's spectrum. They are examples of *absorption spectra*. An element absorbs those wavelengths from the sun's radiation which the same element emits when hot. So elements in the sun's atmosphere can be identified from their absorption spectra.

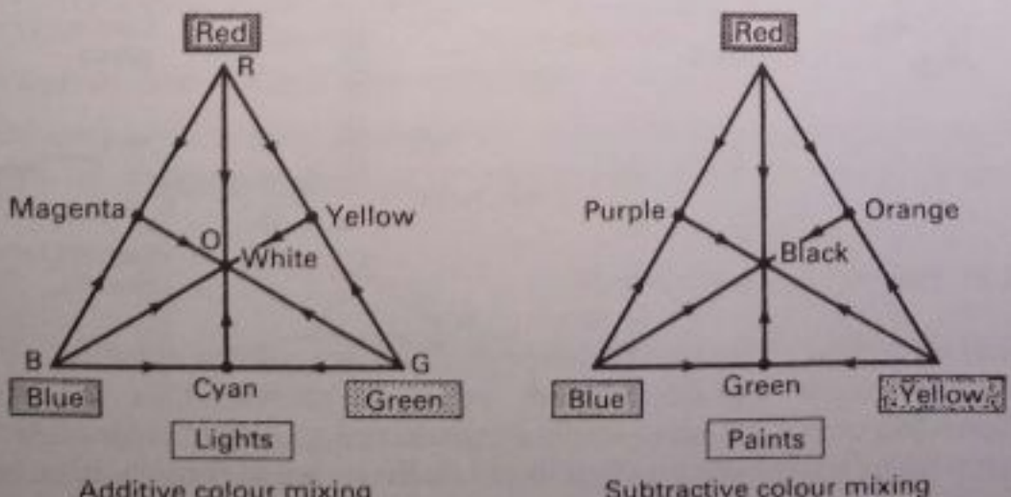
Colours of objects

An object such as paper, which is white in daylight, reflects equally all the colours of the spectrum, red, orange, yellow, green, blue, indigo, violet. When it is illuminated by red light it appears red. An object which appears black in daylight absorbs all the colours and does not reflect at all. So, if it is illuminated by blue light only it appears black. A yellow flower is yellow because its petals absorb most of the other visible colours in white light and reflect only the yellow. So the flower looks dark or black if it is illuminated in a dark room by blue light. A blue tie absorbs all the colours in white light except blue, and so looks black if illuminated by yellow light.

Additive colour mixing · Colour triangle

As we saw on page 330, white light can be produced by adding the *primary colours*, red, green and blue light. The adding of primary colours to produce other colours is known as *additive* colour mixing. Thus red and blue lights on a white paper screen together give a purple colour known as magenta; blue and green give a turquoise or blue-green colour called cyan; and green and red give yellow. The colours which add together to produce white light are called *complementary* colours. Thus magenta is complementary to green; yellow is complementary to blue; and cyan is complementary to red. Colours other than primary colours, magenta, cyan, yellow, are called *secondary colours*.

The colours produced by additive mixing of primary colours can be obtained from a *colour triangle*, RGB (Fig. 21.9a). Each primary colour is placed at the corner of an equilateral triangle. As one moves along a side of the triangle, such



Additive colour mixing (a)

Subtractive colour mixing (b)

Fig. 21.9 Colour triangles

Scanned with CamScanner

as RB, the colours gradually change from one primary, red in this case, to the other, which is blue. Midway between the two primaries will be the colour obtained by mixing or adding equal quantities of the primaries, in this case magenta. White is the centre O of the triangle because this is the point which represents the addition of equal quantities of the three primary colours. Any primary colour, when added to the colour directly opposite to it in the triangle, will produce white as these are complementary colours. Thus

Blue + yellow = white
Red + cyan = white
Green + magenta = white

Subtractive colour mixing

Although the mixing of blue and yellow lights creates whiteness, the mixture of blue and yellow *pigments* (paints) produces a green. This is because the results of paint mixing are due to a *subtractive* process. Blue paint absorbs all light except blue and green. Yellow paint absorbs all light except yellow, orange and green. The only light reflected by both yellow and blue pigments is green, and this is the resulting colour seen. Fig. 21.9b shows a colour triangle for subtractive colour mixing.

Principle of colour television

Additive colour mixing is used in colour television. The light from a particular scene is split up into three coloured components, red, green and blue, by allowing it to fall on special mirrors called dichroic mirrors, built into the television camera (Fig. 21.10). These mirrors have specially prepared layers of transparent

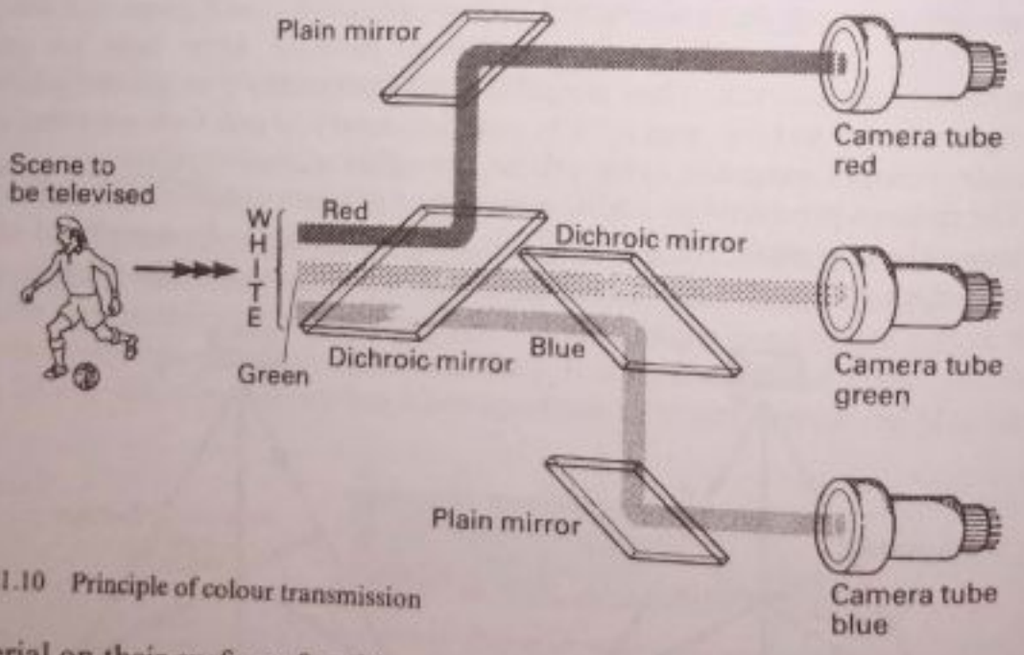


Fig. 21.10 Principle of colour transmission

material on their surfaces for this purpose. The three colours obtained are then incident on three separate camera tubes as shown. This produces a corresponding electrical signal which is transmitted to distant colour television receivers, where the colours are reproduced on the screen by three electron beams and added together.

In high-fidelity reproduction of sound, engineers aim at reproducing accurately three main qualities of the original sound; frequency (pitch),

Scanned with CamScanner

amplitude (loudness or intensity) and tone (or timbre). In accurate reproduction of colour for colour television, engineers aim at reproducing accurately analogous qualities, for example the shade or hue of a colour and its intensity and brightness.

Infra-red and ultra-violet rays

As well as light, the sun also gives out rays you cannot see which make you feel hot. They are called *infra-red* rays. Later it was found that the sun gave out (emitted) invisible rays beyond the violet end of the spectrum also. These cause certain materials to fluoresce and also affect photographic plates. They are known as *ultra-violet* rays.

Radio waves, infra-red rays, the visible rays from red to violet, ultra-violet rays and X-rays are all *electromagnetic waves*. They all travel through a vacuum with exactly the same speed and differ only in having different wavelengths (Fig. 21.11). These range from electromagnetic waves of long wavelength such as radio waves, produced by oscillating electrons in metals, to electromagnetic waves of extremely short wavelength such as gamma (γ)-rays, produced by changes in the nucleus of unstable atoms (see page 564). Fig. 21.11 shows very roughly the family of electromagnetic waves; there is no sharp boundary between the various waves, and overlapping occurs.

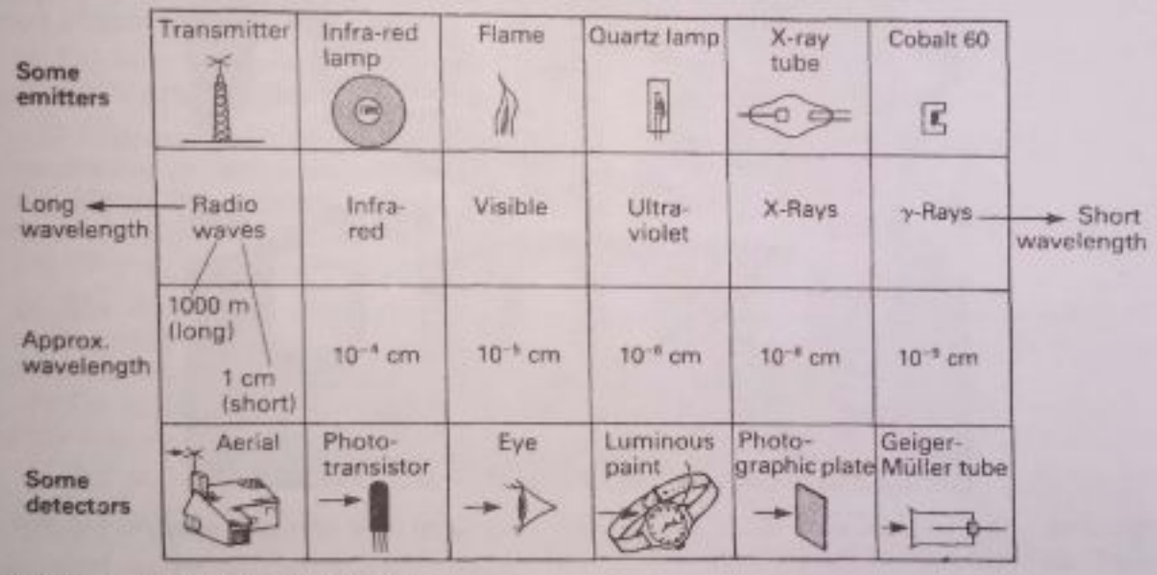


Fig. 21.11 Electromagnetic waves

Detection of infra-red and ultra-violet rays

The presence of infra-red and ultra-violet radiations in the sun's spectrum can be detected by using prisms and lenses of suitable materials and projecting the

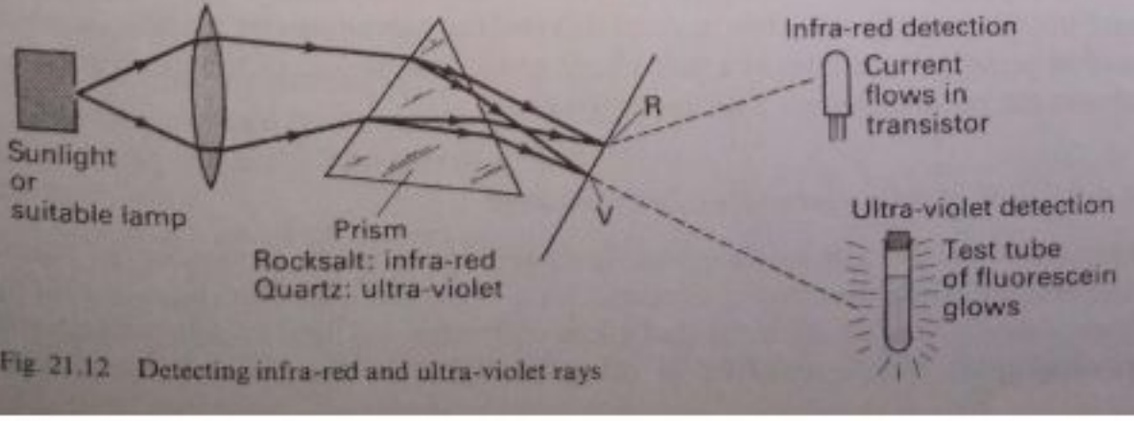


Fig. 21.12 Detecting infra-red and ultra-violet rays

Scanned with CamScanner

spectrum on to a screen in a darkened room. Fig. 21.12 illustrates the principle. Ordinary glass absorbs infra-red and ultra-violet rays. The lens and prism should therefore be made of rock salt for detection of infra-red rays and of quartz for ultra-violet rays.



Plate 21A Thermogram, showing the variation in heat radiated from the walls, windows and roof of a house (*AGA Infrared Systems Ltd*)

Fluorescein solution glows when ultra-violet rays fall on it. Using fluorescein solution in a narrow tube near the violet end of the spectrum, the solution glows when the tube is moved beyond the visible spectrum. This shows the presence of invisible ultra-violet rays.

If a thermopile, or a transistor sensitive to infra-red rays, is moved to the red end of the spectrum and then beyond this end the galvanometer reading increases and becomes a maximum at a point beyond the visible part of the spectrum. This shows the presence of invisible infra-red rays.

Some uses of infra-red and ultra-violet rays

Infra-red rays are not scattered by fine particles as much as rays in the visible spectrum. Clear photographs of scenes have been taken through haze or mist by using camera lenses with filters that allow only infra-red light to pass through and photographic plates sensitive to infra-red light. In the Second World War

Scanned with CamScanner

aeroplanes were equipped at night with infra-red cameras to see enemy planes, and to photograph enemy territory.

Ultra-violet rays cause your skin to tan. A mercury-vapour lamp with a quartz envelope is a source of ultra-violet rays. Dark spectacles must be worn when the body is exposed to the radiation, as the rays are dangerous to the eyes. Various skin infections can be treated by ultra-violet rays. Costumes on the stage, with fluorescent materials on them, produce spectacular effects when the theatre is darkened and ultra-violet lamps are switched on. Ultra-violet light is often used in the laboratory in place of visible light to photograph an object through a microscope, as a sharper definition and more detail is obtained with shorter wavelengths.

Examples on colours

1. Describe and explain the appearance of a green apple on a red cloth when it is viewed through (a) a red filter, (b) a green filter, (c) a blue filter. (All colours referred to may be considered to be pure.) (L)

A green apple looks green because it absorbs all colours except the green which it reflects. In the same way, only red coloured light is reflected from the red cloth.

(a) The red filter allows only red light to pass through it and absorbs all other colours. So the green apple looks black and the red cloth red.

(b) The green filter allows only green light to pass through. So the green apple appears to be on a black cloth.

(c) The blue filter absorbs both the green and the red light. So both the apple and the cloth would appear black.

2. (a) A narrow beam of white light is incident on a glass prism. What is the difference between the *deviation* and *dispersion* produced?
(b) Name the primary colours. What colour is complementary to blue? Give a reason for your answer.
(c) Why do red and green pigments (paints) produce a blackish paint?

(a) The *deviation* is the angle which any colour is turned through from its original direction after refraction by the prism. The *dispersion* is the separation of the colours themselves after refraction through the prism.

(b) The primary colours are red, blue and green. The colour complementary to blue is yellow because white light is produced when these two coloured lights are added together.

(c) Red and green paints together produce black by a 'subtractive' process. Red paint absorbs all the colours of white light except red and orange. Green paint absorbs all the colours of white light except green. Mixed together, therefore, all the colours are absorbed, so the mixture appears black.

Summary

1. The spectrum of white light consists of red, orange, yellow, green, blue, indigo and violet colours.
2. A prism produces dispersion and deviation of colours at its first face, and further deviation of colours at its second face.
3. Colours produced by a prism can be recombined by using another prism with its angle pointing in the opposite direction. Newton's colour disc can be used for adding colours.
4. A pure spectrum of white light can be formed by using a narrow slit, two lenses and a prism. One lens produces a parallel beam of white light, the other collects the different coloured beams emerging from the prism, which are parallel.
5. White light has a continuous spectrum of all colours. Sodium has a

Scanned with CamScanner

characteristic yellow-line spectrum. Mercury vapour has an orange-, green- and blue-line spectrum. Hydrogen has a line spectrum of all colours.

6. A red flower looks red when exposed to red light because it absorbs all the colours except red. For the same reason, it looks black when exposed to blue light.

7. A white screen illuminated by green and red light appears yellow–the two colours are reflected by the screen, and yellow is the sum of the two colours.

8. Pigments (paints) produce a colour by a subtractive process. Blue paint absorbs all the colours except for blue and green; yellow paint absorbs all the colours except for yellow, orange and green. When the paints are mixed green is obtained.

9. Ultra-violet, infra-red and visible rays, together with radio waves and X-rays, are all electromagnetic waves with different wavelengths. Ultra-violet rays have shorter wavelengths than visible rays and produce fluorescence. Infra-red rays have longer wavelengths than visible rays and produce the sensation of heat.

Exercise 21

1. Describe how you could demonstrate that white light is composed of a number of colours. Explain:

(a) the appearance of a red scarf with blue stripes in daylight when the scarf is viewed through a red filter or a sheet of red glass.

(b) why blue and yellow paints (pigments), when mixed, produce a shade of green. (L)

2. Name the three primary colours (not pigments). State why they are called *primary*. What is the complementary colour to yellow? State why. (L)

3. (a) What is meant by complementary colours?

(b) What light is transmitted through a pure red filter and a pure blue filter held together (superimposed) in front of a white light source? (N)

4. A narrow beam of white light is incident upon a triangular glass prism. Draw a clear diagram to illustrate what is meant by (a) deviation, (b) dispersion.

With the help of a well-labelled diagram, describe how a *pure* spectrum may be obtained from white light using such a prism and any other necessary apparatus.

State and explain what form of spectrum would be obtained if (c) a red filter, (d) a green filter were placed in the path of the beam.

Why do coloured objects often appear slightly different in colour when viewed in artificial white light? (L)

5. Shadows of a vertical rod are cast on a vertical white screen by two small coloured lamps, one red and one green, placed at either side of the perpendicular from the screen to the rod. Explain, with a diagram, the positions and colours of the shadows which do not overlap. What will be the colour of the remainder of the screen? (N)

6. Explain why a mixture of blue and yellow pigments appears green when illuminated by white light. What is the appearance of the mixture if viewed through a sheet of red glass? (N)

7. (a) What is a pure spectrum?

(b) In the formation of the spectrum of white light by a prism (i) which colour is deviated least? (ii) which colour is deviated most? (N)

8. Explain the meaning of the terms *deviation, dispersion, pure spectrum.*

Describe how a pure spectrum of white light can be projected on a screen, using two lenses and a prism. Draw a ray diagram to illustrate your answer.

Why does the colour of a body vary with the colour of the light in which it is viewed? (O and C)

9. Distinguish between *deviation* and *dispersion* of a beam of light. Draw a diagram of an optical arrangement suitable for obtaining, on a screen, the spectrum of the light from a slit illuminated by an electric lamp, and show the paths of two rays from a point on the slit to the screen. State the adjustments necessary to produce a pure spectrum.

Describe and explain the appearance of a red tie with blue spots when observed in (a) red light, (b) green light. (L)

10. Describe the appearance of two strips of cloth when a spectrum of white light is

Scanned with CamScanner

projected on them, if one of the strips is pure yellow and the other a mixture of pure yellow and pure green.

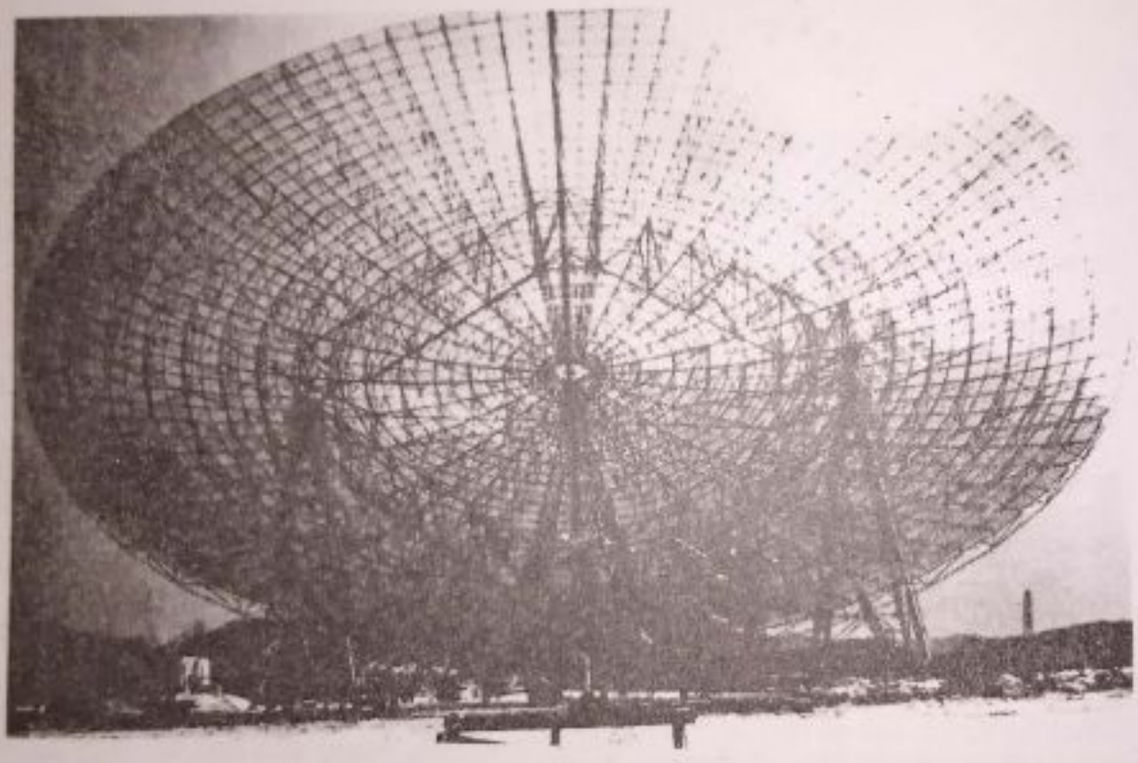
Two football teams have striped jerseys, the first pure green and white and the second black and pure yellow. They were to play a match on a ground floodlit with monochromatic yellow lamps, until it was realised that both team's jerseys would appear to be of the same colours. Describe and explain the appearance of the jerseys in the yellow light. (C)

11. Describe how a pure spectrum from a white-light source may be formed on a screen. How would you show that the colours of the spectrum can be recombined to give white light?

State what you would expect to observe if the spectrum were cast on a sheet of red paper. Give a brief explanation of your statement. (O)

Scanned with CamScanner

WAVES



National Radio Astronomy Observatory in the United States. The diameter of the radio reflector which reflects radio waves to a detector at its focus is over 90 metres. (*Barnaby's Picture Library*)

Scanned with CamScanner

22. Wave Propagation · Wave Effects

We see and hear people by means of light and sound waves. Radio waves travel to aerials through the air from television transmitters. So the study of waves is of great importance. Although we cannot see light, sound and radio waves, they travel across space. Light and heat energy in the form of waves reaches us from the sun after travelling across enormous distances of empty space.

Water waves and ripple tank

Water waves are simple to produce. We can study them with the help of a ripple tank (Fig. 22.1a). This consists of (a) a pool of water W in a shallow

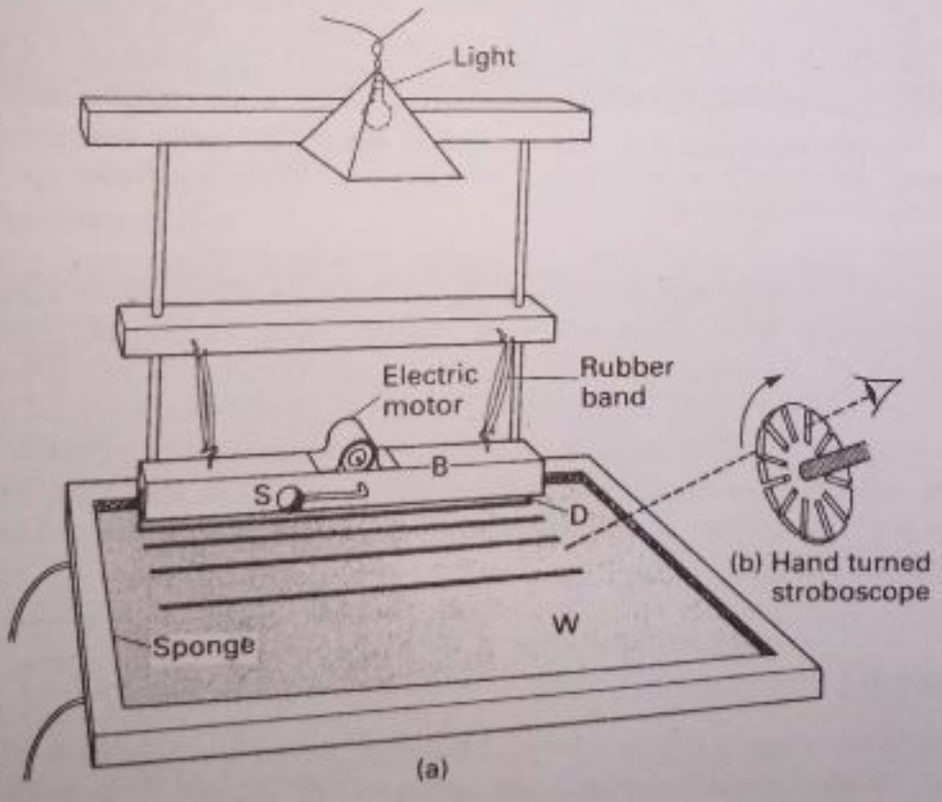


Fig. 22.1 Ripple tank

rectangular dish with a clear glass base, and with edges lined with sponge; (b) a dipper for producing water waves–this may be a vertical plane metal strip D or a small sphere S beneath a bar B; (c) a vibrator in the form of an off-centre weight on the rotating armature of a small electric motor rigidly attached to B–this makes the dipper vibrate up and down on the water; (d) a lamp with a small

Scanned with CamScanner

filament which lights up the water surface from above or below–the ripples can then be seen or they can be projected on a white screen (Plate 22A).

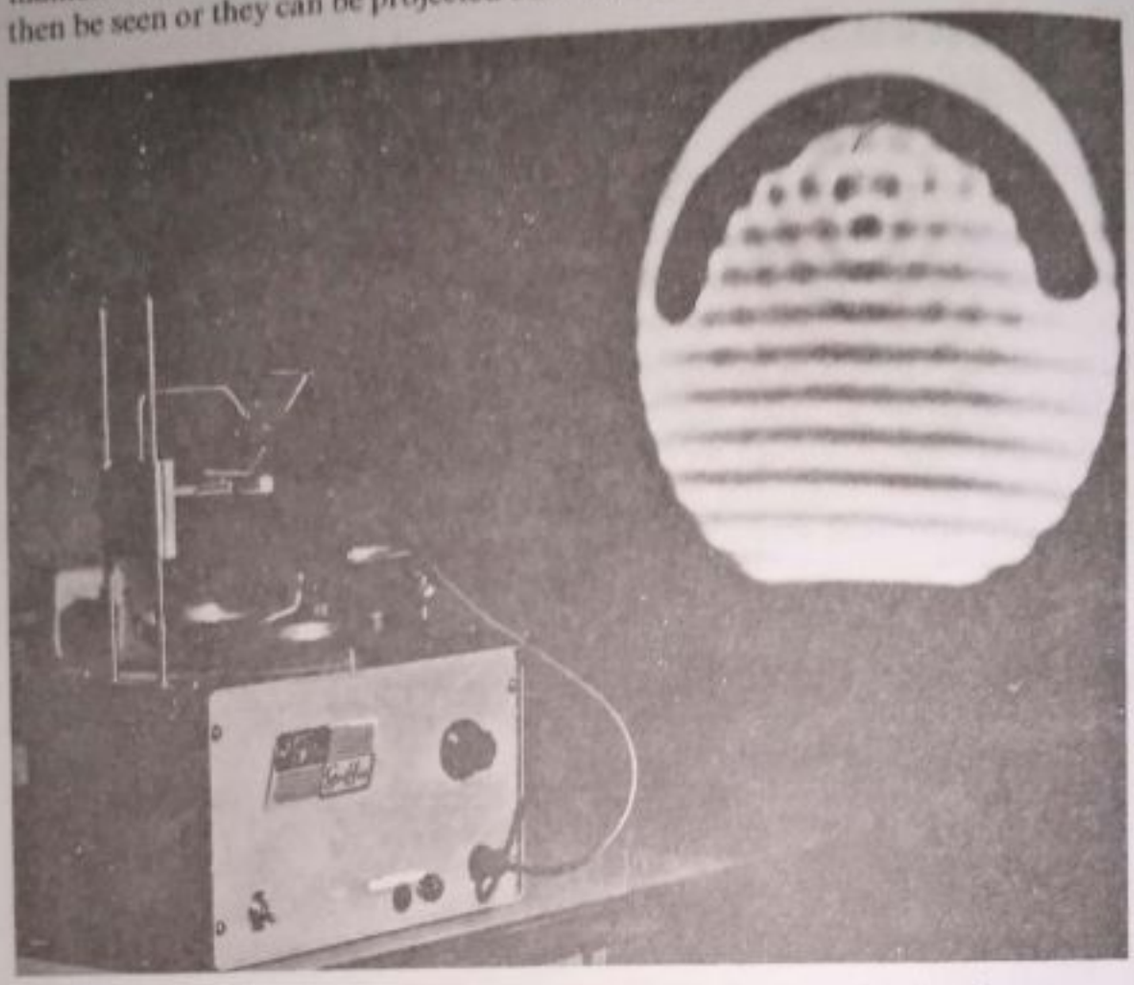


Plate 22A Demonstration ripple tank. Waves formed in a dish of water can be viewed by projection on a white screen. Here incident plane waves are shown focused by reflection at a concave surface. (*Griffin and George*)

When the height of the strip D is adjusted, so that it just dips into the water and the electric motor is switched on, waves which travel across the water are produced. A stroboscope is used to make the waves appear stationary. A simple form of hand stroboscope is a circular disc with slits evenly spaced round it (Fig. 22.1b). It is held by the axle in one hand and spun round with the other hand. On looking at the water surface through the revolving slits the waves can be seen moving across the surface; at one speed the waves appear stationary. They can then be studied closely.

Plane and spherical waves

With the straight edge of the metal strip M dipping into the water, parallel *plane waves* can be seen spreading along the surface (Fig. 22.2a). With a sphere S dipping into the water, *spherical* or *circular waves* spread out from S at their centre (Fig. 22.2b).

Small pieces of cork on the water bob up and down or vibrate as the wave passes. This shows that the water particles themselves do not move from one place to another along the surface as the wave travels. We see that the wave only carries *energy*, the energy of the water movement, along the surface. Water is the medium or material in which the energy travels.

Scanned with CamScanner

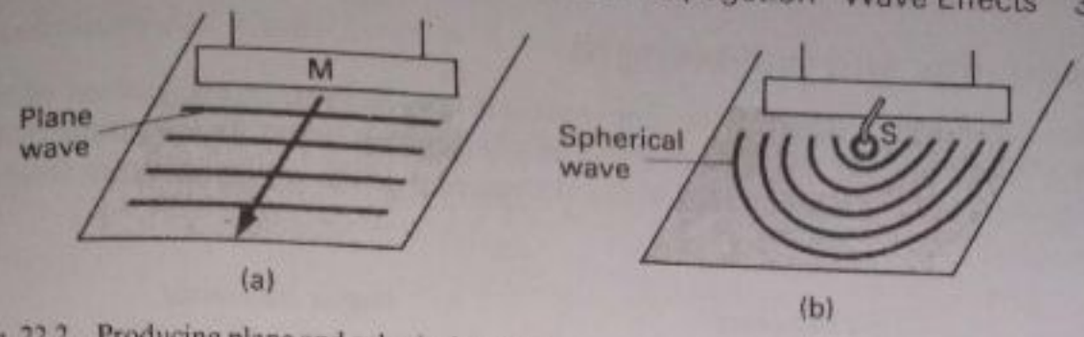


Fig. 22.2 Producing plane and spherical waves

A wave can be produced in a long rope tied at one end to a wall. When the free end is moved quickly up and down, a wave travels along the rope. Ribbons tied all along the rope are seen to move up and down while the wave is travelling. Like the water particles, the rope particles remain at the same place as they vibrate.

With every wave we must distinguish between

(a) the *wave itself*, which travels and carries the energy,

(b) the *vibrations*, which produce the wave.

These vibrations are shifts or displacements about an average or mean position; the maximum displacement in a vibration is called its *amplitude*.

Transverse and longitudinal waves

As we have just seen, the particles of water move up and down vertically when a water wave travels. So the vibrations are vertical. The wave, however, travels in a horizontal direction.

Waves which travel perpendicularly to the direction of the vibrations are called transverse waves. A water wave is an example of a transverse wave. Light waves and radio waves are transverse waves.

Experiment

A long horizontal 'Slinky' coil is supported at one end and the other end is moved up and down in a vertical direction. A transverse wave is seen passing along the coil (Fig. 22.3a).

The long 'Slinky' coil is now spread along a bench or the floor and one end is moved to-and-fro quickly in the direction of the coil axis. A wave is seen travelling along the coil to the other end. This time, however, the vibrations of the coil are *parallel* to the direction of the wave (Fig. 22.3b).

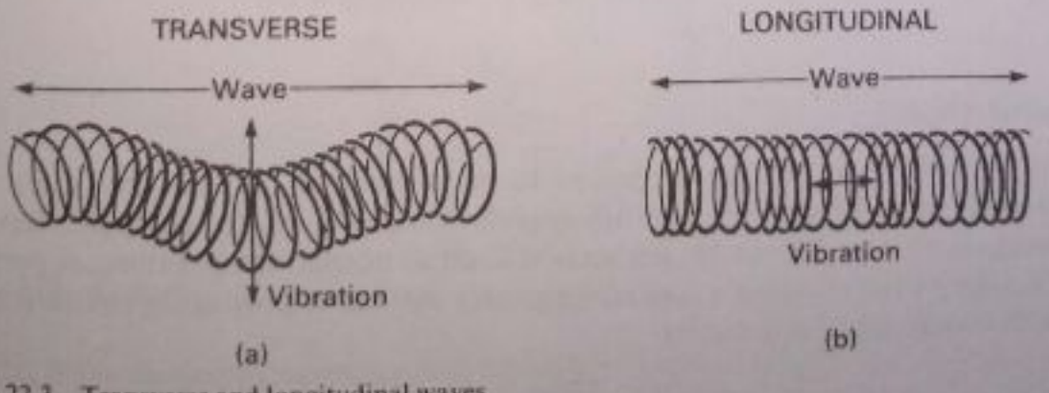


Fig. 22.3 Transverse and longitudinal waves

Waves which travel in a direction parallel to the vibrations are called *longitudinal waves*. As discussed later, sound waves are longitudinal because the vibration of the air particles is in the same direction as the sound wave.

Scanned with CamScanner

Frequency and wavelength

When the electric motor of the ripple tank is speeded up, the vibrations of the dipper increase. The number of wave crests passing a particular point in the water every second then increases and the spacing of the waves becomes shorter (Fig. 22.4).

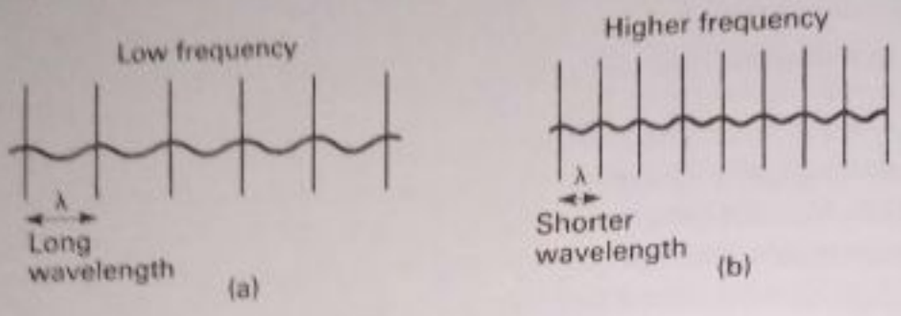


Fig. 22.4 Frequency and wavelength changes

The *frequency* f of the wave is defined as the number of waves passing a given point in 1 second. The hertz, symbol Hz, is the unit of frequency. A frequency of 1 Hz means that one wave per second passes a given point.

Sound waves have much greater frequencies than water waves. A football referee's whistle may have a frequency of 2000 Hz. A frequency of 1000 Hz is called a kilohertz, symbol kHz, so the frequency of the whistle is 2 kHz. A radio broadcasting station may send out radio waves of frequency 1 000 000 Hz, a million hertz. This frequency is called a megahertz, symbol MHz.

The *wavelength* λ of a wave is the distance between successive crests (or troughs) of the wave. When the speed of the wave is constant, the wavelength decreases if the frequency increases. (Fig. 22.4b).

Velocity of a wave

We can obtain a general formula for the wave velocity v using its frequency f and its wavelength λ. Suppose the frequency of a water wave is 4 Hz. Then four waves per second pass a given point. If the wavelength is 5 cm, the distance occupied by four waves in 1 second $= 4 \times 5 = 20$ cm. This is the distance per second travelled by the wave or its *velocity*. So we see that

$$\text{velocity } (v) = \text{frequency } (f) \times \text{wavelength } (\lambda)$$

or

$$v = f\lambda$$

Sound waves

The velocity formula $v = f\lambda$ applies to all waves, no matter how they are produced or in which medium they travel. Sound waves, for example, have a velocity v of about 340 metres per second in air at normal temperature. A person with a deep voice, singing a note of frequency 200 Hz, is producing sound waves whose wavelength λ is given by

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{200 \text{ Hz}} = 1{\cdot}7 \text{ m}$$

If the note rises to 400 Hz, or twice the frequency, the wavelength decreases to 0·85 m, which is half as long as before.

Scanned with CamScanner

Electromagnetic waves

Radio waves have a velocity of about 3×10^8 m/s. Some broadcasting stations send out waves of long wavelength 1500 m. These will have a frequency f given by

$$f = \frac{v}{\lambda} = \frac{3 \times 10^8}{1500}$$
$$= 2 \times 10^5 \text{ Hz} = 0{\cdot}2 \text{ MHz}$$

For radio waves of 300 m wavelength, which are 5 times shorter than 1500 m, the frequency will be five times greater. So $f = 5 \times 0{\cdot}2$ MHz $= 1{\cdot}0$ MHz.

Light waves have the same velocity as radio waves (see below), about 3×10^8 m/s, but their wavelengths are millions of times shorter. A particular yellow light wave has a wavelength 6×10^{-7} m. Its frequency f is therefore given by

$$f = \frac{v}{\lambda} = \frac{3 \times 10^8}{6 \times 10^{-7}}$$
$$= 5 \times 10^{14} \text{ Hz}$$

Radio waves and light waves are members of a family or spectrum of waves called *electromagnetic waves*, because they are produced by electromagnetic vibrations. Infra-red, ultra-violet, X- and gamma-waves are also electromagnetic waves. They all have different wavelengths, whose order of magnitude is given below.

Radio	**Infra-red**	**Visible**	**Ultra-violet**	**X-rays**	**Gamma-rays**
10^{-3} m	10^{-6} m	Red	10^{-8} m	10^{-10} m	10^{-11} m
to		7×10^{-7} m			
1000 m		Violet			
		$4{\cdot}5 \times 10^{-7}$ m			

Example on waves

Fig. 22.5 illustrates part of the displacement–time graph of a wave travelling across water at a particular place with a velocity of 2 m/s. Calculate (a) the amplitude, (b) the frequency, (c) the wavelength of the wave.

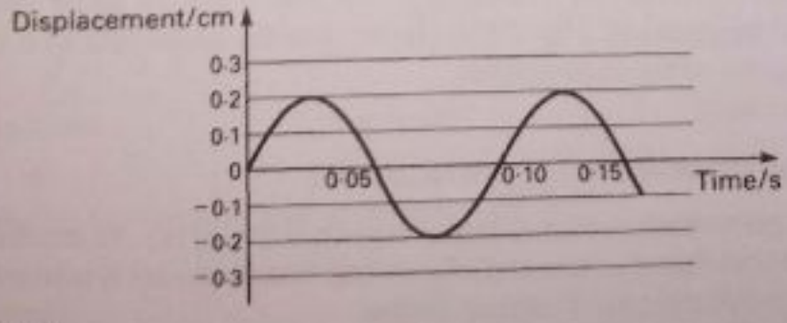


Fig. 22.5 Example

(a) From the graph, maximum displacement $= 0{\cdot}2$ cm $=$ amplitude

(b) From the graph, period T = time for one cycle $= 0{\cdot}1$ s

$\therefore$ frequency = number of cycles per second

$$= \frac{1}{0{\cdot}1} = 10 \text{ Hz}$$

Scanned with CamScanner

(c) Velocity of wave = frequency × wavelength, λ

$$\text{wavelength} = \frac{\text{velocity}}{\text{frequency}} = \frac{2\text{ m/s}}{10\text{ Hz}}$$

So $= 0{\cdot}2\text{ m}$

Forced vibrations and resonance

The pendulum is a simple mechanical system which can be set into vibration. If the bob is pushed gently and then left, it vibrates with a *natural* frequency which depends on the length of the pendulum (page 23). All types of vibrating systems have their own natural frequency of vibration.

Fig. 22.6a shows five pendulums P, Q, R, S, T of different length, each carrying a mass of about 100 g as a bob. They are hung from a horizontal string. Another pendulum A, with a heavier bob such as 1 kg, is also suspended from the string.

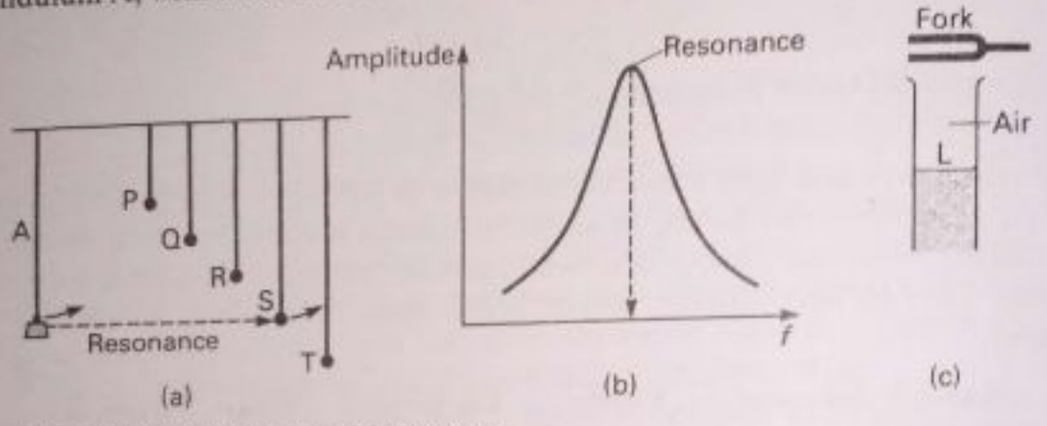


Fig. 22.6 Mechanical and acoustical resonance

A has the same length as the pendulum S. When A is set oscillating on its own, in a short time we see that all the other pendulums except S begin oscillating with small swings. S, however, has a large swing. So because they are all attached to the horizontal string all the pendulums have been forced to vibrate but S has responded considerably. We say that S has been set into a *resonant* vibration by A or that *resonance* has occurred.

S has the same length as A. So it has the same natural frequency as A. We therefore see that a mechanical system can be set into a large amplitude of vibration, or resonance, by a vibrating force which has the same frequency as its own natural frequency, f. Fig. 22.6b shows how the amplitude of a system varies as the frequency of the force varies.

Mechanical, acoustical, electrical resonance

Resonance phenomena occur in many branches of physics. In mechanics, a diver jumping up-and-down at one end of a diving board can set it into resonance and so gain a considerable uplift before diving.

Fig. 22.6c illustrates an example of *acoustical* resonance. Water completely filling a burette is slowly run out. When a vibrating or sounding tuning-fork is held over at the top of the tube as shown, a loud sound can be heard coming from the tube at a certain level L of the water. The air above L in the burette is set into resonance by the fork because its natural frequency is exactly the same as the fork's frequency. A high-pitched note from a singer has been known to break a glass nearby, as the glass is set into resonant vibrations by the sound.

Scanned with CamScanner

A radio receiver is 'tuned' to a broadcasting station when an electrical oscillating circuit inside the receiver is set into resonance by the incoming radio waves.

Reflection of waves

Reflection by plane and curved surfaces

The reflection of water waves at surfaces can be studied using a ripple tank.

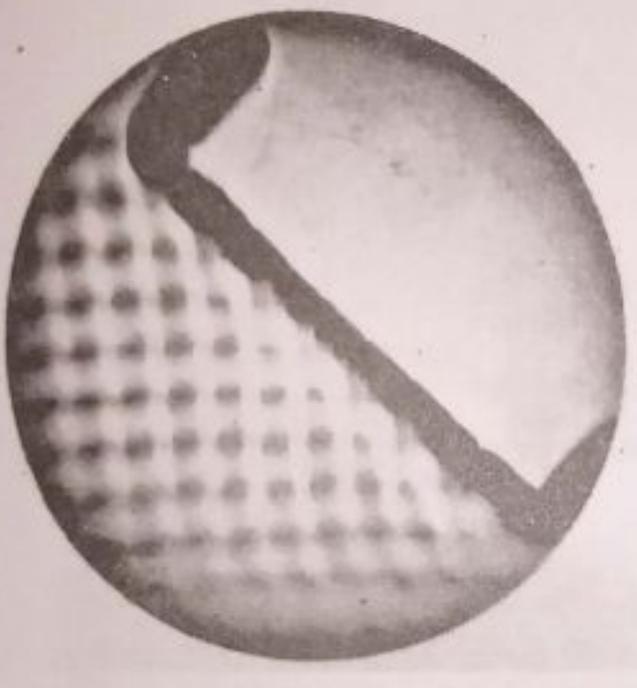


Plate 22B Reflection of plane waves, coming from below, at a plane surface. (*Griffin and George*)

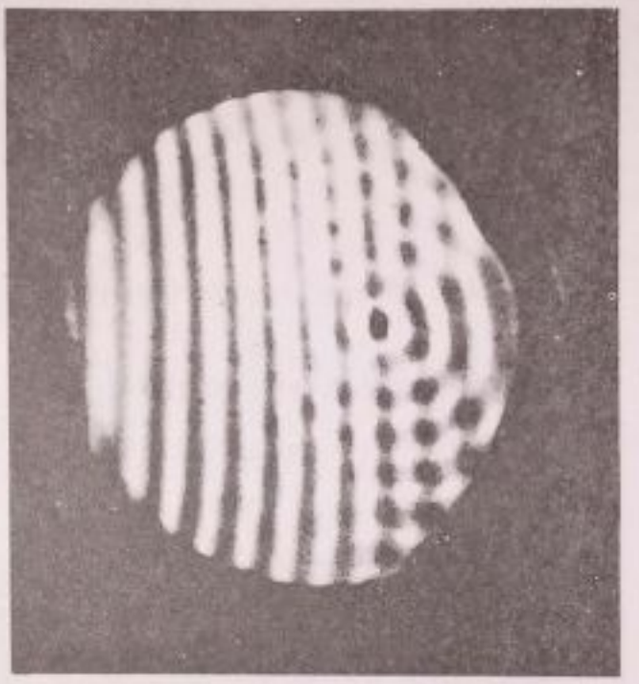


Plate 22C Plane waves, coming from left, reflected to a focus by a parabolic reflector. (*Griffin and George*)

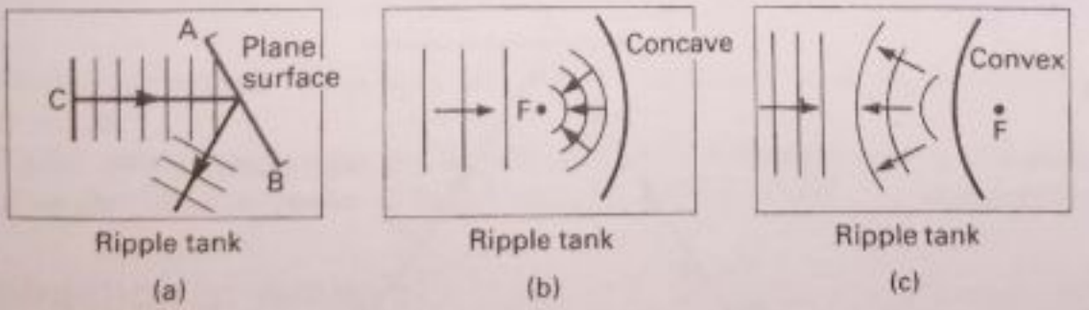


Fig. 22.7 Reflection by plane and curved surfaces in a ripple tank

Experiment

A flat (plane) metal strip AB is placed a short distance from a vibrator C in the water of a ripple tank. Fig. 22.7a. When plane waves are incident on AB, they are reflected in a definite direction. See Plate 22B. The reflection obeys the law angle of incidence = angle of reflection. Light waves are also reflected according to this law (page 262).

When the plane strip AB is replaced by curved metal strips, the reflected waves are as shown in Fig. 22.7b, c and in Plate 22C.

Scanned with CamScanner

Results. We see that plane waves incident on a *concave* surface are reflected as spherical waves converging towards a point or focus F in front of it (Fig. 22.7b). If they are incident on a *convex* surface, they are reflected as spherical waves diverging from a focus F behind it (Fig. 22.7c). We see the same effect when light waves are reflected by curved mirrors (page 269). Fig. 22.8 shows spherical (circular) waves from a point O reflected from a concave and a convex surface. In (a) the reflected waves converge to a point I, in (b) they appear to diverge from a point I behind the surface.

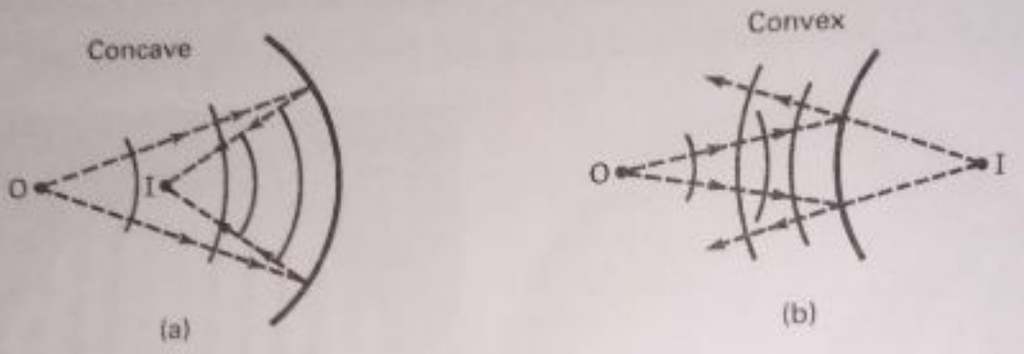


Fig. 22.8 Reflection by concave and convex surfaces

Reflection of sound

Sound waves are reflected in the same way as light waves.

Experiment

A loudspeaker L is placed in front of a hardboard surface or smooth wall, so that sound waves are incident in a particular direction PO (Fig. 22.9). A small microphone connected to a cathode-ray oscilloscope (CRO) is then moved round to locate the direction of the reflected waves.

The results show that the angle of reflection r is equal to the angle of incidence i. This is the same as for light waves reflected from a plane mirror.

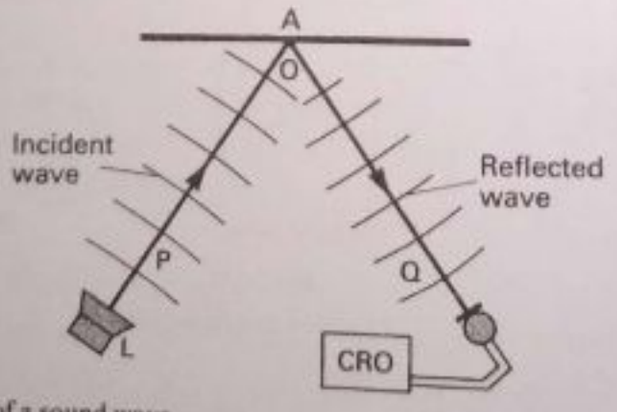


Fig. 22.9 Reflection of a sound wave

The sound heard after reflection of sound waves is called an echo. Sir Isaac Newton, while at Trinity College, Cambridge, is said to have measured the speed of sound by using echoes. He measured the time between repeated echoes as sound travelled backwards and forwards between the walls of a courtyard.

Echoes are used in underwater exploration for gas and oil. They are also used to map the depths of the sea-bed.

Scanned with CamScanner

In water sound travels at a speed of about 1450 metres per second or 5200 kilometres per hour. Thus if sound waves travelled from a ship to the bottom of the ocean and back in 0·5 s, the total distance travelled would be 725 metres.

Reflection of electromagnetic waves

Electromagnetic waves of about 3 cm wavelength, called microwaves, can be generated by a transmitter (Fig. 22.10). A detector or receiver with a sensitive meter G will register a deflection in G if the waves are received.

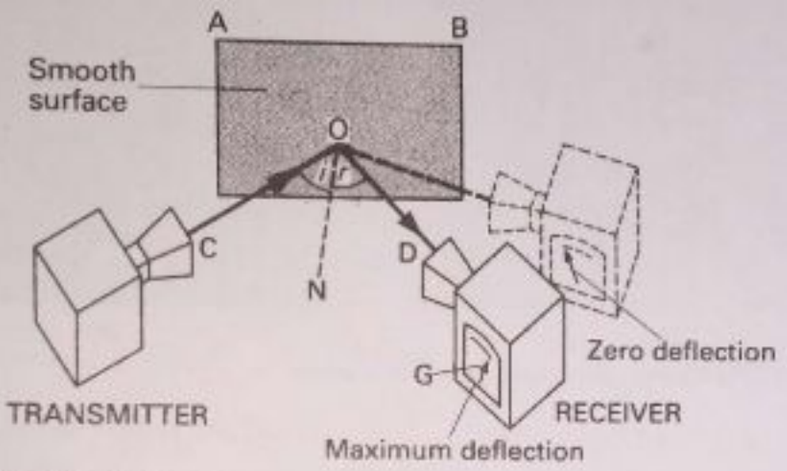


Fig. 22.10 Reflection of microwaves

Experiment

The transmitter is set up in front of a smooth wall, metal sheet or hardboard AB so that waves are incident in the direction CO (Fig. 22.10). The receiver R is then kept pointed at O on the other side of the normal ON, and moved round from AB. At some position such as D the deflection in G will become a maximum. So the incident waves are reflected along the direction OD.

Measurement of the angle of incidence NOC, and the angle of reflection NOD, shows they are practically equal. The same result is obtained when the angle of incidence is varied.

So electromagnetic waves obey the same law of reflection at plane surfaces as light waves.

Radio waves may sometimes be reflected from the plane surface of a metal tank on the roof near an aerial and produce 'interference' in a television receiver.

Refraction of waves

Refraction of water waves

Using a ripple tank, we can study the effect on waves when they travel from one medium to another where their velocity changes.

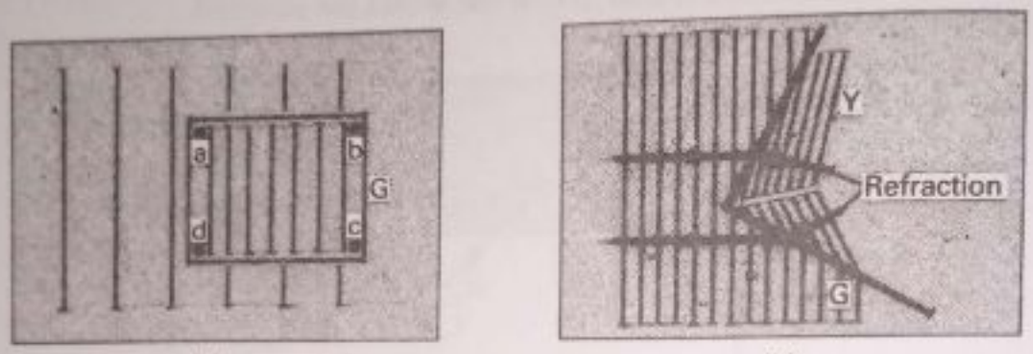
Experiment

A transparent plate G is fixed inside the ripple tank by plasticene, stuck to its corners a, b, c and d, so that it is level and just below (about 1 mm) the surface of the water (Fig. 22.11a).

Scanned with CamScanner

Plane waves are now produced which have low frequency. The region of shallow water above the plate increases the frictional resistance to the oncoming wave, so that its speed is reduced when it passes over G. We then see that the *wavelength is also reduced*, as shown. See Plate 22D.

When the plate G is turned at an angle to the incident waves, we see that the slower waves travel in a *different direction* after passing over the plate. (Fig. 22.11b). The change in direction due to a change in velocity of waves is called *refraction*.



(a) (b)

Fig. 22.11 Velocity change and refraction of water waves

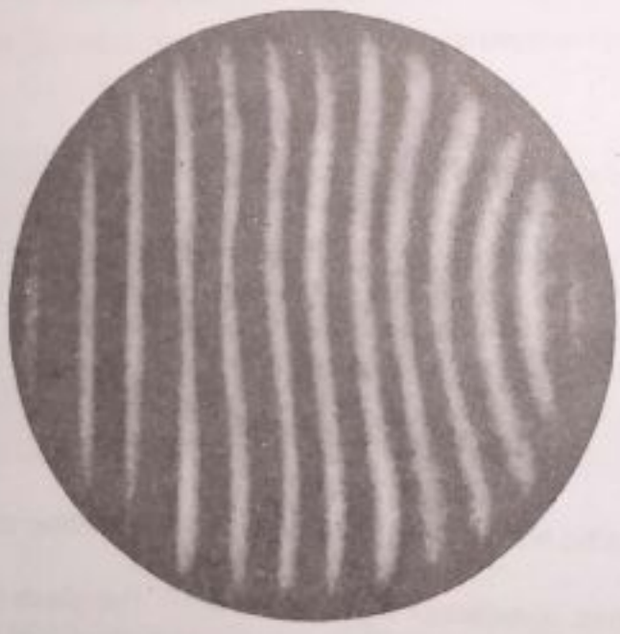


Plate 22D Refraction of plane waves. The waves from the left are slowed down in shallow water by a submerged glass slide. (*Griffin and George*)

Refraction of radio waves (microwaves)

Refraction occurs with all types of waves. Plate 22E shows the refraction of microwaves through liquid paraffin in a hollow prism. The meter in the receiver on the right shows a large deflection. So radio waves from the transmitter on the left have been refracted by the liquid. Large 60° prisms of solid paraffin-wax can be used in place of the liquid to demonstrate the refraction.

Radio waves which travel upwards are refracted by the *Appleton layer*. This region, about 270 km above the earth, contains a high concentration of electrons

Scanned with CamScanner



Plate 22E Apparatus to show refraction of microwaves by a prism containing liquid paraffin (*Rainbow Radio Ltd*)

and ions. The waves sent by a London transmitter, for example, can reach Australia on the other side of the world after their direction is changed by refraction in the Appleton layer.

Refraction of light waves · Refractive index relationship

Light waves travel more slowly when they enter an optically denser medium such as glass or water. The speed of light in air is about 300 000 km/s; in crown glass the speed is about 200 000 km/s.

The directions of the incident and refracted wavefronts have a simple relationship to the speed of light in the two media. Fig. 22.12a shows a plane wavefront, travelling with a speed c in air, which reaches the position AB at the boundary AC with glass. Since the speed of light in glass is two thirds that in air, the wavefronts in glass travel only two thirds the distance of those in air in the same time. So the waves change direction on entering the glass, as shown. In air, the direction XA makes an angle of incidence i with the normal PAQ to the boundary. In glass, the direction AY of the refracted waves makes a *smaller* angle r with the normal, where r is the angle of refraction.

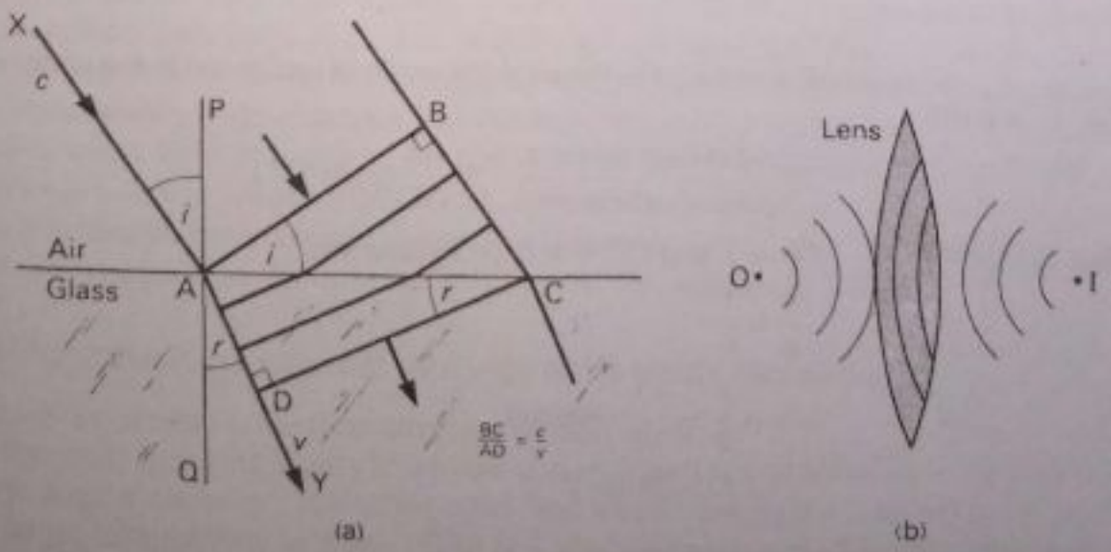


Fig. 22.12 (a) Refractive index and speed of a light wave (b) Refraction through a lens

Scanned with CamScanner

From Fig. 22.12a angle BAC = angle XAP = i and angle DCA = angle YAQ = r. So

$$\frac{\sin i}{\sin r} = \frac{\text{BC/AC}}{\text{AD/AC}} = \frac{\text{BC}}{\text{AD}} = \frac{\text{speed in air}}{\text{speed in glass}} = \frac{3}{2}$$

since the distance AD in glass is two thirds of BC, the distance travelled by the light in air in the same time.

Generally, then, if n is the *refractive index* of light from air to glass (see page 285), and c and v are the respective speeds of light in air and glass,

$$\text{refractive index } n = \frac{\sin i}{\sin r} = \frac{c}{v}$$

Red light travels faster in glass than blue light. So the refractive index of red light is less than that of blue light. This is why the colours making up white light are seen when a glass prism refracts white light, as discussed on page 328.

Fig. 22.12b illustrates how spherical light waves from a point O in air, refracted through a glass converging lens, converge to a point I on the other side. The waves travel slower in glass. So their curvature is changed when they enter and leave the lens, as shown.

Example on speed of light and refraction

Fig. 22.13 shows the refraction of a light wavefront OA travelling from water to air, where it emerges as a wavefront BC. The lengths of OA, OB and BC are respectively 4, 5 and 3 units. If the speed of light in air is 300000 km/s, calculate (a) its speed in

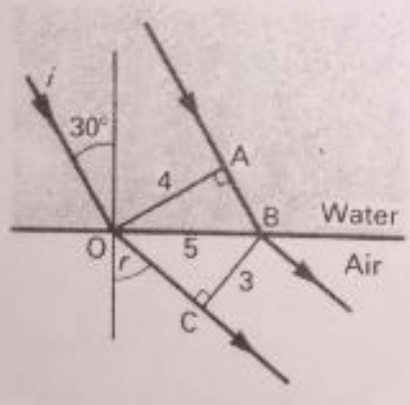


Fig. 22.13 Example

water, (b) the angle of refraction r in the air if the angle of incidence in the water was 30° as shown.

(a)
$$\frac{\text{speed of light in water, } v}{\text{speed of light in air, } c} = \frac{\text{AB}}{\text{OC}} \text{ (page 351)}$$

But AB = 3, and OC = 4, from Pythagoras

$$\therefore \frac{v}{c} = \frac{3}{4}$$

$$\therefore v = \frac{3}{4}c = \frac{3}{4} \times 300000$$

$$= 225000 \text{ km/s}$$

(b) refractive index from water to air $= \frac{v}{c} = \frac{\sin i}{\sin r}$

Scanned with CamScanner

$$\therefore \frac{\sin 30^\circ (i)}{\sin r} = \frac{3}{4}$$

$$\therefore \sin r = \frac{4}{3} \times \sin 30^\circ = \frac{2}{3}$$

$$\therefore r = 42^\circ \text{ (approx.)}$$

Diffraction of waves

We now investigate what happens to waves which pass through wide and narrow openings or apertures.

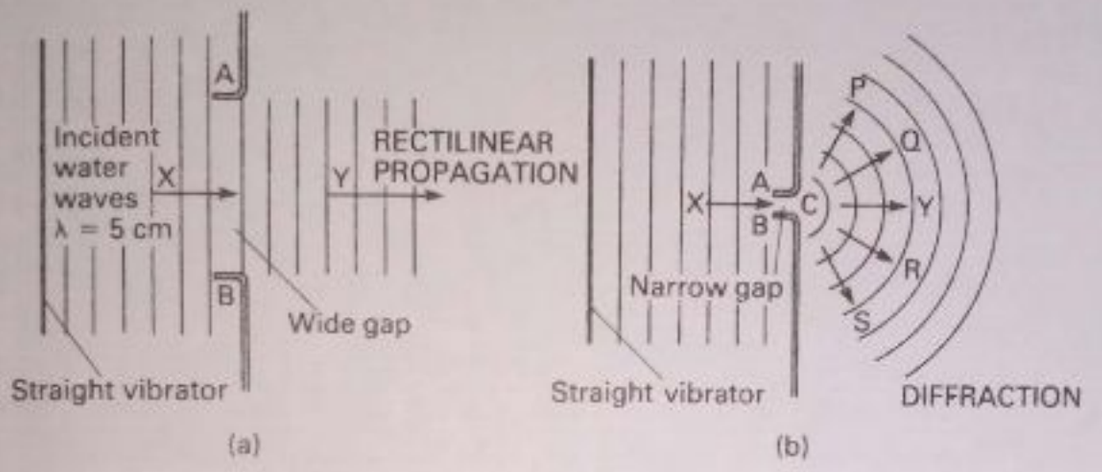


Fig. 22.14 (a) Rectilinear propagation (b) Diffraction

Experiment

Two separated strips A and B are placed in a ripple tank to provide an aperture or opening for plane waves from a vibrator to pass through. In Fig. 22.14a, the width of the opening AB is *much greater*, for example about 20 cm, than the wavelength λ of the incident plane waves. We see that the waves pass through from X to Y in practically the same direction.

Now make the width of the gap C between A and B *very narrow* (about 1 cm), roughly of the same order as the wavelength λ. This time the waves spread round C on passing through the opening–they no longer travel in the same direction from X (Fig. 22.14b). We say the waves are *diffracted* at C. They travel in all directions such as P, Q, Y, R, S.

This explains why light usually travels in straight lines, which is 'rectilinear propagation' (see page 252). The wavelength of light is extremely small; about 6×10^{-7} m or 1/17000 mm. Openings or apertures such as windows, for example, are enormously wide compared to the wavelength. So practically no diffraction occurs when light is incident on the window and the shadows of the window frames are sharp. On the other hand, diffraction of light waves occurs if the hole in a pin-hole camera is made too small. This makes the image blurred as the light waves spread round the hole when they pass through (page 253).

Diffraction of sound waves and microwaves

All waves, whatever their nature, can be diffracted.

A person speaking loudly in a room can be heard round a corner without being seen. Sound waves are therefore diffracted when passing through openings such as doors or windows, perhaps 50 cm or a metre wide (Fig. 22.15). This shows that sound waves have wavelengths of this order (see page 370).

Scanned with CamScanner



Plate 22F Diffraction of water waves round a model of Napier Harbour, New Zealand. (*Hydraulic Research Station*)

The diffraction of microwaves (radio waves) of 3 cm wavelength is easily demonstrated.

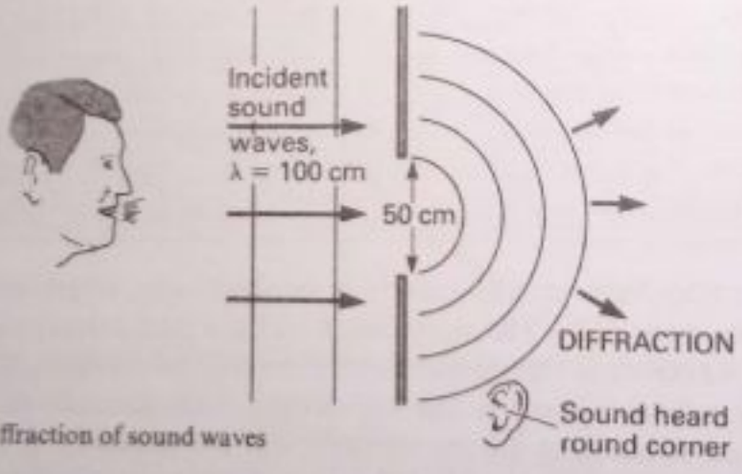


Fig. 22.15 Diffraction of sound waves

Experiment

A microwave transmitter is placed in front of an opening about 1–2 cm wide, formed by the parallel edges of two close hardboard or metal plates (Fig. 22.16). The microwave receiver or detector has a sensitive meter G which produces a deflection when the waves are picked up.

Take the reading on G directly opposite the opening; it is seen to be high. So microwaves pass directly through. When the detector is moved round to a position Y, another high reading is obtained on G. So the microwaves have been diffracted at the opening, as shown.

Scanned with CamScanner

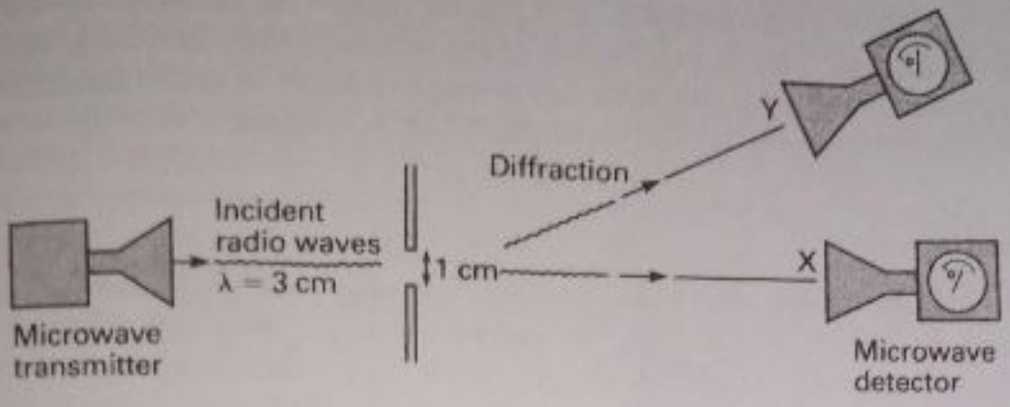
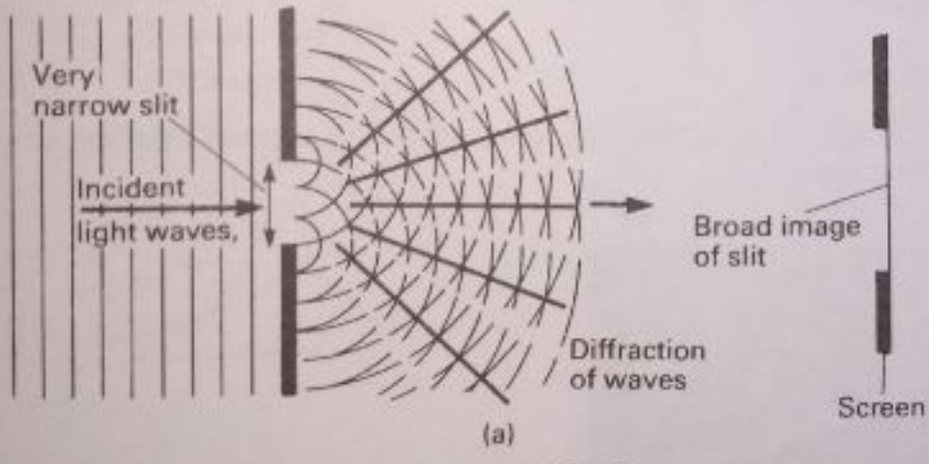


Fig. 22.16 Diffraction of radio waves (microwaves)

Diffraction of light waves

Light passing through a wide opening like a window or letter-box into a darkened place has a sharp boundary where the shadow begins. No diffraction is seen. A narrower opening, such as a slit in a ray-box comb a few millimetres wide, still allows light to pass through in a straight-line path. If, however, an adjustable slit is placed about a metre from a bright overrun ray-box, or motor headlamp bulb, the patch of light on a screen beyond the slit is seen to broaden as the slit is narrowed. Some light has spread round the edges, as shown in the exaggerated sketch in Fig. 22.17a.



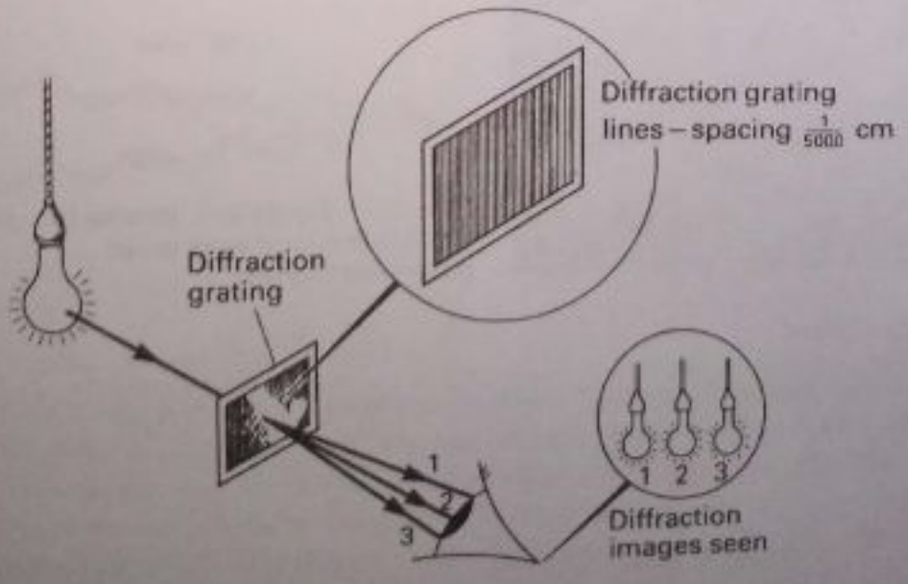


Fig. 22.17 Diffraction of light waves

Scanned with CamScanner

An impressive demonstration of light diffraction is obtained by using a *diffraction grating*. This is a transparent film on which there may be as many as 5000 or more parallel opaque lines occupying a distance of only 1 cm. The clear spaces between the lines are then openings 1/5000 cm apart.

Experiment

Place a fine grating with a few thousand lines per centimetre close to the eye and look at a bright lamp through it (Fig. 22.17b). Observe the appearance of the lamp and the colours. Turn the grating round as you look through it at the lamp. Repeat with a coarse grating which has a few hundred lines per centimetre and note the difference compared with the first grating.

Results: A few lamps are seen with the first grating and many lamps are seen with the second grating. So the light from the lamp has been diffracted at the narrow openings of the grating.

Colours ranging from blue to red, the spectrum of white light, are also seen. Because they are diffracted in different directions, the colours have different wavelengths. Also, since blue light is diffracted less than red light at the narrow openings, this means that the blue light passes more easily through the openings without change of direction. So the wavelength of blue light is less than that of red light.

Interference of waves

Water waves

Waves which spread out from two close vibrating sources interfere at places where the waves overlap.

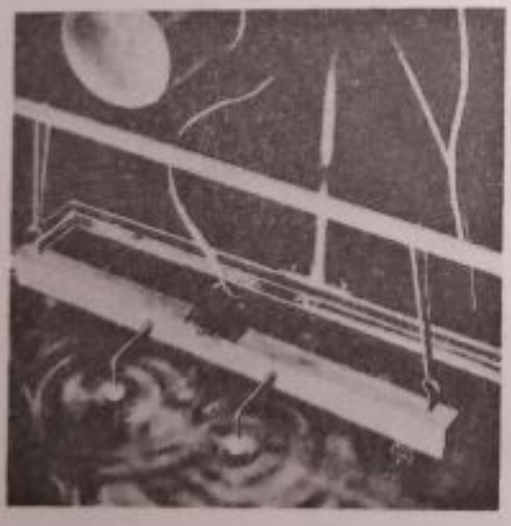


Plate 22G Ripple tank, showing two vibrators and interference of water waves. (*Griffin and George*)

Experiment

As shown, Plate 22G, two small spherical vibrators are placed near each other in a ripple tank so that they just touch the water surface. The two vibrators are then set in motion, and the waves from each overlap or interfere. The water surface is seen to be undisturbed in some directions but higher than normal in others. This 'interference pattern' can be varied by moving the two vibrators closer and then further apart. In each case draw a sketch of the pattern.

Scanned with CamScanner

Explanation An interference pattern is shown in Fig. 22.18a. It can be explained by *adding the two waves* which arrive at any point of the water surface. The vibrators A and B are always in step or 'in phase' with each other and the two waves also have the same wavelength and amplitude. Suppose the point P on the surface is equidistant from A and B, so that AP = BP (Fig. 22.18b). In this case the two waves arrive *in step* at P. So the crests arrive together, forming a crest of twice the height of each, and the troughs arrive together, forming a trough of twice the depth of each. The combined or *resultant* wave at P is therefore twice as big as each wave. We say that the large resultant wave is due to *constructive interference* of the two waves from A and B at P (Fig. 22.18b).

Now suppose we consider a point Q in a direction parallel to AB, where BQ is longer than AQ by $\lambda/2$, half a wavelength. This time the crests of the wave from A say arrive at the same time as the *troughs* of the wave from B, and vice-versa. As shown in Fig. 22.18c, the resultant is zero. This is called *destructive interference* of the two waves.

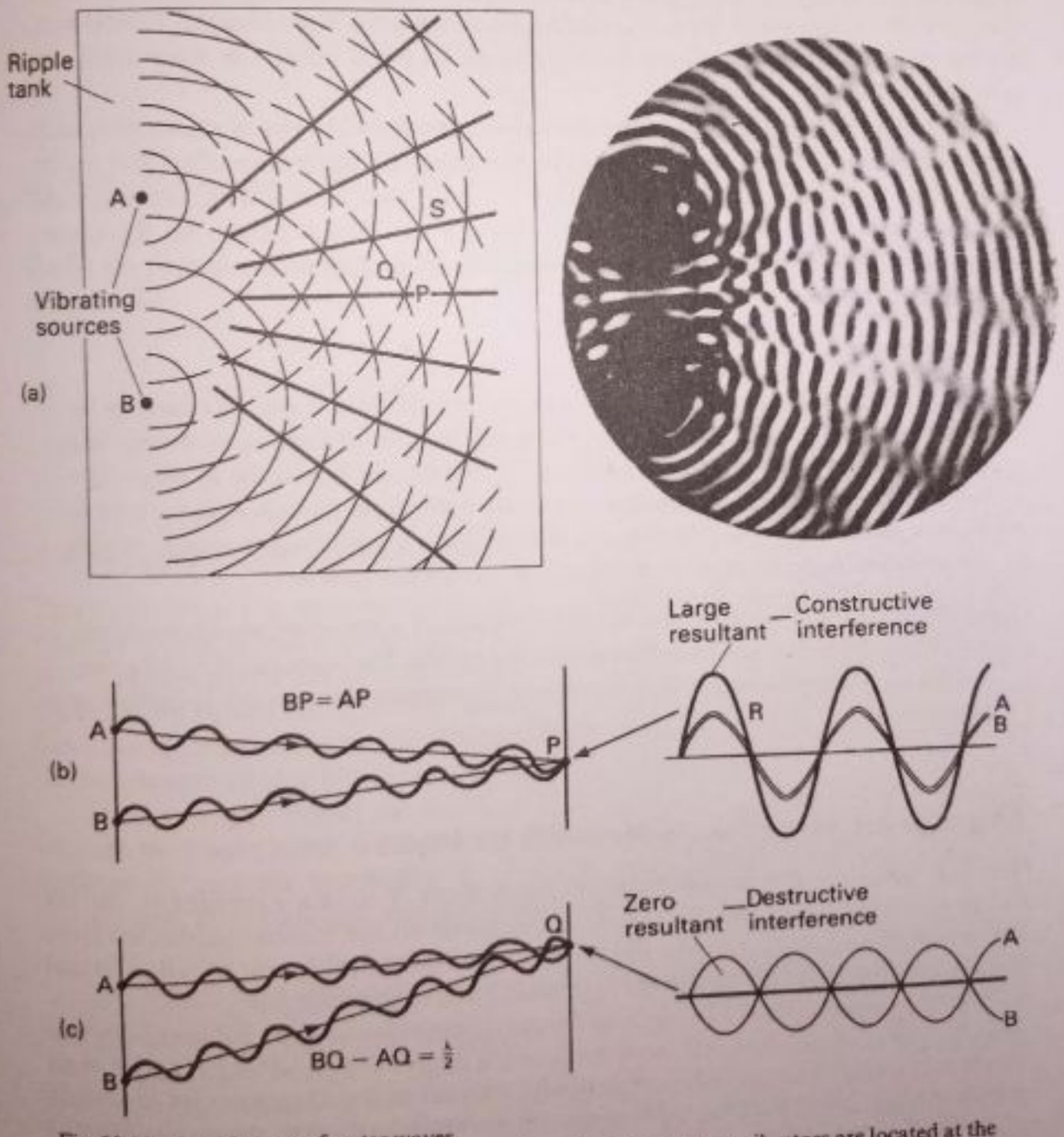


Fig. 22.18 Interference of water waves
The photograph shows interference in an actual ripple tank. The two vibrators are located at the dark patches on the left. (*Griffin and George*)

Scanned with CamScanner

If we move to a point S where BS is longer than AS by λ, one wavelength, the crests and troughs of the two waves arrive in phase at S. So constructive interference occurs at S. Since $BS - AS = \lambda$, measurement of BS and AS enables the wavelength of the water waves to be found. Fig. 22.18a shows how the directions of constructive and destructive interference can be found by drawing circles to represent the waves which spread out from A and B. The interference at any point, X say, depends on the *path difference* between the waves reaching X from A and B, which is $AX - BX$. If this is a whole number of wavelengths, there is constructive interference or reinforcement at X. If the path difference is an odd number of half wavelengths, there is destructive interference or cancellation.

Interference of light waves · Young's two-slit experiment

Interference between two light waves is more difficult to show than with water waves. Firstly, light waves have very short wavelengths, for example, 1/1600 mm. So the two light sources must be very close to each other for their waves to interfere.

Secondly, the two light sources must send out waves always in phase with each other, called *coherent* waves, otherwise a permanent interference effect will not be seen. Two similar lamp filaments, close to each other near a white screen, send out light waves which are always out of phase with each other. So the screen appears bright due to the lamps but no interference effect is seen on it.

Experiment

Fig. 22.19a shows a simple light interference demonstration. It was first done by Sir Thomas Young in 1800. Two narrow slits A and B, about 0·4 mm apart, are cut in a microscope slide coated with an opaque paint. The slits are illuminated by bright light from the vertical filament of a ray-box lamp L; the light passes through a narrow slit S in the box about 1 mm wide. The two slits A and B then act as coherent sources of light waves. Some bright and dark bands are seen on a white screen at W.

Explanation: This is exactly similar to the interference of water waves already discussed. The two waves from A and B interfere where they overlap. The bright bands on the screen are due to constructive interference. The dark bands between them are due to destructive interference.

Measurement of wavelength of light

We can measure the wavelength of light from Young's experiment. In the exaggerated sketch in Fig. 22.19b, O is the point on the screen equidistant from the slits A, B and NO is the normal at N, where N is the midpoint of AB. A bright band is obtained at O because the two waves arrive in phase here.

Now suppose we move from O to the next bright band at P. BP is exactly one wavelength λ longer than AP. So if we cut off a length PQ on BP equal to AP, then BQ is equal to λ. The line PN is practically normal to AQ. The line ON is normal to AB. So the angle PNO or θ between the normals is equal to the angle BAQ.

From triangle PNO, $\tan \theta = y/D$, where y is the distance PO between the bright bands and D is the distance of the screen from the slits. From triangle

Scanned with CamScanner

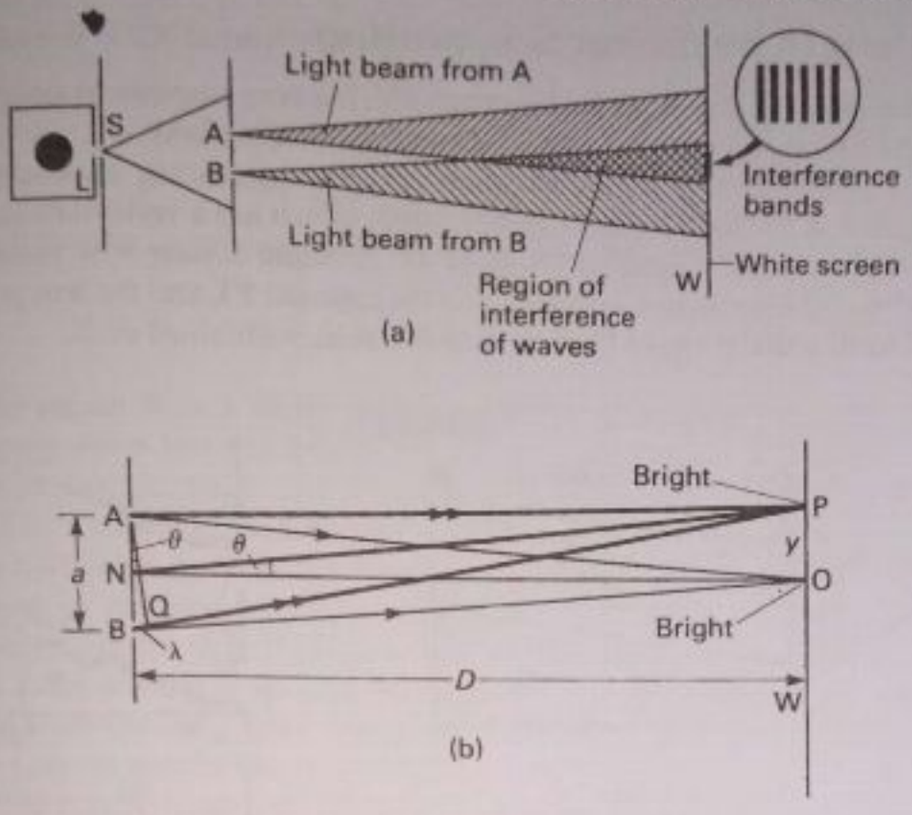


Fig. 22.19 (a) Interference of light waves–Young's experiment (b) Wavelength calculation

BAQ, $\sin\theta = \lambda/a$. The angle θ is so small that its sine is practically equal to its tangent. So

$$\frac{y}{D} = \frac{\lambda}{a}$$

$$\therefore\ y = \frac{\lambda D}{a}, \text{ or } \lambda = \frac{ay}{D}$$

Wavelengths of blue and red light

This result for y, the separation between two bright bands, enables us to find out whether blue light has a longer or shorter wavelength than red light. Place a blue filter and then a red filter in quick succession over the light box slit S (Fig. 22.19a). Observation shows that the blue bands are closer together than the red bands. Since the separation y is smaller for blue light, we see from the formula for y that the wavelength of blue light is shorter than that of red light.

To find a rough value of the wavelength of light, measure the average distance between the bright bands on the screen in Fig. 22.19a using a millimetre scale, the distance D, and the separation a of the slits A and B. In one experiment, $y = 1{\cdot}5$ mm, $D = 1000$ mm and $a = 0{\cdot}4$ mm. So

$$\lambda = \frac{0{\cdot}4 \times 1{\cdot}5}{1000} = \frac{0{\cdot}6}{1000} = 0{\cdot}0006 \text{ mm}$$

$$= 6 \times 10^{-4} \text{ mm} = 6 \times 10^{-7} \text{ m}$$

This is an approximate value for the wavelength of light. As we have seen, the wavelength varies with the colour of the light. The extreme ends of the visible spectrum are about $4{\cdot}5 \times 10^{-7}$ m for violet and $7{\cdot}0 \times 10^{-7}$ m for red light. So the eye is sensitive to a very narrow band of wavelengths.

Scanned with CamScanner

Wavelength by diffraction grating

The diffraction grating, described on page 355, is a very convenient apparatus for measuring the wavelength of light by an interference method.

Fig. 22.20a shows a simple arrangement for measuring approximately a wavelength λ of the light from a ray-box lamp, which has a vertical filament F. A cylindrical lens L is placed in front of F. A white screen S is placed at an appreciable distance from L compared to the distance FL and the lens position is adjusted until a sharp image O of the line filament is obtained on S.

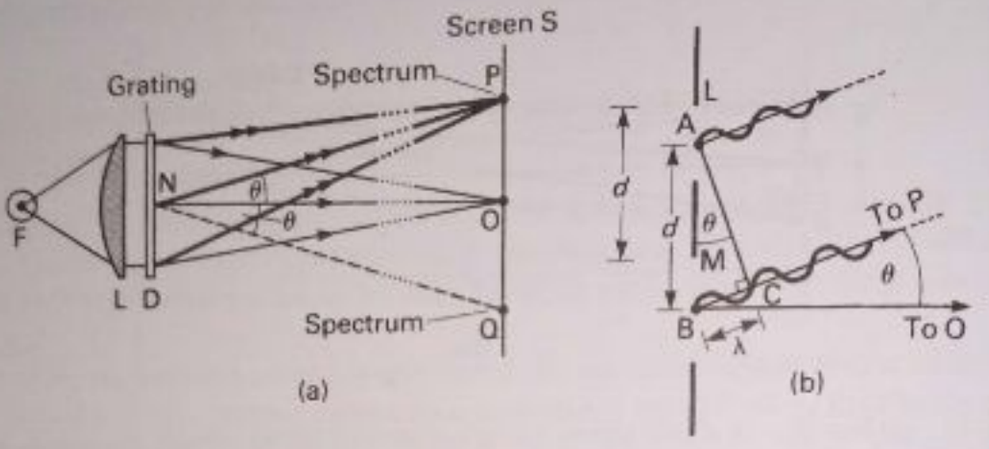


Fig. 22.20 Wavelength of light by diffraction grating

The diffraction grating D is now placed in front of L, as shown. The white sharp line of the filament remains central at O but a coloured spectrum is also obtained at both P and Q on either side. The red is further from O than the blue. So the angle of diffraction θ, which is the angle between the directions NO and NP (or NQ), is greater for red light than for blue light. Theory discussed shortly shows that the angle θ is related to the wavelength λ of the light by

$$d \sin \theta = \lambda$$

where d is the width of the grating 'element', the distance occupied by one slit and one line. Since θ is greater for red light, we deduce that the wavelength of red light is longer than that of blue light.

Measurements and calculation

To find a value for the wavelength of red light, the distance PQ between the positions of the red end of the spectrum at P and Q and the distance NO are both measured. The angle θ can then be found, since

$$\tan \theta = \frac{\frac{1}{2}\text{PQ}}{\text{NO}}$$

In one such measurement, the grating used had 6000 lines per cm, PQ was 22 cm and NO was 26 cm. So

$$\tan \theta = \frac{\frac{1}{2} \times 22}{26} = \frac{22}{2 \times 26} = 0{\cdot}423$$

Scanned with CamScanner

From tables, $\theta = 23°$ approximately. Also, $d = 1/6000$ cm. So

$$\lambda = d \sin\theta = \frac{1}{6000} \sin 23° = 6{\cdot}5 \times 10^{-5} \text{ cm}$$
$$= 6{\cdot}5 \times 10^{-7} \text{ m}$$

This result is only an approximate value for the wavelength.

Theory

Since the screen S is a fairly long way from the lens, we assume that the overlapping waves travel in a fairly parallel direction after diffraction.

Consider two points A and B separated a distance d in neighbouring clear spaces (Fig. 22.20b). If the path from B to the screen is one wavelength λ longer than the path from A, there will be constructive interference. All the waves from two points in neighbouring spaces, separated a distance d, will have a path difference of λ in the same direction. So a *bright image* is obtained at P.

If AC is the normal from A to BC, then BC is the path difference. Now BC = AB sin θ from the right-angle triangle ABC, since angle BAC is equal to θ. So $d \sin\theta = \lambda$, as we used in the calculation.

Another bright image is produced at Q on the other side of the normal NO for an angle of diffraction θ, as shown.

A bright image is also obtained on the screen where the path difference is 2λ. This is called the 'second order' diffraction image: the path difference λ produces the 'first order' image. The angle of diffraction θ' is now greater than that for the first order image and is given by

$$d \sin\theta' = 2\lambda$$

Interference of microwaves

Experiments similar to Young's experiment can be carried out very easily with microwaves (radio waves).

Fig. 22.21 shows the arrangement. S sends out microwaves, which pass through two narrow gaps A and B between screens P, Q and R. At X, equidistant from A and B, the overlapping waves produce constructive interference; similarly at Z, where the path difference from A and B is λ. At Y, where the path difference is $\lambda/2$, there is destructive interference. By measurement, BZ – AZ = λ = 3 cm. Poor television reception is sometimes due to interference between radio waves received directly by the aerial and those received after reflection from a neighbouring metal tank or a wall.

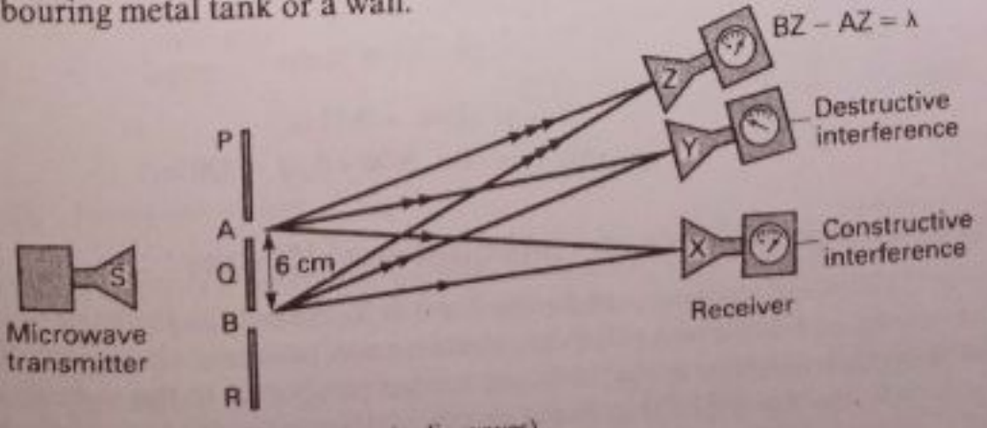


Fig. 22.21 Interference of microwaves (radio waves)

Scanned with CamScanner

Interference of sound waves

Fig. 22.22 shows an arrangement similar to Young's experiment, but for sound waves. A and B are two loudspeakers joined in parallel to the output terminals of an audio-oscillator. Two sound waves of the same frequency are then emitted from A and B.

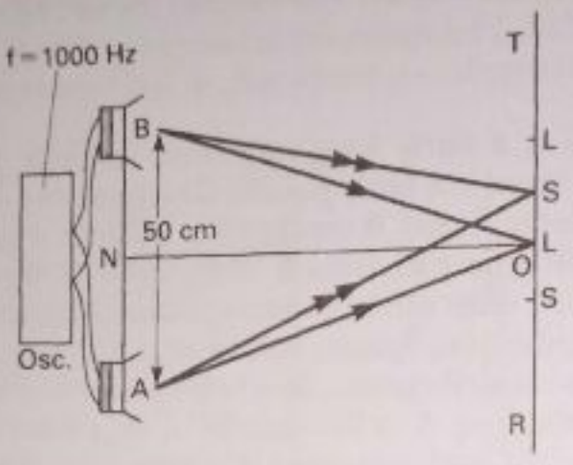


Fig. 22.22 Interference of sound waves

If the waves from A and B are in phase, a loud sound (constructive interference) is heard at O, equidistant from A and B, and at points marked 'L' along RT where the path difference from A and B is λ, 2λ, etc. Between these points, soft sounds are heard at points marked 'S' where the path difference is $\frac{1}{2}\lambda$, $1\frac{1}{2}\lambda$, etc. A loud sound is heard at all points along ON, the perpendicular bisector of AB, since there is no path difference here.

If the connections from the oscillator to one speaker, A say, is reversed, the sound waves from A and B then start in *antiphase*. A crest of the sound wave from A now reaches O at the same time as a trough from B. Destructive interference thus occurs at O. In this case, therefore, practically no sound is heard at O. Further, no sound is heard at all points along ON, unlike the previous case when the two waves from A and B started in phase from the speakers.

Example

In Fig. 22.22, the distance AS is 98 cm and BS is 82 cm, where S is the first position of minimum sound. Calculate the velocity of sound if the oscillator frequency is 1000 Hz.

$$\text{Path difference AS} - \text{BS} = \frac{\lambda}{2}$$

So

$$\frac{\lambda}{2} = 98 - 82 = 16\,\text{cm}$$

$$\therefore\ \lambda = 32\,\text{cm} = 0{\cdot}32\,\text{m}$$

$$\text{velocity, } v = f\lambda = 1000 \times 0{\cdot}32 = 320\,\text{m/s}$$

Polarisation of Waves · Mechanical Waves

In Fig. 22.23(i), YX is a horizontal rope fixed at X and held in the hand at Y. If Y is moved up and down and sideways, *transverse* waves move along from Y to X. The vibrations which produce the waves are perpendicular to the wave direction and in this case the vibrations occur in many different planes as shown by the

Scanned with CamScanner

arrows in the plane at Y.

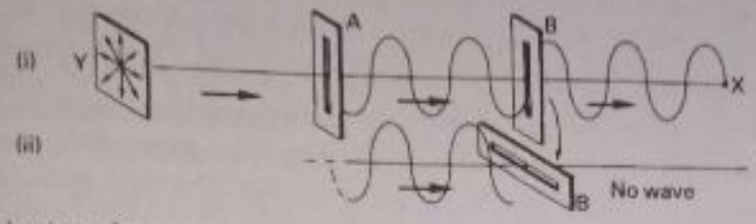


Fig. 22.23 Polarisation of transverse waves

Suppose two parallel slits A and B are now placed along the rope as shown. When the experiment is repeated, the transverse waves in many different planes still travel from Y to A (not shown) but the slit A only allows through a transverse wave whose vibrations are parallel to the slit. We call the wave from A to B a *plane-polarised* because it is due to vibrations in one plane only. At Y the vibrations are in all planes, so the waves travelling from Y to A are not polarised.

The plane-polarised wave travels through the slit B parallel to A but not when slit B is turned *perpendicular* to A, Fig. 22.23(ii). No wave is now obtained along BX.

Polarisation of waves can only be obtained if the waves are transverse. If the rope in Fig. 22.23 is replaced by an elastic cord, longitudinal waves can be produced at Y by pulling and then releasing the cord. The longitudinal waves travel through A and B in Fig. 22.23(i) and also when B is turned perpendicular to A as in Fig. 22.23(ii). So *polarisation can never be produced with longitudinal waves.*

Polarisation of light

Polaroid is the name of a crystalline material which can be made into thin sheets. Due to its crystalline structure, Polaroid allows light waves to pass through it if the vibrations of the waves are parallel to a certain axis of the crystal. In Fig. 22.24(i), suppose OR is the 'allowed' direction in a Polaroid X held up to the window W in daylight.

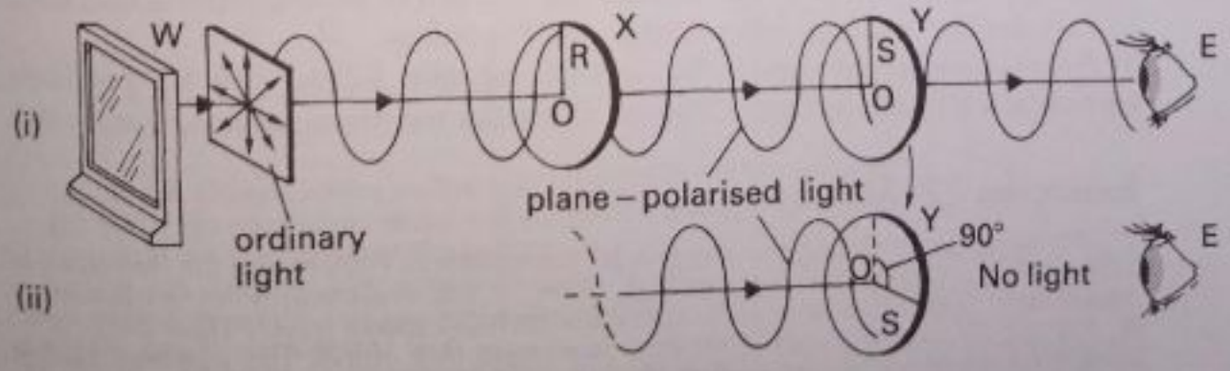


Fig. 22.24 Polarisation of light

Holding another Polaroid Y parallel to X, and with its allowed direction OS parallel to OR, an observer at E sees light passing through X and Y. But if X is kept still and Y is turned round slowly in its vertical plane, E sees the light passing through Y becoming less strong. *No light* passes through Y when its allowed direction OS is perpendicular to OR, Fig. 22.24(ii).

Scanned with CamScanner

This experimental result can only be explained if light waves are *transverse* waves. If light waves were longitudinal, E would still be able to see light passing through Y in Fig. 22.24(ii). Compare Fig. 22.23 with Fig. 22.24.

Experiments show that *all electromagnetic waves*, such as radio waves, for example, *are transverse waves*. Sound is a longitudinal wave—it can not be polarised. Both light and sound waves can produce reflection, refraction, diffraction and interference. But only light waves produce the phenomenon of polarisation.

Summary

1. Water and sound waves are due to vibrations of layers or particles in the medium concerned. Light and radio waves are due to electromagnetic vibrations.
2. Sound waves are longitudinal waves–the wave travels in the same direction as the vibrations. Water, light and radio waves are transverse waves–the waves travel at right angles to the vibrations.
3. For all waves, *velocity* = *frequency* × *wavelength*. Light waves have a very short wavelength, such as $\frac{1}{16\,000}$ cm, and travel with a velocity of 300 million metres per second in air. Sound waves have a long wavelength, such as 50 cm, and travel with a velocity in air of about 340 m/s at normal temperatures.
4. All waves can be *reflected* and *refracted*. Refraction is due to the change of speed when the wave travels from one medium to another.
5. *Resonance*. A vibrating system produces its largest amplitude of vibration when the frequency of an external force is equal to the natural frequency of the system. Mechanical, acoustical, electrical and optical resonances can all be produced.
6. *Interference*. Two close sources, vibrating in step (coherent sources), produce interference effects. In Young's experiment bright and dark bands are produced. from which the wavelength of light can be measured.

A stationary wave is due to interference of two waves travelling in opposite directions.

7. *Diffraction*. All waves may spread round corners or be diffracted. If the opening is very large compared with the wavelength the incident wave travels straight through without change of direction (rectilinear propagation). If the opening is similar in size to the wavelength, diffraction occurs. Light is diffracted by very narrow openings and sound by wide openings.
8. *Polarisation*. Light (and other electromagnetic waves) can be polarised because it is a transverse wave. Sound, a longitudinal wave, can not be polarised.

Exercise 22 (Answers, page 642)

1. The wavelength of signals from a radio transmitter is 1500 m and the frequency is 200 kilohertz (kilocycles per second). What is the wavelength for a transmitter operating at 1000 kHz and at what speed do the radio waves travel? (O and C)

2. Fig. 22.25 shows a snap-shot of a waveform in a string. The numbers on the diagram show scales of centimetres. The speed of travel of the waves is 8·0 metres per second. With reference to this wave-motion:

(a) What is the wavelength? (b) What is the amplitude? (c) What is the frequency?

3. The strip in Fig. 22.26 represents part of the electromagnetic spectrum extending

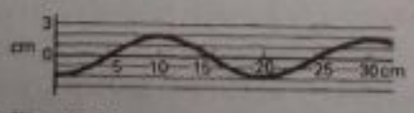


Fig. 22.25

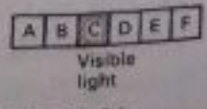


Fig. 22.26

Scanned with CamScanner

on both sides of the region of visible light, C. If A is the region of shortest wavelengths and F the region of longest wavelengths, name which section will represent (a) the ultra-violet region, (b) the X-rays, (c) the infra-red region.

State one difference in the behaviour of ultra-violet and X-rays. (L)

4. Explain clearly, with diagrams, the difference between a transverse and a longitudinal wave. State *four* ways in which light waves differ from sound waves. (N)

5. A parallel beam of light in air is refracted at a plane glass surface as shown in Fig. 22.27. Find, by using construction lines which are to be shown on the diagram, the ratio of the speeds of light in air and glass. (C)



Fig. 22.27

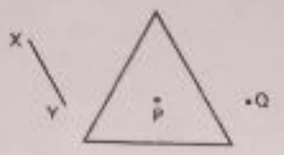


Fig. 22.28

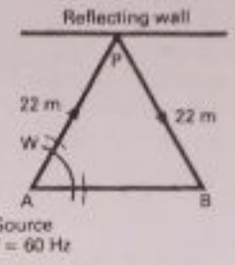


Fig. 22.29

6. Use the fact that light waves travel more slowly in an optically denser medium to explain, with the aid of a diagram, the refraction of light on entering glass. (N)

7. A sound of wavelength 70 cm in still air travels 840 metres in 2·5 seconds. Calculate (a) the speed, (b) the frequency of the sound. (C)

8. XY is a wavefront of a parallel beam of light incident upon a glass prism (Fig. 22.28). Show on a diagram the approximate positions of the wavefront as it passes through the points P and Q, and mark in by an arrow at each wavefront the direction in which the wavefront is travelling. (N)

9. Explain the difference between a transverse and a longitudinal wave. Give an example of each.

Plane waves are incident on a narrow gap. Draw the appearance of the waves or draw the wavefronts before and after they have passed through the gap, (a) when the wavelength of the waves is large, (b) when the wavelength is very small. What name is given to this effect?

Describe an experiment which shows that sound waves will not travel through a vacuum but light waves will. (L)

10. The table below represents a blank chart of the whole spectrum of electromagnetic waves, with the wavelengths increasing from left to right. It is not drawn to scale, and when completed is merely intended to show the correct order in which the different radiations occur. Letters E, F, C and D represent different types of radiation.

	E	F	C	Visible spectrum	D
Short wavelengths		A		B	Long wavelengths

(a) Which colours of the visible spectrum appear at A and at B?

(b) What are the names of the radiations in the regions C and D?

(c) What are the names of the radiations in the regions E and F?

(d) Which regions of the spectrum contain radiations capable of ionising the air in their path and which region of the spectrum contains the radiation which cannot excite visible fluorescence? (C)

11. In Fig. 22.29, A is a source sending out sound waves of frequency 60 Hz (cycles per second) in all directions. At B, sound waves which have travelled directly along the path AB and those which have been reflected along the path APB are both received. The speed of sound in air is 330 m/s.

(a) What is the wavelength of the sound waves?

(b) Considering a given wavefront, such as W, what is the time interval between the arrival at B of the part that has travelled along AB and the part that has been reflected along APB?

(c) What is the path difference, in wavelengths, between the two wave paths from A to B?

Scanned with CamScanner

(d) Why is the sound which is received at B louder than it would be if the receiver were moved a metre or two nearer to A along the line BA? (O)

12. Explain the terms *interference* and *diffraction* of waves.

Describe an experiment to demonstrate interference or an experiment to demonstrate diffraction.

How may the following observation be accounted for?

When two identical loudspeakers pointing towards each other and several metres apart in open air are connected in parallel to the same source of a pure tone, the sound detected by a microphone at a point equidistant from the speakers is either louder than the sound from a single loudspeaker *or* zero.

If, along the line joining the two loudspeakers, successive maxima of sound can be detected 10·0 cm apart, what is the frequency of the sound? (Take the velocity of sound in air as 330 m/s.) (O and C)

13. A diffraction grating having 6500 lines/cm is placed at right angles to a parallel beam of white light. In what directions will light of wavelength 6×10^{-5} cm have a maximum intensity after passing through the grating? (O and C)

14. Why are the lines on a diffraction grating regularly spaced? Why are they closely spaced?

A grating of 6000 lines per cm is illuminated normally by yellow light of wavelength $6{\cdot}0 \times 10^{-5}$ cm.

(a) Why do 'interference' and 'diffraction' occur,

(b) at what angle to the normal are the first and second order images obtained,

(c) if infra-red rays are incident normally on the grating, what wavelength will coincide with the second order image? ($d \sin\theta = \lambda$ gives 'first order' image direction; $d \sin\theta = 2\lambda$ gives 'second order' image direction.)

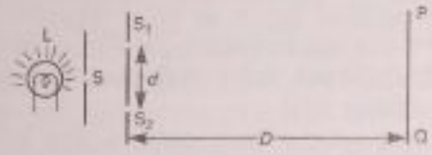


Fig. 22.30

15. Fig. 22.30, not drawn to scale, represents an experiment for showing Young's fringes.

(a) What would be a likely size for the distance d between the slits S_1 and S_2? What would be a likely size for the distance D from the slits S_1, S_2 to the screen PQ?

(b) Draw a labelled diagram to show what you would see on the screen PQ.

(c) Mention *three* difficulties you might encounter doing this experiment.

(d) If the lamp L were replaced by one giving light of a single colour, what would you find out about the light from the experiment?

(e) What do you think would happen if the slit S_2 were covered leaving S and S_1 unchanged? (O and C)

Multiple-Choice Questions

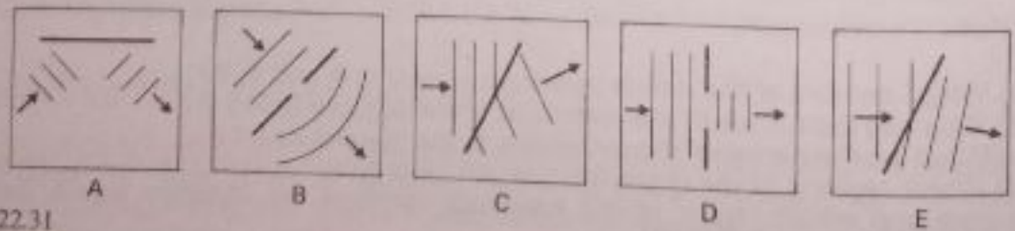


Fig. 22.31

In Fig. 22.31, which of the wave diagrams A to E best illustrate:

16. water waves travelling from shallow to deep water

17. diffraction of waves

18. light waves travelling from water to air.

19. Plane waves travelling parallel to the principal axis are incident on a concave mirror. The waves are reflected

A towards the centre of curvature of the mirror B as plane waves

C with the same wavelength D towards infinity

E as circular waves diverging from a point behind the mirror

Scanned with CamScanner

SOUND



A recording studio. The acoustic properties of the studio are improved by absorbent material on the walls. (*Electrical Musical Industries Ltd*)

Scanned with CamScanner

23. Basic Principles of Sound · Strings and Pipes

Sound waves and matter

Sound waves travel through gases, liquids and solids. When a train approaches a station, the rails can be heard vibrating before the train comes into view. The metal of the rail carries the sound, which is then transmitted to our ears through the air. 'Depth-sounding' uses the fact that sound waves travel in water (page 348).

Experiment

To demonstrate that sound waves cannot pass through a vacuum, the apparatus shown in Fig. 23.1 is assembled. Thick grease round the cork and the base of the bell-jar provides an air-seal.

When there is air in the vessel and the battery is connected to the leads of the suspended bell, the bell is heard ringing. The pump is switched on to remove the air, the sound dies down and the bell becomes fairly silent. The clapper, however, can be seen vibrating although no sound is heard, showing that light waves travel through a vacuum although sound waves do not. When air is allowed to return to the bell-jar the sound is heard once more.

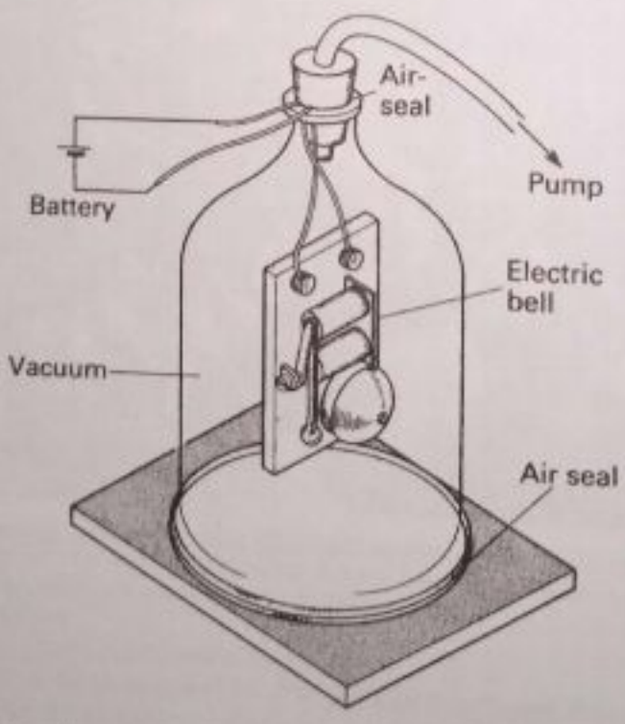


Fig. 23.1 Effects of air and a vacuum on sound waves

Scanned with CamScanner

There is little atmosphere on the moon to carry sound waves. For this reason astronauts on the moon talk to each other by radio even when they are close together.

How sound waves travel

Sound waves are carried through air by the displacements of air particles. Fig. 23.2a shows the undisturbed layers of air. Fig. 23.2b shows their displacements at a particular instant when sound passes through the air if we could 'freeze' the wave. Fig. 23.2c shows the wave pattern when we plot the displacements at the different places–a displacement to the right is positive and to the left is negative. This pattern changes as the wave travels. Since the vibrations of the air are in the same direction as the wave, a sound wave is a *longitudinal* wave (see page 343).

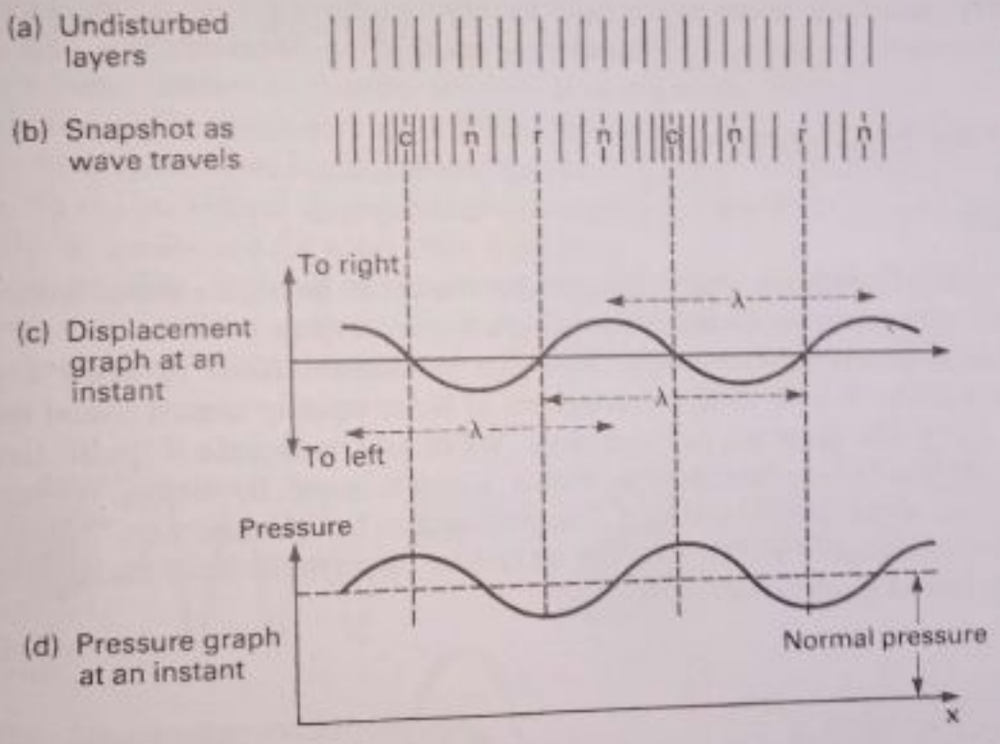


Fig. 23.2 Sound wave in air

When the layers of air are more crowded together at a particular place, as at c in Fig. 23.2b, the air pressure is above normal. We call this a *compression*. When the layers are less crowded, as at r in Fig. 23.2b, the air pressure is below normal. This is called a *rarefaction*. Compressions and rarefactions in sound waves are detected by the carbon microphone in a telephone (page 460).

Velocity of sound waves

The *amplitude* of a wave is the maximum displacement from its undisturbed position. The *frequency* is the number of waves per second passing a given point; or the number of cycles per second of its vibrating particles (the unit of frequency is the hertz, Hz). The *wavelength* is the distance between successive places of maximum displacement (similar to crests of water waves) or of successive compressions.

Scanned with CamScanner

If f is the frequency and λ is the wavelength, then, as shown on page 344, the velocity is given by

$$v = f\lambda$$

In air at normal temperature, the velocity of sound is about 340 m/s. The wavelength λ when a note of frequency f equal to 1000 Hz is played is therefore

$$\lambda = \frac{v}{f} = \frac{340}{1000} = 0{\cdot}34 \text{ m}$$

The velocity of sound does not depend on the air pressure. The velocity is the same at the foot of a mountain as at the top.

The velocity of light is enormously greater than the velocity of sound. So in a thunderstorm, distant observers see the lightning before they hear the thunder produced at the storm-centre. The velocity of sound is about $\frac{1}{3}$ km/s. Neglecting the very short time taken by the light to travel to the observer, a time interval of 6 s between the lightning and thunder means that the storm-centre is 2 km away.

Sound characteristics

Pitch

The sounds from boys or girls at sporting events can be distinguished from those of older spectators by their relatively high pitched voices.

Pitch depends on frequency. This can be demonstrated by using a metal circular plate W with different numbers of holes equally spaced round its axle (Fig. 23.3). The plate is a form of siren. When air from a tube T 'puffs' through the holes and W is turned fast by a motor, a note is heard. By turning W faster the frequency of the 'puffs' increases. The pitch is then heard to increase. At the same speed of rotation of W, blowing the air harder through the holes makes the note louder but its pitch remains the same.

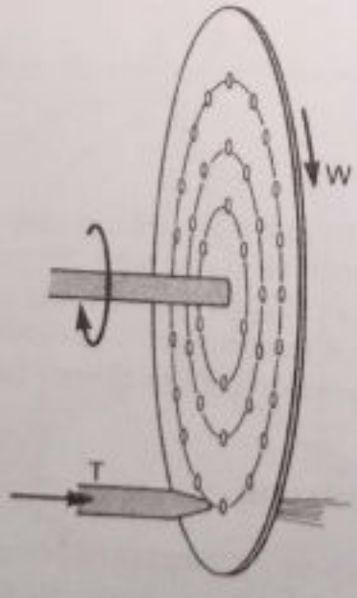


Fig. 23.3 Pitch and frequency

From this it follows that the pitch of a note depends on its frequency. A note of high pitch such as that from a baby crying, has a high frequency; a whistle may have a frequency of about 1000 Hz. A note of low pitch such as that from a gruff

Scanned with CamScanner

voice has a low frequency; a low hum may have a frequency of about 100 Hz (Fig. 23.4). Human beings cannot hear notes of frequencies greater than about 20000 Hz but some animals, including dogs and cats, can hear much higher notes. Dolphins are known to produce sounds of over 100000 Hz, which enable them to find each other by echoes under water.

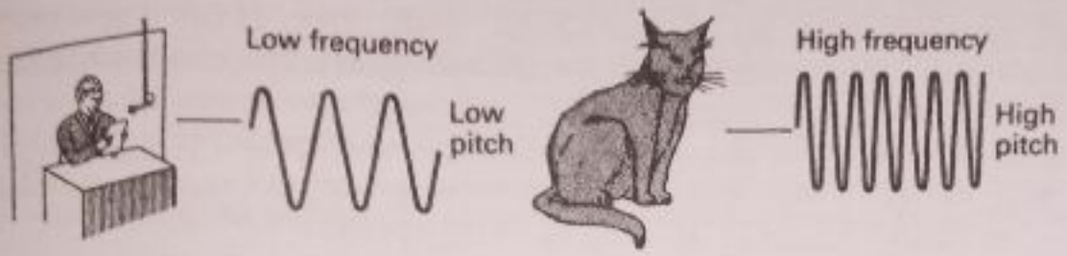


Fig. 23.4 Pitch depends on frequency

Loudness and intensity

If the volume control of a radio receiver is accidentally turned up so that the sound suddenly becomes very loud, light articles on or near it begin to jump. The energy from the receiver is much more than before, and this disturbs the air layers more. So the maximum displacement or amplitude increases. We say that the *intensity* or *loudness* of the sound has increased.

Loudness, then, depends on the amplitude of the wave. As shown in Fig. 23.5, a porter and a child may both shout a note of the same frequency but the note from the porter is louder because the amplitude of the sound wave is larger.



Fig. 23.5 Loudness depends on amplitude

Generally, the greater the mass of air which can be set into vibration the louder will be the sound. The prongs of a vibrating tuning-fork, for example, make only a small mass of air vibrate. The sound is soft but if the end of the fork is placed on

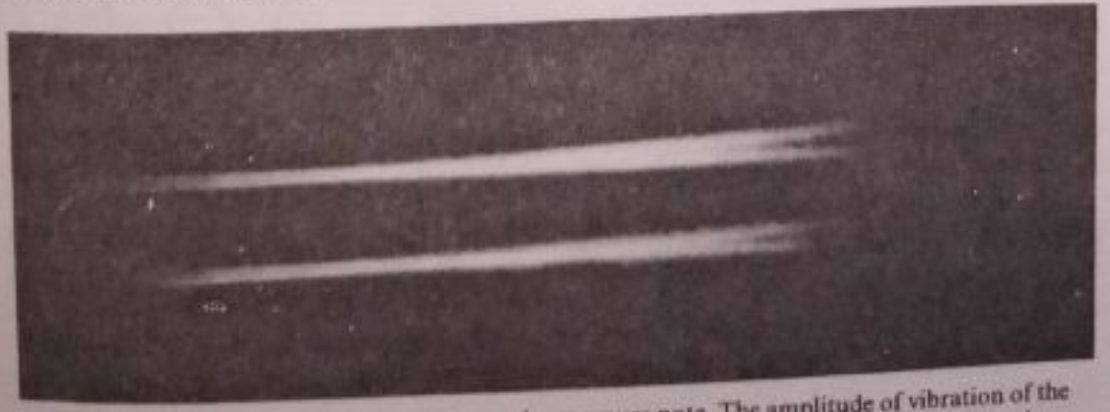


Plate 23A Vibrating tuning fork, which produces a pure note. The amplitude of vibration of the metal prongs increases to a maximum at the free (right hand) end of the fork. (*Crown copyright. Science Museum, London*)

Scanned with CamScanner

a table, the sound is much louder. The large mass of air in contact with the surface of the table is then set into vibration. Very little air is set into motion directly by a violin string, which has a small surface area, but it is attached to a sounding box of large surface area. When the box vibrates it moves a large amount of air.

For similar reasons, the sound from a telephone earpiece is heard distinctly only when the ear is placed close to the earpiece (Fig. 23.6a). The vibrating circular metal plate inside has only a small area. Thus only a small mass of air is set into vibration. On the other hand, a loud sound may be heard from a small transistor set or a television set. This is due to the loudspeaker (Fig. 23.6b). The loudspeaker has a vibrating cone with a relatively large surface area and a large mass of air in contact is therefore set into vibration (page 473).

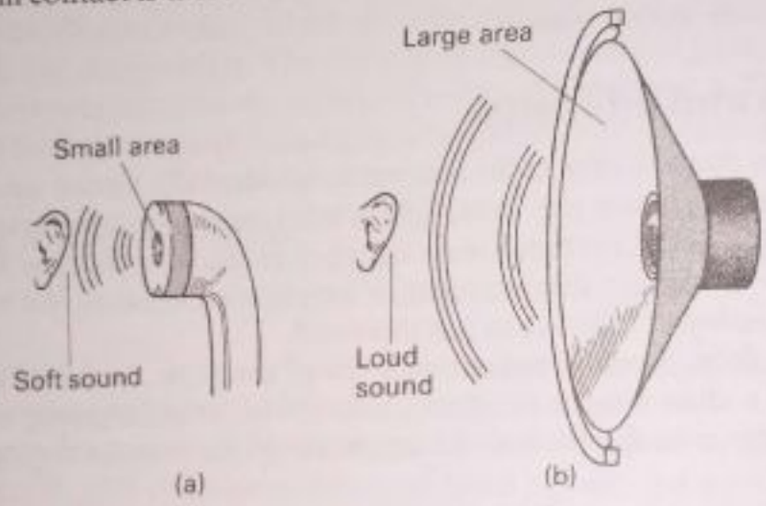


Fig. 23.6 Loud and soft sounds

Quality or tone (timbre)

A well-made tuning-fork can produce a 'pure' note, that is, one with a single frequency. A musical instrument such as a piano or guitar never produces a note of a single frequency. In addition to the main frequency, *overtones* of higher frequency may be present. This is due to the characteristic *waveform* of the note produced by an instrument, as shown in Fig. 23.7. A note of 300 Hz played on a

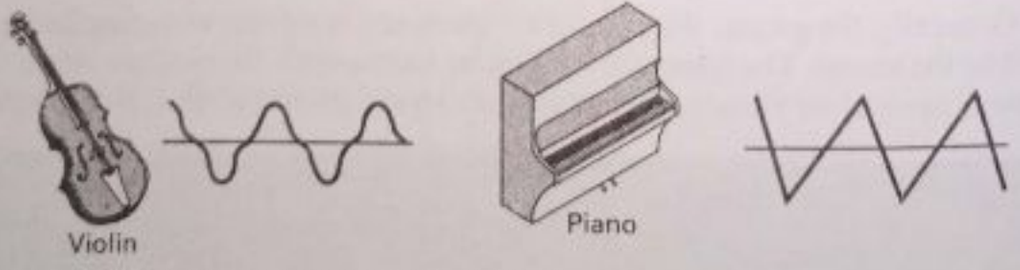


Fig. 23.7 Quality (or tone or timbre) depends on overtones or waveform

violin, for example, may be accompanied by overtones of frequencies 900, 1500 and 2100 Hz. Though quieter than the main frequency, the different overtones produced by different instruments provide a 'background' which helps us to distinguish between the same note played on a violin and a piano, for example.

The quality or timbre of a note, then, depends on the overtones or waveform of the note.

Harmonics are frequencies which are multiples of the fundamental frequency from a musical instrument. If the fundamental frequency of a plucked string is

Scanned with CamScanner

400 Hz, harmonics have frequencies which are two, three, four times as high, at 800, 1200, 1600 Hz. Some harmonics may not be present in the overtones from a sounding instrument.

An *octave* of a note is a note of twice the frequency. The octave of 256 Hz is 512 Hz, the octave of 512 Hz is 1024 Hz. The *musical interval* is the ratio of two frequencies; it is the same in the two cases.

Measurement of velocity of sound

Experiment

1. *Echo method* The velocity of sound in air can be roughly measured outside by echoes.

Stand some distance away from, and directly in front of, a large high wall. Clap your hands sharply and listen for the echo. Now clap repeatedly and try to arrange the next clap to coincide with the echo heard. When this is done, time a number of successive claps. Find the time between two successive claps.

Measure the perpendicular distance from where you stand to the wall. The sound takes half the time between successive claps to reach the wall. So velocity of sound = distance from wall/time, and this is calculated.

2. *Stationary wave method* The velocity of sound in air can also be found from measurements on a stationary wave in the laboratory. An oscillator is connected through an amplifier to a loudspeaker, which is turned to face a large smooth board or metal reflector about a metre away (Fig. 23.8). A frequency of 2000–3000 Hz is suitable. Stationary waves are then set up between the loudspeaker and the reflector (page 374). A small microphone, connected to the Y-plates of a cathode-ray oscilloscope, is now moved along a ruler towards the board or away from it. Wave traces are seen which diminish to a minimum (node) and then increase to a maximum (antinode). The wavelength λ is twice the average distance between successive nodes or antinodes, as explained on page 375; this distance is measured.

Calculation: Knowing the frequency f of the sound and wavelength λ, the velocity is calculated from $v = f\lambda$.

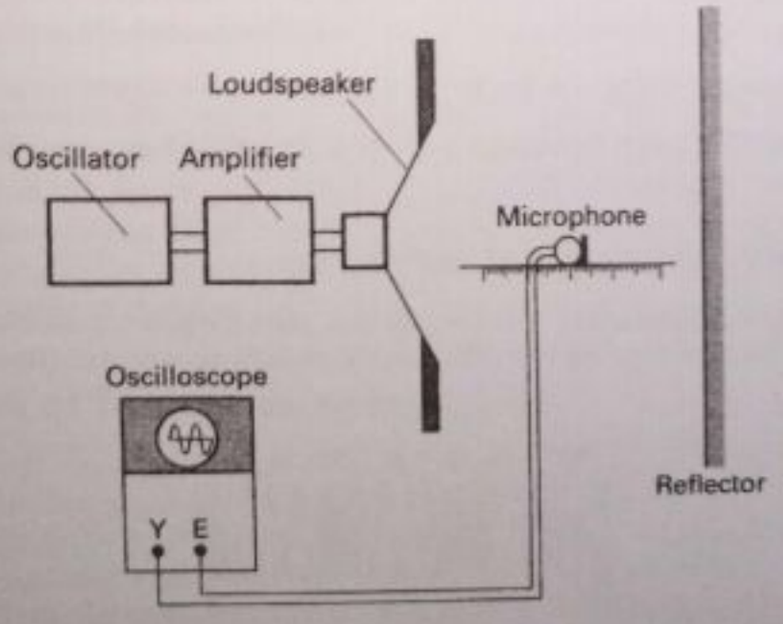


Fig. 23.8 Determination of velocity of sound

Scanned with CamScanner

By changing the frequency, it can be shown experimentally that the velocity of sound is independent of frequency. This is also shown by listening to an orchestra. Although different instruments send out simultaneously sound waves of different frequencies, when the sound reaches us we hear the orchestra playing in time.

Waves in strings and pipes

Stationary transverse wave

When a stringed instrument such as a violin is played, a transverse wave travels along the vibrating string. At the fixed end of the string the wave is reflected back. We then have two waves travelling in opposite directions along the string and so they interfere.

The resultant wave can be seen by attaching one end of a thread to the clapper C of an electric bell and passing the thread over a pulley P (Fig. 23.9). A scale-pan is connected to the thread at the other end. When weights are placed on the scale-pan and adjusted, the thread can be seen vibrating with stationary loops along its length. The successive wave positions are shown by 1, 2, 3, 4, 5. This type of wave is called a *stationary transverse wave*.

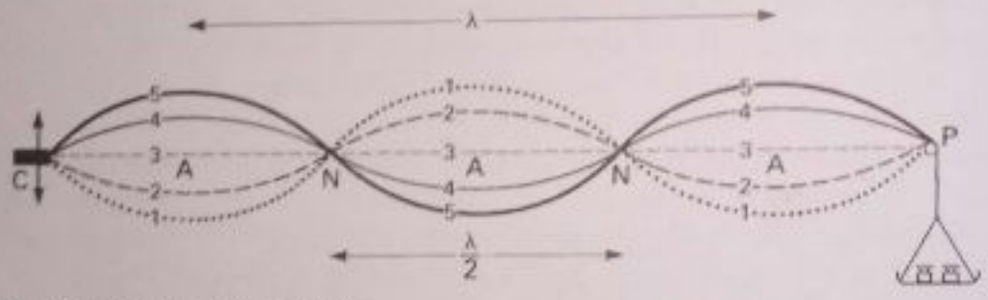


Fig. 23.9 Stationary transverse waves

In a stationary wave, some points N are always at rest; these are called displacement nodes. Other points between the nodes are vibrating with varying amplitude. The points A half-way between the nodes are vibrating with maximum amplitude; these are called displacement antinodes. We see from Fig. 23.9 that if λ is the wavelength of the wave travelling along the string,

the distance NN or AA between successive nodes or antinodes is $\lambda/2$

the distance NA between a node and the nearest antinode is $\lambda/4$.

Stationary longitudinal wave

Fig. 23.10 shows a stationary wave set up in a 'slinky' coil by attaching one end Y to a fixed block and moving the other end Z rapidly to-and-fro (in-and-out) in a

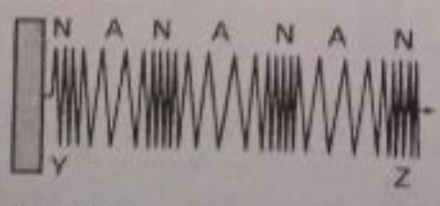


Fig. 23.10 Stationary longitudinal wave

Scanned with CamScanner

horizontal direction. This is a stationary *longitudinal* wave because the vibrations are in the same direction as the wave produced. The distance between two antinodes A, or two nodes N, is equal to $\lambda/2$, where λ is the wavelength of the wave along the coil.

The stationary wave is an important wave as it occurs in many different branches of physics. In sound, it is produced along the vibrating string of a stringed instrument, as we have seen. It is also produced along the vibrating air column inside an organ pipe or wind instrument, discussed later.

The sonometer

The frequency of the notes obtained from a stringed instrument can be studied with the aid of a sonometer. A thin wire is attached to a hollow sounding box at one end F (Fig. 23.11a). At the other end it is passed over a grooved wheel P and kept taut by a heavy weight W of several kg. The bridge at C is adjusted to change the length l of wire which vibrates.

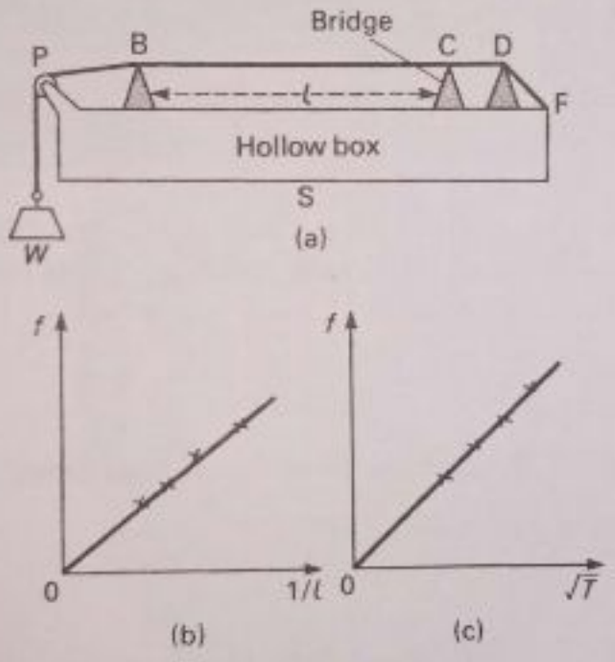


Fig. 23.11 Sonometer experiment and results

As we see shortly, the sonometer can be used to find out how the frequency of a note is altered when the length of string is altered under a constant tension. This is done by a violinist, for example, by placing a finger firmly on the violin string at different places. It can also be used to investigate how the frequency of a fixed length of string varies with the tension. Violin strings are tuned by altering their tension by means of the pegs.

Variation of frequency with length

Preliminary If the length BC of string (shown in Fig. 23.11a) is made say 20 cm by moving the bridge, the note obtained by plucking the wire gently in the middle is heard to be fairly high in pitch. If the bridge is moved so that BC increases to 30 cm, a lower note is obtained when the wire is now plucked in the middle. At a length of 40 cm, the note obtained is recognised as one octave lower in pitch than at 20 cm, and lower in pitch than the note obtained at 30 cm. We therefore

Scanned with CamScanner

conclude that, at constant tension, increasing the length l of string *lowers* the frequency f of the note produced.

Experiment

Relationship between f and L. This can be investigated using six tuning forks of known frequency between 256 Hz and 512 Hz. With a constant weight W to keep the tension constant, the wire BC is tuned in turn to each sounding fork by moving the bridge (Fig. 23.11a).

To help in tuning, if the sounding fork is placed with its lower end on the sonometer box or on one bridge, an inverted V-shaped paper rider, placed in the *middle* of the wire, will vibrate and be thrown off when the wire is tuned to the fork's frequency. This is because the fork, through the wooden box or bridge, makes the wire vibrate in resonance. Alternatively, the length may be adjusted until a throbbing note is heard when the wire is plucked and the fork is sounding at the same time. The rise and fall or throbbing results from interference between the waves reaching the ear from the wire and the fork when their frequencies differ very slightly from each other and produce a so-called 'beat' note.

Measurements

f/Hz	l/cm	$1/l$

Results: A graph of frequency f plotted against $1/l$ results in a straight line passing practically through the origin (Fig. 23.11b). So $f \propto 1/l$.

This is an *inversely-proportional* relationship between f and l, that is, if the length is doubled the frequency is halved, and if the octave (double frequency) is required the length must be halved.

Example

Under constant tension, the note produced by a plucked string is 300 Hz when the length is 0·90 m.

(a) At what length is the frequency 200 Hz?

(b) What frequency is produced at a length of 0·30 m?

(a) Since $f \propto 1/l$,

$$\frac{1/l}{1/0{\cdot}90\text{ m}} = \frac{200\text{ Hz}}{300\text{ Hz}}$$

$$\therefore \frac{0{\cdot}90}{l} = \frac{200}{300}$$

$$\therefore l = \frac{300}{200} \times 0{\cdot}90\text{ m} = 1{\cdot}35\text{ m}$$

(Alternatively, the frequency 200 Hz is lower than the frequency 300 Hz, so the length l must be *longer* than 0·90 m. So $l = 0{\cdot}90\text{ m} \times (300/200) = 1{\cdot}35\text{ m}$).

(b) The length of 0·30 m is one third of 0·90 m. So the frequency f is 3 times the frequency at 0·90 m.

Hence

$$f = 3 \times 300\text{ Hz} = 900\text{ Hz}$$

Variation of frequency with tension and thickness of wire

1. Keeping a *constant length* of wire between the bridges of a sonometer, the tension T in the wire can be altered by altering the weight W in Fig. 23.11. The

Scanned with CamScanner



Plate 23B A lyre guitar, showing the different thicknesses of the strings and the tuning pegs at the tops of them for altering the tension. (*Barnaby's Picture Library*)

frequency f of the note obtained by plucking the wire then alters and experiment shows f is *directly proportional* to $\sqrt{T}$ or $f \propto \sqrt{T}$.

This means that the tension must be increased *four* times to double the frequency or obtain the octave. If the tension is doubled, the frequency will only increase $\sqrt{2}$ or 1·4 times.

2. With the *same tension and length*, experiment shows that (a) a thinner wire emits a note of *higher* frequency and (b) a wire of the same thickness but greater density gives a note of *lower* frequency. More detailed experiments show, in fact, that $f \propto 1/\sqrt{m}$, where m is the mass per metre length of the wire.

Scanned with CamScanner

Violins have four strings of different densities. So m, the mass per metre, is different for each. This helps the violinist to obtain a wide range of frequencies from the strings.

Generally, then, the frequency f of a plucked wire obeys the following laws: (a) $f \propto 1/l$ (constant tension and wire), (b) $f \propto \sqrt{T}$ (constant length and wire), (c) $f \propto 1/\sqrt{m}$ (constant tension and length).

Example

The frequency obtained from a plucked string is 400 Hz when the tension is 2 N. Calculate (a) the frequency when the tension is increased to 8 N, (b) the tension needed to produce a note of frequency 600 Hz.

(a) Since $f \propto \sqrt{T}$ for a constant length and string, then

$$\frac{f}{400\,\text{Hz}} = \sqrt{\frac{8\,\text{N}}{2\,\text{N}}} = \sqrt{4} = 2$$

$$\therefore f = 2 \times 400 = 800\,\text{Hz}$$

(b) Suppose T is the tension required. Then

$$\sqrt{\frac{T}{2\,\text{N}}} = \frac{600\,\text{Hz}}{400\,\text{Hz}} = \frac{3}{2}$$

Squaring both sides

$$\frac{T}{2\,\text{N}} = \frac{9}{4}$$

$$\therefore T = \frac{9}{4} \times 2\,\text{N} = 4{\cdot}5\,\text{N}$$

Fundamental frequency formula

These experimental results can be explained by considering the stationary wave set up along a string. The lowest possible note or *fundamental frequency* is produced by plucking the string gently in the middle, which sets up the simplest stationary wave, shown in Fig. 23.12. A displacement node N is obtained at each end of the string because these points cannot move, and a displacement antinode A is obtained in the middle where the string is most free to move.

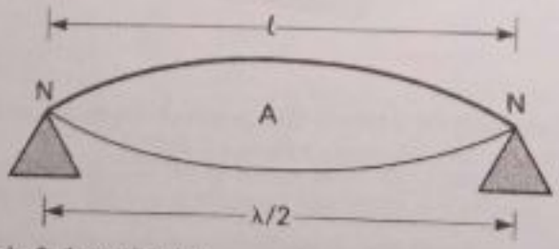


Fig. 23.12 Fundamental of a bowed string

We now see that the length of the string = NN = $\lambda/2$, where λ is the wavelength of the wave produced. So if the length l = 20 cm, for example, then $\lambda/2$ = 20 cm or λ = 40 cm. Now on page 344 it was shown that, for any wave, $v = f\lambda$, where v is the velocity of the wave and f is the frequency of vibration. So

$$\text{frequency } f = \frac{v}{\lambda} = \frac{v}{40}$$

Scanned with CamScanner

Generally it can be seen that, for a string or wire of length l,

$$f = \frac{v}{\lambda} = \frac{v}{2l}$$

or, for a given velocity v,

$$f \propto \frac{1}{l}$$

This explains the experimental result obtained with the sonometer.

The velocity v of the wave along the wire depends on its tension T and its mass per unit length m. The exact relationship is $v = \sqrt{T/m}$. Since, from above, $f = v/2l$, it follows that

$$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

From this relationship we see that (a) $f \propto 1/l$ (constant tension and wire) and (b) $f \propto \sqrt{T}$ (constant length and wire).

Harmonics in a stretched string

The lowest note obtained from a bowed string is called its *fundamental* and when it is produced, the string vibrates in one loop as shown in Fig. 23.12. Higher frequencies than the fundamental can be obtained from a bowed string, however, and these are called harmonics.

Experiment

Fig. 23.13a shows how a harmonic is obtained using a sonometer wire. Suppose the length of the wire between the bridges is 96 cm. The wire can be 'damped' at a point B one-third along the wire from the bridge O, so that OB is 32 cm. Damping can be done by touching the wire at B lightly with a finger or with a rolled piece of blotting paper. Small paper riders R, in the form of an inverted V, are placed at points further along the wire, 48, 64 and 80 cm from O, to show any vibration at these points of the wire.

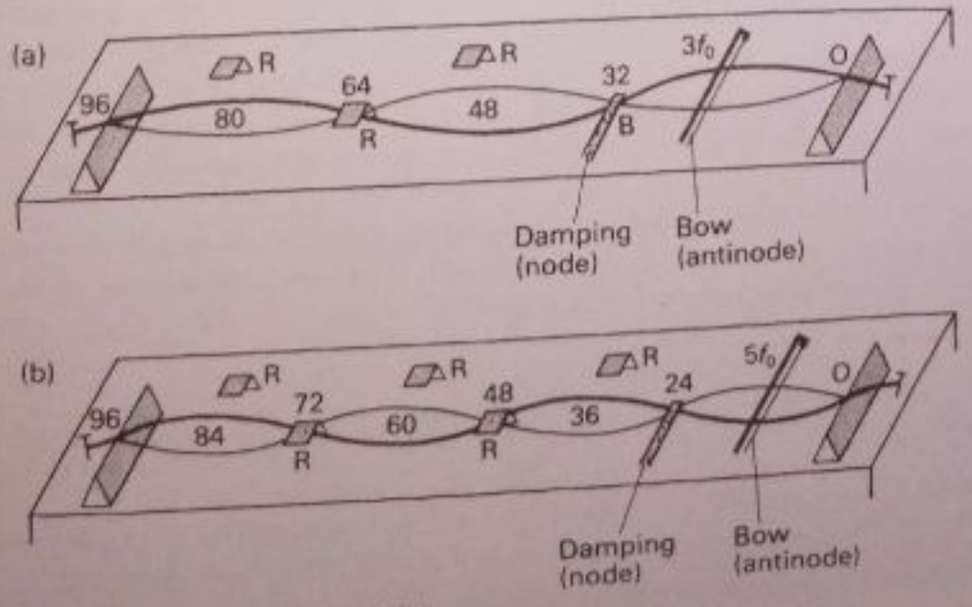


Fig. 23.13 Harmonics in a vibrating string

Scanned with CamScanner

When the part OB of the wire is bowed gently in the middle while B is damped, the paper riders remain on the wire at 64 cm but are thrown off at 48 cm and 80 cm. So at 64 cm there is a node of the stationary wave in the wire and at 48 cm and 80 cm there are antinodes. The wire now vibrates in three loops, as shown. So the frequency of the note produced is $3f_0$ where f_0 is the fundamental frequency.

Fig. 23.13b shows how a harmonic of $4f_0$ is produced. The wire is damped at a point one-quarter of the length of wire from O and this part of the wire is bowed in the middle. Paper riders are now thrown from the wire at points 36, 60 and 84 cm from O, showing antinodes, but remain on the wire at points 48 and 72 cm from O, showing nodes. Four vibrating loops of wire are now obtained, as shown.

When the wire is bowed in the middle, the fundamental is accompanied by a softer note of frequency $3f_0$, where f_0 is the fundamental frequency. This higher frequency is an *overtone* (page 372).

Longitudinal waves in pipes

Organ pipes, flutes and trombones all produce their characteristic notes when the air inside them is made to vibrate by blowing. Longitudinal waves then travel up and down the air column inside the instrument.

We can see roughly how the frequency of the note depends on the length of air column by filling two similar test-tubes with water to different levels. The air column of shorter length produces the note of *higher* frequency when we blow gently across the top of each tube.

Resonance tube experiment

A more exact investigation into the variation of the frequency f of the note obtained with the length l of the air column can be carried out with the apparatus shown in Fig. 23.14a.

Experiment

A long narrow tube B is placed inside a vessel A nearly filled with water (Fig. 23.14a). An air column is then obtained in B above the water level. Starting with a very small air column, a tuning fork F of 512 Hz is sounded gently above B. The tube is then slowly raised to increase the length of air column while F is sounding and the sound coming from the air column is heard to increase. When the sound is loudest, the air has been set into *resonance* by F. So the natural frequency of the length of air column is equal to the frequency of F.

The length of l of the air column is measured and recorded with the value of f in a table of measurements. The experiment is repeated with four other tuning-forks of known frequency, and the results for f and l recorded.

Measurements

f	l	$1/l$

Calculation: The reciprocal of l, $1/l$, is calculated for each value and a graph of $1/l$ is plotted against f.

Results: When a graph of $1/l$, the reciprocal of l, is plotted against f a straight line is obtained which nearly passes through the origin (Fig. 23.14b). So, approximately,

$$f \propto 1/l$$

Scanned with CamScanner

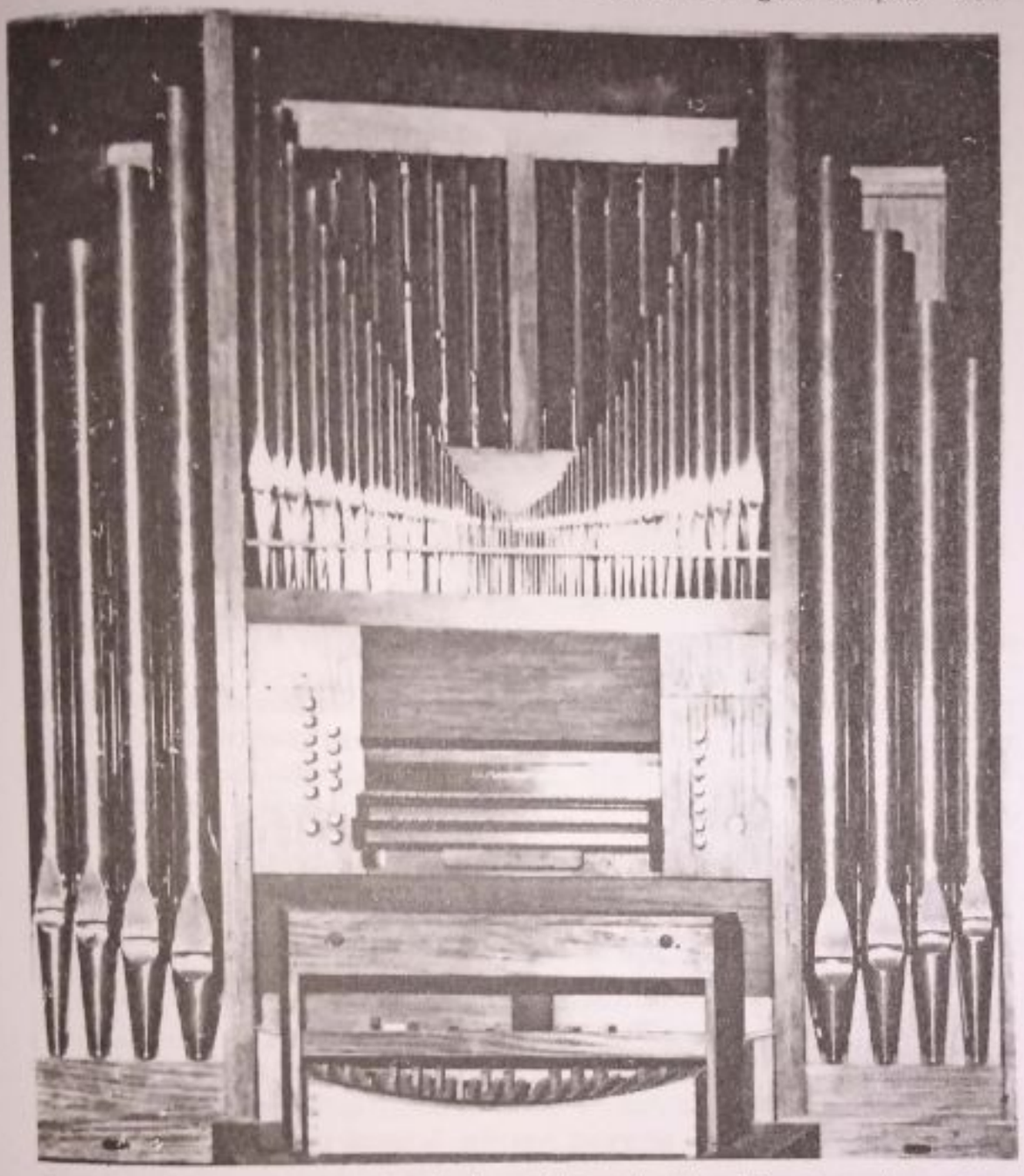


Plate 23C A chamber organ showing the pipes of different lengths and diameters used to produce different notes. (*Royal Festival Hall, London*)

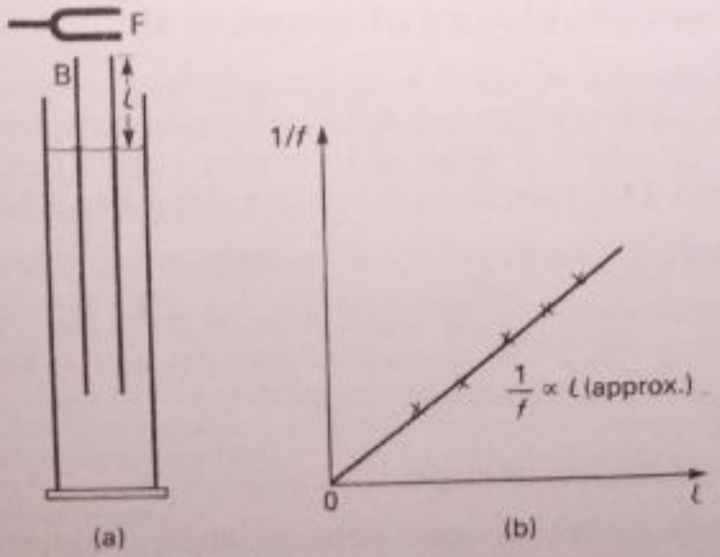


Fig. 23.14 Resonance tube experiment

Scanned with CamScanner

Frequency of a closed pipe

We can explain this experimental result by considering the simplest stationary wave set up in the air column when it is blown gently. The layer of air at the water cannot move. So this is a displacement node N of the wave (Fig. 23.15). The air at

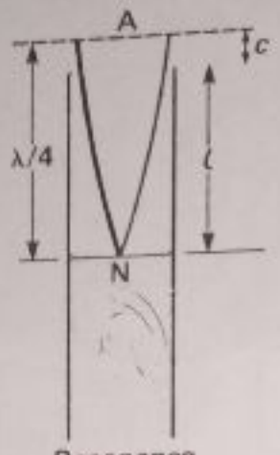


Fig. 23.15 Resonance of a closed pipe

the open end of the tube is free to move. So this is a displacement antinode A. Thus the length of the air column $= \lambda/4$, where λ is the wavelength (page 374). The antinode A is actually a small distance c outside the end of the tube, as shown in Fig. 23.15, and c is called the 'end-correction'. If we ignore c for the present, then if l is the length of the pipe

$$\frac{\lambda}{4} = l, \text{ or } \lambda = 4l, \text{ approximately}$$

Now $v = f\lambda$, where f is the frequency and v is the velocity of sound in the air.

$$f = \frac{v}{\lambda} = \frac{v}{4l}$$

Since v is constant, it follows that $f \propto 1/l$, as we found in the experiment with the resonance tube. This is not an exact relation, because the antinode is formed beyond the end of the pipe. In contrast, the relation $f \propto 1/l$ in the sonometer is an exact one, because there is no end-correction in this case.

Measurement of velocity of sound in air

The velocity of sound in air can be measured approximately by using a fork of known frequency such as 512 Hz and the resonance tube. Suppose the length l of the resonating air column is found to be 16·0 cm when a fork of frequency $f = 512$ Hz is used. Then, from above, $\lambda = 4l = 4 \times 16 = 64$ cm. Using $v = f\lambda$

$$\text{velocity of sound } v = 512 \times 64 = 33000 \text{ cm/s} = 330 \text{ m/s}$$

This is a value for the velocity of sound in the air inside the tube. The sound wave in the tube is affected by the walls of the tube. So this value is slightly different from the velocity of sound in 'free air'.

Overtones of a closed pipe

A pipe of length l produces some notes of higher frequencies than the fundamental when sounded. These are the overtones of the pipe.

Scanned with CamScanner

Fig. 23.16a shows the stationary wave formed in the air when the fundamental note is produced, and Fig. 23.16b, c show the stationary wave when the first overtone, frequency $3f_0$, and second overtone, frequency $5f_0$, are obtained. In each case the open end of the tube, where the air is most free to move, is an antinode of displacement, and the bottom of the tube, where the air cannot move, is a node.

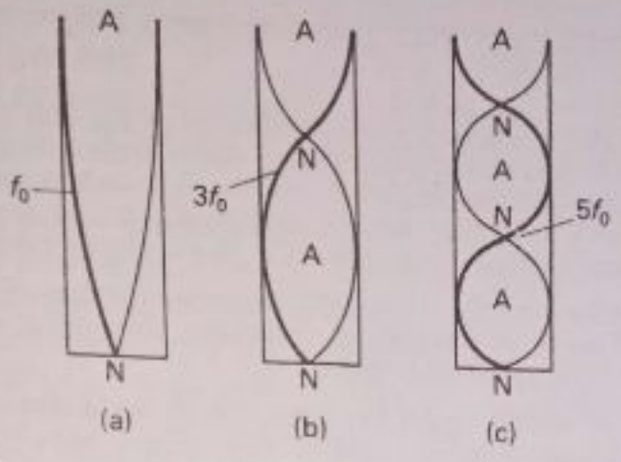


Fig. 23.16 Overtones

In Fig. 23.16b, the wavelength is one third that in Fig. 23.16a. So the frequency, which is inversely proportional to the wavelength, is three times that in Fig. 23.16a or $3f_0$. In Fig. 23.16c, the wavelength is one fifth that in Fig. 23.16a, so the frequency is $5f_0$.

Example

A pipe closed at one end has a length of 10 cm. If the velocity of sound in the air of the pipe is 340 m/s, calculate the frequency of (a) the fundamental, (b) the first overtone. Give *one* reason why the frequency of the fundamental may alter during the day.

(a) *Fundamental* Here the length of the pipe is $\lambda/4$ approximately, where λ is the wavelength. So

$$\frac{\lambda}{4} = 10\,\text{cm}$$

$$\therefore\ \lambda = 4 \times 10 = 40\,\text{cm} = 0{\cdot}4\,\text{m}$$

$$\therefore\ \text{The fundamental frequency } f_0 = \frac{v}{\lambda} = \frac{340}{0{\cdot}4} = 850\,\text{Hz}$$

(b) *First overtone* Frequency $f = 3f_0 = 3 \times 850 = 2550\,\text{Hz}$

If the temperature changes during the day, the velocity of sound v changes since v depends on the air temperature. So the frequency, which is v/λ, will change.

Examples on waves in strings and pipes

1. PQ is a metal wire held in tension on a hollow box. A and B are fixed bridges and C is movable. Light paper riders are placed at X, Y and Z (Fig. 23.17).

(a) When the wire is plucked gently between A and C, the riders at X and Z fall off but that at Y stays on. Explain this and name the point Y.

(b) The same result can be obtained by putting the stem of a suitable vibrating tuning fork on the box near AC. Name this effect, stating why it occurs and give another example of the same effect.

(c) If a tuning fork of slightly higher frequency were used explain what you would do to achieve the same result.

(d) State exactly where you would put the bridge C so that the rider at X will not fall off when the mid-point of AC is plucked lightly. (L)

Scanned with CamScanner

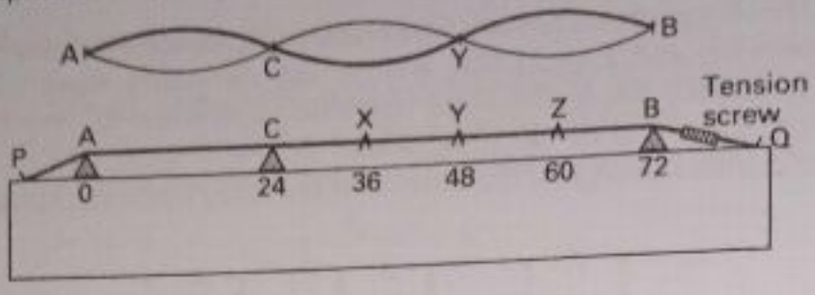


Fig. 23.17 Example

(a) When the wire AC is plucked, a wave travels along the wire and is reflected at the fixed points A, C and B. As a result of interference between the waves, a stationary wave is set up, as shown at the top of Fig. 23.17. AC = CY = YB. So Y is a displacement node of the stationary wave; this is why the rider does not fall off at Y. The displacement antinodes (maximum amplitude of vibration) are at X and Z; this is why the riders at X and Z fall off.

(b) The wire is set into vibration through the box when the stem of the fork is placed on the box near AC. This is an example of resonance. Resonance can be shown by a pendulum experiment, as explained on page 346.

(c) If a tuning fork of slightly higher frequency is used, the bridge at C would be moved slightly left. The natural frequency of AC is then slightly higher since the frequency increases when the length is shorter.

(d) To make the rider at X not fall off, X must now become a displacement node. So AC must equal CX. This means that C must be moved to 18 cm from A or 6 cm to the left.

2. The shortest length of the air column in a resonance tube (with one end closed and one end open) which resonates to a note of frequency 500 Hz is found to be 160 mm. What is the wavelength of this sound in air?

What is the shortest length of the column of air which resonates in similar conditions to a note of frequency 800 Hz? (Neglect end-corrections in each case.) (O and C)

(a) Since length of column = 160 mm $= \frac{\lambda}{4}$, wavelength $\lambda = 4 \times 160 = 640$ mm

(b) The velocity of the wave in the air column

$$v = f\lambda$$

$$\therefore \quad v = 500 \times 640 \text{ mm/s}$$

So wavelength λ for frequency 800 Hz is given by

$$\lambda = \frac{v}{f} = \frac{500 \times 640}{800}$$

$$= 400 \text{ mm}$$

$$\therefore \text{ shortest length} = \frac{\lambda}{4} = 100 \text{ mm}$$

3. Explain why (a) strings of different thickness are used on a stringed instrument such as a violin, (b) notes of the same pitch played on a violin and a flute sound different, (c) the lowest pitched note played on a flute is lower than that played on a piccolo (a similar but smaller wind instrument). (L)

(a) The frequency or pitch of a note from a bowed violin string depends on the length of the string, its tension, its mass per unit length or thickness, and the material. The violinist changes the pitch of the note played by varying the length of one of the strings, and strings of different thickness enable him or her to obtain a greater range of pitch.

(b) Notes on the violin and flute have a different quality (or timbre) because each fundamental note is accompanied by overtones and the overtones of a string instrument are different from those of a wind instrument. This gives the different waveforms, which determine the quality of the note.

(c) The length of pipe producing the lowest note is greater on the flute than the piccolo. The wavelength of the note is therefore longer for the flute. Since the frequency is inversely proportional to the wavelength, the note from the flute is lower in frequency or pitch.

Scanned with CamScanner

Summary

1. Sound waves in air produce compressions (pressure above normal) and rarefactions (pressure below normal) of the air.
2. Sound is produced by a vibrating object. The loudness of a note depends on the amplitude of vibration; the pitch depends on the frequency (number of vibrations per second); the timbre or quality depends on the overtones present.
3. The velocity of sound in air increases with rise in temperature but is independent of the pressure.
4. If two waves of equal frequency and amplitude travel in opposite directions along a string or air in a closed pipe, a stationary wave is set up.
5. Nodes are points on a stationary wave which are at rest. Antinodes are points where the amplitude of vibration is greatest.

The wavelength is twice the distance between successive nodes or four times the distance between adjacent nodes and antinodes.

6. The fundamental frequency f of a given plucked string varies as follows: $f \propto 1/l$ and $f \propto \sqrt{T}$, where l is length of string and T is tension. The greater the density of the string material or the greater its diameter, the lower its frequency.
7. The fundamental frequency of a closed pipe $= v/4l$ (approx.), where l is the length and v is the velocity of sound in air.

A resonance tube can be used to measure approximately the speed of sound in air.

Exercise 23 (Answers, page 642)

1. State *three* ways in which two musical notes may differ from each other. Name and explain the physical factors which cause these differences.

Describe what you would do, by using a microphone connected to a cathode ray oscilloscope, to demonstrate these differences, and draw diagrams of the traces you would expect to see on the screen. (N)

2. Explain the following:

(a) the sound is louder when the stem of a vibrating tuning fork is held against the top of a table, (b) the pitch of the note from a sounding organ pipe changes as the temperature rises, (c) the same note has a different quality when produced by different musical instruments. (N)

3. Distinguish between progressive and stationary sound waves and give an example of each.

A long tube whose length can be varied is closed at one end. The air in it can be set in vibration by a tuning fork held near the open end. Describe what will be heard as the length is increased, and explain how the wavelength of the note produced by the fork may be determined.

By means of short lines, arrowed at each end, indicate on a diagram the directions and amounts (exaggerated) of movements, due to the sound, of the air layers at different points along the tube. (O and C)

4. Define *frequency*, *amplitude* as applied to sound waves. What characteristics of a musical note are altered when these quantities are varied?

Describe an experiment to determine the frequency of a tuning fork, being provided with any apparatus you require, including a fork of known frequency.

Calculate the wavelength in air of (a) a note of 400 Hz, and (b) a note an octave higher, when the velocity of sound in air is 330 m/s. (L)

5. (a) Describe an experiment to determine the velocity of sound in air. Indicate the sources of error and show how they are likely to affect your result.

(b) Explain why more than one echo may be heard from a sound made inside a large room. (N)

6. What is meant by *frequency*, *overtone*?

How does the frequency of the note obtained by plucking a stretched wire depend on (a) the length of the wire; (b) its tension?

Describe an experiment to verify the statement given in *either* (a) *or* (b).

Scanned with CamScanner

Why is the intensity of the sound emitted by a plucked wire increased when it is mounted on a board? (L)

7. (a) Describe how you would determine the velocity of sound in air by one method. Show clearly how the result is obtained from the readings taken.

(b) Describe one experiment in each case to demonstrate the production of (i) a progressive wave and (ii) a stationary wave in *either* a stretched elastic cord *or* a spring. Draw a diagram to show the displacement of the cord or spring in case (ii) and indicate on it (iii) the position of a node, (iv) the wavelength of the motion and (v) its amplitude. (L)

8. Explain what is meant by *amplitude* and *frequency* of a vibrating wire. How would you increase (a) the amplitude, and (b) the frequency of vibration of a stretched wire, and what would you expect the result to be on the sound heard in each instance?

Describe how you would investigate the relation between the frequency of vibration of a stretched wire and its length when under constant tension. Indicate how you would use your readings to obtain the relationship between the two quantities.

Explain how it is possible for a vibrating wire to emit a fundamental note and overtones (harmonics) of higher frequencies at the same time. (L)

9. A stretched wire adjusted to a length of 48 cm produces the same note when plucked as a fork of frequency 256 Hz. If the wire is then adjusted to 32 cm and the tension kept constant, what is the frequency of the fork which would then be in tune with the wire? (N)

10. How would you find by experiment the frequency of vibration of a tuning fork?

The frequency of the note emitted by a sonometer wire vibrating transversely is 300 Hz. What will be the frequency of the note when (a) the length of the wire is reduced by one third without changing the tension; (b) the tension is increased by one third in the original length of wire? (N)

11. How would you attempt to test in the laboratory the truth of the statements (a) that sound requires a material medium to transmit it, (b) that the speed of sound in air is independent of frequency?

Draw a graph to illustrate the changes of pressure with time at a fixed distance from a loudspeaker emitting a steady pure tone of 200 Hz. Draw a second graph to show the effect of increasing the intensity, keeping the frequency constant.

Show on a third graph what happens if the tone is changed to one an octave higher in pitch but having the same amplitude as the original tone. (O and C)

Multiple-Choice Questions

12. A note X from a guitar produces a wave of amplitude 2 mm and frequency 800 Hz. A note Y from a whistle produces a similar waveform of amplitude 1 mm and frequency 1000 Hz. If the two notes X and Y are compared,

A Y is louder than X
B X has a higher pitch than Y
C X and Y have the same quality or timbre
D Y is less loud than X
E Y has a greater speed than X

13. Which of the statements A to E is NOT correct? Sound waves can be

A diffracted B travelling faster in a vacuum than in air
C reflected D refracted E absorbed

14. The note from a plucked guitar string will have a LOW pitch if the string is

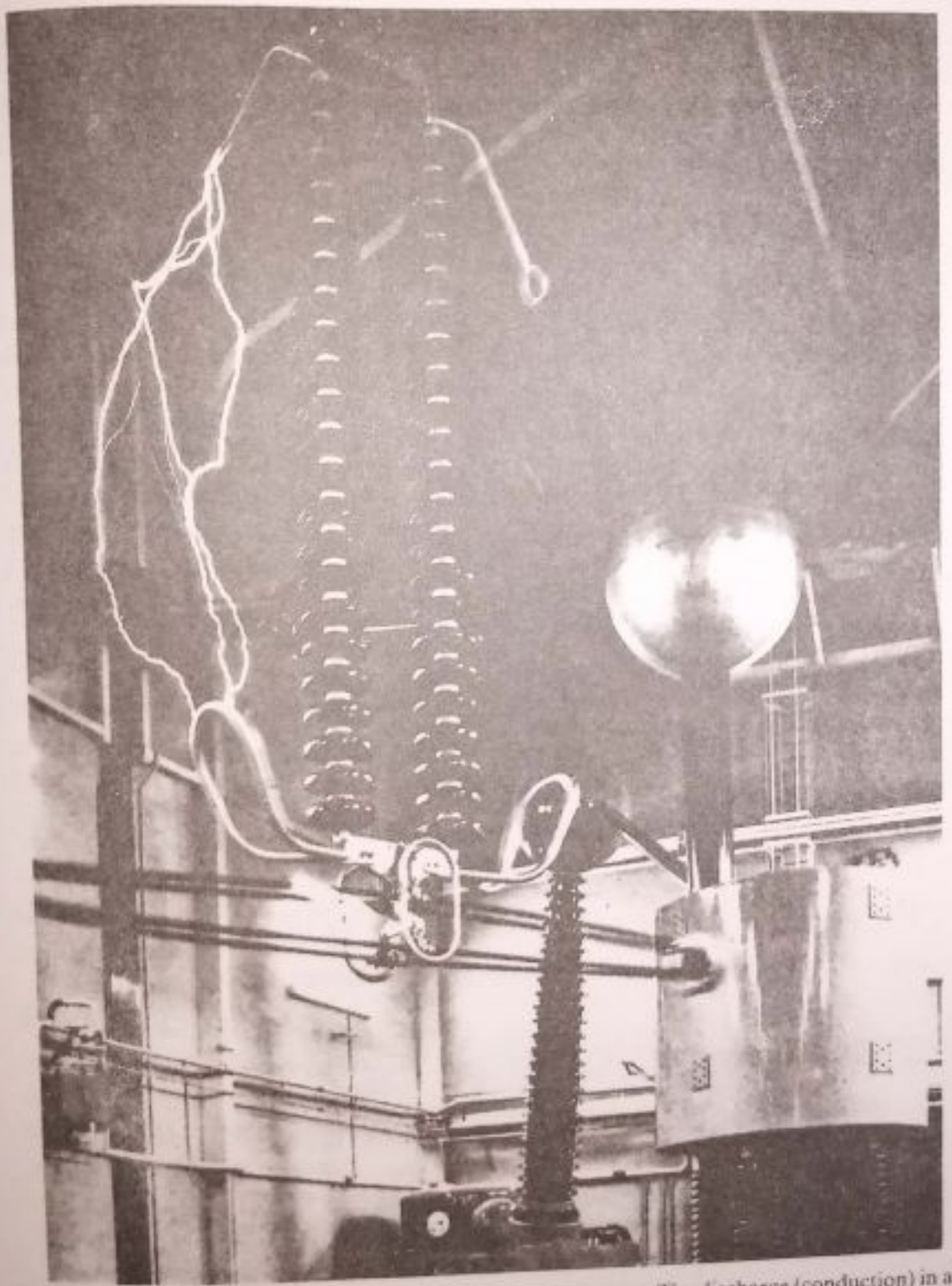
A thick and long B thick and slack C thin and short
D thin and long E thick and tight

15. When a pipe is played, a sound wave X is produced inside the pipe and a sound wave Y is heard outside the pipe. Then

A X is transverse and Y is longitudinal
B X and Y are both transverse waves
C X is stationary and Y is progressive
D Y is stationary and X is progressive
E X travels slower than Y

Scanned with CamScanner

ELECTRICITY



Testing high-voltage insulators used in the National Grid System. The discharge (conduction) in air between the metals when a high voltage is applied shows that the insulating column can withstand the voltage. (*Doulton Insulators Ltd*)

Scanned with CamScanner

24. Current, Potential Difference, Resistance

In this modern age, the science of Electricity holds one of the key positions. Electrical devices of one kind or another are needed in television, computers, satellites, telephones and commercial lighting, to name a few examples.

The battery · Electric current

Electrical phenomena had been known for thousands of years, but few useful effects were obtained, because it was static or stationary electricity (see

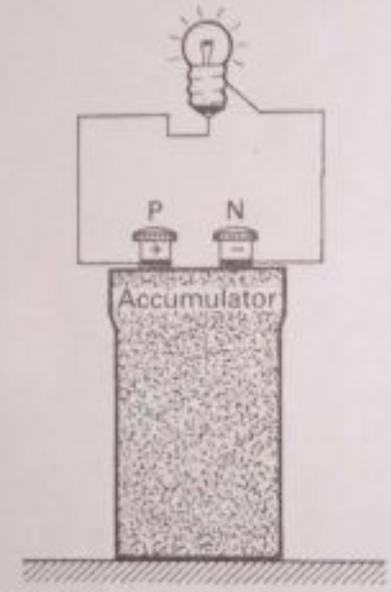


Fig. 24.1 Battery (accumulator) and its heating effect

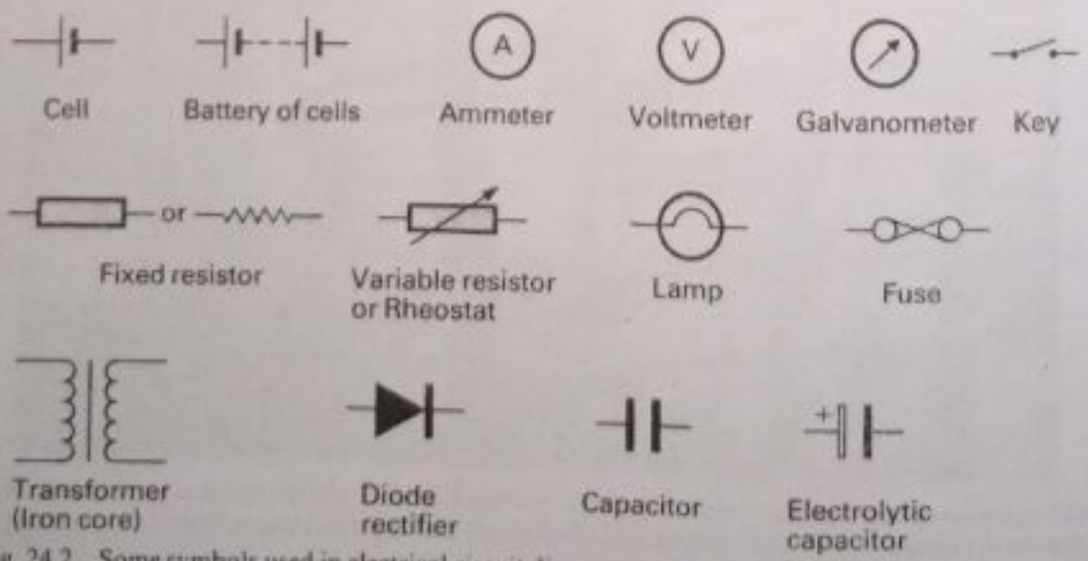


Fig. 24.2 Some symbols used in electrical circuit diagrams

Scanned with CamScanner

Electrostatics, page 509). About 1800, however, an Italian scientist called Volta discovered that suitable metals and chemicals were able to keep electricity moving. We call this arrangement a *cell*. A group of cells is called a battery. Later, the accumulator was invented. This is a very efficient cell; it keeps an electric current flowing for a long time. To distinguish between the poles or terminals of a cell, battery or accumulator, one is called the positive (+) pole or terminal and the other the negative (−) pole or terminal. These are shown in Fig. 24.1 by P and N.

Fig. 24.2 shows some of the electrical symbols used in electric circuits.

Conductors and insulators

Substances which allow electricity to flow through them are called *conductors*. Those which do not allow electricity to flow are called *insulators*. Examples are

Conductors: Metals, salt and inorganic acid solutions.

Insulators: Air, plastic materials, rubber, wood, paper, ebonite, pure water, organic acid solutions.

Silver is the best of the metallic conductors; copper is the next best. Pure copper inside flex and plastic materials, is widely used for connecting wire for domestic and commercial electricity supplies. When impurities, however small, are mixed with pure copper an alloy is produced of much lower conductivity or, to put it the other way, of much higher electrical *resistance*. Nichrome and manganin are two copper alloys used in circuits as *resistance wire* (see page 403). The insulating properties of materials can break down under certain conditions and they then become conductors. For example, air at normal pressure is usually an insulator but flashes of lightning show their conducting path during electrical storms.

Charge carriers

The electrical current is carried through solids, liquids or gases by moving tiny particles carrying charges. A 'charge' is a quantity of electricity which may be negative (−) or positive (+), as we see later. The charge carriers in some materials, discussed later, are:

Metals: electrons (−)
Salt and acid solutions: ions (+ and −)
Gases: electrons (−) and ions (+)
Semi-conductors: electrons (−) and holes (+)

Electric current

If a steady electric current flows in a wire, a constant quantity of electricity or charge per second passes every section of the wire. Quantity of electricity or charge is measured in units called *coulombs*. So if 4 coulombs pass a section of the wire in 2 seconds

$$\text{current} = \frac{\text{quantity}}{\text{time}} = 2 \text{ coulombs per second}$$

A steady current of water in a pipe is measured in a similar way. For example, suppose 60 cm^3 of water flows past a section of the pipe in 10 s. Then

$$\text{current of water} = \frac{\text{quantity}}{\text{time}} = 6 \text{ cm}^3 \text{ per second}$$

Scanned with CamScanner

If the electric current is increased, 6 coulombs may pass a section in 2 seconds. In this case the current is 3 coulombs per second.

The carriers of the current in a wire are electrons (page 389). Each electron carries the very tiny but definite amount of electricity, $1{\cdot}6 \times 10^{-19}$ coulomb approximately. A current of 1 coulomb per second in a wire is due to a flow of about 6×10^{18} electrons per second past any section of it.

All electrical quantities have symbols or letters which represents them. Q is the symbol used for quantity of electricity or charge and I is the symbol for electric current. So if Q coulombs flow past a section of wire in t seconds

$$I = \frac{Q}{t}$$

So

$$Q = I \times t$$

This means that if the current I is 4 coulombs per second, the quantity Q passing a section in 10 s is $4 \times 10 = 40$ coulombs.

The ampere · Current meters

The unit of electric current is called the *ampere* symbol A. 1 coulomb is defined as the quantity of electricity or charge passing a section of a wire in 1 second when the current is 1A. So a current of 2A means that 2 coulombs per second pass a section of the wire.

The current in a 120 W electric lamp filament on our mains is about 0·5 A. A 1 kW electric fire element carries a higher current of 4 A. Currents vary in size from the very small in transistor radio receivers, such as a few thousandths of an ampere, to the very large in powerful electric motors, such as 500 A. Small units of current are

1 milliampere (mA) = one-thousandth ampere $= \frac{1}{1000}$ A

1 microampere (μA) = one-millionth ampere $= \frac{1}{10^6}$ A

Meters that measure electric current may be ammeters, milliammeters or microammeters, depending on the size of the current to be measured. A galvanometer can measure or detect currents thousands of times smaller than a microampere.

To measure a current, an ammeter must always be placed in a circuit so that the current measured flows directly through it. In Fig. 24.3, for example, the

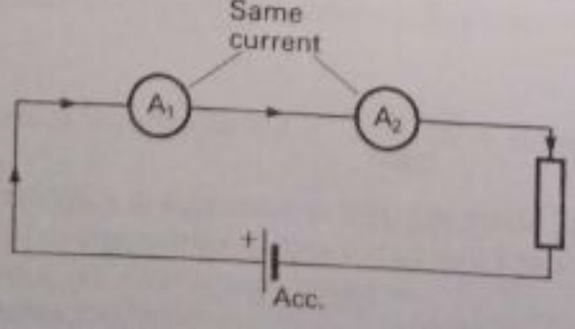


Fig. 24.3 Position of the ammeter in circuits

Scanned with CamScanner

current flows through the ammeter A_1 and through the ammeter A_2. Both measure the current in the circuit; and as the current does not flow faster at different points, the readings on the two meters are the same. An ammeter such as A_1 or A_2 is said to be placed *in series* in the circuit.

In Fig. 24.4 the ammeter A measures the current through the battery B or the 'main' current. The ammeter A_1 measures the current in the wire P. The ammeter A_2 measures the current in the wire Q.

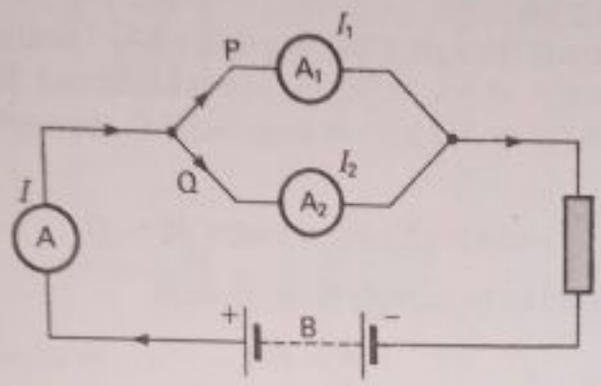


Fig. 24.4 Division of current between parallel resistors

Potential difference

Electric current is commonly used to produce heat and light energy. So the current carriers, the electrons, in the heating or lighting coil must provide energy when they move from one end to the other.

The movement or flow of electricity along a wire is similar to the flow of water along a pipe. Water will flow from one end of the pipe to the other when the two ends have a difference in level or height. In a similar way, electricity will flow along a wire when the two ends have an electrical difference in level. We call this an electrical *potential difference* (or p.d. for short). The potential difference between two points in a circuit is defined as the *energy per coulomb* released when electricity moves from one point to the other. The point at a higher potential may be said to have a 'positive' potential and the point at a lower potential a 'negative' potential.

The volt · Electrical energy

The volt is the practical unit of potential difference. It is defined by reference to the joule, the practical unit of energy, as follows:

1 volt is the potential difference between two points if 1 joule of energy is obtained when 1 coulomb moves between the points.

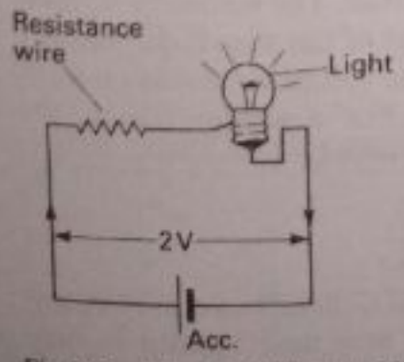


Fig. 24.5 Potential difference 2 V

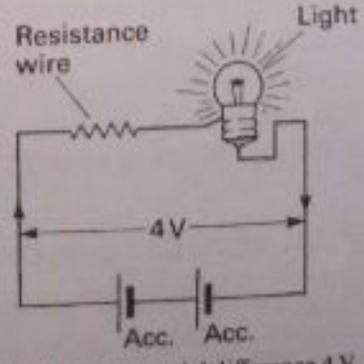


Fig. 24.6 Potential difference 4 V

Scanned with CamScanner

A lead-acid accumulator has a p.d. at its terminals of about 2 volts (2 V) (Fig. 24.5, 24.6). Batteries used in cars usually have a p.d. of 12 V. At power stations the p.d. at the generators is about 11000 V. X-ray tubes may be operated at 40000 V. A generator built specially for nuclear-energy experiments has the enormously high p.d. of 7000 million V. Units of p.d. other than volts are:

1 microvolt (μV) = one-millionth volt
1 millivolt (mV) = one-thousandth volt
1 kilovolt (kV) = 1000 volts

From the definition of the volt, it follows that when 5 coulombs move between two points having a p.d. of 1 V, 5 J of energy are released. If 5 coulombs move through a p.d. of 2 V, then 10 J are released. So if Q coulombs move through V volts

$$\text{energy released in joules, } W = QV$$

Since $Q = I \times t$, where I is the current flowing, then

$$W = IVt$$

We shall consider later in more detail the topic of energy in electricity (page 424).

The voltmeter

Potential differences are measured with a meter called a voltmeter. Since it measures the difference in potential between two points, it must be connected in parallel between the points. So in Fig. 24.7 the voltmeter V_1 measures the potential difference across the wire P by connection to A and B. The voltmeter V_2 measures the potential difference across the wire Q by connection to C and D.

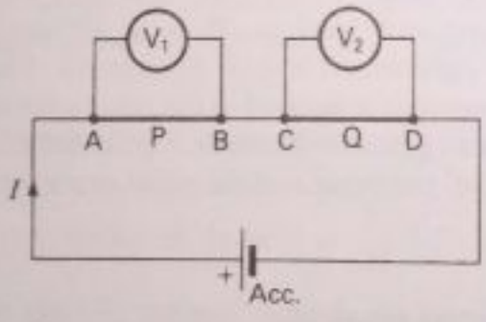


Fig. 24.7 p.d. across series resistors

The way a voltmeter is connected should be compared with the way an ammeter is connected. Ammeters measure current. They must therefore be connected in series in the circuit, so that the current flows through them. Voltmeters are connected in parallel, for the reason given. The voltmeter V_1 must have a large electrical resistance compared with that of the wire P. In this way only a very small amount of current flows through the voltmeter to make it work and practically all the current flows through P. For a similar reason, the voltmeter V_2 must have a large resistance compared with Q.

P.D. across resistance

A voltmeter helps us to find out how the p.d. is shared or distributed in electrical circuits. In Fig. 24.8, for example, the voltmeter V_1 may read 2 V and V_2 may read 4 V. A voltmeter connected to X and Y across both resistances then reads

Scanned with CamScanner

6 V. If the current is altered by the rheostat S, V_1 may now read 1·5 V and V_2 read 3·0 V, and the p.d. across X and Y is then 4·5 V. Generally, then, for resistances in series, such as P and Q

total p.d. across resistances = sum of individual p.d.s.

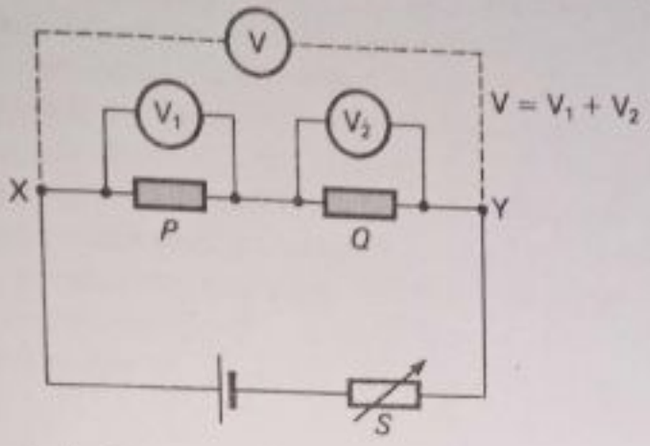


Fig. 24.8 p.d. in a series circuit

We can investigate how the potential difference V across a resistance R varies with the magnitude (size) of R when the current in the resistance is constant.

Experiment to investigate p.d. across a resistance

1. V across uniform wire

Place a metre length of resistance wire, such as one made of constantan, along a metre rule AB (Fig. 24.9). Use crocodile clips or another method to keep the whole wire stretched tightly across the rule, so that lengths of it can be read off on the scale.

Connect a 1·5 V dry battery or 2 V accumulator (S) to A and B. Join one terminal of a suitable voltmeter V (such as 0–2 V) to A, and the other terminal to a long wire with a crocodile clip at one end.

By making contact with the clip at points on the wire such as C or D, measure the p.d. V across a length AC, or l. Record the values of V when l = 10, 20, 30, ... 90 cm and enter the results in a table.

Measurements	l/cm	V/V

Graph: Plot a graph of V against l, using a zero origin for both axes.

Conclusion: Since a straight line graph passing through the origin is obtained, $V \propto l$. So $V \propto R$, the resistance of the length of wire, when the current is constant.

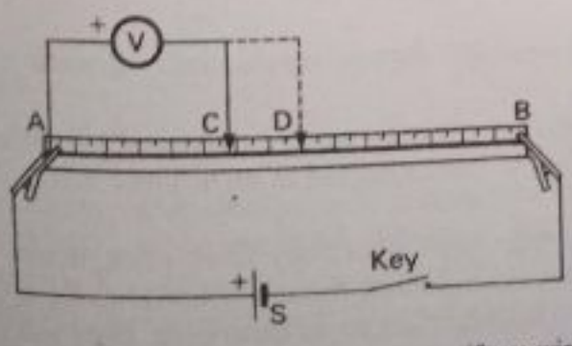


Fig. 24.9 Variation of p.d. across a uniform wire

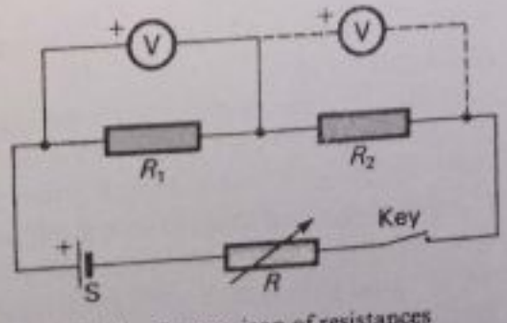


Fig. 24.10 Comparison of resistances

Scanned with CamScanner

2 Comparison of resistances

Connect two resistances R_1 and R_2, each of a few ohms, in series with a small rheostat R and a suitable dry battery or accumulator S (Fig. 24.10). Place a suitable voltmeter V, such as 0–2 V, across R_1 as shown. The +ve terminal of V must be the same side of the circuit as the +ve terminal of S.

With the current flowing, alter R so that a high reading is obtained in V and record the value V_1. Without altering the current, transfer the voltmeter across R_2 and record the value V_2. (If the reading is now too high for the voltmeter, alter the rheostat R and re-measure V_1.)

Replace the voltmeter across R_1, alter the rheostat a little, and repeat the measurements of V_1 and V_2 across R_1 and R_2 for the new value of current flowing. Enter the results in a table. Repeat the experiment for other changes in the rheostat, so that more values of V_1 and V_2 are obtained.

Measurements	V_1/V	V_2/V	V_1/V_2

Calculation: Work out the ratio V_1/V_2 for each pair of values. Enter the result in the table.

Result: Since from our previous experiment (1) $V_1/V_2 = R_1/R_2$, the ratio of the two resistances R_1/R_2 can be obtained.

Additional: To measure the actual value of an unknown resistance R_1 using the same circuit, place a known resistance R_2 in series with R_1 and obtain corresponding values of V_1 and V_2 when the same current passes through them. Then $R_1/R_2 = V_1/V_2$, so

$$R_1 = \frac{V_1}{V_2} \times R_2$$

Ohm's law

In 1826 Ohm found by experiment that, for good conductors, *the electric current in a given conductor is directly proportional to the potential difference applied, provided that the temperature of the conductor and its other physical factors remain constant.*

This is known as Ohm's law. It means that when the temperature of a given wire is constant, and a p.d. of 1 V, 2 V and 3 V is connected in turn across the wire, the currents flowing on each occasion may be respectively $\frac{1}{2}$ A, 1 A and $1\frac{1}{2}$ A. The ratio

$$\frac{V}{I}, \text{ where } V \text{ is the p.d. applied and } I \text{ is the current}$$

is then constant and equal to 2 units and does not depend on the size or direction of the p.d. used.

A thinner wire of the same material and length may carry currents of $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{3}{8}$ A respectively, when a p.d. of 1 V, 2 V and 3 V is connected in turn. The constant ratio V/I for this wire is then 8 units.

Although good conductors such as metals and alloys obey Ohm's law, many circuit components widely used in industry do not obey the law. Metal rectifiers, diode radio valves and voltage-dependent resistors are some examples. They will be discussed later.

Scanned with CamScanner

Resistance

Ohm defined the resistance of a wire as the ratio V/I, where V is the p.d. applied to it and I is the current produced. We see that the greater the p.d. needed to produce a given current in a wire, the greater is the resistance of the wire.

The symbol used for resistance is R and its unit is the *ohm* (Ω, Greek letter 'omega' = ohm). One ohm is therefore the resistance of a wire if a current of 1 ampere flows when a p.d. of 1 volt is connected to it.

The magnitude of resistance ranges from very small values, such as 1/100 Ω for a piece of copper wire, to the very large values used in radio and television sets, such as a million ohms. A small rheostat in the laboratory may have a total resistance of a few ohms. A rotary rheostat or 'potentiometer' used in radio may have a total resistance of a thousand ohms. The 1 kW element of an electric fire made of nichrome has a resistance of about 60 Ω. The tungsten filament of a 60 W electric lamp has a resistance of nearly 1000 Ω (Fig. 24.11).

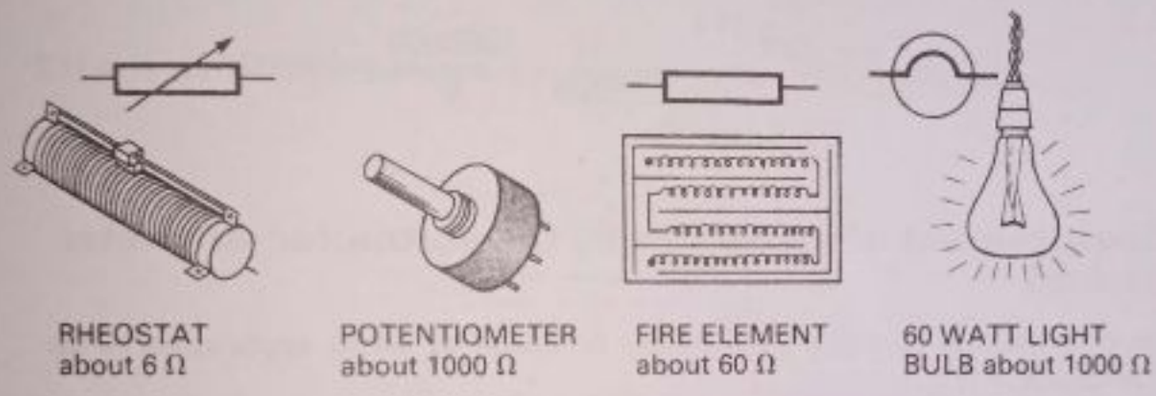


Fig. 24.11 Some resistors: their circuit symbols are shown above them

Small and large units of resistance are:

1 microhm ($\mu\Omega$) = one-millionth ohm
1 kilohm (kΩ) = 1000 ohms
1 megohm (MΩ) = one million (10^6) ohms

Fundamental formulae

Electrical and radio engineers have particular need of formulae for calculating current, potential difference and resistance. We have seen that, by definition, resistance = p.d./current. Thus if R, V and I represent the magnitude of the quantities

$$R = \frac{V}{I}$$

Hence

$$I = \frac{V}{R}$$

and

$$V = I \times R$$

The current formula $I = V/R$ might be required, for example, in estimating the right fuse wire to use in an electrical circuit when V and R are known. The p.d. formula $V = IR$ might be required, for example, in estimating the right voltmeter range to use in testing a radio component with a voltmeter.

In these formulae R must be in *ohms*, V in *volts* and I in *amperes*.

Scanned with CamScanner

Examples on formulae

1. Suppose a 2 V accumulator is connected to a 10 Ω wire. Then the current flowing in the wire is given by

$$I = \frac{V}{R} = \frac{2}{10} = 0{\cdot}2\,\text{A}$$

2. A current of 6 mA flows in a radio with a resistance of 20 kΩ. The p.d. V across the resistor is given by

$$V = IR = \frac{6}{1000}(\text{A}) \times 20000\,(\Omega) = 6 \times 20 = 120\,\text{V}$$

3. The p.d. across a resistance wire is 12 mV, and the current flowing is 4 μA. Since 1 μA = 1-millionth A

$$R = \frac{V}{I} = \frac{\frac{12}{1000}(\text{V})}{\frac{4}{1000000}(\text{A})} = \frac{12}{1000} \times \frac{1000000}{4} = 3000\,\Omega$$

Measurement of resistance by the voltmeter–ammeter method

The resistance R of a coil of wire can be found by direct application of the definition $R = V/I$.

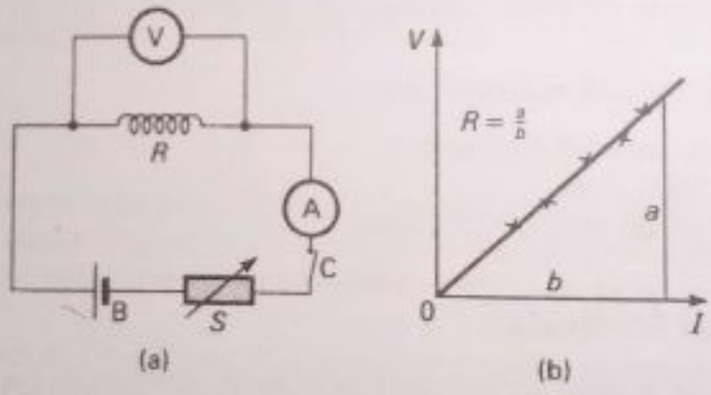


Fig. 24.12 Voltmeter–ammeter measurement of resistance

Experiment

Connect in series a resistor R of a few ohms, such as a length of resistance wire, a battery B such as a 2 V accumulator, a switch C, a rheostat S of a few ohms, and an ammeter A such as 0 to 2 A. (Fig. 24.12a). Then connect a voltmeter V such as 0 to 3 V across R, as shown. Check that the + ve side of the ammeter A and the + ve side of the voltmeter V are both on the + ve side of the battery B.

Switch on the current. Adjust the rheostat to obtain five widely different values of V and corresponding values of current I. To help accuracy, adjust the rheostat each time so that the ammeter pointer (or the voltmeter pointer) is exactly on a division of the meter scale before taking your readings.

Measurements	V/V	I/A

Graph: Plot V against I. The origin will be a point on the graph since $I = 0$ when $V = 0$. Draw the best straight line through the origin and all the points plotted.

Calculation: The best value of R is found from the gradient a/b of the straight lines, as shown (Fig. 24.12b). Otherwise, calculate R from $R = V/I$ for each pair of values V and I, and take the average.

In this experiment, high currents are unsuitable, as the wire may get too hot and its temperature vary, in which case its resistance may change appreciably.

In this method of measuring R, some current is always diverted through the voltmeter to make it work. So the p.d. across R is reduced. This may lead to error in the value of R calculated from V/I. If the voltmeter resistance is very high compared with R–at least ten times as large–only a very small current flows through the voltmeter and the error is then low.

Effect of temperature on resistance of materials

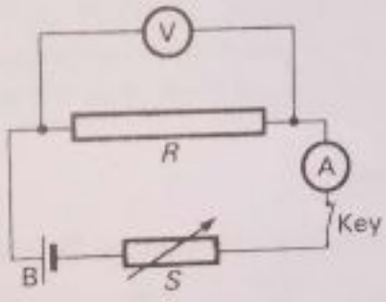


Fig. 24.13 Variation of resistance with temperature

Experiment

Set up the circuit as shown in Fig. 24.13, using a motor headlamp or ray-box lamp for R, a rheostat S which produces an appreciable change in the light from the lamp when adjusted from maximum to minimum, an ammeter A and voltmeter V which can read the values of I and V with fair accuracy.

Switch on the current. Vary the rheostat S and record the values of V and I from the minimum value of V to the maximum.

Now repeat the experiment, using in place of the tungsten filament R (a) a length of constantan wire of a few ohms and then (b) a thermistor, a special form of semiconductor material sensitive to temperature change. Replace the battery and meters by others if necessary; check the meter readings are suitable before starting to record values of V and I.

	Tungsten			Constantan			Thermistor		
Measurements	V/V	I/A	$R = V/I$	V/V	I/A	$R = V/I$	V/V	I/A	$R = V/I$

Calculations and graph: Calculate the resistance R from $R = V/I$ for each pair of values. Plot a graph of (a) I against V, (b) R against V from the results for the tungsten, constantan and thermistor material respectively (Fig. 24.14).

Scanned with CamScanner

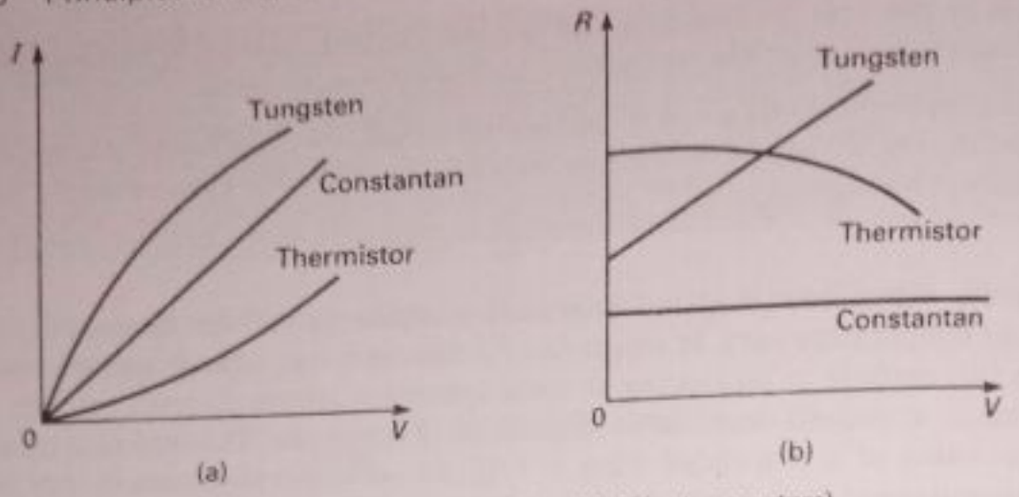


Fig. 24.14 (a) Variation of I with V (b) variation of R with V (or temperature)

Results: Since increasing V will increase the heat given to the material and therefore its temperature, we see that, as V or the temperature rises:

(a) constantan, an alloy, has a resistance which remains practically constant; this is why 'standard' resistances are made from this material;

(b) tungsten, a pure metal, increases in resistance; so a tungsten filament resistance changes when the lamp is switched on and it becomes hot;

(c) the thermistor, made from a special form of semiconductor material, decreases in resistance; so a thermistor in a circuit will allow the current to rise as it becomes warmer.

Resistances in series

In electrical circuits, several resistances, such as R_1 and R_2 in Fig. 24.15, may be in *series*. In this case

(a) the current I in R_1 and R_2 is the same,

(b) the p.d. across both resistors, $V = V_1 + V_2$, where V_1 is the p.d. across R_1 and V_2 is the p.d. across R_2.

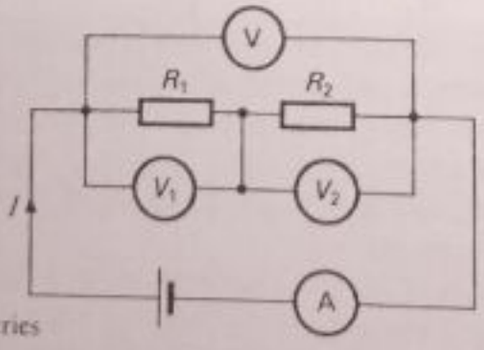


Fig. 24.15 Resistances in series

If R is the combined or total resistance, then $V = IR$ and $V_1 = IR_1$, $V_2 = IR_2$. So from (b),

$$IR = IR_1 + IR_2$$

from which

$$R = R_1 + R_2$$

So the total resistance of 4Ω and 6Ω in series is 10Ω.

Resistances in parallel

In domestic lighting, lamps must be wired as in Fig. 24.16a. Then switching one lamp off or on does not affect the others. The lamps are said to be in parallel with

Scanned with CamScanner

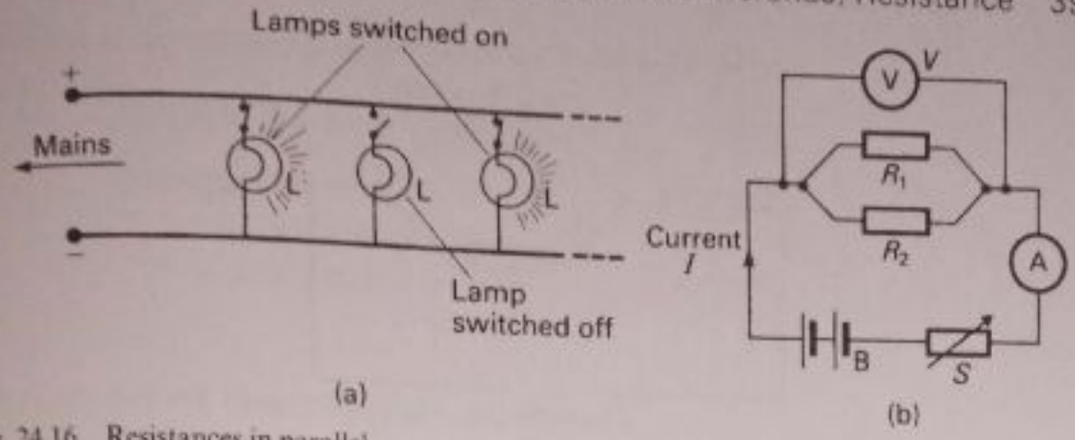


Fig. 24.16 Resistances in parallel

each other. Similarly, the resistors R_1 and R_2 in Fig. 24.16b are in parallel. Since resistors in parallel are connected in the same points, the *p.d. across* each resistor is the same. For resistors in series the *current through* each resistor is the same.

Fig. 24.16b shows two resistances R_1 and R_2 in parallel in a circuit, with a rheostat S, an ammeter A and a battery B. Part of the current I from the battery, I_1 say, flows through R_1; the rest of the current, I_2, flows through R_2. So

$$I = I_1 + I_2$$

Now $I = V/R$, where R is the combined or total resistance of R_1 and R_2, and $I_1 = V/R_1$, $I_2 = V/R_2$.

So from $$I = I_1 + I_2$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

Cancelling V $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

So the combined resistances of 3 Ω and 6 Ω in parallel are given by

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6}, \text{ or } R = \frac{6}{3} = 2\Omega$$

Examples on series and parallel circuits

The following examples illustrate how currents and p.d.s. are calculated in series and in parallel circuits, and should be studied carefully.

1. A resistance of 6Ω is in series with one of 4Ω and a p.d. of 20V is applied across the whole arrangement. Calculate the current in each wire and the p.d. across each.

20 V is the p.d. across *both* wires together.

The combined resistance $= 6 + 4 = 10\Omega$

$$\therefore \text{ current } I = \frac{V}{R} = \frac{20}{10} = 2\text{A}$$

Across the 6Ω wire alone $V_1 = IR_1 = 2 \times 6 = 12\text{V}$.
Across the 4Ω wire alone $V_2 = IR_2 = 2 \times 4 = 8\text{V}$.

Note: A common error is to state that the current in the 6Ω wire is 20/6 A. This is *not* the case, because 20 V is not the p.d. across the 6Ω wire but across both wires.

2. A circuit consists of a 1Ω wire in series with a parallel arrangement of 6Ω and 3Ω, and a p.d. of 12V is connected across the whole circuit (Fig. 24.17). Calculate the currents in each of the three wires and the p.d. across each.

Scanned with CamScanner

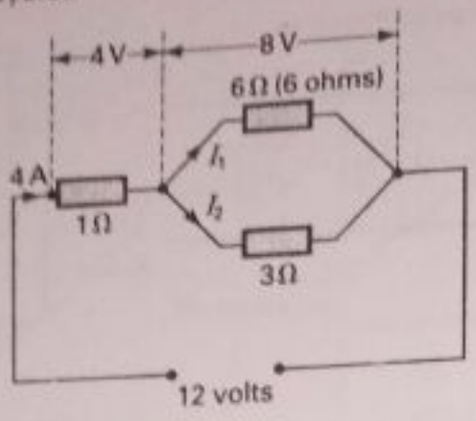


Fig. 24.17 Example

Since 12 V is the p.d. across the whole circuit, we must first find the combined resistance, S, of the 6 Ω and 3 Ω in parallel and then add the result to 1 Ω.

We have $$\frac{1}{S}=\frac{1}{6}+\frac{1}{3}=\frac{3}{6}, \text{ so } S=\frac{6}{3}=2\,\Omega$$

$$\therefore \text{ total circuit resistance } R=2+1=3\,\Omega$$

$$\therefore \text{ current flowing } I=\frac{V}{R}=\frac{12}{3}=4\,\text{A}$$

This is the current in the 1 Ω wire because it flows from the 12 V supply.

To find the current I_1 in the 6 Ω wire we need to know the p.d. V_1 across the 6 Ω and 3 Ω wires in parallel. Now p.d. across the 1 Ω wire $= IR = 4\times 1 = 4$ V. So

$$\text{p.d. across parallel wires, } V_1 = 12\,\text{V}-4\,\text{V}=8\,\text{V}$$

$$\therefore \text{ current in 6 Ω wire } I_1=\frac{V_1}{6}=\frac{8}{6}=1\tfrac{1}{3}\,\text{A}$$

$$\text{Similarly, current in 3 Ω wire } I_2=\frac{V_1}{3}=\frac{8}{3}=2\tfrac{2}{3}\,\text{A}$$

Thus:

(a) the currents are respectively 4 A, $1\frac{1}{3}$ A, $2\frac{2}{3}$ A

(b) the p.d.s. are respectively 4 V, 8 V, 8 V.

Note The current of 4 A flowing through 1 Ω is divided in the ratio 6:3 in the parallel resistors of 6 Ω and 3 Ω. The larger share of current flows through the *smaller* resistance, 3 Ω. So

$$\text{current in } 3\,\Omega=\frac{6}{6+3}\times 4\,\text{A}=\frac{6}{9}\times 4\,\text{A}=2\tfrac{2}{3}\,\text{A}$$

and

$$\text{current in } 6\,\Omega=\frac{3}{6+3}\times 4\,\text{A}=\frac{3}{9}\times 4\,\text{A}=1\tfrac{1}{3}\,\text{A}$$

This is a quick way of finding the currents in the parallel resistors.

3. Two wires of 10 Ω and 5 Ω are in parallel, and the arrangement is in series with a 20 Ω wire (Fig. 24.18). If the current in the 5 Ω wire is 2 A, calculate the current in the 10 Ω wire and the total p.d. V across the whole circuit.

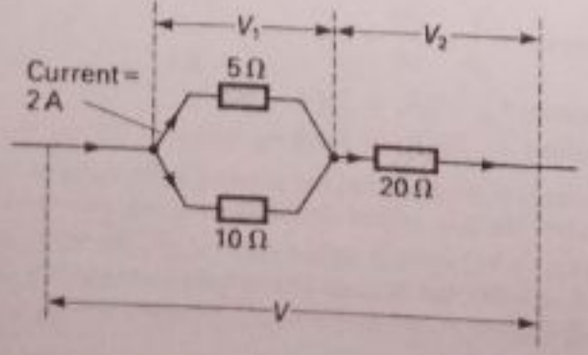


Fig. 24.18 Example

Scanned with CamScanner

The p.d. V_1 across the 5 Ω wire $= IR = 2 \times 5 = 10$ V

$\therefore$ current in 10 Ω wire $= \dfrac{V_1}{10} = \dfrac{10}{10} = 1$ A

$\therefore$ current in 20 Ω wire $= 1 + 2 = 3$ A

$\therefore$ p.d. V_2 across 20 Ω wire $= I \times R = 3 \times 20 = 60$ V

$\therefore$ total p.d. V = p.d. across parallel arrangement + 60 V

$= 10 + 60 = 70$ V

Variation of resistance with length and area

The resistance R of a length of wire depends on its length l, its cross-sectional area A and its material.

Experiment

1. *Variation of R with l*

To find how R varies with l, connect a length l of resistance wire such as constantan in series with a suitable battery B, ammeter A, rheostat S and switch C, and join a voltmeter V across the terminals X, Y of the wire (Fig. 24.19a). Check that the + ve terminals of A and V are both on the + ve side of the battery. The length l measured is the length of the wire between the terminals X, Y, or between the crocodile clips if these are used.

Start with a length l such as 50·0 cm. Connect the circuit and, by means of the rheostat, obtain two pairs of suitable values of V and I. Repeat for lengths of 100·0 cm, 150·0 cm and 200·0 cm, for example.

Measurements	l/cm	V/V	I/A	$R = V/I$

Calculation and graph: Calculate the respective average values for R for each length l from $R = V/I$. Plot a graph of R against l.

Results: Typical results show that $R \propto l$. (Fig. 24.19b)

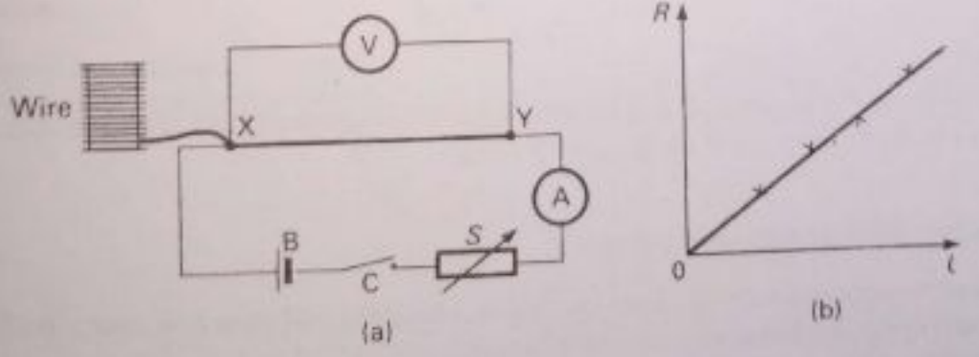


Fig. 24.19 Variation of resistance R with length l

2. *Variation of R with A*

To find how the resistance R varies with A, the cross-section area, *equal* lengths of constantan wire are cut from two reels of different diameter or thickness. The two diameters d_1, d_2 are measured with a micrometer gauge (page 8) and the two resistances R_1, R_2 are found from the circuit in Fig. 24.19a.

Calculation: The ratio R_1/R_2 and the ratio d_1^2/d_2^2 are calculated. The ratio of the areas of cross-section is proportional to the ratio of the square of the diameters, so $d_1^2/d_2^2 = A_1/A_2$ where A_1, A_2 are the respective cross-section areas.

Scanned with CamScanner

Conclusion: We find that $R_1/R_2 = A_2/A_1$, the *inverse*-ratio of the areas. So $R \propto 1/A$ where A is the cross-section area.

Example

A wire has a length of 1·20 m, a diameter 0·64 mm and a resistance of 2·4 Ω. Calculate the resistance R of a wire of the same material of length 0·80 m and diameter 0·32 mm.

Since $R \propto l$ and $R \propto 1/A \propto 1/d^2$, it follows *by proportion* that

$$R = 2{\cdot}4\,\Omega \times \frac{0{\cdot}80\,\text{m}}{1{\cdot}20\,\text{m}} \times \left(\frac{0{\cdot}64\,\text{mm}}{0{\cdot}32\,\text{mm}}\right)^2$$

$$= 2{\cdot}4 \times \frac{2}{3} \times \left(\frac{2}{1}\right)^2$$

$$= 2{\cdot}4 \times \frac{2}{3} \times 4 = 6{\cdot}4\,\Omega$$

Resistivity

The two results of the previous experiment on resistance can now be combined into one. If l is the length of a given material and A is the cross-sectional area, then, if R is the resistance,

$$R \propto \frac{l}{A}$$

Thus
$$R = \frac{\rho l}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where ρ is a constant known as the *resistivity* of the material.

Since, from (1), $R = \rho$ when $l = 1$ and $A = 1$, resistivity can be defined as the resistance of unit length of material of unit cross-sectional area.

From (1),
$$\rho = \frac{R \times A}{l} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

In SI units, l is in metres, A in metre^2, and R in ohm in (1) or (2) above. The unit of resistivity is then 'ohm metre', symbol Ω m.

Experiment to measure resistivity

To measure the resistivity of a material such as constantan, nichrome or copper, cut off a length of the wire from a reel and then measure:

(a) the length l with a metre rule;

(b) the diameter d by a micrometer gauge;

(c) the resistance R by a voltmeter-ammeter method (page 396).

Result: Suppose $l = 1{\cdot}00$ m, $d = 0{\cdot}64$ mm $= 0{\cdot}64 \times 10^{-3}$ m, $R = 3{\cdot}6\,\Omega$.

$$\rho = \frac{R \times A}{l} = \frac{R \times \pi r^2}{l} = \frac{R \times \pi\left(\frac{d}{2}\right)^2}{l} = \frac{R \times \pi d^2}{4l}$$

$$\therefore \rho = \frac{3{\cdot}6 \times \pi \times (0{\cdot}64 \times 10^{-3})^2}{4 \times 1{\cdot}0}$$

$$= 1{\cdot}2 \times 10^{-6}\,\Omega\,\text{m}$$

Scanned with CamScanner

Resistivity values

The resistivities of various metals and other materials are listed in physical tables. Copper, used in cables, has a resistivity of $1{\cdot}7 \times 10^{-8}\,\Omega\,m$. Nichrome (resistance wire) has a much higher resistivity of about $1 \times 10^{-6}\,\Omega\,m$. Nichrome is used as the material for electric fires, because it does not oxidise when white-hot. Constantan or manganin is used for making coils of standard resistances, whose resistances are known to a high degree of accuracy, because these materials show very little change of resistance when their temperature rises.

Once the resistivity of a material is known, a length of it can always be cut off to make a particular resistance for an appliance. As an illustration, suppose a length of nichrome wire is needed for making a $50\,\Omega$ electric fire element. If the diameter of the wire used is 0·56 mm, the cross-sectional area $A = \pi r^2 = \pi \times (0{\cdot}28 \times 10^{-3})^2\,m^2$. Assuming the resistivity ρ of nichrome is $10^{-6}\,\Omega\,m$, then, from $R = \rho l/A$

$$\text{length } l = \frac{R \times A}{\rho} = \frac{50 \times \pi \times (0{\cdot}28 \times 10^{-3})^2\,m}{10^{-6}}$$

$$= \frac{50 \times 3{\cdot}14 \times 0{\cdot}0784 \times 10^{-6}}{10^{-6}} = 12{\cdot}3\,m$$

So a length of 12·3 metres of the wire will form a coil of $50\,\Omega$ resistance.

The variation of resistance with temperature is an important factor taken into account in the design of electric appliances. The resistivity of copper, used in cables and in transformers, increases when it becomes hot. The resistivity of tungsten, used in electric-lamp filaments, also increases when hot. Conversely, carbon and non-metals decrease in resistance with temperature rise. At very low temperatures near the absolute zero the electrical resistance of metals vanishes and this is called *superconductivity*. A current established in the metal can then circulate on its own for days.

Example

2 metre of resistance wire, area of cross-section 0·50 mm^2, has a resistance of $2{\cdot}20\,\Omega$. Calculate (a) the resistivity of the metal; (b) the length of the wire which, connected in parallel with the 2 metre length, will give a resistance of $2{\cdot}00\,\Omega$.

(a)

$$R = \frac{\rho l}{A}, \text{ or } \rho = \frac{R \times A}{l}$$

Now

$$A = 0{\cdot}50\,mm^2 = 0{\cdot}50 \times 10^{-6}\,m^2$$

$$\therefore\ \rho = \frac{2{\cdot}20 \times 0{\cdot}50 \times 10^{-6}\,\Omega\,m}{2}$$

$$= 5{\cdot}5 \times 10^{-7}\,\Omega\,m$$

(b) Suppose R is the resistance of the wire required.

Then

$$\frac{1}{2{\cdot}00} = \frac{1}{R} + \frac{1}{2{\cdot}20}$$

$$\therefore\ \frac{1}{R} = \frac{1}{2{\cdot}00} - \frac{1}{2{\cdot}20} = \frac{1{\cdot}1}{2{\cdot}2} - \frac{1}{2{\cdot}2} = \frac{0{\cdot}1}{2{\cdot}2} = \frac{1}{22}$$

$$\therefore\ R = 22\,\Omega$$

Since

$$R \propto l$$

by proportion

$$l = \frac{22}{2{\cdot}2} \times 2\,m = 20\,m$$

Scanned with CamScanner

Measuring resistance by metre bridge

Pointer instruments such as ammeters and voltmeters usually have errors in them; for example, before any current is passed into an ammeter, its pointer may not read zero. So the ammeter–voltmeter method is not used when measuring resistance very accurately.

The *Wheatstone bridge* is a circuit widely used for accurate measurement of resistance. One form is a metre bridge circuit shown in Fig. 24.20. AB is a one metre length of resistance wire. AC, DF and HB are brass strips (good conductors), which have two gaps between C, D and F, H.

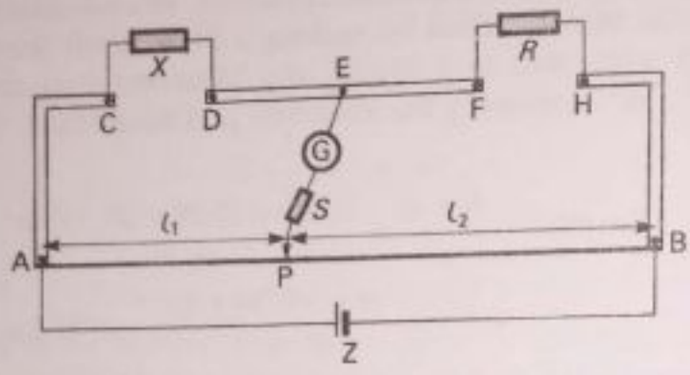


Fig. 24.20 Metre bridge experiment

Experiment

The unknown resistance X to be measured is connected in one gap CD. A *known* resistance R, which must be the same order as X, is placed in the other gap FH. A battery Z is joined to A, B. A galvanometer G with a centre-scale zero is joined between E and a slider, which is a metal rod with a knife-edge so that it can make contact with any point on AB. As an excessive current may flow through G when the circuit is made, a high resistance S is often placed in series with G to protect it. It is removed when the current in G is very low.

The slider is placed lightly in contact with the wire AB and moved until *no current flows in G*, that is, a balance is obtained. The lengths l_1, l_2 of AP, PB respectively are then noted where P is the balance point. The two resistances X and R are now interchanged in position and the experiment is repeated. This helps to get average values for l_1 and l_2, which reduces error.

Calculation: The Wheatstone bridge theory shows that

$$\frac{X}{R} = \frac{l_1}{l_2}$$

or $$X = R \times \frac{l_1}{l_2}$$

So suppose $R = 5{\cdot}0\,\Omega$ $l_1 = 42{\cdot}1$ cm and $l_2 = 57{\cdot}9$ cm

Then $$X = 5{\cdot}0 \times \frac{42{\cdot}1}{57{\cdot}9} = 3{\cdot}6\,\Omega$$

Summary

1. In metals the charge carriers are electrons (– ve particles, extremely light).
2. Connecting wire is made of pure copper, which has a very low resistance. Resistance wire is made of alloys, such as constantan.

Scanned with CamScanner

3. Ohm's law states: The ratio p.d./current is a constant for a given conductor, provided the physical conditions of the conductor, such as temperature, remain constant. Non-ohmic conductors, such as a thermistor, do not obey this law.
4. $I = V/R$, $V = IR$, $R = V/I$
5. The ammeter, measuring current, is placed in series in a circuit and has a low resistance. The voltmeter, measuring potential difference, is placed in parallel with the circuit, and should always have a high resistance compared with the component across which it is connected.
6. In a circuit containing resistances *in series*: (a) the current is the same in each resistance; (b) the total p.d. = the sum of the individual p.d.s.; (c) combined resistance = sum of individual resistances.
7. In a circuit containing resistances *in parallel*: (a) the p.d. across each resistance is the same; (b) the combined resistance is given by $1/R = 1/R_1 + 1/R_2 + 1/R_3$, if there are three wires of resistances R_1, R_2, R_3 respectively.
8. Resistance can be measured by a voltmeter–ammeter method.
9. Resistivity (ρ) is the resistance of a unit length with unit cross-sectional area of the material concerned. SI unit of resistivity: *ohm metre*.

The resistance of a wire is given by $R = \rho l/A$.

10. The resistance of a pure metal increases with temperature rise. 'Standard resistances' are made with alloys, such as constantan, which have a very low resistance change with temperature rise.

Exercise 24 (Answers, page 642)

1. (a) The p.d. across a resistance of 10 Ω is 4 V. Calculate the current flowing. If the current increases to 0·5 A, what is the new p.d. across the resistor?

(b) Find the combined resistance of 6 Ω and 12 Ω in *parallel*. A battery of 12 V and negligible internal resistance is connected across the arrangement. Find the current in each wire and in the battery.

(c) A current of 2 A flows through a resistance R in series with an 8 Ω resistor. The total p.d. across the two resistors is 20 V. Calculate R.

(d) Two resistance wires are made from the same material. One wire X has half the length and half the diameter of the other, Y. What is the ratio of the resistance of X to that of Y?

2. A resistor of 500 Ω, a voltmeter and a 2·00 V accumulator of negligible resistance are all connected in series. If the voltmeter reads 1·20 V, calculate its resistance. (C)
(*Hint*: The voltmeter records the p.d. across its own resistance.)
3. Fig. 24.21 shows an electric circuit in which some of the quantities are represented by numbers, others by letters. Calculate the values of x and r. (C)

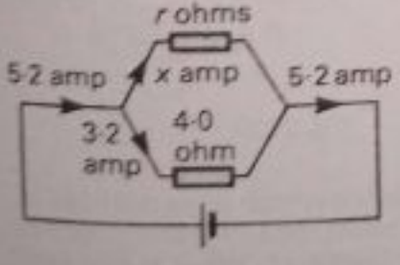


Fig. 24.21

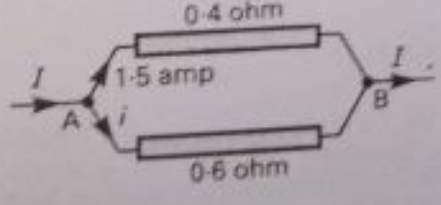


Fig. 24.22

4. A cell of electromotive force 2·00 V and negligible internal resistance is connected in series with a resistance of 3·50 Ω and an ammeter of resistance 0·500 Ω. Calculate the current in the circuit. When a resistance of 0·100 Ω is connected in parallel with the ammeter, the current changes. Calculate the new current: (a) in the cell; (b) in the ammeter. (C)
5. State *Ohm's law* and show how it leads to a definition of resistance.

Scanned with CamScanner

Describe an experiment to determine how the resistance of a uniform wire depends on its length. Give a diagram of the arrangement of the apparatus and state the result expected.

A resistance of 6 Ω is connected between two points A and B and resistances of 3 Ω and 4 Ω in parallel between B and a point C. If a potential difference of 9 V is applied across A and C, what is (a) the current in each resistance; (b) the potential difference between B and C? (L)

6. State Ohm's law and define *electrical resistance*.

Distinguish between *resistance* and *resistivity*.

Show on a diagram and explain how you would connect up a voltmeter and an ammeter to an electric lamp in order to measure the resistance of the lamp. (O and C)

7. Fig. 24.22 shows two resistors, whose resistances are 0·4 Ω and 0·6 Ω, connected in parallel. They are part of a circuit which carries a current I, and the current in the 0·4 Ω resistor is 1·5 A. Calculate (a) the potential difference between the points marked A and B in the diagram; (b) the current i in the 0·6 Ω resistor; (c) the current I in the circuit. (C)

8. Values of the current I passing through a coil for corresponding values V of the potential difference across the coil, as measured on a voltmeter, are shown below.

The voltmeter has a zero error which has not been allowed for in the readings given:

I amperes	0·05	0·20	0·35	0·50	0·65
V volts	0·85	2·80	4·74	6·70	8·65

Plot a graph to show the relation between V as ordinate (y-axis) and I as abscissa (x-axis).

Use the graph to determine (a) the resistance of the coil, (b) the correction which must be applied to the voltmeter readings, and give the correct value of the first p.d. Explain how each is determined from the graph. (N)

9. Describe how you would investigate the relationship between the current through a length of resistance wire and the potential difference between its ends. Indicate how you would use your readings to show how the two quantities are related. Give one special precaution you should take and state why.

In Fig. 24.23, V is a voltmeter of resistance 200 Ω and A is an ammeter of negligible resistance which indicates a reading of 0·01 A. If the cell of e.m.f. 1·5 V has negligible internal resistance, calculate the value of R.

If, by mistake, the voltmeter were connected in *series* with R, the cell and the ammeter, what would be the readings of the ammeter and voltmeter respectively? (L)

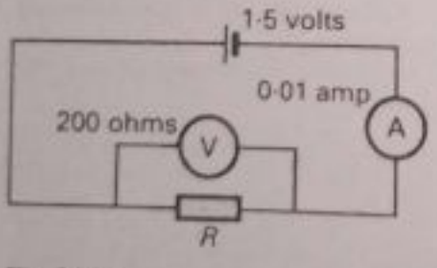


Fig. 24.23

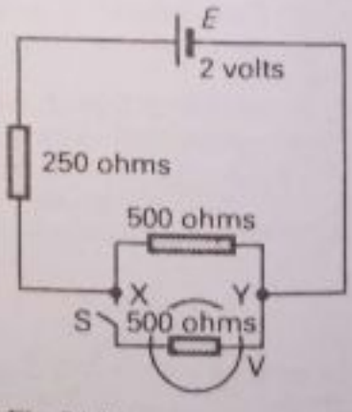


Fig. 24.24

10. State Ohm's law and mention *one* current-carrying device which does not obey it.

A wire is 200 cm long, 0·50 mm in diameter and has a resistance of 3·0 Ω. Another wire of the same material is 300 cm long and 0·60 mm diameter. What is the total resistance of the two wires when they are connected in series?

If wires of resistance 3·0 Ω and 4·0 Ω are put in parallel and a current divides between them so that 8·0 A pass through the first, what current will be flowing in the second wire? What p.d. will be required to cause these currents?

From the total current passing through both, and the p.d. required, deduce their combined resistance. (O and C)

11. State Ohm's law, and define *resistance*.

How does the resistance of a piece of wire depend on its length, its diameter and the material of which it is made?

Scanned with CamScanner

In Fig. 24.24, E is a 2 V cell of negligible internal resistance, and V a voltmeter of resistance 500 Ω that can be connected across XY by means of the switch S. Find the current given by the cell, and the p.d. between X and Y (a) with S open, (b) with S closed so that the voltmeter is in circuit. (O)

12. Find the resistance of a piece of wire 100 cm long and area of cross-section 0·25 mm^2. Resistivity of the material of the wire is 10^{-6} Ω m.

13. A wire of length 1 m and uniform diameter has a resistance of 1·05 Ω. What length of wire of the same material, but having half the diameter, would be needed to make a 5 Ω coil? (N)

14. A wire 150 cm long and diameter 1·0 mm is made of an alloy of resistivity 44×10^{-8} Ω m. What is the resistance of the wire? (N)

15. Define *electrical resistance*, *resistivity*.

How would you determine the resistance of an electric lamp while it is in use?

Calculate the resistance of a 5·46 m length of lead strip, 0·450 cm wide and 1·25 mm thick. What length of similar strip, but made of copper, would have to be put in parallel with the lead strip for the effective resistance of the combination to be 0·100 Ω? (Resistivity of lead $= 20{\cdot}6 \times 10^{-8}$ Ω m; of copper $= 1{\cdot}72 \times 10^{-8}$ Ω m.) (C)

16. Explain what is meant by the *resistivity* of a substance. In what units is resistivity measured? Give one reason why a knowledge of the resistivity of a substance can be of practical use.

Given a voltmeter and an ammeter and a suitable battery, how would you measure the resistivity of a sample of a material in the form of a thin wire? Draw a circuit diagram of the electrical apparatus and state what other measurements you would need to make and the apparatus required for them.

Two wires, A and B, are made of the same material. A has twice the length of B and twice its diameter. What is the ratio of the resistance of A to that of B? (O and C)

Multiple-Choice Questions

17. Three wires each of resistance 12 Ω can be arranged to form a combined resistance of

A 48 Ω B 32 Ω C 18 Ω D 12 Ω E 6 Ω

18. A resistance is doubled in value and the potential difference across it is reduced to one-third its value. Then the ratio *new current/old current* is

A 6/1 B 3/2 C 2/3 D 1/3 E 1/6

19. When the resistance R of a wire is measured using an ammeter, voltmeter, battery and rheostat, then

A the ammeter is in parallel with R
B the voltmeter is in series with R
C a graph of V against I can give an accurate value of R
D a graph of I against V has a gradient equal to R
E the rheostat is in parallel with R

20. The length of a resistance wire is doubled and its diameter is reduced to one-third of its value. The ratio *new resistance/old resistance* is

A 36/1 B 18/1 C 6/1 D 2/3 E 1/3

21. A solid metal cube has each side doubled to make a solid cube of eight times the volume. The ratio *resistivity of new cube/resistivity of old cube* is

A 8/1 B 6/1 C 1/1 D 1/6 E 1/8

Scanned with CamScanner

25. Batteries, E.M.F. and Circuits

Voltaic cell

An invention by Volta at the end of the 18th century led to the development of batteries. Galvani, a scientific friend, had told him that a dead frog's leg twitched when a knife was lightly placed in contact with it while the frog lay on a zinc plate. Volta came to the conclusion that this was an electrical phenomenon, due to two different metals with the chemical material in the frog's leg between them.

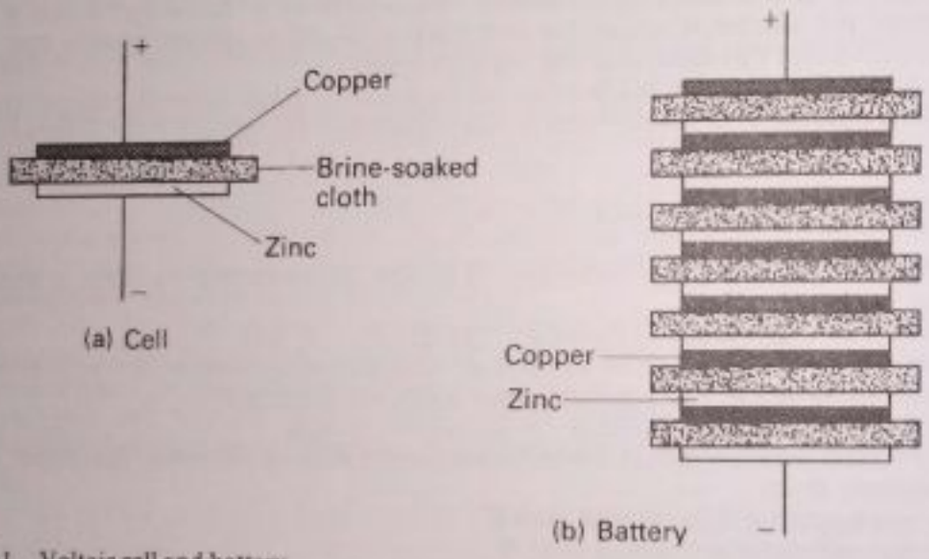


Fig. 25.1 Voltaic cell and battery

After some experiments he placed cloth soaked in brine alternately between a number of discs of zinc and copper (Fig. 25.1). One unit of zinc and copper with brine between them is called a *cell* (Fig. 25.1a). Several cells together, which produce a greater effect than one cell, are called a *battery* (Fig. 25.1b). It was soon found that a wire became warm when joined to the ends of the battery. An electric current was then flowing in the wire.



Plate 25A An electrically-powered car. This Berkeley sports car is driven by nickel-alkaline batteries in place of petrol. (*A. T. Freeman*)

Scanned with CamScanner

Basically, the cell or battery transforms chemical energy to electrical energy. So the chemical materials are gradually used up when the cell is used.

In a cell, one metal plate or pole, called the *positive* (+ve) *pole*, is at a higher electrical potential than the other, which is called the *negative* (−ve) *pole*. This is due to chemical actions at the two plates with which we are not concerned here. So a current flows when the two poles are joined by a wire. The terms 'positive' and 'negative' have no mathematical significance.

Simple cell

An early cell, known as the simple cell, used a copper and zinc plate in dilute sulphuric acid (Fig. 25.2).

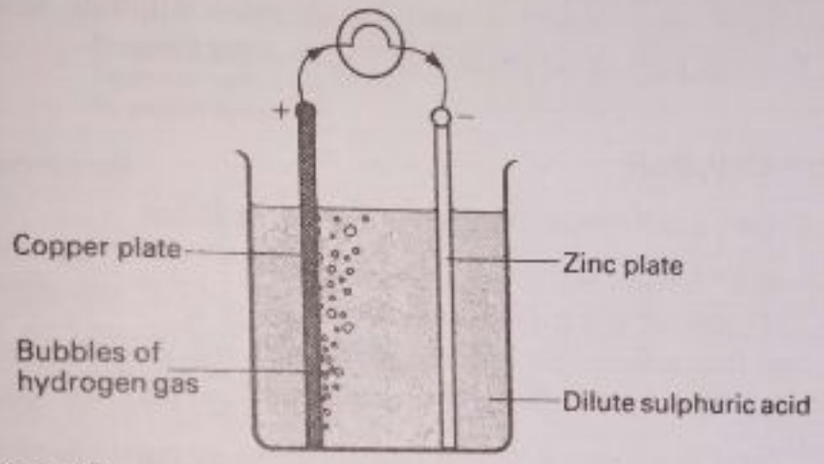


Fig. 25.2 Simple cell

When a voltmeter is connected between the plates, the instrument shows that copper is at a higher potential than zinc. So copper is the positive and zinc the negative pole or terminal. A small electric bulb joined to the plates lights up, showing current flows through its filament. Observation of the plates inside the cell shows that a gas collects on the copper plate. It proves to be hydrogen. The light in the bulb goes out after a short time, indicating that the cell is no longer producing a current.

Electromotive force (e.m.f.)

The name *electromotive force*, *e.m.f.*, was given years ago to the p.d. at the terminals of a cell when it is on 'open circuit', that is, its terminals are not connected to any resistance or apparatus. At the time it was thought that a force existed inside a cell which kept the electric current flowing. Now we consider that a p.d. is set up by contact between copper and acid, and between zinc and acid, due to movement of electric charges. This results in a net p.d. between copper and zinc and the name 'electromotive force' is given to it when the cell is on open circuit.

Note that the term 'e.m.f.' can be used only for a *generator* of electrical energy, that is, a device which transforms energy of some kind to electrical energy. So we can refer to the e.m.f. of a cell, which changes chemical to electrical energy, or to the e.m.f. of a dynamo, which changes mechanical to electrical energy. We never use 'e.m.f.' in connection with a wire which carries a current, because the wire, by itself, does not generate electrical energy.

Scanned with CamScanner

Defects of the simple cell

As already stated, bubbles of hydrogen gas collect on the copper plate when the simple cell is working.

Now hydrogen and zinc in the cell also have an e.m.f. between them, as they are unlike elements. This e.m.f. acts in the opposite direction to that between the copper and zinc, and so it is known as a 'back e.m.f.' The back e.m.f. due to hydrogen reduces the current in the circuit in a short time, and when the cell stops working it is said to be *polarised*. So the hydrogen at the copper plate produces polarisation.

Polarisation of cells can be reduced by adding a suitable chemical, which is known as the *depolariser*. This should not mix with the main chemical of the cell, such as the acid, called the *electrolyte*.

The *Daniell cell* (no longer in use) has copper sulphate solution as a depolariser. This cell provides a small current for some time.

Leclanché dry cell

Leclanché designed a cell still in use today (Fig. 25.3). It has

(a) a +ve pole of carbon in the form of a rod;

(b) a −ve pole of zinc; this is the containing vessel;

(c) an electrolyte of ammonium chloride (NH_4Cl); this is made into a paste or jelly of ammonium chloride with flour and gum so that the electrolyte is reasonably dry;

(d) a depolariser of manganese dioxide (MnO_2), mixed with powdered carbon; this is in a muslin bag round the carbon pole.

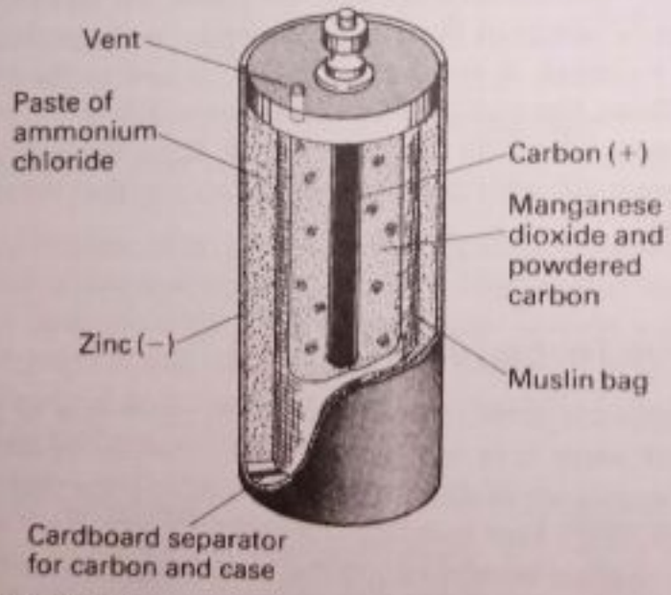


Fig. 25.3 Dry (Leclanché) cell

A cardboard disc is used at the bottom to prevent the carbon +ve pole touching the zinc −ve container, which would short-circuit the cell. As the e.m.f. of one Leclanché cell is 1·5 V, a 9 V dry battery has six cells inside it in series. A high-tension battery of 120 V has eighty cells in series.

Fig. 25.4 shows cells of the layer type, which form a 9 V battery. *Layer cells*, made with more pure manganese dioxide and with the electrolyte in a paper lining, provide greater current for a longer time than the round cell.

Scanned with CamScanner

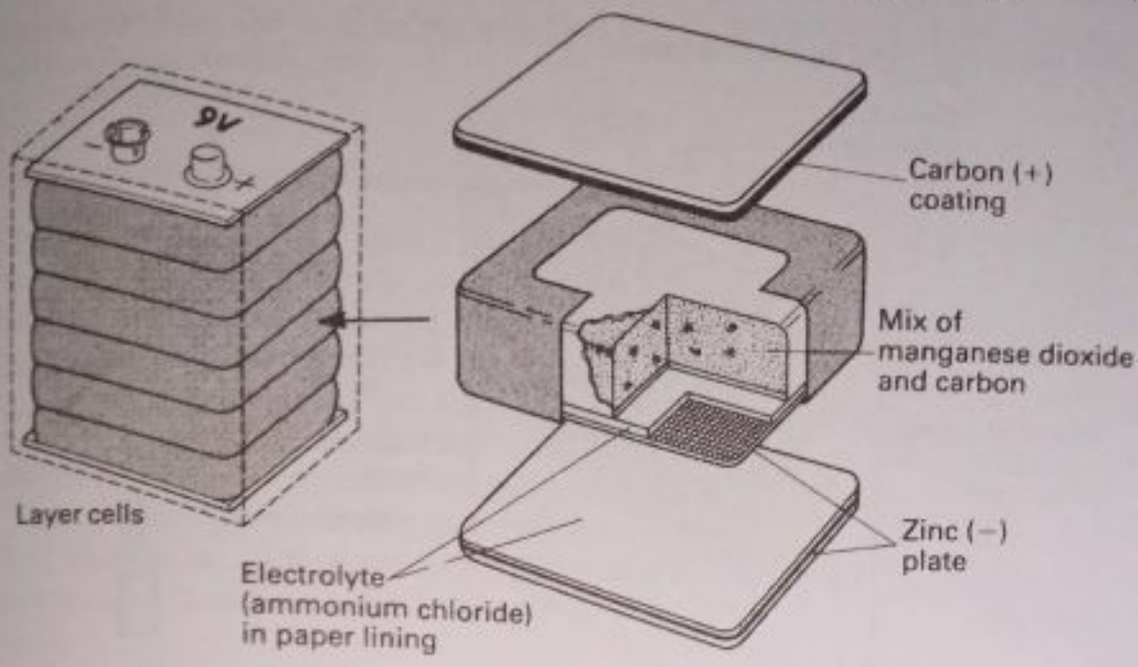


Fig. 25.4 Layer dry cell

The advantage of the Leclanché cell is the cheapness of its chemicals and the relatively high e.m.f. The disadvantage is that manganese dioxide is a slow depolariser and more hydrogen is produced than it can react with. The cell recovers if it is disconnected from the circuit concerned and left alone. It is therefore used in bell circuits, for example, where a current is required only at intervals. The depolariser, manganese dioxide, is a solid and a poor conductor, and it is therefore mixed with powdered carbon which is a good conductor.

Accumulators

The disadvantage of the *primary cells*, as the Leclanché and Daniell cells are called, is that the chemicals in them are gradually used up as they are used. Accumulators, however, called *secondary cells*, can be recharged; that is their chemicals can be restored on the plates by a suitable current (page 413). They can therefore last for a long time. Further, unlike a primary cell, an accumulator can maintain a large current for a long time without polarisation. Accumulators are used in cars to provide the energy for the spark igniting the petrol in the engine (page 209), and they are stored at telephone exchanges to provide the current required in telephone cables when a call is made.

Lead–acid accumulator

There are now a number of different types of accumulators. One of the most widely used is the lead–acid accumulator (Fig. 25.5a). Another is the nickel–alkaline or 'Nife' cell. We shall deal only with the lead–acid type. This has

(a) a +ve pole of lead peroxide–chocolate brown in colour;
(b) a −ve pole of lead–slate-grey in colour;
(c) an electrolyte of dilute sulphuric acid.

The lead peroxide and lead are placed inside several grid plates made of a lead–antimony compound. All the positive plates are joined together and are interleaved with, but separated from, the negative plates (see Plate 25B).

Scanned with CamScanner

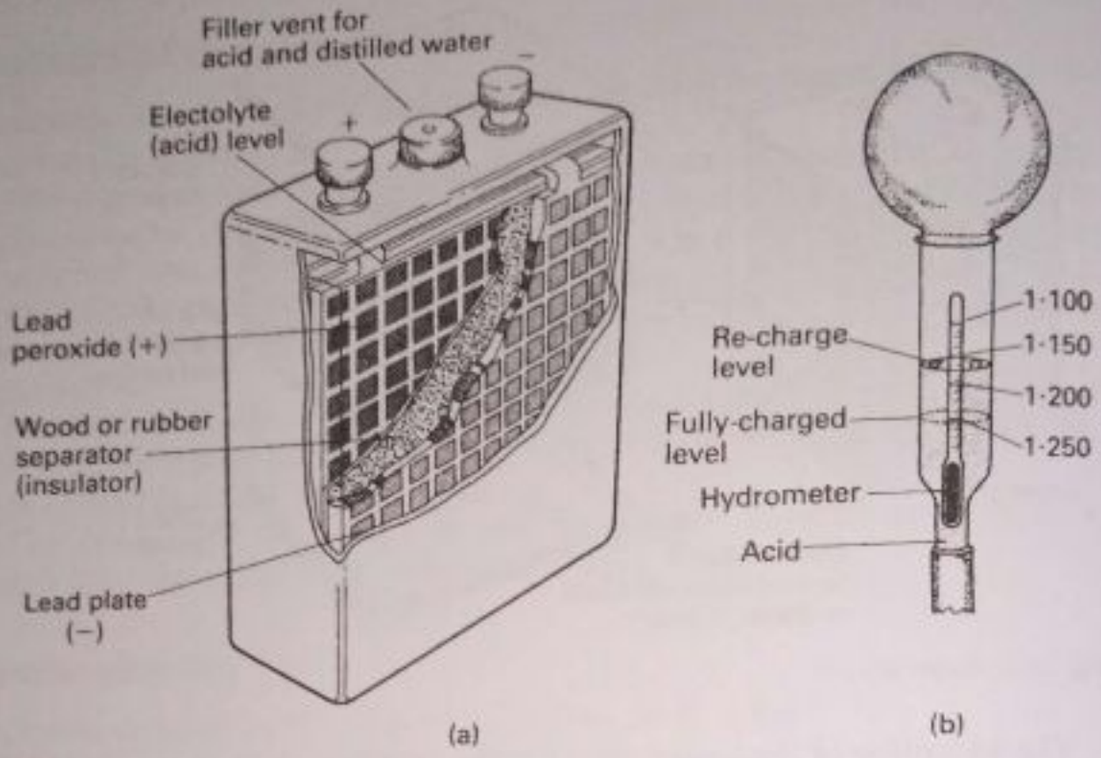


Fig. 25.5 (a) Lead–acid accumulator (b) Battery hydrometer

When the accumulator is freshly made

(a) it has an e.m.f. of about 2 V;

(b) the acid has a relative density (specific gravity) of about 1·25, depending on the manufacturer's specification.

The capacity of an accumulator is classified in 'ampere–hours'. An accumulator of 30 amp–hours could maintain a current of 1 A for 30 h, or a current of 2 A for 15 h, before recharging was necessary. As higher currents are



Plate 25B Lead-acid accumulator parts; note that the plates are interleaved. (*Chloride Storage Co*)

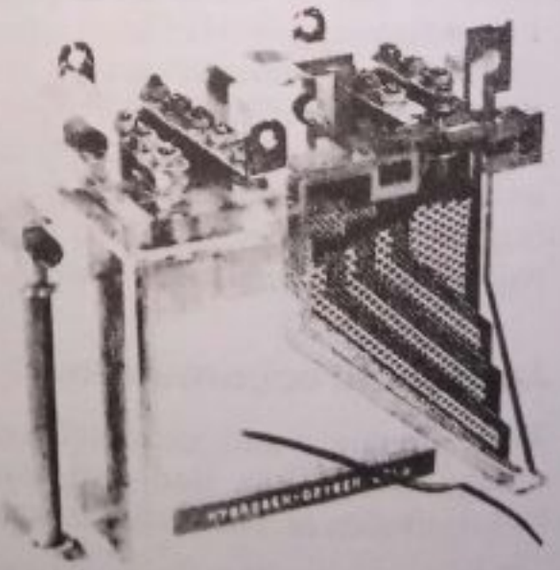


Plate 25C A fuel cell. Unlike the dry cell and accumulator, these cells obtain electrical energy from fuel such as hydrogen and oxygen fed into them continuously. They are very efficient, deliver high power, and could be used in the future to power cars and other vehicles. (*Chloride Storage Co*)

Scanned with CamScanner

drawn, however, the time before recharging becomes less than that calculated from the theoretical figure.

Using (discharging) the accumulator

When the accumulator is used a little lead sulphate is formed at both plates and the e.m.f. and relative density of the acid slowly decreases. The e.m.f. and relative density should not be allowed to fall below the figures recommended by the manufacturer, for example, 1·9 V may be the minimum for the e.m.f. and 1·18 may be the minimum for the relative density. If they fall below these values, excessive lead sulphate, a hard white substance, is formed on the plates, and the accumulator may be permanently damaged as it is then difficult to recharge.

So accumulators need care and maintenance if they are to last for some time. They should be 'topped-up' regularly with distilled water, for example. The accumulator is said to be discharging when it is used.

Recharging accumulators

An accumulator is recharged by passing a current through it in the opposite direction to the current it supplies (Fig. 25.6). A current in one direction, called direct current (d.c.), is needed. If the mains supply is used it must first be changed from an alternating current (a.c.) to a direct voltage (d.c.) supply by a rectifier in the battery-charger.

Care must always be taken to connect the positive terminal of the charging supply to the positive (red terminal) side of the accumulator, and the negative mains terminal to the negative (black or blue terminal) side of the accumulator (Fig. 25.6). The current then passes into the accumulator in the opposite way to when the accumulator was used and recharges it.

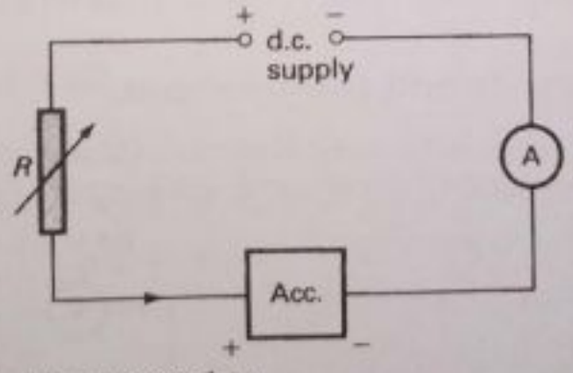


Fig. 25.6 Charging circuit for an accumulator

If a mistake is made and the positive mains terminal is joined to the negative pole side of the accumulator, the accumulator would help the mains to supply current. This would discharge the accumulator and is therefore harmful. A rheostat R is needed in the charging because the acid has a very low resistance, about $1/50\,\Omega$. An ammeter A is necessary so that the charging current can be read.

As the accumulator is charged and restored to a usable condition, the ammeter reading falls. This is because of the rise in e.m.f. of the accumulator. It occurs when the lead sulphate, which was formed on the plates while the accumulator was discharging, is changed back to lead peroxide and lead. The relative density

Scanned with CamScanner

of the acid also rises. The charging is near the end of its process after 'gassing' occurs inside the accumulator, as the current begins also to decompose the water present. When the relative density reaches the recommended value, such as 1·25, which is checked by a hydrometer (Fig. 25.5b) the accumulator is fully recharged. The e.m.f. is then above 2 V; it soon falls to this figure after short use.

Car battery

Some cars have an ammeter as one of the panel instruments, which is in series with the car battery, usually a 12 V one. When the car is in motion a dynamo, driven by the fan-belt, recharges the battery. On a long journey in daylight, when lights are not used, the battery thus becomes fully charged, as shown by the ammeter reading. When the engine is idling the ammeter indicates a discharge. The dynamo is now automatically disconnected from the battery.

The ammeter has a maximum range of 30 A on either side of the zero. When a car is started, the starter motor draws a very large current from the battery for a short time. Frequent short journeys by car tend to 'drain' the battery because a large current flows each time the starter is operated. A home battery-charger is useful in this case.

Electromotive force and circuits

E.m.f. and internal resistance

We now discuss the relationship between current, potential difference and resistance in electrical circuits. The cell or battery used has an electromotive force, E, and an internal resistance, r, due to the chemicals inside it. As stated before, E is equal to the p.d. at the terminals of the cell on open circuit, that is, when the cell is not used to supply a current.

(The 'energy' definition of the e.m.f. of a cell is given on page 425.)

External resistance and terminal p.d.

Suppose a cell X is connected to an outside or external resistance R, with an ammeter A for measuring the current and a voltmeter V for measuring the p.d.

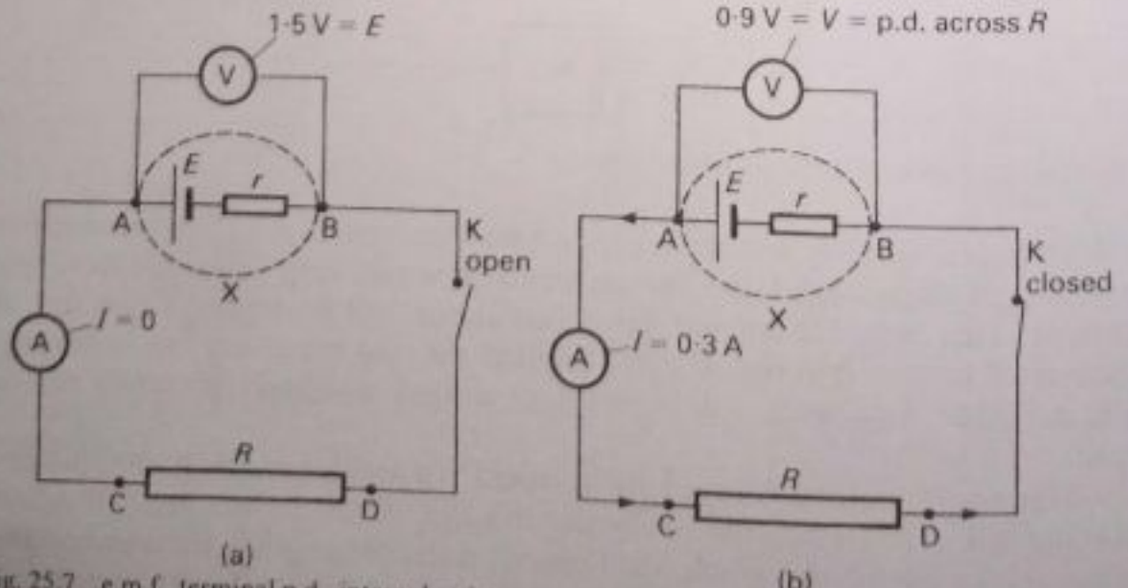


Fig. 25.7 e.m.f., terminal p.d., internal resistance

Scanned with CamScanner

across the cell terminals A, B (Fig. 25.7a). A key K is included for breaking the circuit, in which case the cell is on 'open circuit'.

(a) *Key open* Since the cell is on open circuit, the voltmeter reading, say 1·5 V, is the e.m.f. E. The ammeter reading is zero.

(b) *Key closed* A current I flows round the circuit through R and r, say 0·3 A. The voltmeter reading may then be 0·9 V (Fig. 25.7b).

It is important to realise that the p.d. V across the terminals A, B is the p.d. across the external resistance R if we neglect the small ammeter resistance. So if the voltmeter was taken from A, B and connected across C, D, the reading would again be 0·9 V. Remember that

$$\text{terminal p.d.} = \text{p.d. across } \textit{external} \text{ resistance}$$

Since e.m.f. E 'drives' the current round the whole circuit, 1·5 V is the p.d. across R *and* r, the internal resistance. Now 0·9 V, or V, is the p.d. only across R. So

$$\text{p.d. across } r = E - V = 1{\cdot}5 - 0{\cdot}9 = 0{\cdot}6\,\text{V}$$

Circuit formulae

(a) Since E is the p.d. across the total resistance $(R+r)$, the current I flowing in the circuit is given by

$$I = \frac{E}{R+r} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

So if $E = 20\,\text{V}$, $R = 8\,\Omega$ and $r = 2\,\Omega$,

$$I = \frac{20}{8+2} = \frac{20}{10} = 2\,\text{A}$$

(b) If the p.d. across R, or terminal p.d., is V, then

$$I = \frac{V}{R} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

So if $V = 16\,\text{V}$ and $R = 8\,\Omega$

$$I = \frac{16}{8} = 2\,\text{A}$$

(c) If the p.d. across $(R+r)$ is E, and the p.d. across R alone is V, then

$$\text{p.d. across } r = E - V$$

So

$$r = \frac{E-V}{I} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

If $E = 20\,\text{V}$, $V = 16\,\text{V}$ and $I = 2\,\text{A}$, then

$$r = \frac{20-16}{2} = \frac{4}{2} = 2\,\Omega$$

Measurement of internal resistance

Experiment

Connect in series a dry battery X (such as 1·5 V), an ammeter A (such as 0–1 A), a rheostat R of a few ohms and a key K (Fig. 25.8). Join a voltmeter V (such as 0–2 V) across the battery terminals.

Scanned with CamScanner

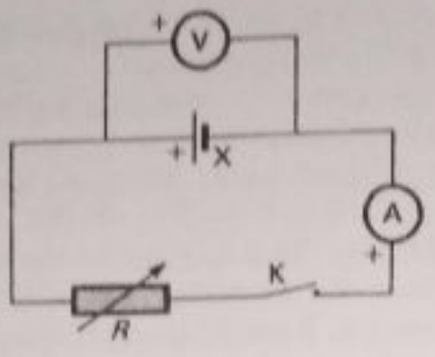


Fig. 25.8 Measurement of internal resistance

(a) With the key K open, record the voltmeter reading. This is the e.m.f., E.

(b) With K closed, adjust the rheostat R so that a small current I flows.

Adjust the rheostat so that the ammeter pointer is on an exact division of the scale so that I can be read accurately. Record the readings of I and V in a table. Now increase the current, and record several more pairs of values of V and I. After each pair of readings, open the key K and check if the e.m.f. E has the same value as at the beginning of the experiment.

Measurements	V/V	I/A	$r = (E-V)/I$

Calculation: For each pair of values V and I, calculate r from $(E-V)/I$ and enter the values in the table.

Graph: Plot V against I. The equation $V = E - Ir$ shows that the graph is a straight line of negative slope or gradient equal to r, which can therefore be found.

Note: If too much current is taken from the battery, its e.m.f. and internal resistance change appreciably.

Internal resistance values

In the case of the lead–acid accumulator, the reading of a voltmeter across its terminals is 2·0 V on open circuit. So $E = 2{\cdot}0$ V. When a resistance of a few ohms is connected across the terminals, the voltmeter reading V is only slightly less than 2·0 V. So the p.d. across the internal resistance r (which is $E - V$) is very low even when a current flows. We therefore deduce that r is very low for the accumulator. This is because the acid inside has a very low resistance.

With a dry battery, however, there is an appreciable difference between E and V when a small resistance is connected across the terminals. So the internal resistance of the dry battery is appreciable, a few ohms for example.

Circuit calculations

1. A cell of unknown e.m.f. E and internal resistance 2 Ω is connected to a 5 Ω resistance. If the terminal p.d. V is 1·0 V, calculate E.

Terminal p.d. $V = 1{\cdot}0$ V = p.d. across 5 Ω

$\therefore$ current in 5 Ω wire $I = \dfrac{V}{R} = \dfrac{1{\cdot}0}{5} = 0{\cdot}2$ A

$\therefore$ p.d. across internal resistance r of 2 Ω $= Ir = 0{\cdot}2 \times 2 = 0{\cdot}4$ V

e.m.f. E = p.d. across R + p.d. across r

$= 1{\cdot}0 + 0{\cdot}4 = 1{\cdot}4$ V.

Scanned with CamScanner

Alternatively The current flowing, $I = \frac{V}{R} = \frac{1{\cdot}0}{5} = 0{\cdot}2\,\text{A}$

Now $$I = \frac{E}{R+r} = \frac{E}{5+2} = \frac{E}{7}$$

So $$E = I \times 7 = 0{\cdot}2 \times 7 = 1{\cdot}4\,\text{V}$$

2. A battery of e.m.f. 40 V and internal resistance 5 Ω is connected to a resistance of 15 Ω. Calculate the terminal p.d.

Since 40 V = E = p.d. across the whole circuit

$$I = \frac{E}{R+r} = \frac{40}{15+5} = 2\,\text{A}$$

Terminal p.d. = p.d. across $R = IR = 2 \times 15 = 30\,\text{V}$

3. The terminal p.d. of a battery is 12 V when an external resistance of 20 Ω is connected and 13·5 V when an external resistance of 45 Ω is connected. Calculate the e.m.f. and the internal resistance of the battery.

For the whole circuit $$I = \frac{E}{R+r}$$

For the terminal p.d., or p.d. across the external resistance R

$$I = \frac{V}{R} = \frac{12}{20} = \frac{E}{20+r} \quad \ldots\ldots\ldots\ldots (1)$$

and $$I = \frac{V}{R} = \frac{13{\cdot}5}{45} = \frac{E}{45+r} \quad \ldots\ldots\ldots\ldots (2)$$

From (1), $$E = \frac{12}{20}(20+r) = 12 + \frac{3}{5}r$$

From (2), $$E = \frac{13{\cdot}5}{45}(45+r) = 13{\cdot}5 + \frac{3}{10}r$$

So $$12 + \frac{3}{5}r = 13{\cdot}5 + \frac{3}{10}r$$

$$\therefore \frac{3}{5}r - \frac{3}{10}r = 13{\cdot}5 - 12, \text{ or } \frac{3}{10}r = 1{\cdot}5$$

$$\therefore r = \frac{10 \times 1{\cdot}5}{3} = 5\,\Omega$$

From (1), $$\frac{12}{20} = \frac{E}{20+5}$$

$$\therefore E = \frac{12}{20} \times 25 = 15\,\text{V}$$

Cells in series

To obtain a bigger e.m.f., cells can be arranged in series to help each other (Fig. 25.9). In this case

(a) total e.m.f. between terminals A and B = sum of individual e.m.f.s;

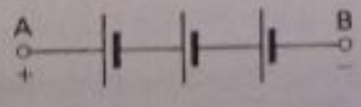


Fig. 25.9 Cells in series

Scanned with CamScanner

(b) total internal resistance = sum of individual internal resistances. So if the three cells shown in Fig. 25.9 are similar, and each has an e.m.f. of 2 V and internal resistance 6 Ω,

$$\text{total e.m.f. } E = 3 \times 2 = 6\text{ V},$$
$$\text{total internal resistance } r = 3 \times 6 = 18\,\Omega$$

If the external resistance $R = 12\,\Omega$ the current flowing is

$$I = \frac{E}{R+r} = \frac{6}{18+12} = \frac{6}{30} = \frac{1}{5} = 0{\cdot}2\text{ A}$$

$\therefore$ p.d. across terminals A and B = p.d. across R
$= IR = 0{\cdot}2 \times 12 = 2{\cdot}4\text{ V}$

The p.d. across the total internal resistance $= 6 - 2{\cdot}4 = 3{\cdot}6\text{ V}$.

Cells in parallel

Suppose three identical cells, each of e.m.f. 2 V and internal resistance 6 Ω, are arranged in parallel (Fig. 25.10). A voltmeter connected to the terminals A and B shows that the total e.m.f. is only 2 V, the e.m.f. of one cell. This is explained by the fact that the three cells together act like a single cell whose materials are the sum of those in the individual cells. Now the e.m.f. of a cell depends only on the nature of the chemicals used and not on their amount. So the e.m.f. is 2 V, the e.m.f. of one of the cells.

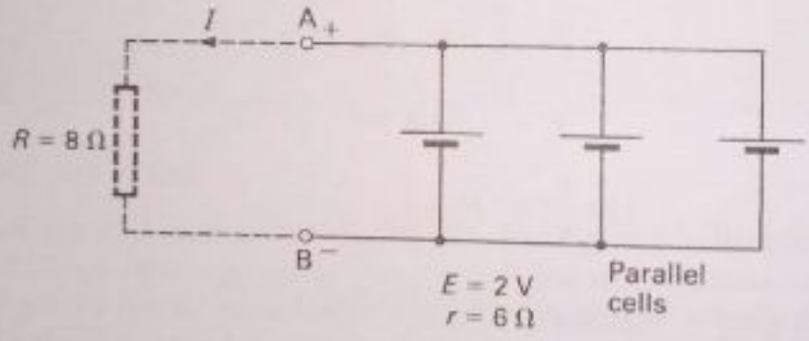


Fig. 25.10 Cells in parallel

The internal resistance of the battery is different from that of a single cell. From Fig. 25.10 we see that the internal resistances of the three cells are in parallel. So the total internal resistance r is given by

$$\frac{1}{r} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$$
$$\therefore\ r = \frac{6}{3} = 2\,\Omega$$

So the battery has an e.m.f. E of 2 V and an internal resistance of 2 Ω. If the external resistance is 8 Ω,

$$I = \frac{E}{R+r} = \frac{2}{8+2} = 0{\cdot}2\text{ A}$$

This is the current in the 8 Ω wire. The current through each cell is one third of 0·2 A or about 0·07 A.

Scanned with CamScanner

Examples on resistivity and electric circuits

1. The coil A (Fig. 25.11) has resistance of 5 Ω and is connected to the terminals of a cell of constant e.m.f. E volt and internal resistance r ohm. A uniform wire of length L cm is joined across the ends of the 5 Ω coil to produce a parallel arrangement of resistance 4Ω.

Determine (a) the resistance of the wire L and (b) its length if the resistivity of its material is $5 \times 10^{-7}\,\Omega$ m and its cross-sectional area is $4 \times 10^{-7}\,m^2$

The current drawn from the cell when the wire is connected across A, as shown, is 0·3 A but this is reduced to 0·25 A when the wire is removed, leaving A alone connected to the cell. Determine (c) the e.m.f. of the cell, (d) the internal resistance of the cell and (e) the electrical energy dissipated in the 5Ω resistor when the current of 0·25 A flows through it for a period of 4 minutes. (L)

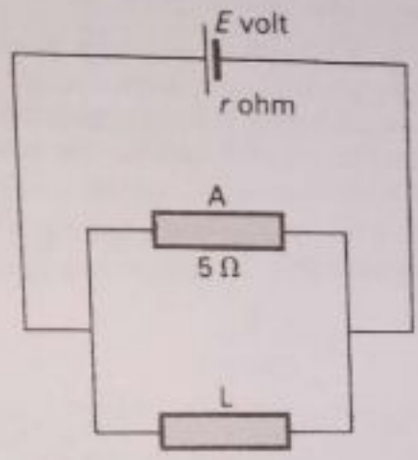


Fig. 25.11 Example

(a) Suppose R is the resistance of the wire L. Then, from the parallel resistance formula,

$$\frac{1}{R}+\frac{1}{5}=\frac{1}{4}$$

$$\therefore\ R = 20\,\Omega$$

(b) From $R = \rho l/A$,

$$l = \frac{R \times A}{\rho} = \frac{20 \times 4 \times 10^{-7}\ \text{m}}{5 \times 10^{-7}}$$

$$= 16\ \text{m}$$

(c) With L present, total circuit resistance $= 4 + r$

$$\therefore\ I = 0{\cdot}3 = \frac{E}{4+r}$$

$$E = 0{\cdot}3\,(4+r). \quad \ldots\ldots\ldots\

2. Describe, giving a circuit diagram, an experiment which would show whether a particular cell has an appreciable internal resistance. Indicate briefly what readings would be needed in order to estimate its value.

Explain why a check against a short circuit should be made when using secondary cells such as lead plate accumulators, whereas this is not so important when dry cells are used.

Resistances of 2Ω and 3Ω are connected in series with a cell. A high resistance voltmeter connected across the 3Ω resistor reads 1·0 V, but this increases to 1·2 V when an extra 2Ω resistor is connected in parallel with the first 2Ω resistor. Calculate the electromotive force and internal resistance of the cell. (L)

Using the circuit in Fig. 25.7, the e.m.f. E when the cell is on open circuit, and the terminal p.d. V when the cell is connected to a low resistance are both measured. The cell has an appreciable internal resistance when $(E-V)$ has an appreciable difference. The internal resistance can be measured from $r = (E-V)/I$.

The internal resistance of a lead–acid accumulator is very low, say 1/50 Ω. So any accidental short circuit will produce a very large current and this may damage the accumulator. A dry cell has a much higher internal resistance, say a few ohms. So a short circuit will produce a much smaller current than for the accumulator, which would not damage the dry cell to the same extent.

Since the p.d. V across 3Ω is 1·0 V, the current I flowing $= V/R = 1/3$ A. But if E is the e.m.f. and r is the internal resistance (Fig. 25.12)

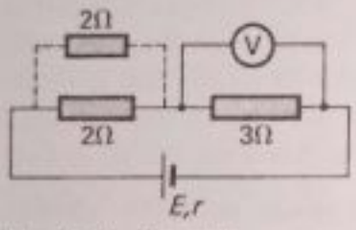


Fig. 25.12 Example

$$I = \frac{E}{R+r}$$

$$\therefore \frac{1}{3} = \frac{E}{2+3+r} = \frac{E}{5+r}$$

$$\therefore E = \frac{5+r}{3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

When the 2Ω is placed in parallel with the original 2Ω, the combined resistance R is given by

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{2} = 1, \text{ so } R = 1\,\Omega$$

Also, the new current flowing

$$I = \frac{V}{R} = \frac{1{\cdot}2}{3} = 0{\cdot}4\text{ A}.$$

From $I = E/(R+r)$

$$0{\cdot}4 = \frac{E}{1+3+r} = \frac{E}{4+r}$$

$$\therefore E = 0{\cdot}4(4+r) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

From (1) and (2), then

$$\frac{5+r}{3} = 0{\cdot}4(4+r) = 1{\cdot}6 + 0{\cdot}4r$$

$$\therefore 5+r = 4{\cdot}8 + 1{\cdot}2r$$

$$\therefore 0{\cdot}2r = 0{\cdot}2, \text{ or } r = 1$$

From (1)

$$E = \frac{5+r}{3} = \frac{6}{3} = 2\text{ V}$$

The potentiometer

When a voltmeter is connected directly across the poles of a cell to measure its e.m.f., a very small current flows in the meter. The voltmeter does not then give the e.m.f. to a high accuracy because the e.m.f. is the p.d. between the poles when

Scanned with CamScanner

no current flows, that is the cell is on 'open circuit' (page 415). Another disadvantage of the voltmeter is that it has to use a pointer, which could cause errors in reading (see page 469).

The potentiometer provides an accurate method of measuring e.m.f. It consists of a length AB of uniform resistance wire, that is one with constant diameter, with a constant current flowing through it from an accumulator C (Fig. 25.13). The length of AB may be one metre and any length from A is read from a metre rule below AB.

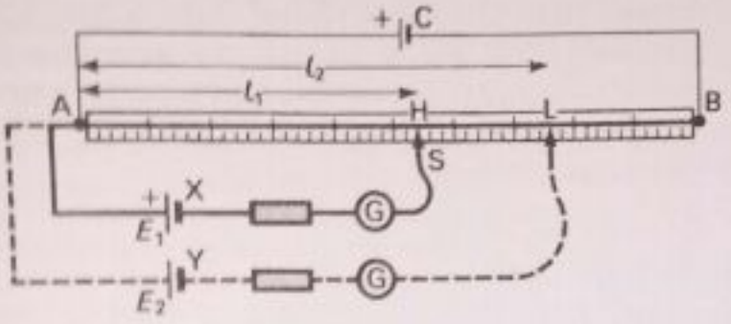


Fig. 25.13 Potentiometer. Comparison of e.m.f.s

Experiment

To compare the e.m.f.s of two cells, X and Y say, (a) join the +ve pole of X to A, the same terminal to which the +ve pole of C is joined, (b) join the −ve pole of X to a slider S through a sensitive galvanometer G. The galvanometer must have a centre-scale zero and a high resistance in series to protect it from excessive current.

By tapping gently on the wire with S, find the point H when no current flows in G. H is said to be the 'balance point'. Read off the length AH or l_1 from the metre rule.

Remove the cell X and replace it with the second cell Y. As before, obtain the new balance point L and read off the length AL or l_2.

Theory explained below shows that

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

If AH = 52·4 cm = l_1 and AL = 74·6 cm = l_2 in an experiment, then

$$\frac{E_1}{E_2} = \frac{52{\cdot}4}{74{\cdot}6} = 0{\cdot}70$$

Thus the two e.m.f.s can be compared.

If the actual e.m.f. E_1 of the cell X is required, then the cell Y must be one whose e.m.f. E_2 is accurately known.

Theory Since no current flows in G at the balance point H, the e.m.f. E_1 of cell X balances exactly the p.d. V_1 between A and H on the wire. Hence $E_1 = V_1$. Similarly, if E_2 is the e.m.f. of cell Y and V_2 is the p.d. between A and L, then $E_2 = V_2$.

$$\therefore \frac{E_1}{E_2} = \frac{V_1}{V_2}$$

Now if the wire AB carries a constant current, the p.d. V across any length l is directly proportional to l. So $V_1/V_2 = l_1/l_2$. Therefore $E_1/E_2 = l_1/l_2$.

Scanned with CamScanner

Summary

Cells

1. A simple cell has a +ve copper and −ve zinc pole, and an electrolyte of dilute sulphuric acid. Defect: polarisation due to hydrogen at the copper pole produces back e.m.f. and high internal resistance.
2. A Leclanché dry cell has a +ve carbon and −ve zinc pole, an electrolyte of ammonium chloride paste and a solid depolariser of manganese dioxide mixed with powdered carbon. The hydrogen produced at the carbon pole is oxidised to water. The e.m.f. is about 1·5 V, and a current is maintained for a short time.
3. An accumulator of lead–acid type has +ve lead peroxide and −ve lead poles and electrolyte of dilute sulphuric acid. E.m.f. about 2 V, very low internal resistance. It can be recharged, unlike primary cells.

E.m.f. and circuits

4. The e.m.f. of a cell is the total p.d. across the external and internal resistances (see page 425 for energy definition of e.m.f.).
5. When a resistance R is joined to the cell, current $I = E/(R+r)$. The p.d. at terminals = p.d. across $R = IR$.
6. Internal resistance can be investigated by using a voltmeter and ammeter.
7. Cells in series: total e.m.f. is the sum of individual e.m.f.s, and total internal resistance is the sum of individual internal resistances.

Cells in parallel: with *similar* cells, total e.m.f. = e.m.f. of one cell, and total internal resistance is given by the parallel resistance formula.

Exercise 25 (Answers, page 643)

Cells

1. What is the value of the electromotive force of (a) a lead accumulator; (b) a simple (copper–zinc) voltaic cell; (c) a Leclanché cell? (N)
2. Name *two* advantages which a lead accumulator has over a dry cell. (N)
3. Give a diagram showing the structure of a Leclanché cell, naming its components. State one advantage and one disadvantage of the cell and explain on what factors its internal resistance depends. (L)
4. Draw a circuit diagram for the re-charging of an accumulator from a direct-current supply, at a definite charging-current; show clearly the polarity of the accumulator and of the supply terminals. If the polarity of the supply terminals was not marked, how would you determine the positive terminal by a magnetic method? (C)

E.m.f. and circuits

5. (a) A battery of e.m.f. 3 V has an internal resistance of 2 Ω. Calculate the current flowing when it is connected to an 8 Ω resistor. What is the p.d. across the battery terminals?

(b) A wire of 4 Ω is connected to a battery of e.m.f. 6 V and internal resistance r. The current flowing is 1 A. Calculate r.

(c) A 10 Ω resistor is connected to a battery of internal resistance 5 Ω. The p.d. across the terminals of the battery is 2 V. Find (i) the current flowing, (ii) the e.m.f. of the battery.

(d) A battery of e.m.f. 2 V and internal resistance 1 Ω is joined in series with a battery of e.m.f. 4 V and internal resistance 3 Ω so that they assist each other. A resistor of 8 Ω is joined to this arrangement. Calculate the current flowing.

6. Two resistors of 20 ohm and 30 ohm are connected in parallel across the terminals

Scanned with CamScanner

of a Leclanché cell of e.m.f. 1·5 V and internal resistance 3·0 ohm. Calculate the potential difference across the terminals of the cell. (N)

7. A battery consists of two cells joined in parallel. If each cell has an e.m.f. of 1·5 V and internal resistance 5 Ω, what current will flow through an external resistance of 5 Ω? (N)

8. Describe one form (either wet or dry) of the Leclanché cell.

Explain what is meant by polarisation. Show how depolarisation is achieved in the form of cell you have described.

A 120 V dry battery sends a current of 0·24 A through an external circuit of resistance 440 Ω. What is the internal resistance of the battery? What current would pass if the external resistance were reduced to 240 Ω? (O)

9. You are supplied with an accumulator, ammeter, voltmeter, rheostat, key, copper wire and an unknown resistance X. Describe, with the aid of a circuit diagram, how you could use this apparatus to determine the resistance X.

Explain why the method does not lead to an accurate result.

A cell of e.m.f. 1·5 V and internal resistance 2·0 Ω is connected in series with an ammeter of resistance 0·5 Ω and a 5·0 Ω resistance. What will be (a) the ammeter reading, and (b) the potential difference across the terminals of the cell? (N)

10. Define *the ampere* and *the volt*. How would you investigate the way in which the current through a wire depends on the potential difference between its ends?

Three cells, each of e.m.f. 1·5 V and internal resistance 0·6 Ω are joined in series to form a battery. What current passes when the battery is connected across a 5 Ω resistance, and what is the p.d. between the terminals of the battery? (O)

11. What is meant by the internal resistance of a cell?

You are supplied with a cell, an ammeter of negligible resistance, two standard resistances, a switch and connecting wire and no other instruments. How would you use this apparatus to find the internal resistance of the cell?

A cell of e.m.f. 1·5 V is connected in series with a coil of resistance R and an ammeter of resistance 0·20 Ω. The difference of potential between the terminals of the cell is 1·35 V and the ammeter reading is then 0·3 A. Find the value of R and the internal resistance of the cell. (N)

12. Draw a labelled diagram of a common dry cell and describe what takes place inside it when it is producing a current.

Two identical cells, each of e.m.f. E V and internal resistance r Ω are joined (a) in series; (b) in parallel. What is the e.m.f. and the internal resistance of the arrangement in each case?

A cell is joined in series with a resistance of 2 Ω, and a current of 0·25 A flows through it. When a second resistance of 2 Ω is connected in parallel with the first the current through the cell increases to 0·3 A. What is (c) the e.m.f.; (d) the internal resistance of the cell? (N)

13. Fig. 25.14 shows four resistors, AB of 2 Ω, BC of 4 Ω, AD of 1 Ω and DC of 2 Ω, connected to a 6 volt accumulator of negligible internal resistance. What is (a) the current in ABC, (b) the current in ADC, (c) the potential difference between A and B?

What would be the reading of a voltmeter connected to B and D? (O and C)

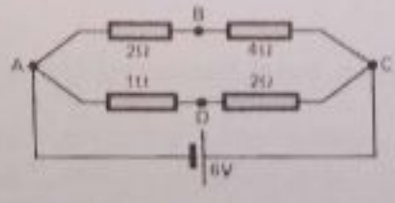


Fig. 25.14

14. A cell has an e.m.f. of 1·08 V and an internal resistance of 0·5 Ω. When it is connected in series with a resistor of R Ω, the potential difference between the terminals falls to 0·96 V. What was the value of R? (O and C)

Scanned with CamScanner

26. Electrical Energy and Power

Electrical energy can be transformed to many other useful forms of energy. In electric motors, for example, most of the electrical energy supplied is changed to the mechanical energy of rotation. In a loudspeaker most of the electrical energy is transformed into sound energy.

When an electric current passes through a resistance wire there is a transfer of energy from electrical to heat energy. Basically, this is due to the moving electrons 'colliding' with the vibrating atoms of the metal, and giving up their energy to the atoms. The heating effect of an electric current is widely used in the home and in industry; electric fires and light bulbs use the heating effect of a current, for example.

Quantity of electrical energy

As we explained on page 391, electrical energy is obtained when a quantity of charge moves between two points which have a potential difference. Thus, from the definition of the volt, 1 J (joule) of energy is obtained when 1 coulomb moves between two points at a p.d. of 1 V. So if 20 coulombs move between two points having a p.d. of 4 V, 80 J of energy are obtained. Generally, if Q coulombs move through two points having a p.d. of V volts, the energy W in joules is given by

$$W = QV. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

A more useful formula for energy is one which contains current, I. Now $Q = I \times t$, where t is the time for which the current flows (page 390). So, from (1), the energy W in joules is given by

$$W = IVt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Electrical machines and energy used

The formula $W = IVt$ tells us how much electrical energy is supplied to an appliance with a p.d. V across it in a time t when a current I flows. In Fig. 26.1a, a battery is connected to an electric motor, which produces mechanical energy. When a battery is connected to a resistor as in Fig. 26.1b, heat energy is produced. In both cases the energy used by the motor and the electric fire is given by $W = IVt$.

An electric motor transfers most of the electrical energy supplied to mechanical energy. In the case of an electric fire, however, *all* the electrical energy is transformed to heat energy in the fire element resistance. So, for a resistance coil,

$$\text{heat energy } H = IVt$$

Scanned with CamScanner

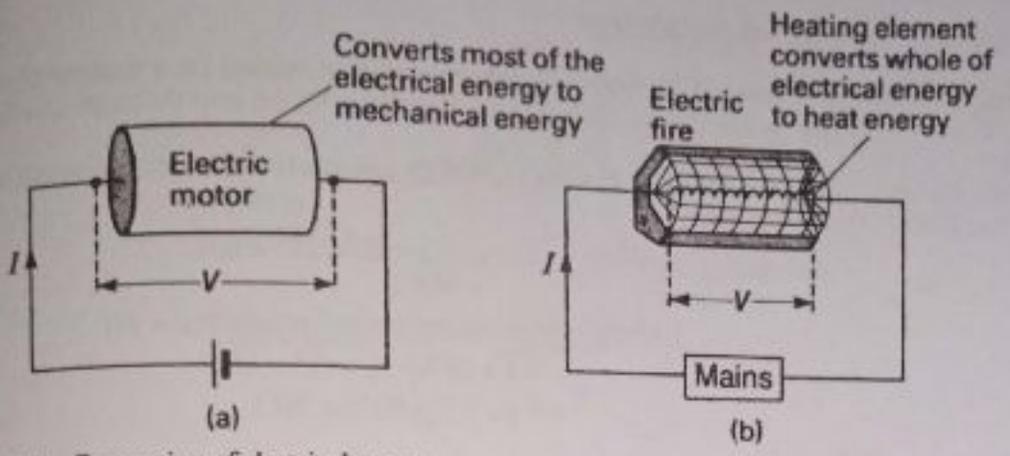


Fig. 26.1 Conversion of electrical energy

Electrical power

Power is defined generally as the *rate of energy transfer* (page 58). So power may be calculated from

$$\text{power } P = \frac{\text{energy transferred}}{\text{time taken}}$$

In the case of electrical machines, the energy transferred in a time t seconds is IVt. So the power P used is given by

$$P = \frac{IVt}{t}$$

$$\therefore \quad P = IV$$

Here P is in *watts*, symbol W, when I is in amperes and V is in volts. 1 W is the rate of working at 1 J/s (page 58).

An electrical appliance, which takes a current I when the p.d. across its terminals is V, uses power IV. An electric motor, which is connected to a 50 V supply and takes a current of 5 A, uses power of 50×5 or 250 W. An electric fire carrying a current of 4 A when the mains p.d. is 240 V uses a power of 960 W. An electric lamp filament carrying a current of $\frac{1}{4}$ A when the mains p.d. is 240 V uses a power of 60 W.

E.m.f. of a battery

A battery or cell is a device for producing electrical energy. We may therefore define the e.m.f., E, as *the total energy per coulomb* it produces. E can also be defined as the *total power per ampere* it produces. So if the battery current is I when it is connected to a resistance R, the total power produced by the battery is EI.

Part of the power is delivered to the resistance R and the rest to the internal resistance r of the battery. So

$$EI = \text{power in } R + \text{power in } r$$
$$= IV + Iv$$

where V is the p.d. across R and v is the p.d. across r. Dividing by I

$$E = V + v$$

or $E =$ p.d. across external resistance + p.d. across internal resistance.

Scanned with CamScanner

Example on energy from a battery

Define electromotive force. Calculate the total energy provided by a battery of e.m.f. 2·50 V when it causes a steady current of 0·40 A to flow for 15 min through an electric bulb.

If the battery had an internal resistance of 2·00 Ω, calculate the heat dissipated in the electric bulb in that time. (L)

$$\text{Total energy provided} = IEt = 2{\cdot}5 \times 0{\cdot}4 \times (15 \times 60) = 900\ \text{J}$$

$$\text{Energy dissipated in internal resistance} = Ivt$$
$$= 0{\cdot}4 \times (0{\cdot}4 \times 2) \times (15 \times 60)$$
$$= 0{\cdot}4 \times 0{\cdot}8 \times 900 = 288\ \text{J}$$

since $v = Ir = (0{\cdot}4 \times 2)\ \text{V}$

$\therefore$ Energy dissipated in electric bulb $= 900 - 288 = 612\ \text{J}$

Electric filament and fluorescent lamps

An electric lamp produces light (luminous) energy from electrical energy. Most of the electrical energy, however, is transformed to heat energy.

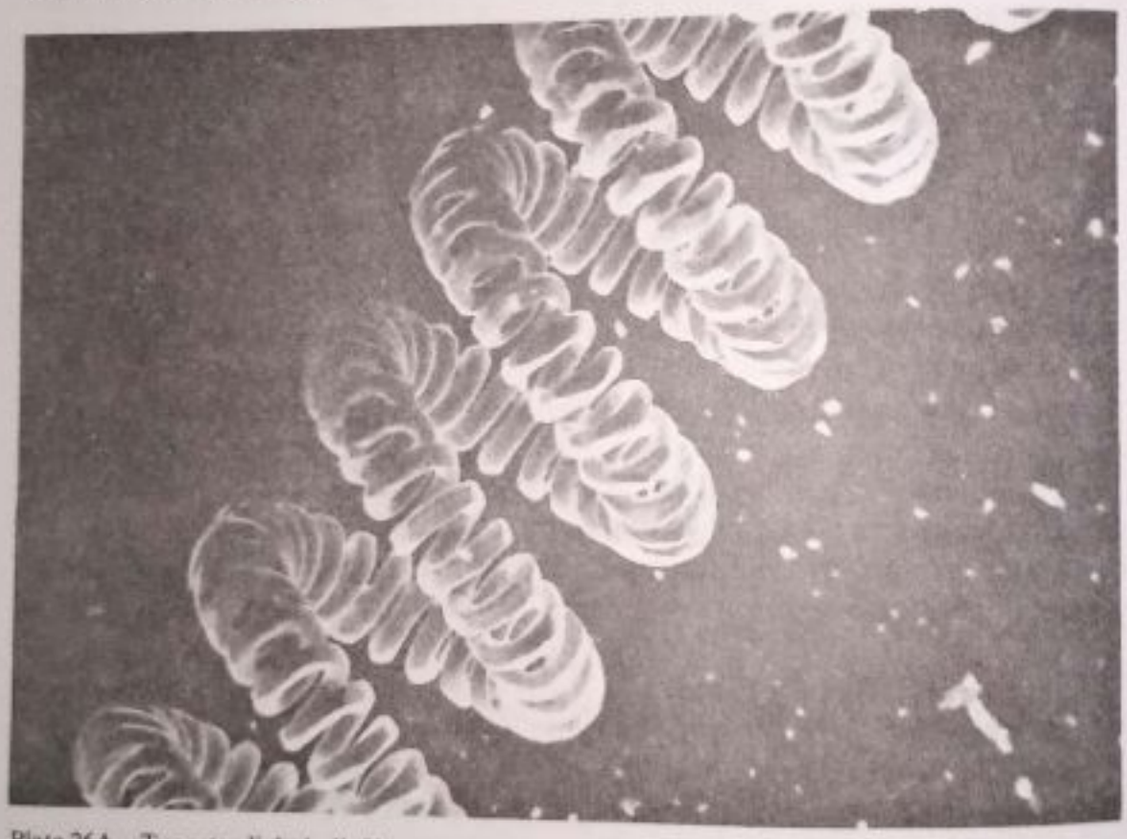


Plate 26A Tungsten light bulb filament magnified 35 times, showing the coiled coil which gives greater light for a given power. (*Dr Tony Brain/Science Photo Library*)

Fig. 26.2 shows a tungsten filament lamp that you might find in your home. Tungsten is used because this metal has a high melting point. The lamp contains a small quantity of the inert gas argon to prevent the hot filament from evaporating. Air is not used as this would oxidise the tungsten.

Fluorescent lamps contain a gas at low pressure inside a long glass tube. When a high voltage is connected between the metal terminals at each end of the tube, a gas in the tube, such as mercury vapour, glows and produces invisible ultra-violet rays. These rays fall on a fluorescent coating inside the lamp, which then glows with a bright colour. The colour of the light can be altered by changing the type of

Scanned with CamScanner

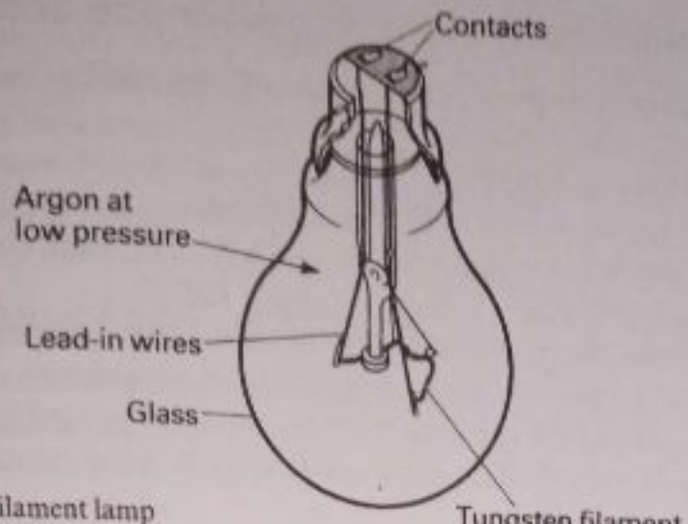


Fig. 26.2 Electric filament lamp

fluorescent coating used. Fluorescent lamps have advantages over the filament type; they are more efficient and cheaper to run, and the light produced is softer.

Power and resistance

Generally, the power consumed in any electrical machine or appliance is given by $P = IV$. In the case of an electric lamp or electric fire the p.d. $V = IR$, where R is the resistance of the element in the lamp or fire.

$$P = IV = I \times IR = I^2R$$

Also
$$P = IV = \frac{V \times V}{R} = \frac{V^2}{R}$$

So in addition to $P = IV$, the relations $P = I^2R$ and $P = V^2/R$ can be used to calculate the power dissipated as heat in resistance wires.

An electric lamp may be rated by the manufacturer as '60 W, 240 V', for example. This means that when 240 V is connected to the lamp, the power used is 60 W. (If the voltage connected is not 240 V, the filament resistance is altered because it depends on temperature, and the power used is then not 60 W.) From $P = IV$, the current I flowing in the lamp when used normally is given by

$$60 = I \times 240$$

$$\therefore\ I = \frac{60}{240} = \frac{1}{4}\text{A}$$

$$\therefore\ \text{Resistance of filament } R = \frac{V}{I} = \frac{240}{1/4} = 240 \times \frac{4}{1} = 960\,\Omega$$

This result can also be obtained directly by using the relation $P = V^2/R$. In this case,

$$60 = \frac{240^2}{R}$$

$$\therefore\ R = \frac{240^2}{60} = \frac{240 \times 240}{60} = 960\,\Omega$$

The relation $P = V^2/R$ shows that, since V is the same for lamps in parallel across the mains, a 100 W lamp filament has a *smaller* resistance than a 60 W filament. So the *current* in the 100 W lamp is greater than in the 60 W lamp when the lamps are used.

Scanned with CamScanner

Commercial units of electric power and energy

Large amounts of electric power are used in industry and large units are therefore more convenient to use.

The kilowatt (kW) = 1000 W.

The megawatt (MW) = 1000000 W = 1000 kW.

One horse-power (h.p.) is about 746 W or $\frac{3}{4}$ kW. The small electric motor used in one type of vacuum cleaner is $\frac{1}{6}$ h.p. or about 125 W.

When commercial electricity companies present their accounts for the cost of lighting and heating premises they are concerned with the amount of electrical energy used. The unit of energy used by the companies in the United Kingdom is the Board of Trade unit, the *kilowatt-hour* (kWh). It is the energy used in 1 hour when the power is 1 kilowatt.

The energy consumed by six 60 W lamps in a room, burning continuously for 8 h

$$= 6 \times 60 \text{ W} \times 8 \text{ h} = 2880 \text{ watt-hour}$$
$$= 2{\cdot}88 \text{ kWh} = 2{\cdot}88 \text{ units.}$$

If the cost is 3p per unit, the total cost of using the lamps is 8·64p.

Electrical energy consumption

Some examples of power consumption in the home are approximately as follows.

Television and radio receivers	100 W (watts)
Small domestic vacuum cleaner	120 W
Electric kettle	750 W
Electric fire	2 kW (kilowatts)
Immersion heater	3 kW
Electric cooker	5 kW

A small house may use on average in winter:

Lighting	400 W	Electric fires	2 kW
Entertainment	100 W	Refrigerator	100 W

Total power used = 2600 W or 2·6 kW. Assuming 5 h use per day, total energy = $2{\cdot}6 \times 5 = 13$ kWh

$\therefore$ total energy used in 10 weeks = $(10 \times 7) \times 13 = 910$ kWh.

If 1 kWh, or Board of Trade unit, costs 3p, then charge

$$= 910 \times 3 = £27.30$$

Heating effect of a current

Joule was the first person to investigate the heating effect of an electric current. His conclusions, known as *Joule's laws of electrical heating*, were:

The heat energy W developed in a wire is proportional to:

(a) the time t (for a given resistance and current);

(b) the square of the current or I^2 (for a given resistance and time);

(c) the resistance R of the wire (for a given current and time).

As we have seen previously, the power or energy per second in a resistance R carrying a current I is $P = I^2R$. So if t is the time, the energy or heat produced, $W = I^2Rt$. For resistances in *series*, I is the same in each resistance. So a large resistance produces more heat than a small resistance.

Scanned with CamScanner

For resistances in *parallel*, however, the p.d. across each is the same. Now we have shown previously that $P = V^2/R$, so the energy or heat produced, $W = V^2t/R$. It follows that, for a fixed p.d., the *smaller* resistance produces more heat than a larger resistance. Lamps in parallel in lighting circuits have the same p.d. Since the power P in the lamp is inversely proportional to R, a 120 W lamp filament has a *smaller* resistance than a 60 W lamp one.

Measurement of coil resistance by heating effect

If only an ammeter is available, the resistance of a coil can be found by using the heating effect of a current.

Experiment

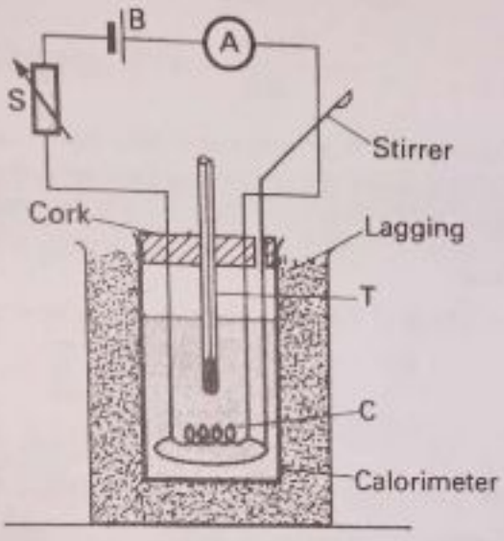


Fig. 26.3 Resistance of a coil by its heating effect

The circuit is connected as in Fig. 26.3. C is a low resistance coil of constantan or nichrome inside a lagged calorimeter, in series with a 6 V accumulator B, a rheostat S and an ammeter A. The calorimeter is weighed, sufficient water to cover the coil C completely is then added, and the water and calorimeter weighed so that the mass m of water is known.

With the coil removed from the water, the circuit is made and the rheostat is adjusted to give a suitable current for heating. The coil is then placed back in the water, the initial temperature is noted, the current is switched on again, and simultaneously a stopclock is started. The current I is passed for a measured time t to produce a suitable temperature rise θ, read from the thermometer T after the water is stirred. The constant value I of the current is read from the ammeter.

Calculation: Assuming negligible loss of heat to the surroundings, then

$$\text{heat produced} = (mc + C)\theta = \text{energy supplied} = I^2Rt$$

where c is the specific heat capacity of water, 4200 J/kg K, C is the heat capacity (mass × specific heat capacity) of the calorimeter, R is the coil resistance and t is the time in seconds. So

$$R = \frac{(mc + C)\theta}{I^2t}$$

Scanned with CamScanner

The value of R given by this calculation can be compared with the value obtained by the voltmeter–ammeter method (page 396). If the two values of R differ widely, a list of possible errors in the heating experiment would usefully account for the difference.

Examples on electrical heating

1. A current of 3 A flows for 2 min through a wire of resistance 20 Ω. If the wire is totally immersed inside 0·1 kg of water in a can of heat capacity 40 J/K, calculate the temperature rise of the water ($c = 4200$ J/kg K for water).

$$\text{Heat supplied} = I^2Rt = 3^2 \times 20 \times 120 = 9 \times 2400 = 21\,600 \text{ J}$$

since $I = 3$ A, $R = 20\,\Omega$, $t = 2$ min $= 120$ s (Note t is in *seconds*).
If x°C is the temperature rise,

$$\text{heat gained by water and calorimeter} = (0{\cdot}1 \times 4200 \times x) + (40 \times x) = 420x + 40x = 460x$$

$$\therefore\ 460x = 21\,600$$

$$\therefore\ x = \frac{21\,600}{460} = 47\text{°C (approx.)}$$

2. Define the *watt* and the *joule*. If an electric kettle contains a 960 W heating unit, what current does it take from 240 V mains? How long will the kettle take to raise 2 kg of water at 15°C to the boiling point, if 90% of the heat produced is used in raising the temperature of the water? How much would this cost if the charge is 3p for 1 kWh? ($c = 4200$ J/kg K for water)

The *watt* is the rate of working at 1 J/s. 1 J = 1 newton × 1 metre.

(a) From $P = IV$

$$960 = I \times 240$$

$$\therefore\ I = \frac{960}{240} = 4 \text{ A}$$

(b) Heat to raise water to boiling point $= 2 \times 4200 \times (100 - 15) = 714\,000$ J

Now energy in joules supplied in t seconds $= 960t$

$\therefore$ heat to raise water to boiling point $= 90\%$ of $960t = 864t$

$$\therefore\ 864t = 714\,000$$

$$\therefore\ t = \frac{714\,000}{864} = 826 \text{ s}$$

(c)

$$\text{Energy used} = 960 \text{ W} \times 826 \text{ s} = 0{\cdot}96 \text{ kW} \times (826/3600) \text{ h}$$

$$= 0{\cdot}22 \text{ kWh}$$

So

$$\text{cost} = 0{\cdot}22 \times 3\text{p} = 0{\cdot}66\text{p}$$

Hot wire ammeter

A *hot wire ammeter* is an ammeter which uses the heating effect of current in its operation. Fig. 26.4 illustrates the principle.

When a current flows along the wire AB, it becomes hot and expands, so that P falls. A fine thread passes from T, a point on a second wire PQ, round a pulley X to a spring S. The spring thus takes up the slack in the wire PQ, turning the pulley X at the same time. So the pointer Y attached to X moves over the scale.

The scale is calibrated by passing known currents through the meter. The scale is not uniform; it spreads out from the zero. See Plate 26B. This is because the heat produced in the wire is proportional to the *square* of the current. The hot wire ammeter can measure alternating current (a.c.) as well as direct current (d.c.), since heat is produced in a wire in both directions of current.

Scanned with CamScanner

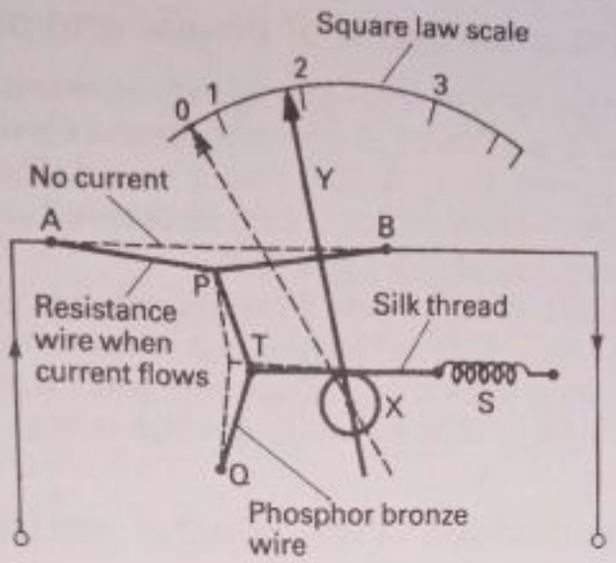


Fig. 26.4 Hot wire ammeter

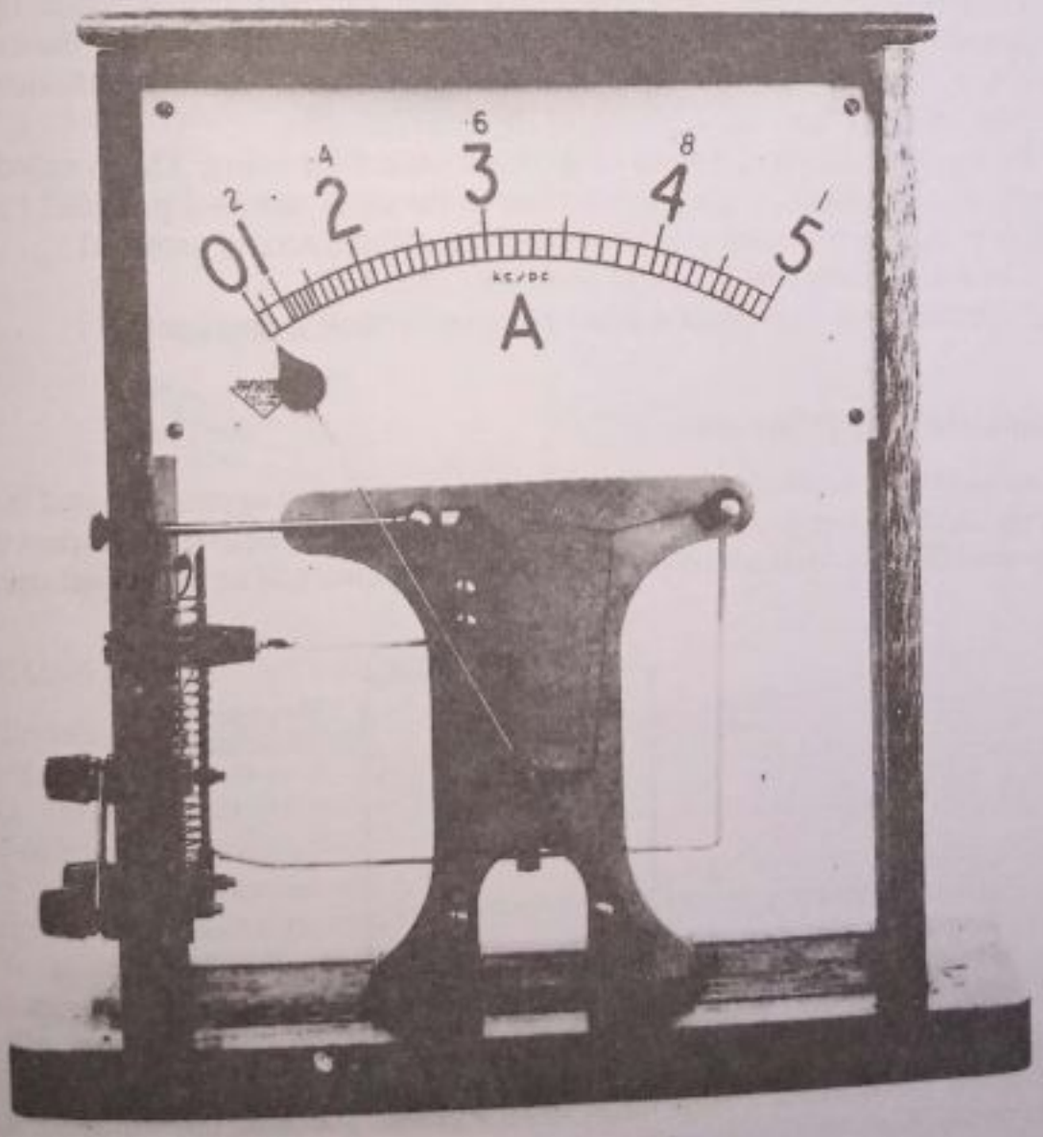


Plate 26B Demonstration hot wire ammeter, with a square-law scale of 0–5 A (*White Electrical Instrument Co Ltd*)

Scanned with CamScanner

High voltage transmission of power and power loss

Electrical power must be sent from power stations, where it is produced, to homes and factories. Since $P = IV$, it is possible to send a given amount of power either with a small current I and a high voltage V, or with high current and small voltage. For example, 1000 W of power may be obtained with a current of 4 A and 250 V or with a current of 100 A and 10 V.

The loss of electrical power as heat in the cables, however, is I^2R, where R is the resistance of the cables. So for small heat losses it is best to transmit power at low current and high voltage. This is why commercial power is transmitted by 'high tension (h.t.) transmission'; high tension means high voltage.

Electrical installations · Live, neutral, earth

Electricity is an extremely convenient and clean source of power. Electricity is distributed throughout the United Kingdom by a nationwide system of power stations, interconnected by cables and other necessary appliances such as transformers. This is called the *Grid system* and is discussed on page 491.

The domestic supply in the United Kingdom is mainly 220/240 V a.c. This is supplied by two cables from a local sub-station. One cable is called the live or line, L and is 240 V a.c. relative to the other cable which is known as the neutral, N. The neutral is earthed originally at the power station.

In domestic supplies, a third cable is introduced for safety. This is called the earth, E, as it makes a good connection to the earth terminal provided by the electrician. The potential of this earth may be called a zero of potential

The colour code now in use for domestic cables is

brown (*live*), light blue (*neutral*), green and yellow stripes (*earth*).

Earthing appliances

As a safety precaution, the metal casings of all electrical appliances used in the home, such as electric fires, are connected to earth by the earth cable E when they are used (Fig. 26.5). If a fault occurred in the appliance and no earth were used, a

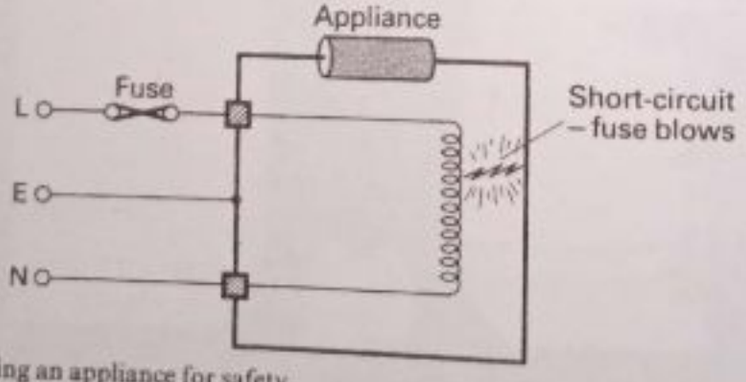


Fig. 26.5 Earthing an appliance for safety

dangerously high current from the 240 V live cable, touching the casing by accident, would pass through the body of a person touching the appliance into the earth. With the casing earthed, however, the current would pass directly to earth. So a person touching the appliance would be safe from injury. Of course, the fuse is blown if the mains is 'shorted' to earth and the appliance is then cut off from the mains.

Scanned with CamScanner

Fuses

The fuses used in the mains 13 A plug are *cartridge fuses*. Each consists of a length of fuse wire connected to metal caps on the end of a short tube (Fig. 26.6). The fuse is connected in the live cable which passes to the appliance (as in Fig. 26.5). So the current taken by the appliance passes through the fuse. Two cartridge fuse ratings available are:

Current rating	*Recommended power range*
3 A, coloured blue	0–500 W, e.g. for record player
13 A, coloured brown	500 W–3 kW, e.g. electric fire

The fuse wire is made of a tin–lead alloy. If its current rating is exceeded, the wire becomes hot and melts, so breaking the circuit. Appropriate fuses, to take the powers used, should be fitted in the plug.

For example, a small domestic vacuum cleaner has a motor of about $\frac{1}{6}$hp or 110 W when running. The appliance takes a current given by power = current × mains p.d., so that current = power/mains p.d. = 120 W/240 V = $\frac{1}{2}$A. A 3 A fuse may therefore be used in the vacuum cleaner plug. On the other hand, a 2 kW or 2000 W electric fire takes a current given by 2000 W/240 V = 8 A approximately. So a 13 A fuse should be used; a 3 A fuse would be too low in power rating and would 'blow'.

Fig. 26.6 also shows a conventional fuse, as used in some old fuse boxes.

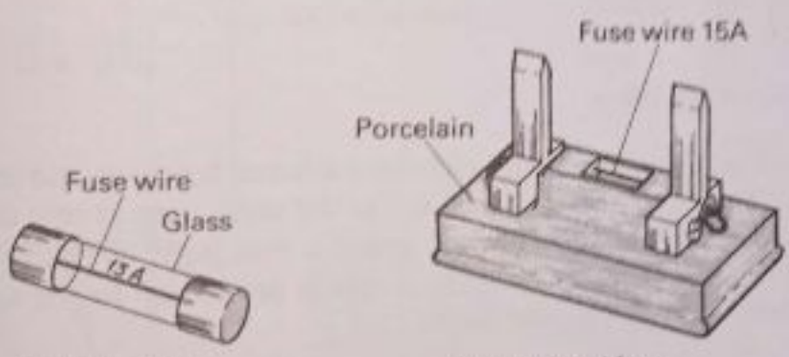


Fig. 26.6 Fuses

House electrical system

A British house electrical system is shown simplified in Fig. 26.7. The mains cables L and N enter at A. A mains service fuse in a box is placed in the live (L) lead. This mains cut-out operates at a high current value such as 60–100 A, and is sealed by the Electricity Board.

A meter measures the units (kWh) of energy used in the whole house. The supply is joined to a distribution box B. This contains an on-off switch and a set of fuses such as F_1, F_2, F_3 for each circuit in the house. The fuses are always connected in the live (L) lead of a circuit. If they were placed in the neutral lead, and a fault occurred by accidental shorting of the live cable L to earth, no protection would be given by the fuse.

Only small currents are needed in lighting circuits. So thin cables can be used. The protective fuse wire is normally rated at 5 A. For heating circuits, however, large currents and therefore thicker cables are needed. Fuse wire of 15 A rating is used. The total current used in a house may be as much as 50 A or more and this current flows in the mains cable.

Scanned with CamScanner

Ring circuit

Electric cookers, central heating units and immersion heaters use relatively large amounts of power. These appliances have their own separate circuits, each with a separate fuse (Fig. 26.7).

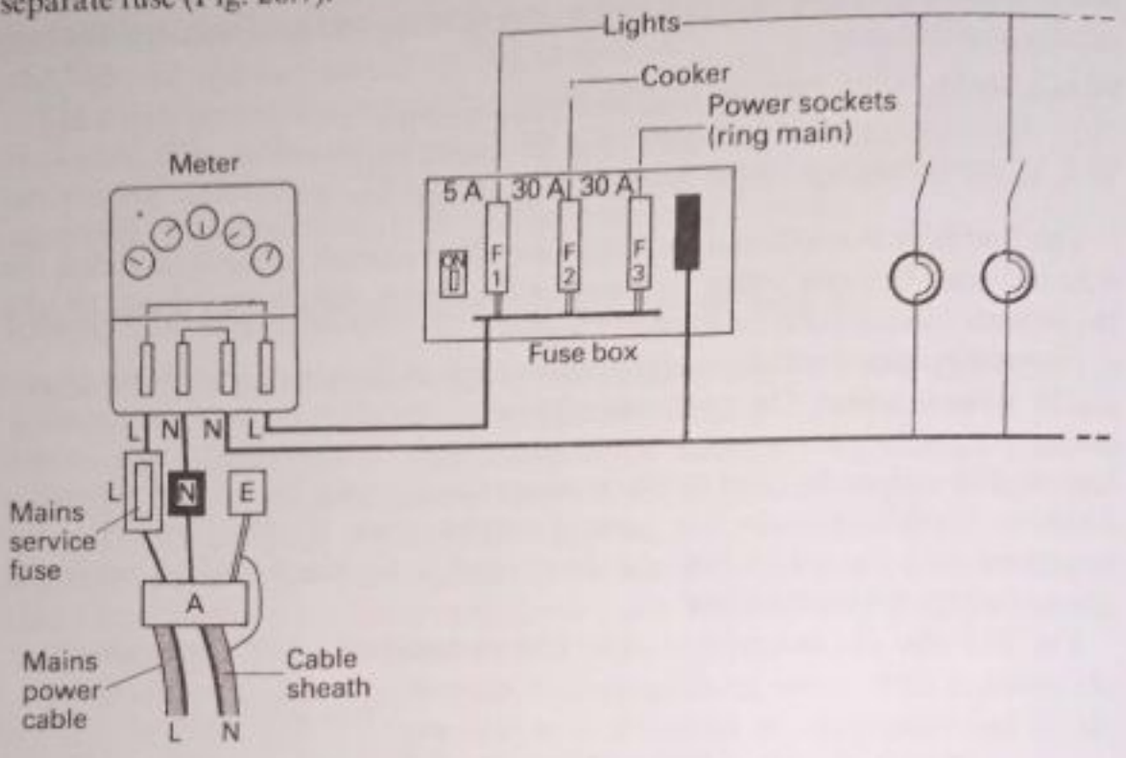


Fig. 26.7 A house wiring system

Television sets, vacuum cleaners and electric irons, however, use relatively low power. They may therefore be connected at the same time in one circuit, using different socket outlets. This circuit is called a *ring main circuit* (Fig. 26.8). It consists of a loop of cable, both ends of which are joined to the supply at the distribution box on the left of the figure.

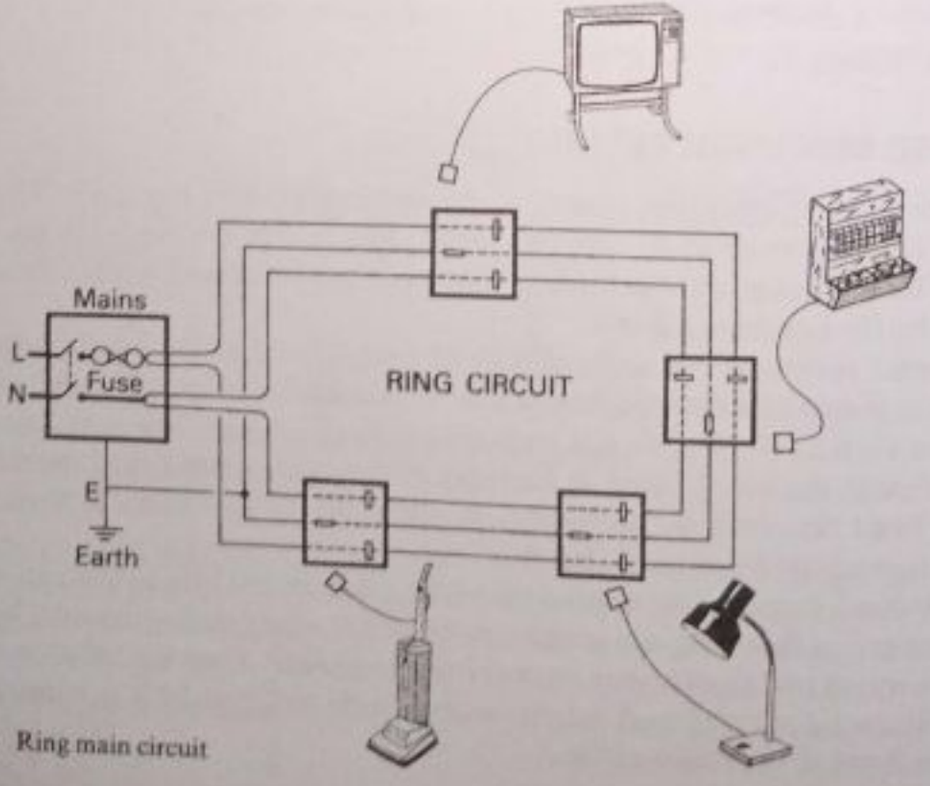


Fig. 26.8 Ring main circuit

Scanned with CamScanner

As shown, the power needed for separate sockets for domestic appliances can be taken off any part of the ring. The current carried by each cable section is only half of what it would be if the circuit were joined straight to the distribution box. So the cable needed for a ring main has a lower rating than that required for direct connection to the mains and the cost is therefore less. Another advantage is that new sockets can be joined in the ring main circuit very easily. Fig. 26.8 shows a circuit with four points.

Fused plugs

To safeguard appliances used on a ring circuit, each plug connected to it has its own fuse, that is the appliance is separately fused. The plug, with a 13 A fuse, is shown in Fig. 26.9. It has three pins of rectangular cross-section and can only be pushed one way, the right way, into its socket. The socket has safety shutters over the live and neutral connections. These open only when the plug is pushed in and are a safeguard against small children. The plug has a plastic insulating case, terminals N and E which are set at the top of their respective pins, and a terminal A. The top of the live pin L has a clip which holds one end of a cartridge fuse F. The other end of the fuse is joined to a clip on the top of the terminal A.

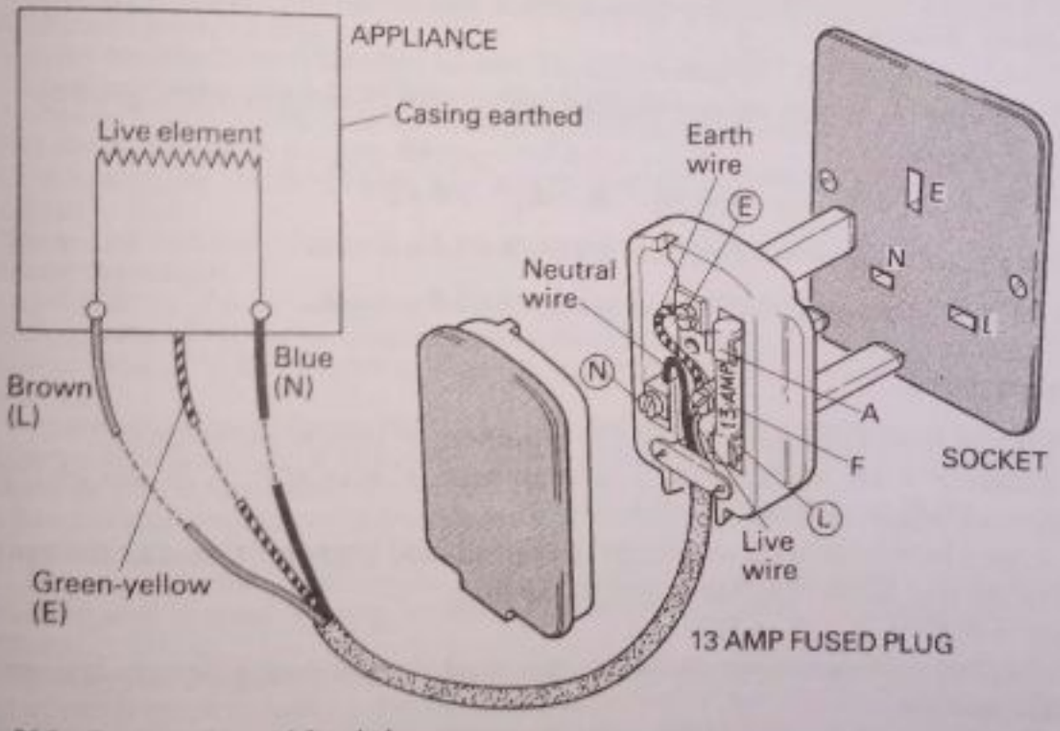


Fig. 26.9 Power socket and fused plug

When the plug is fitted on to an appliance such as an electric heater, the three wires inside the cable of the appliance must be connected as shown. The green-yellow striped or earth wire goes to pin E. The blue or neutral wire goes to pin N. The brown or live wire goes to terminal A, and then, through the fuse, to pin L.

Example on heating coils

Two heating coils dissipate heat at the rate of 40 W and 60 W respectively when connected in parallel to a 12 V d.c. supply of negligible internal resistance. Calculate the resistances of the coils. Assuming that these resistances remain constant, what would be their rates of dissipation of heat when connected together in series with the same supply as before? (L)

Scanned with CamScanner

In Fig. 26.10a, the resistors R_1 and R_2 are in parallel. Using the power formula $P = V^2/R$

$$40 = \frac{12^2}{R_1} = \frac{144}{R_1}, \text{ so } 40R_1 = 144$$

$$R_1 = \frac{144}{40} = 3{\cdot}6\,\Omega$$

Also

$$60 = \frac{12^2}{R_2} = \frac{144}{R_2}, \text{ so } 60R_2 = 144$$

$$R_2 = \frac{144}{60} = 2{\cdot}4\,\Omega$$

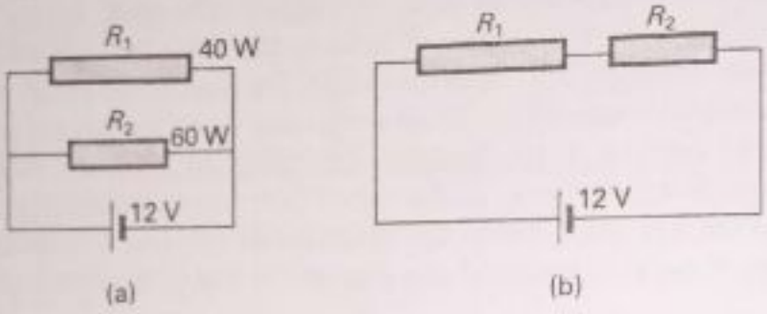


Fig. 26.10 Example

In Fig. 26.10b, R_1 and R_2 are now in series. The current I flowing in each is given by

$$I = \frac{V}{R_1 + R_2} = \frac{12}{R_1 + R_2} = \frac{12}{3{\cdot}6 + 2{\cdot}4} = 2\,\text{A}$$

So for R_1, power $P = I^2R_1 = 2^2 \times 3{\cdot}6 = 4 \times 3{\cdot}6 = 14{\cdot}4\,\text{W}$

and for R_2, power $P = I^2R_2 = 2^2 \times 2{\cdot}4 = 4 \times 2{\cdot}4 = 9{\cdot}6\,\text{W}$

Summary

1. Energy $W = IVt$ joules, where I is in ampere, V in volt, t in second.
2. Power $P = IV$ watts, where I is in ampere, V is in volt. A Board of Trade unit $= 1\,\text{kWh} = 1\,\text{kW} \times 1\,\text{h} = 3{\cdot}6 \times 10^6\,\text{J}$.
3. Joule's laws of heating state: The heat produced is proportional to the square of the current, to the resistance and to the time.
Heat $= I^2Rt = V^2t/R = IVt$.
4. The hot wire ammeter measures a.c. and d.c.; it has a square-law (non-uniform) scale.
5. Fused plugs have three terminals, L, N, E (earth connection to appliance casing for safety) and fuses of 3 A (for lighting or low power appliances) or 13 A (higher power).
6. Lighting circuits use relatively small currents and small fuses. Power circuits use relatively high currents and large fuses.

Exercise 26 (Answers, page 643)

1. An electric lamp is marked 12 V, 36 W. Calculate (a) the resistance of the lamp when in use; (b) the energy in joules expended each minute. (N)
2. An electric lamp is marked 100 W, 250 V. If the lamp is connected to a 250 V mains, calculate (a) the current taken; (b) the cost of using the lamp for 100 hr at 1p per kilowatt hour (Board of Trade unit). (N)
3. Define *watt, kilowatt-hour.*

Scanned with CamScanner

A washing machine for use on 240 V mains has a $\frac{1}{3}$ h.p. motor and a heating element rated at 2 kW. What current does it take when in use, and what is the cost of using it for 40 min each week for a period of 12 weeks if the cost of electricity is 2p per unit? (Assume 1 h.p. = 0·75 kW.) (L)

4. Fig. 26.11 shows a three-pin plug which fits into an electric socket and is connected by the three wires shown to an electric kettle.
(a) What do E and L each represent?
(b) Why are the connecting wires covered with insulating material?
(c) Where are the far ends of the three wires connected at the kettle end? (L)

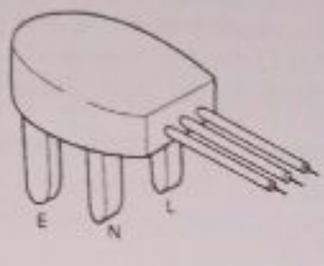


Fig. 26.11

5. Give an expression for the rate at which heat is produced in a conductor carrying an electric current, stating the physical quantities involved and the units in which they are measured.

Water in an electric kettle connected to a 240 V supply took 6 min to reach its boiling point. How long would it have taken if the supply had been one of 210 V? (L)

6. A current of 0·5 A flowing through a wire produces 21 J of heat in $\frac{1}{2}$ min. Find the resistance of the wire. (N)

7. (a) An electric lamp is labelled 12 volt, 60 watt. Given this information state:
(i) one necessary precaution when using the lamp,
(ii) the energy the lamp will dissipate each second when correctly used,
(iii) the current then passing through the lamp.

Give a diagram of the circuit you would use to check the value of the current obtained in (iii).

Name one desirable property of the material used for the filament of a lamp and indicate the reason.

(b) If 100 cm of wire were used in the heating element of a 12 V, 60 W heater what length of wire of the same material having double the cross-sectional area would be required for a 24 V, 60 W heating element, assuming that the final temperature is the same? (L)

8. An electric kettle is rated at 1000 W, 250 V. (a) What is the resistance of the heating element when in use, (b) if electricity is charged at $1\frac{1}{2}$p per kilowatt hour and the kettle is used for 20 min each day, what does it cost per week? (N)

9. Describe the structure and give the action of a hot wire ammeter. Why is it suitable for measuring both alternating and direct current? (L)

10. Describe briefly experiments, *one* in each case, to illustrate the chemical, magnetic and thermal effects by means of which a current of electricity may be detected.

A battery sends a current through two wires of resistance 10 and 20 Ω. Compare the rate of production of heat in the 10 Ω wire with that in the 20 Ω wire when they are connected (a) in series, (b) in parallel, across the battery. (N)

11. Why is a fuse often included in an electrical circuit? Describe, as fully as you can, the principles upon which the action of a fuse depends. (L)

12. Describe an experiment to determine how the electrical energy expended in a resistance wire when a current flows through it is related to the heat produced in the wire.

A hot water tank containing 40000 g of water is heated by an electric immersion heater rated at 3 kilowatt, 240 volt. Calculate (a) the current in the heating element, (b) the time required to raise the temperature of the water from 20° C to 60° C assuming that 80 per cent of the heat supplied is retained by the water. (Sp. ht. capacity of water = 4200 J per kg per K.) (N)

13. Describe an experiment to show that when the same current is passed through different wires the rates of production of heat are directly proportional to the resistance of the wires.

Two wires X and Y of the same material and the same length are joined in parallel to the terminals of a battery. The diameter of X is double that of Y. Find the ratio of the heat developed in X to that developed in Y during the same time. (N)

Scanned with CamScanner

14. It is required to raise the temperature of 20 litres of water from 15° to 75°C in 30 min by means of an electric heater. What must be the power of the heater and what current will it take if it is designed for use on 240 V mains? Explain why, in practice, the power must be greater than that calculated above. (L)

15. State the factors which determine the rate of heat production in a resistance due to the passage through it of a steady electric current.

The heating element of an electric kettle, containing 1 litre of water, initially at a temperature of 20°C, is connected to a 250 V d.c. supply and the water commences to boil in 9 min. If the current through the heating element is 4 A, calculate (a) the thermal capacity of the kettle; (b) the cost involved when the price of electrical energy is 6p per kilowatt hour. Neglect heat losses. (O and C)

16. Define the *volt*, the *watt* and the *kilowatt-hour*.

Explain how a fuse wire inserted in an electrical circuit protects the rest of the circuit against excessive current.

A small house with a mains supply at 250 V has two 2 kW electric heaters and six 100 W lamps. The power and the lighting circuits are entirely separate, and each has its own main fuse. What current passes through each of the fuses when both heaters and all the lamps are in use? (O)

17. Define *volt*, *ampere* and *kilowatt-hour*.

How would you test the statement that the rate of production of heat in a given resistor is proportional to the square of the current passing through it?

A resistor is immersed in liquid in a calorimeter, the thermal capacity of the whole being 1260 J/K. When a steady current of 7 A passes for 5 min, the temperature rises by 16 degC. Find (a) the p.d. across the resistor; (b) its resistance in ohms. (O)

18. A 3 kilowatt fire is designed to operate from a 240 V supply. Calculate the resistance of the fire. The fire is connected to the supply by long leads of resistance 0·8 Ω. Assuming their resistance to remain unaltered, determine (a) the current in the leads, and (b) the power dissipated in the leads.

Explain why a single pole switch (cutting one wire only) should be placed so that it cuts the live wire. (L)

Multiple-Choice Questions

19. When a 2 kW electric fire is connected to a 240 V mains, the fuse in the plug is

A 1 A B 3 A C 5 A D 7 A E 13 A

20. The maximum power which a 100 Ω resistor can absorb is 4 W. The maximum voltage across the resistor is

A 5 V B 10 V C 20 V D 25 V E 400 V

21. Three heating coils P, Q and R of equal resistance are used in a small mains electric cooker. Which arrangement in the Table, from A to E, will give the three correct settings for 'low heat', 'medium heat' and 'high heat'.

	Low	*Medium*	*High*
A	P only	Q only	R only
B	P only	P, Q parallel	P, Q, R parallel
C	P only	P, Q series	P, Q, R series
D	P, Q, R, series	P, Q series	P only
E	P, Q, R series	P only	P, Q parallel

Scanned with CamScanner

27. Magnetism

Magnets and magnetic materials are used in many important pieces of apparatus or machines. For example, hi-fi loudspeakers must have a very powerful magnet to produce sound of high quality and sufficient volume. Transformers, used in power transmission and in many electronic devices, must have a magnetic core with special magnetic properties. A television tube has a shield made of magnetic material. And tapes in cassettes are coated with a special magnetic powder.



Plate 27A Magnetic field round magnetic tape after use in a tape recorder, showing formation of small magnets. (*3M company*)

Fundamental properties of magnets

Magnetism was first observed over 2000 years ago in China. It was then known that a mineral called lodestone, which is a magnetic iron oxide, always pointed approximately north and south when it was freely suspended. Lodestone is a

Scanned with CamScanner

natural magnet. Iron and steel were found to be attracted to the ends of the magnet.

The first person to make a serious study of magnetism was Dr Gilbert. About 1600 he wrote a book called *De Magnete* in which he gave an account of his researches into magnets and their properties.

Experiment shows that

(a) iron filings cling mainly round the ends of a bar magnet (Fig. 27.1a);

(b) a bar magnet, suspended so as to swing freely in a horizontal plane, always comes to rest with its axis pointing roughly north–south (Fig. 27.1b).

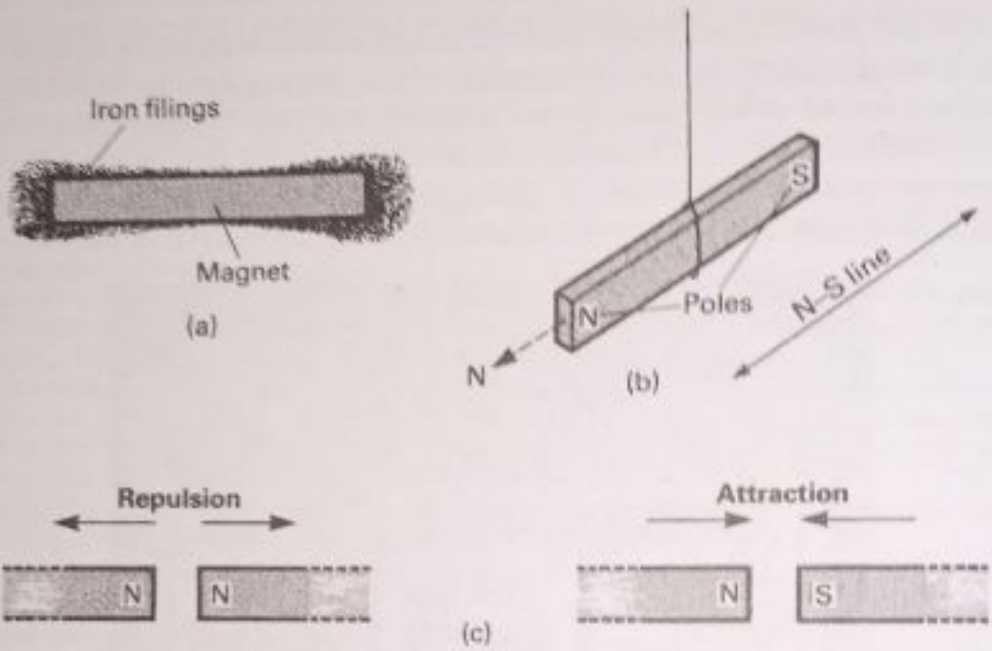


Fig. 27.1 Properties of magnets

The ends of the magnet, where the attracting power is greatest, are called its *poles*. The pole which points towards the north is called the 'north-seeking' or north (N) pole of the magnet; the other is called the south (S) pole.

(c) If a N pole of a magnet is brought near to the N pole of another, suspended, magnet repulsion occurs. If the N pole is brought near to the S pole attraction occurs (Fig. 27.1c).

So *like or similar poles* (that is, two N or two S poles *repel*, and *unlike poles* (that is, an N and S pole) *attract*.

Methods of making magnets

Electrical method

The quickest or industrial way of making a magnet is to use the magnetic effect of an electric current (page 457). If a bar magnet is needed, a steel bar is placed inside a *solenoid*, a long coil of insulated copper wire, to which a battery can be connected (Fig. 27.2a). A current is now switched on for a few seconds and then switched off. On testing, the steel bar X is found to be a magnet with N and S poles as shown. We shall see later that a rule for polarity is:

If the current flows *clockwise* in the coil when one end of it is viewed, then the bar X has a *south* (S) pole at this end. If the current flows *anticlockwise* this end of X is north (N).

Scanned with CamScanner

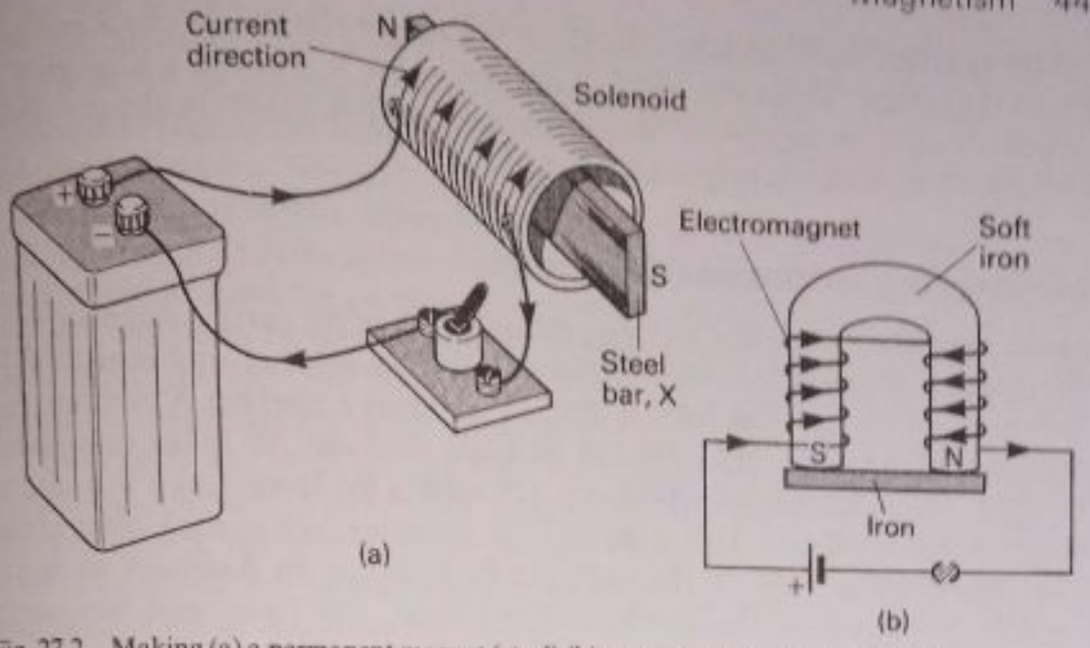


Fig. 27.2 Making (a) a permanent magnet (steel) (b) a temporary magnet (soft iron)

If a ring or horse-shoe magnet is required, an insulated solenoid is wound all round the material, and a steady current is switched on and then off (Fig. 27.2b).

Single touch

A steel bar X can be magnetised by placing it on the table and then stroking it repeatedly with one end Y of a magnet. As shown in Fig. 27.3, the magnet is raised each time it reaches the end of the steel bar X. Experiment shows that the end of X last touched by Y in the stroking process has a pole opposite to that used. In Fig. 27.3a, for example, the N pole of Y is used and the end of X last touched by this pole becomes an S pole.

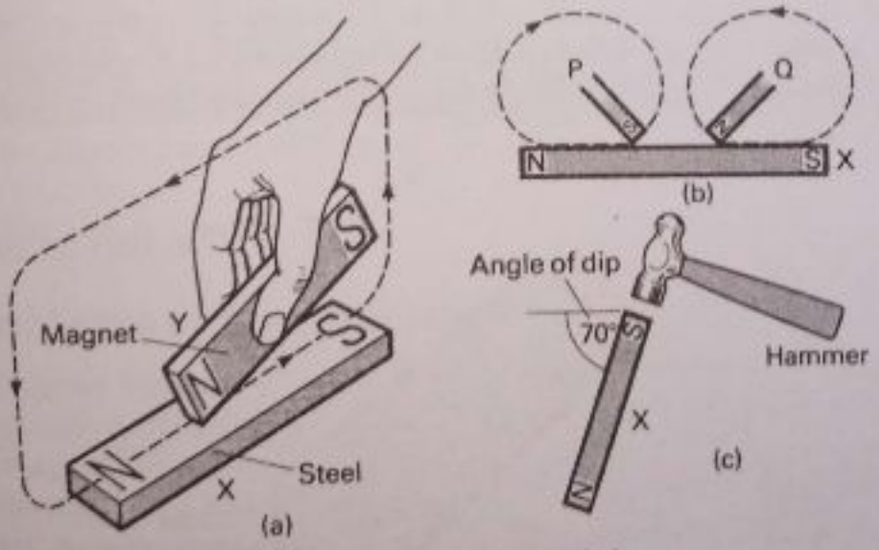


Fig. 27.3 Making weak magnets; angle of dip about 70° in Britain.

Divided touch

In this process each half of the steel specimen X is stroked repeatedly in opposite directions by the *opposite* poles S and N of two bar-magnets, P and Q (Fig. 27.3b). The stroking begins in the middle of X each time as shown. The rule for the polarity of X is the same as for the method of single touch.

Scanned with CamScanner

If the same poles of P and Q are used, an N pole appears at each end X and an S pole in the middle. These are called *consequent poles*. X is not then a 'magnet' in the normal sense of the word. If it is suspended it will not settle in any particular direction, as its resultant magnetism is zero.

Hammering in the earth's field

A weak magnet can be made by using the influence of the earth's magnetic field. This field acts at an angle to the horizontal called the *angle of dip* (see page 451), which is about 70° in Great Britain. Fig. 27.3c shows a steel bar X pointing in a north–south direction and inclined at an angle of about 70° to the horizontal. The upper end is hammered repeatedly. On testing the lower end it is found to have weak north polarity. This is the case for countries such as the United States or Russia, which are also in the northern hemisphere. In Australia or South America, which are in the southern hemisphere, the lower end has south magnetic polarity.

Demagnetisation

Magnets can be partially demagnetised by hammering them hard when they are pointing east–west, that is, at 90° to the earth's magnetic field direction; heating magnets strongly also partially demagnetises them.

Experiment

The most effective method of demagnetisation is to use an alternating current. A low mains voltage *V*, such as 12 V a.c., is connected in series with a suitable rheostat *R* and a solenoid S (Fig. 27.4). The solenoid is placed with its axis pointing east–west. The object to be demagnetised, a bar magnet or wrist-watch, for example, is then placed inside S and the alternating current is switched on. After a few seconds the object is slowly withdrawn from the solenoid to a long distance away. It will then be demagnetised.

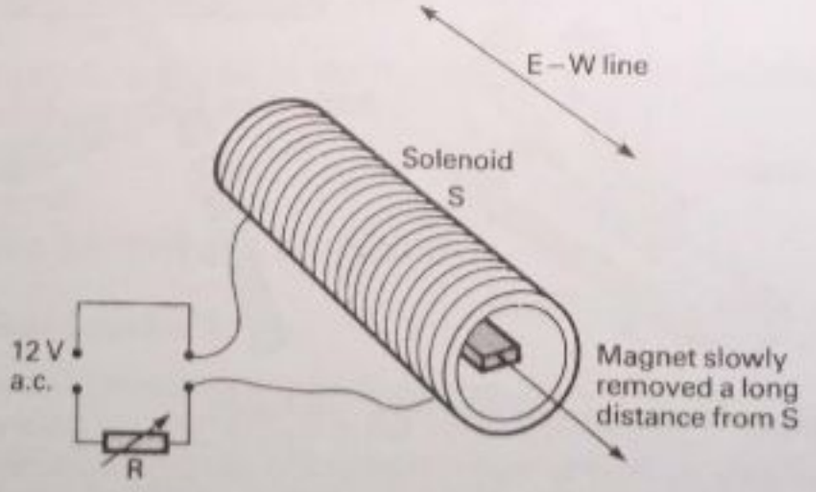


Fig. 27.4 Demagnetisation

Magnetic and non-magnetic materials

Iron, nickel, cobalt and certain alloys of these metals can be made into strong magnets. These strongly magnetic materials are called *ferromagnetic* materials.

Scanned with CamScanner

If a bar of pure or soft iron is placed inside a solenoid and the current is switched on, the iron becomes a powerful magnet which can pick up heavy pieces of iron. When the current is switched off, however, the iron falls to the ground. *Temporary magnets* are therefore made of soft iron. Electromagnets, which are temporary magnets, use soft iron.

Steel is an alloy of iron, made by adding a small percentage of carbon to pure iron. It is a much harder metal than pure or soft iron. If a bar of steel is placed inside a solenoid and the current is switched on, the steel becomes a magnet and can pick up pieces of iron. When the current is switched off the iron remains attracted to the steel. So *permanent magnets* are made of steel. Steel is more difficult to magnetise than iron, but loses its magnetism much less easily.

Today, iron and steel are not used for making very powerful temporary or permanent magnets. Alloys of these and other materials are used instead. For electromagnets, for example, alloys named Mu-metal and Permalloy may be used. For producing powerful permanent magnets alloys of cobalt and nickel such as Ticonal or Alcomax may be used.

Elements other than iron, nickel and cobalt can be made into very feeble magnets, over a million times weaker than those made from ferromagnetic materials. We call them *non-magnetic* substances. Copper and brass, for example, are non-magnetic.

Ferrites

Ferrites are a special class of strongly magnetic materials. They are made by the chemical combination of a metal oxide such as zinc oxide with iron oxide (Fe_2O_3). Ferrites can be magnetised strongly and, even more important, they are not demagnetised easily once they are magnetised. Ferrite rods, which are black and hard, are used in aerials of transistor receivers. They are particularly useful for reception of microwaves, which are radio waves of very high frequency (page 349). Usually there are considerable power losses in magnetic materials due to circulating electric currents inside them (see page 488). Ferrites have a high electrical resistance, so the current and power loss are extremely small.

A fine iron oxide powder is often used to coat the tapes used in tape recorders.

Magnetic fields

Lines of force or magnetic flux

Fig. 27.5 shows how magnetic compass needles, placed at a, b, c, d near a bar magnet, point in different directions. So there is a magnetic effect in the space round a magnet. We call a region where magnetic influence is felt a *magnetic field.* A compass needle is a sensitive way of finding the direction of the magnetic field at different points. The 'intensity' or 'strength' of a magnetic field is a vector quantity because it has both direction and magnitude.

The direction of the magnetic forces in an area of the field can be mapped out by marking the north and south ends of a compass needle with dots at l and m, say, on a sheet of paper. The compass is then moved until the S pole is at l and the new position of the N pole is marked at p. By moving the compass in this way and repeating the procedure, the paper can be covered by a series of dots. These are then joined by a smooth curve, called a *line of magnetic force.* Other lines of force are drawn in the same way.

Scanned with CamScanner

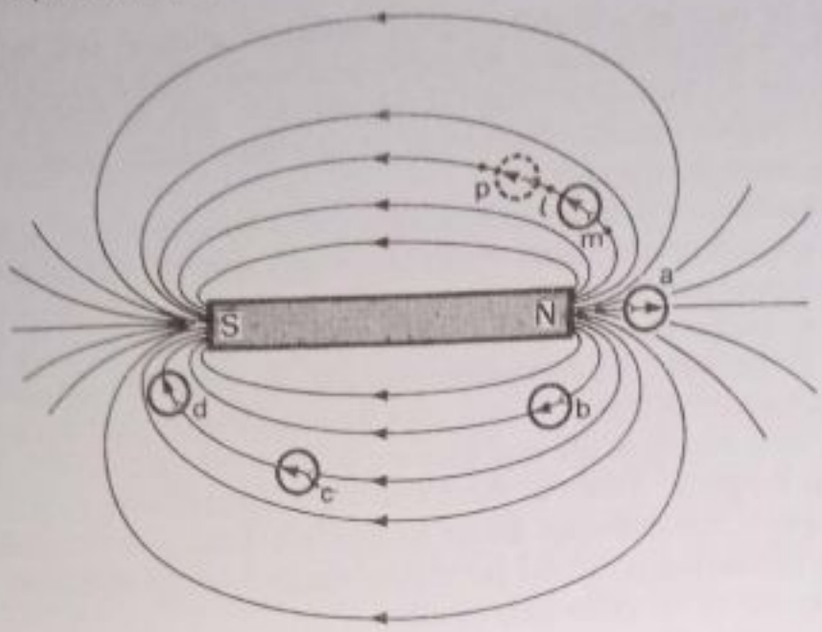


Fig. 27.5 Plotting a magnetic field

Lines of force never cross; otherwise the magnetic field would have two possible directions at the point of intersection. The direction of the field at a point is taken as the direction which a *north* pole would tend to move if placed at that point. A line of force can be defined as either

(a) a line such that the tangent to it at any point is the direction of the field at this point; or

(b) a line along which a free N pole would tend to move.

The area all round the magnet, in three dimensions, can be imagined full of these lines. They are called the *magnetic flux*. Magnetic fields and magnetic flux are used by electrical engineers in designing machines, for example, and astronomers use them in theories of the universe.

Patterns of magnetic fields

The field patterns can be quickly seen by using iron filings. To obtain the field pattern round a magnet a piece of paper is placed on top of it and iron filings are sprinkled lightly on to the paper. When the paper is tapped gently the iron filings turn and line up in definite directions. The pattern of field lines may then be seen. Iron filings behave in this way because they become induced magnets in the field of the magnet, and when free, they behave like miniature compass needles and settle along lines of force.

Some field patterns are shown in two dimensions in Fig. 27.6. The fields actually exist in three dimensions in space. Points worth noting are as follows.

(a) *Bar magnet* The lines are oval in shape and come out of the N pole into the air and enter the S pole (Fig. 27.6a). The lines are continuous; one should imagine them inside the magnet, passing from the south pole S to the north pole N through the magnetic material.

(b) *Unlike and like poles* (Fig. 27.6b) The field between the N and S poles has many lines and is therefore strong. In contrast, there are no lines round P between the N and N poles facing each other, and so there is no magnetic force or field here.

(c) *Earth's field* Locally, the field consists of parallel lines pointing northwards (Fig. 27.6c). So the density of the lines at any part of the field is constant. The

Scanned with CamScanner

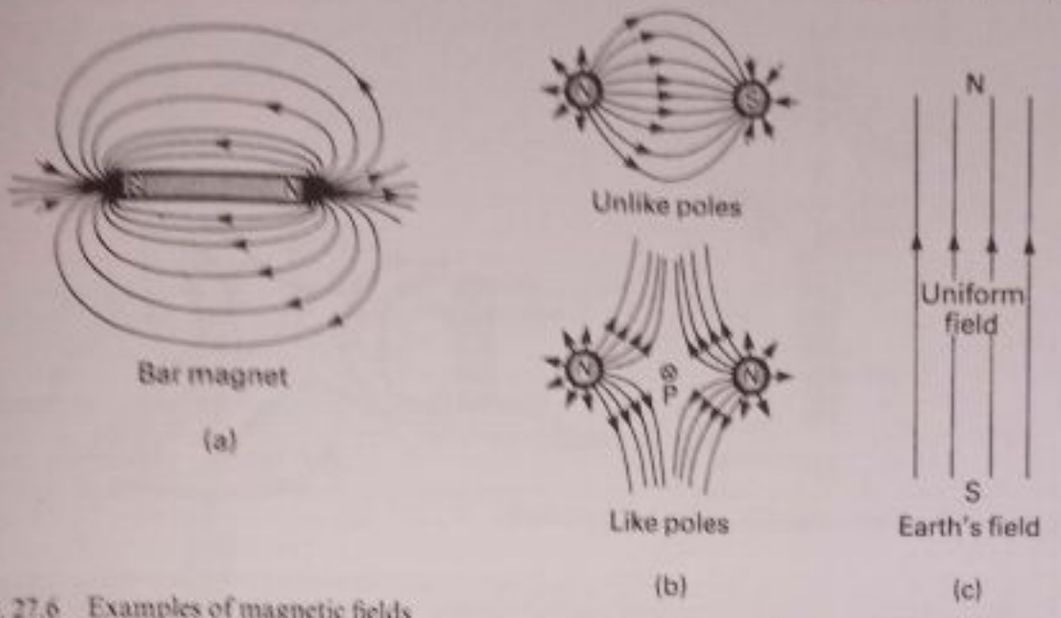


Fig. 27.6 Examples of magnetic fields

earth's field is an example of a 'uniform field', one whose direction and strength is constant. The field round a bar magnet is non-uniform.

(d) *Neutral points* The earth's magnetic field F_E neutralises completely the field F_M due to a bar magnet at certain points. These points are therefore called *neutral points*. Fig. 27.7 and Fig. 27.8 show examples of neutral points. At each point, note that the direction of the force F_E due to the earth's field on a north pole of a compass needle placed there is exactly equal and opposite to the force F_M due to the magnet.

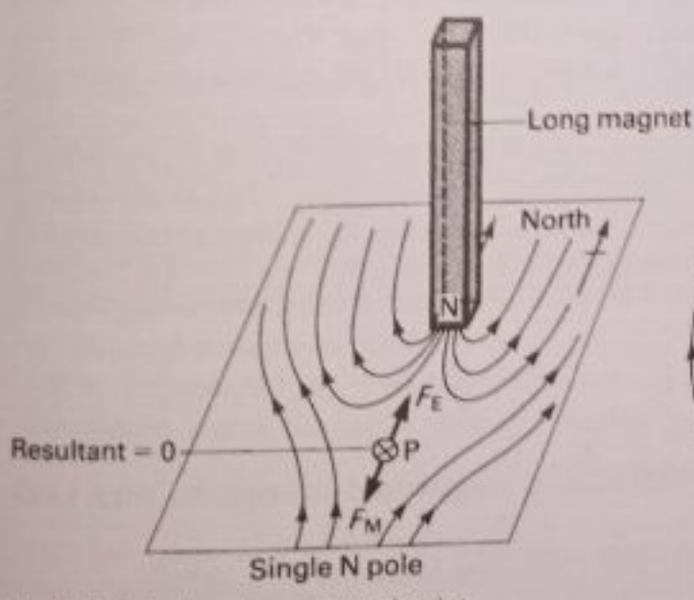


Fig. 27.7 Single pole and neutral point

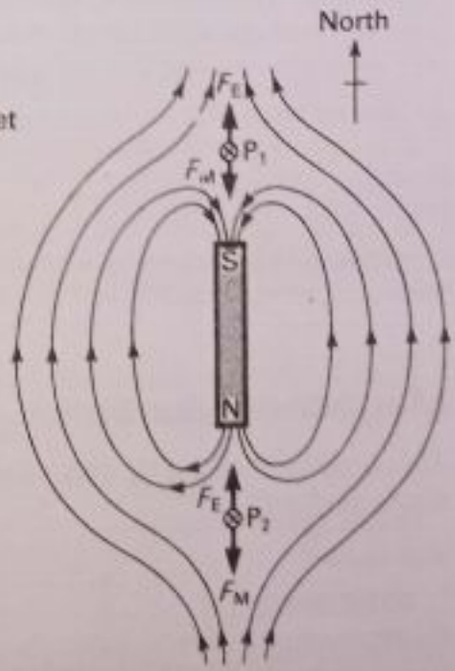


Fig. 27.8 Bar magnet in earth's field and neutral points

Experiment to investigate the combined magnetic and earth's fields

(a) Bar magnet S pole pointing north Pin a white sheet of paper to a drawing board. Using a compass needle in the middle of the paper, draw the magnetic north–south line by marking with a pencil the N and S pole positions of the compass.

Scanned with CamScanner

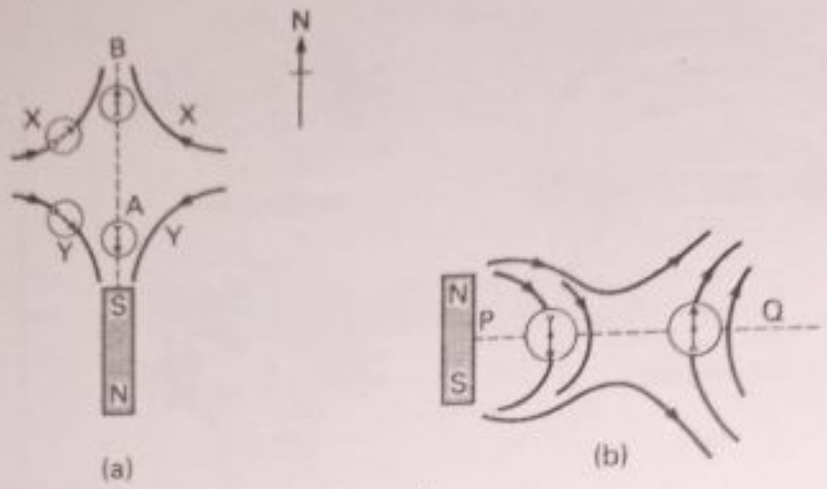


Fig. 27.9 Combined fields of a magnet and the earth

Place a bar magnet in the magnetic north direction, with its S pole pointing north (Fig. 27.9a). Draw the outline of the magnet with a pencil. Place a compass needle close to the S pole, on the north–south line as at A, so that the needle points directly towards the S pole. Here the magnet's field is much stronger than the earth's field. Move the compass back slowly to B along the north–south line, where the needle turns round and its N pole now points north. Here the earth's field is stronger than that of the magnet. Between B and A, the two fields cancel each other out at the neutral point. If the neutral point is off the paper, alter the magnet's position to get the point on the paper.

Using the compass needle as explained on page 443, plot the magnetic lines of force or magnetic flux pattern all round the neutral point area. In Fig. 27.9a, X and Y are parts of the lines required. (Remember that lines of force do *not* cross.) Mark the position of the neutral point as best as you can in the middle of the 'empty' area.

Now draw the magnetic flux pattern below the N pole of the magnet and locate the neutral point here in the same way.

(b) *Bar magnet N pole pointing north* Place the bar magnet on a sheet of paper with its N pole pointing north (Fig. 27.9b). As in (a), find if there is a neutral point along the perpendicular bisector PQ of the magnet. Draw the magnetic flux pattern all round the neutral point if there is one and mark its rough position.

Magnetic induction

Mu-metal is a magnetic alloy which can be magnetised strongly by very weak magnetic fields.

Experiment

If a long rod of mu-metal is held pointing downwards at an angle of about 70° to the horizontal (in the U.K.) the lower end A repels the N pole of a suspended magnetic needle (Fig. 27.10a). The other end repels the S pole of the needle. So the rod is a magnet.

A surprising change occurs when the rod is turned round so as to point in an east–west direction and held horizontally. This time the end A attracts both the N pole of the magnetic needle and its S pole (Fig. 27.10b). The other end of A also attracts both the S and N poles of the needle. This shows that the rod is unmagnetised. On turning the rod round to lie again in the north–south direction, it becomes a magnet once more.

Scanned with CamScanner

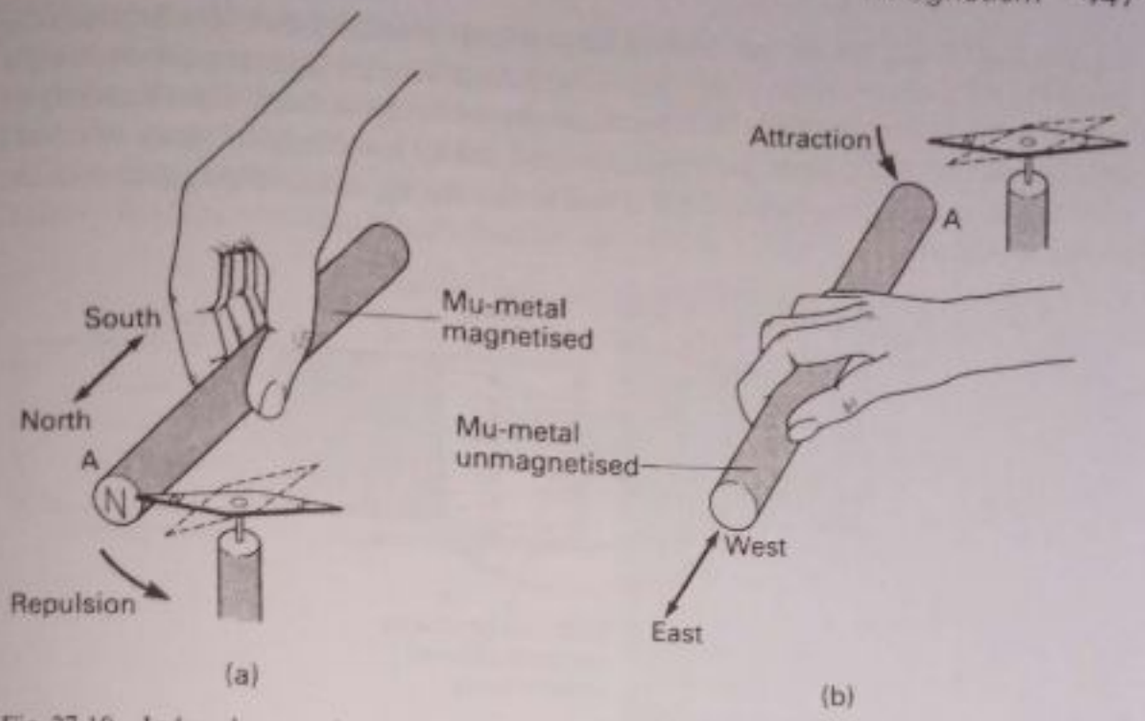


Fig. 27.10 Induced magnetism

Explanation: When the rod is placed in the direction of the earth's magnetic field, that is north–south, the earth's field magnetises it. The rod is not in contact with a magnet, as in the method of single touch, so the magnetism in the rod is called *induced magnetism*. The phenomenon is called *magnetic induction*. The earth's field has no effect or component in a direction at right angles to itself. So if the rod is placed east–west, the induced magnetism is zero.

Other examples of induction

When a magnet is placed near an unmagnetised steel pin, the pin first becomes magnetised by induction. It is then attracted to the magnet (Fig. 27.11a, b, c). So *induction takes place before attraction*. This explains why unmagnetised iron and steel are attracted to magnets. Steel objects, such as pliers or railings or girders in buildings, are often found to be magnetised (Fig. 27.11d). This is due to induction by the earth's magnetic field.

The appearance of the lines of force when mu-metal or soft iron X is introduced into a magnetic field can be shown by sprinkling iron filings in the

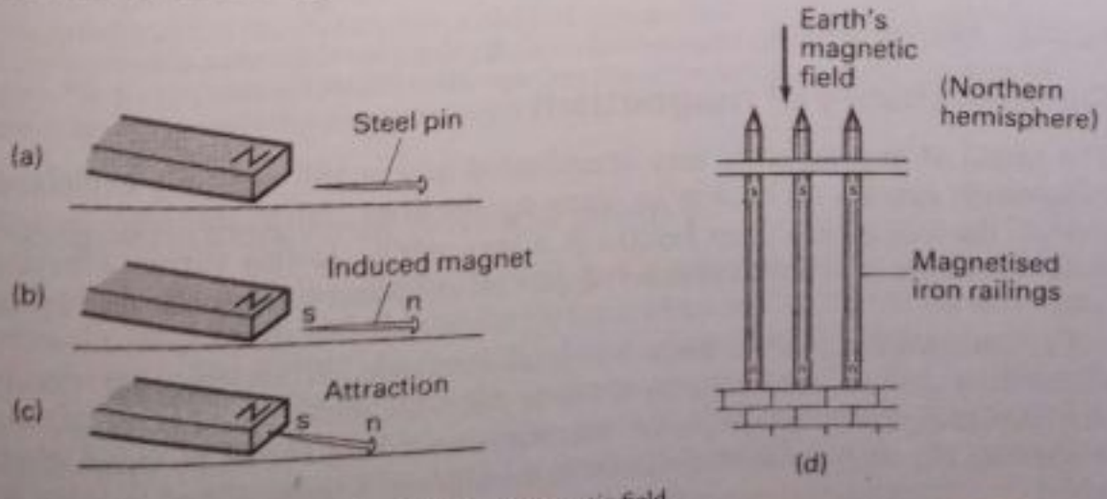


Fig. 27.11 Induction by magnet and earth's magnetic field

Scanned with CamScanner

region. Fig. 27.12 shows the lines of force when some soft iron X is between the poles N and S of two strong magnets. Compared with the appearance when air is between the poles, the lines of force now crowd into the iron. The crowding or concentration of the lines may be increased many hundreds of times with soft iron alloys such as mu-metal. This is due to the strong induced magnetism in the metal, as shown.

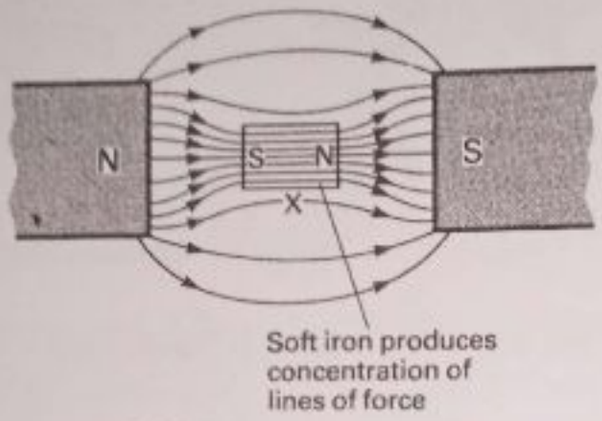


Fig. 27.12 Soft iron in a magnetic field

Magnetic shield

The concentration of lines of force in soft iron can be used to shield objects from unwanted magnetic fields. Fig. 27.13a shows an object X with a soft iron ring round it and a powerful magnet Y outside. All the lines of force pass through the ring. None passes into the air inside the ring. So X is shielded from the magnet. The electron beam inside cathode-ray tubes in television receivers is shielded from external magnetic fields by placing a soft iron cylinder round the neck of the tube (Fig. 27.13b).

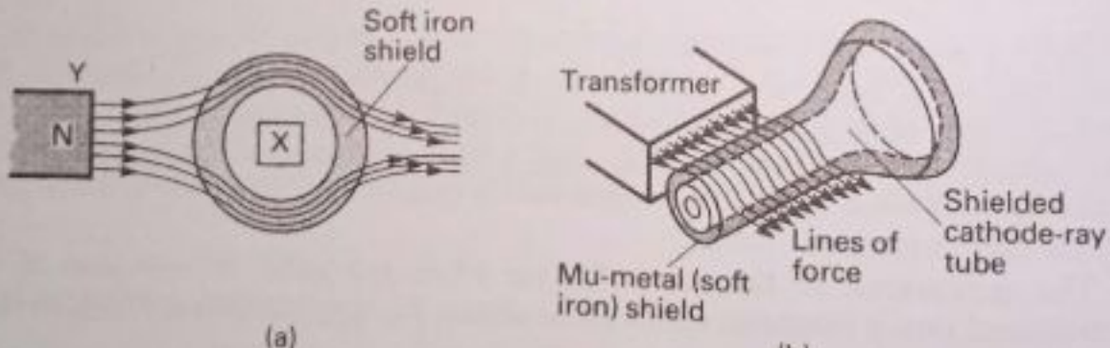


Fig. 27.13 Magnetic screening

Domain theory of magnetism

The origin of magnetism is very complicated and is still not fully explained. Magnetism appears to be due to spinning electrons. Since electrons carry a charge, the spin makes them behave in a way similar to tiny current-carrying coils. As we see in the next chapter, a current produces a magnetic field (page 454).

Ferromagnetic materials have a crystal structure. When they are closely examined numerous small regions of the tiny crystals appear to have very strong resultant magnetism, although the material as a whole shows no magnetism. These tiny regions are called *domains*. In each domain the resultant magnetism points in different and random directions.

Scanned with CamScanner

The magnetism in a domain can be represented like a vector (see page 29). We therefore draw an arrow pointing in the direction concerned. In an unmagnetised material the resultant magnetism of all the domains is zero. Fig. 27.14a shows how this occurs in a single crystal. The long arrows show the magnetism in a so-called 'easy direction of magnetisation' of the crystal. You can see from the arrows that the *resultant* magnetism in the crystal is zero.

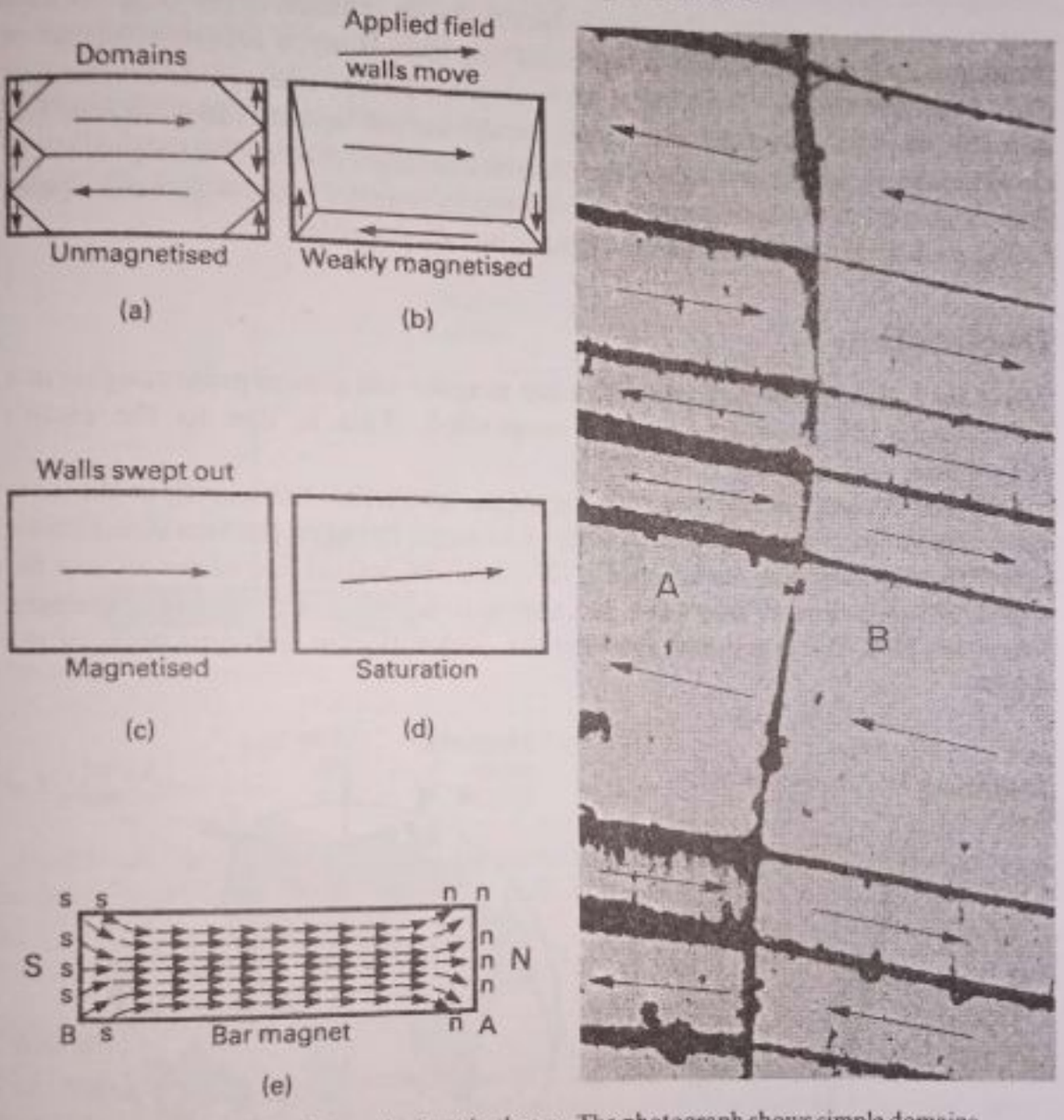


Fig. 27.14 and Plate 27B Magnetic domain theory. The photograph shows simple domains (magnified × 225) of two crystal grains A and B of unmagnetised iron alloy (3·2% silicon–iron transformer core material). Neighbouring domains have opposite magnetisation so their resultant is zero. The domain walls are parallel to the easy direction of magnetisation of the crystal and the grains have been orientated along this direction.

Magnetisation and domain theory

When a magnetic field is applied to a crystal at an angle to the easy direction, the domain walls move and the magnetism increases in the field direction. This is illustrated in Fig. 27.14b. As the applied field is increased, the domain walls move further until the whole of the crystal becomes magnetised (Fig. 27.14c). When the field becomes strong, the magnetism turns towards the field direction (Fig. 27.14d). The specimen now is magnetically 'saturated', that is, the magnetism cannot be increased further.

Scanned with CamScanner

Fig. 27.14e shows the domains in an ideal saturated bar magnet. The magnetism in them all points one way. Thus a resultant N pole is obtained at one end A and a resultant S pole at the other end B. Similar poles in neighbouring domains repel each other. So poles are produced at the sides as shown. This is why iron filings cling to the sides of a bar magnet as well as to its ends.

If no precautions are taken, the domain walls slowly return to the pattern of Fig. 27.14a, because this is a more stable pattern than Fig. 27.14c. Demagnetisation then occurs. To prevent this happening, a soft iron 'keeper' is placed between the poles of a horse-shoe magnet. This produces a closed chain of domains round the magnet and keeper, as the keeper becomes magnetised. The closed chain is stable so it keeps the domains in alignment in the magnet itself. For the same reason bar magnets are stored in parallel pairs with unlike poles facing each other, and with keepers placed across the poles.

Declination

As we have seen, a magnetic needle or bar magnet will always point roughly in a north–south direction when freely suspended. This is due to the earth's magnetism.

At a particular place on the earth, the magnetic north is not usually in the same direction as the geographic north there. The angle between the two directions is called the *declination* or *variation* (Fig. 27.15a). The declination varies all over the world. Seamen need to know the declination to navigate, by magnetic compass. Maps are therefore prepared showing the declination at different parts of the world.

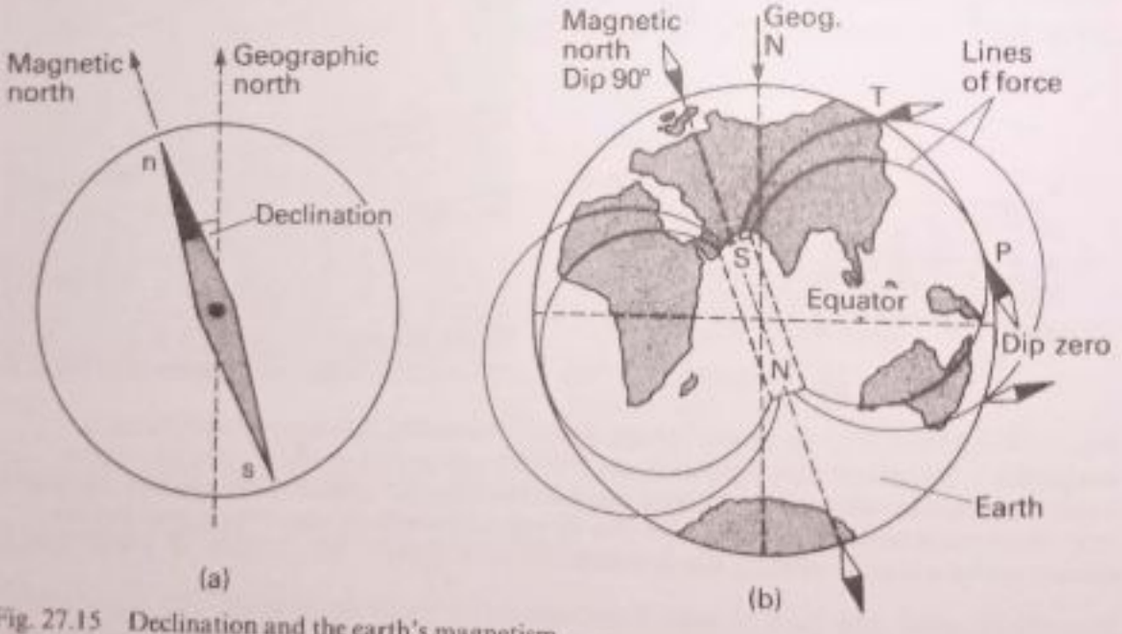


Fig. 27.15 Declination and the earth's magnetism

Magnetic poles of the earth

To explain the magnetic behaviour of the earth, we may imagine a bar magnet pointing roughly in the direction shown in Fig. 27.15b. Because unlike poles attract, there must be an S magnetic pole near the north geographic pole since the N pole of a suspended magnet points towards this pole; the opposite is also true.

The earth's magnetism is believed to be due to currents generated inside the molten hot core of the earth. The earth's external magnetic field, however, is

Scanned with CamScanner

similar to that produced by the imaginary magnet in Fig. 27.15b although no actual magnet could exist inside the hot core of the earth.

Angle of dip · Magnetic elements

A uniform brass bar M, which is non-magnetic, will stay horizontal if suspended at its midpoint (Fig. 27.16a). However, if a magnetic needle is pivoted at its centre of gravity O (Fig. 27.16b) so that it can move in a vertical magnetic north–south plane, the needle points *downwards* at about 70° to the horizontal in Britain. This shows that the resultant earth's magnetic field acts at an angle of 70° to the horizontal in the magnetic north–south plane. This angle is called the *angle of dip*. In Nigeria, its value is about 10°.

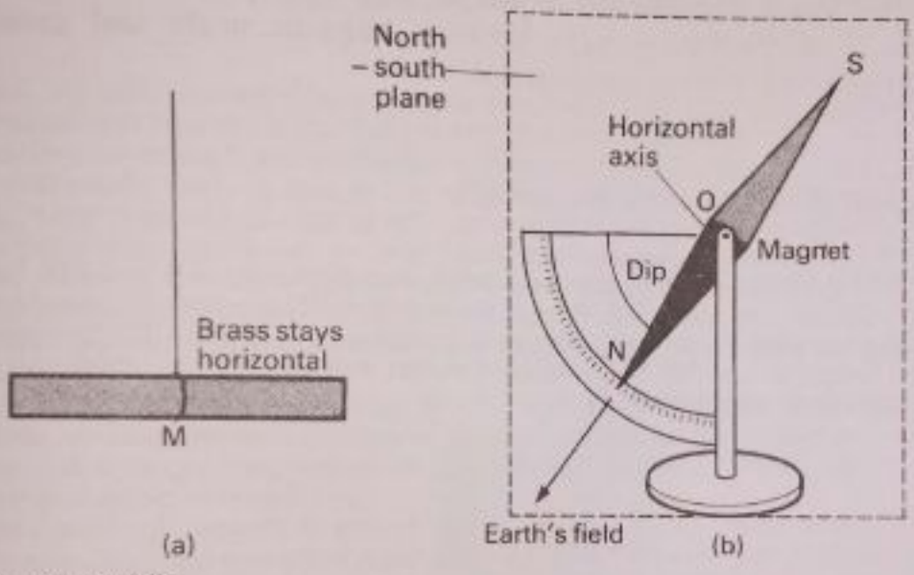


Fig. 27.16 Angle of dip

In Britain, the north pole of the dip needle points downwards. In Australia, on the other side of the world, the south pole of the dip needle points downwards. See Fig. 27.15b.

Variation of angle of dip

The angle of dip varies all over the earth's surface. It is 90° near the geographic poles and 0° near the equator. This can be explained by considering the field of the imaginary bar magnet shown in Fig. 27.15b. At the magnetic north and south poles the lines of force are vertical. Thus the angle of dip here is 90°. At the magnetic equator the lines of force are tangential to the earth's surface, that is, parallel to the surface, so that the angle of dip is 0° as shown at P. The dips at other points are shown in Fig. 27.15b.

The angle of dip, declination and the earth's horizontal field strength are called *magnetic elements*. Their values at a particular place such as London or Lagos enable the direction and magnitude of the earth's field to be found there.

Summary

1. Like magnetic poles repel. Unlike poles attract.
2. Magnets can be made (a) by the electrical method (clockwise current = south

Scanned with CamScanner

pole); (b) by single or double touch (end last touched has opposite polarity to stroking pole); (c) by hammering or induction in the earth's field. (In Britain and Nigeria, the end of the bar pointing downwards is an N pole.)

3. Soft (pure) iron is more easily magnetised than steel (iron alloy with carbon), and loses its magnetism more easily.

4. A *magnetic field* is a region in which a magnetic force may be detected. A *line of force* is a line along which an N pole would tend to move if it were free. Magnetic fields are vector quantities and their directions can be plotted with the aid of a compass needle. The appearance of the whole field can be seen by using iron fillings. Neutral points are obtained where the earth's magnetic field is exactly equal and opposite to the field due to a bar magnet.

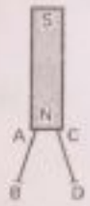
5. Angle of dip = angle made with horizontal by earth's total magnetic field. It is 90° at the magnetic poles and zero at the magnetic equator.

6. Angle of declination = angle between magnetic north and geographical north directions at the place concerned.

Exercise 27 (Answers, page 643)

1. Fig. 27.17 shows what is observed when two steel pins, AB and CD, hang from the north pole of a magnet, NS. Why is the distance BD greater than the distance AC and what does this indicate regarding the condition of AB and CD? Describe briefly how you would show that the condition stated was correct, assuming you may use normal laboratory apparatus. (L)

Fig. 27.17



2. Describe *three* different methods of magnetising similar steel bars indicating, in each instance, the polarity of the magnet formed.

How would you expect the three magnets to compare in strength, and what test would you make to determine which is the strongest? (L)

3. Explain the following

(a) Two steel needles hang from the lower end of a vertical bar magnet, but do not hang vertically.

(b) A horse-shoe magnet is often supplied with a 'keeper'.

(c) When a bar magnet is placed on a table there are two points near it where there is no horizontal magnetic field. (N)

4. What is meant in magnetism by *line of force* and by *neutral point*?

Why can iron filings be used to plot the lines of force in a strong magnetic field, but are unsuitable if the field is weak?

A bar magnet stands vertically on a table with its N pole downwards. Describe how you would plot the lines of force round the end of the magnet. Give a diagram of the field and mark the position of the neutral point. (L)

5. Explain, with diagrams, the statements (a) the angle of dip is 68°, (b) two neutral points can be located when a bar magnet is placed horizontally with its axis in the magnetic meridian and its S pole pointing north. (N)

6. You are given two similar rods, one being magnetised steel and the other unmagnetised steel. Explain the fact that you could distinguish between them by touching an end of one against the centre of the other; and that by touching an end of one against an end of the other you could not distinguish between them, nor would the first method then be usable for the purpose. (C)

7. Explain what is meant by the term 'neutral point' in magnetism. Sketch diagrams of the magnetic fields in a horizontal plane when (a) a bar magnet is placed with its axis

Scanned with CamScanner

in the magnetic meridian with its south-seeking pole pointing north; (b) a bar of unmagnetised iron which has the same shape as the magnet in (a) replaces that magnet.

State two uses of the effect seen in (b) and explain one of them. (N)

8. Two cylindrical rods, AB, of soft iron, and CD, of steel, rest side by side on the flat base of a horizontal solenoid of rectangular cross-section, Fig. 27.18. State what happens to each of the rods and how they react to each other (a) when a large direct current flows in the solenoid from the battery, E; (b) when the current is switched off by the switch, S. (L)

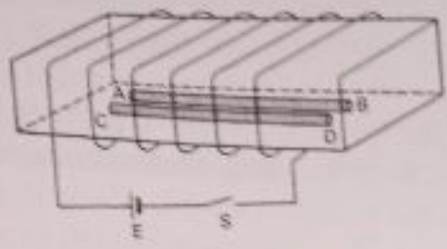


Fig. 27.18

9. Explain (a) what is meant by magnetic induction; (b) why one end of a bar of unmagnetised soft iron may attract either end of a compass needle.

Draw a diagram of the magnetic field in a horizontal plane when (a) a bar magnet is placed horizontally with its axis in the magnetic meridian and its N pole pointing north; (b) a bar magnet and a bar of soft iron are placed horizontally in line with a gap between them (neglect the influence of the earth's magnetic field). (N)

10. What is the angle of dip at (a) the magnetic equator; (b) the magnetic poles? (N)

11. Define *declination* (magnetic variation), *dip*.

Describe how you would obtain the value at a given place of *either* the declination *or* the dip.

Explain why a vertical iron girder is often found to be magnetised. State the polarity of the girder. In what position must an unmagnetised iron girder be placed in order (a) to become as strongly magnetised as possible by the earth's field; (b) to remain unmagnetised along its length? (L)

12. A powerful bar magnet is placed horizontally with its poles in line with the magnetic meridian, and two small regions are found on the axis of the magnet where a small plotting compass sets in a random manner. With the aid of a diagram on which the poles of the magnet are marked, account for this effect.

On another diagram show how similar regions could be produced on a horizontal line through the mid-point of the magnet and perpendicular to its axis. (L)

Multiple-Choice Questions

13. A bar of soft iron in a solenoid reaches a maximum amount *M* of magnetisation when the current in the solenoid is increased from zero to 5 A. If the current had been increased to 10 A instead of 5 A, the amount of magnetisation in the iron would have been

A $10M$ B $5M$ C $4M$ D $2M$ E M

14. Fig. 27.19 shows a bar magnet and a compass-needle T on the perpendicular bisector PQ of the magnet. Which of A to E shows the correct direction of the compass T?

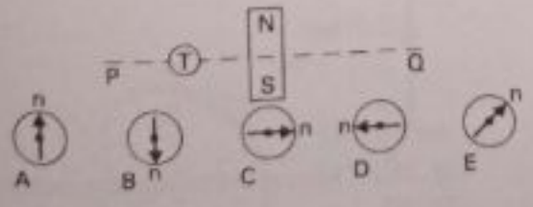


Fig. 27.19

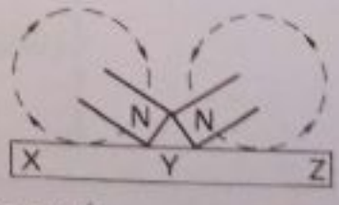


Fig. 27.20

15. In Fig. 27.20, the N-poles of two magnets stroke a steel bar. The ends of the bar are X and Z and the middle is Y. Which of A to E gives their correct polarities?

A X = N, Y = N, Z = N B X = N, Y = S, Z = N
C X = S, Y = N, Z = S D X = S, Y = S, Z = N
E X = N, Y = N, Z = S

Scanned with CamScanner

28. Magnetic Effect of a Current

The magnetic effect of an electric current has great practical importance. For example electric motors, dynamos and telecommunications depend on it.

Oersted's discovery

The magnetic effect was first noticed by Oersted in 1820.

Experiment

Oersted's experiment can be repeated by connecting a straight length of insulated wire AD to a battery B, with a small rheostat *R* and a key K in the circuit (Fig. 28.1).

When AD is held parallel to and over a magnetic compass needle NS (that is, the wire is placed in a north–south direction) the needle is deflected. When the current is increased using *R* the deflection increases. For large currents the deflection is nearly 90°, that is, the needle points nearly perpendicular to the current direction. If a current-carrying wire is placed under the magnetic needle instead of above it, and in line with its axis, the needle deflects the opposite way (Fig. 28.2).

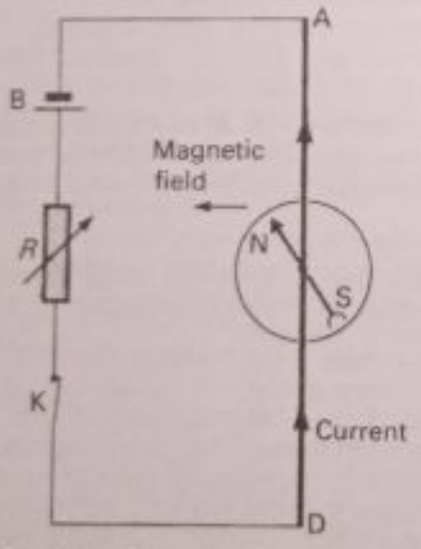


Fig. 28.1 Magnetic effect–current above needle

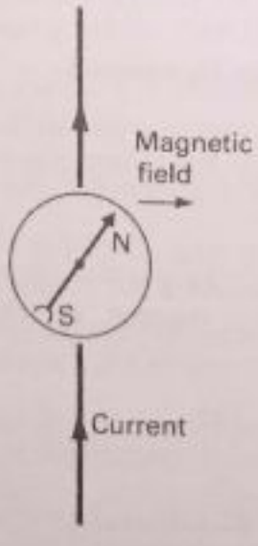


Fig. 28.2 Current below needle–deflection reverses

From the direction of movement of the compass needle, we see that the current produces magnetic forces all round the wire which are *perpendicular* to the current direction. The magnetic field direction is the direction of the magnetic

Scanned with CamScanner

force on an N pole and this is shown above and below the wire in Fig. 28.1 and Fig. 28.2. The magnetic field of the current, then, appears to circulate round the wire in a plane perpendicular to the current, as we now discuss further.

Magnetic field pattern round a straight conductor

Using iron filings or a magnetic compass needle, we see that the magnetic field pattern round a straight current-carrying conductor consists of *circles* with the wire as centre (Fig. 28.3). This means that the field is symmetrical round the wire, as we may expect when there are no other magnetic influences. Note that the magnetic field at any point acts along the tangent to the circle passing through the point. So a compass needle due north of the wire points east or west, depending on the current direction; a needle due east of the wire points north or south. Fig. 28.3 shows the direction of the magnetic field at these and other points.

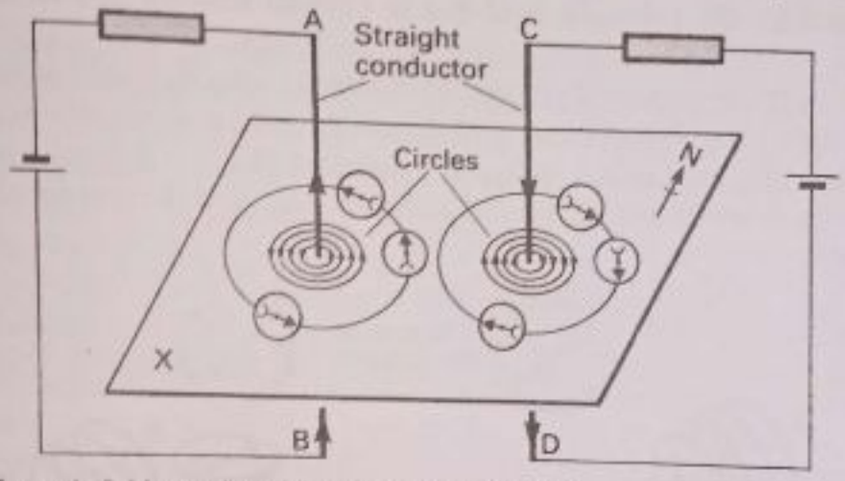


Fig. 28.3 Magnetic field round straight current-carrying wires

In Fig. 28.3, the current in AB is opposite in direction to that in CD. So the magnetic fields round the currents are in opposite directions, as shown by the arrows on the lines of force. The field round a wire occurs in every plane perpendicular to the wire and not just in one plane. One form of memory store in computers uses the magnetic field due to straight current-carrying wires.

Maxwell's corkscrew rule

Several rules have been given for the direction of the magnetic field around a straight current-carrying conductor. This is the direction of the force on an N pole in the field. One of the most useful is *Maxwell's corkscrew rule*. This states *if*

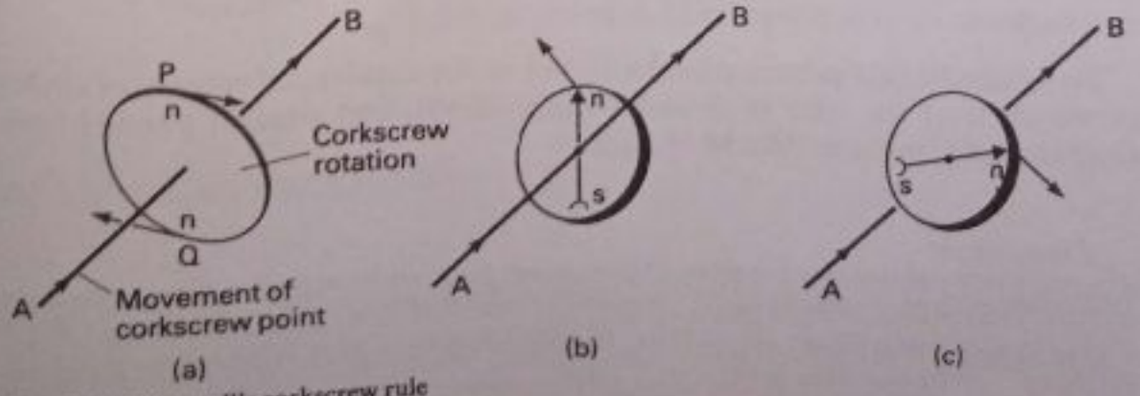


Fig. 28.4 Maxwell's corkscrew rule

Scanned with CamScanner

a right-handed corkscrew is turned so that its point travels along the current direction, the direction of rotation of the corkscrew gives the direction of the magnetic field.

Fig. 28.4a shows a straight current-carrying wire AB. At a point P above it an N pole moves from left to right, as shown, according to Maxwell's rule. This is the direction of the magnetic field at P. At a point Q below it an N pole moves from right to left in the opposite direction. If the corkscrew rule is applied to the wire AB (Fig. 28.4b), the N pole of the needle below it moves to the left as at Q in Fig. 28.4b. The S pole moves in the opposite direction. If the magnetic needle is placed above the wire AB (Fig. 28.4c), the N pole moves to the right, as at P in Fig. 28.4a, and the S pole moves to the left.

Magnetic field around a circular coil

We now consider the magnetic field due to current flowing in a circular coil of wire.

A simple narrow circular coil can be made by winding thin insulated wire round the finger a number of times and then sliding the coil off. When a current is passed through the coil and a magnetic compass needle is brought near to one end A, the N pole is attracted (Fig. 28.5a). So the end A of the coil acts like a *south* magnetic pole.

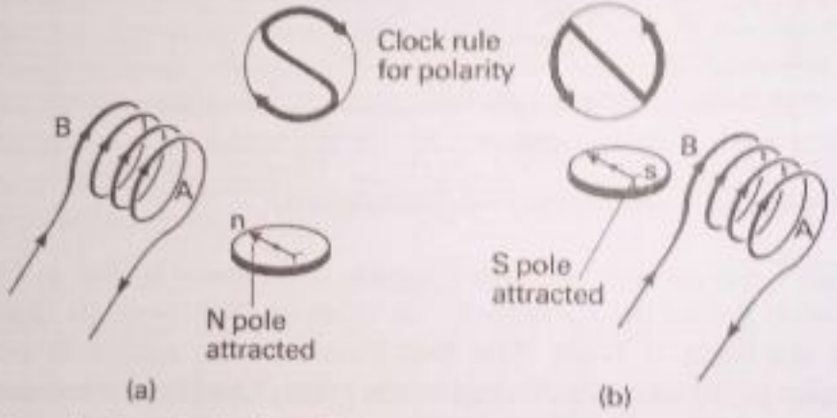


Fig. 28.5 Magnetic effect of a circular coil

The other end B of the circular coil is found to have *north* polarity. See Fig. 28.5b. So the circular coil acts like a short magnet. Fig. 28.5 shows at the top a simple 'clock rule' for finding the polarity of the ends of a circular coil when a current flows in it. Briefly,

clockwise current produces an S pole,
anticlockwise current produces an N pole.

The magnetic field pattern round a *narrow* or *flat* circular coil consists of circles round parts of the wire at A and B (Fig. 28.6a), and straight parallel lines (uniform field) in the middle M of the coil.

Experiment

The ends of a small circular coil C are joined to some copper and zinc respectively (Fig. 28.6b). The metals are placed inside a small beaker V, which can be floated on water in a trough D using cork, as shown.

Scanned with CamScanner

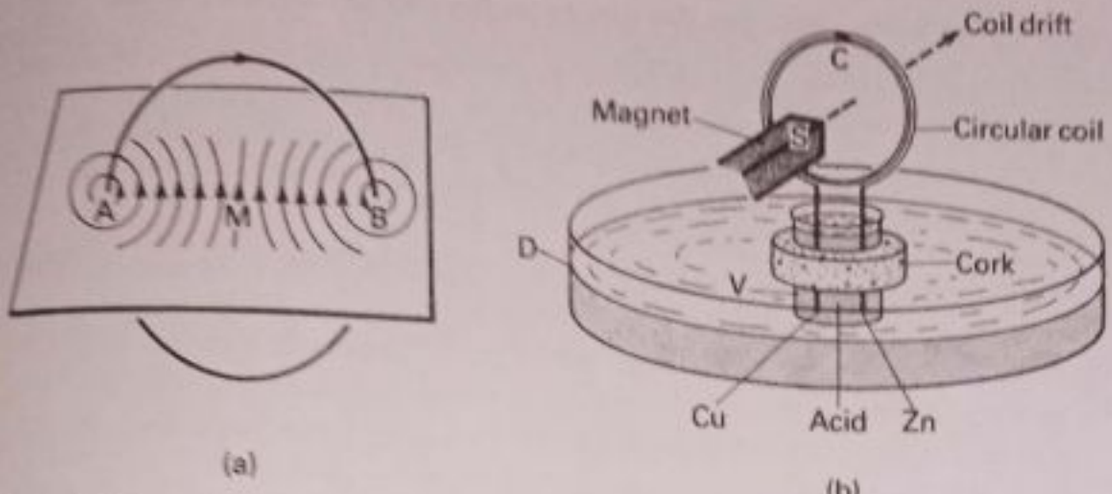


Fig. 28.6 Magnetic effect of a narrow circular coil

Some dilute sulphuric acid is then poured into the beaker. A simple cell is formed and a current flows in the coil C. With the cell in the middle of the water, an S pole of a magnet is brought near to one face of C. The coil is seen to drift away from the pole. So this face acts like an S pole since like poles repel. When an N pole is brought near the same face of the coil, it is now seen to be attracted.

So a current-carrying coil of wire acts like a magnet. This can also be seen by suspending the coil from a wooden stand, passing a current through it from a battery and then bringing up an N and an S pole in turn to one end of the coil. Attraction and repulsion are seen.

Magnetic effect of a solenoid

The magnetic effect of solenoids, which are long coils of closely-wound wire, is widely used in industry. When a magnetic compass needle M is placed near one end B of a current-carrying solenoid, the S pole is attracted to B in the case shown in Fig. 28.7. When the needle is taken to the other end A, the N pole is attracted.

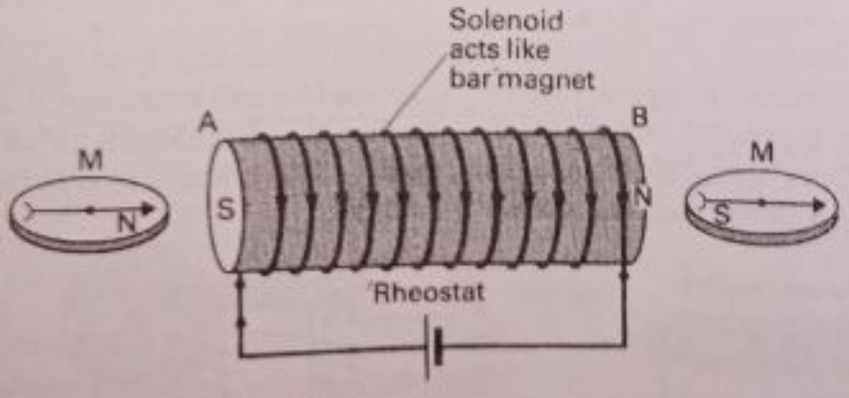


Fig. 28.7 Magnetic effect of a solenoid

So the solenoid acts like a bar magnet. The rule for magnetic polarity is the same as for the narrow circular coil, that is, if the current flows clockwise when one end is viewed that end acts like an S pole. If it flows anticlockwise that end acts like an N pole.

Scanned with CamScanner

Experiment to investigate the magnetic field pattern of a solenoid

Fig. 28.8a shows how the magnetic field pattern *outside* a current-carrying solenoid can be quickly seen. Iron filings are sprinkled lightly on the wooden board surrounding the solenoid and after the board is tapped, the filings settle in the pattern shown. The pattern is similar to that round a bar magnet. This is not surprising, because the ends of the solenoid act like an N and an S pole.

Fig. 28.8b shows how the magnetic field pattern *inside* the solenoid can be seen. This time a perspex board, widely-spaced turns and a row of compass needles inside the coil are used. The board is turned so that the axis of the coil points east–west and when the current is switched on, the needles are deflected from their north–south direction and point along the axis of the coil as shown. The field is practically constant in direction and strength over most of the length of the solenoid. See also Fig. 28.8c.

If a soft iron core is placed inside the solenoid it becomes magnetised strongly by the field when the current is a few amperes.

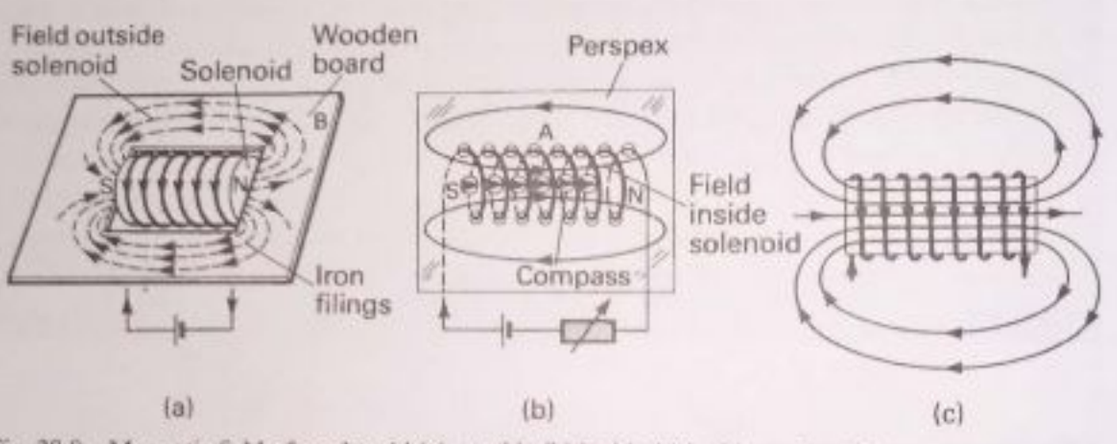


Fig. 28.8 Magnetic field of a solenoid (a) outside (b) inside (c) inside and outside

Making magnets

The field inside a solenoid can be used for making *permanent magnets* such as bar magnets. We simply place a steel bar inside the solenoid, switch on the current for a moment, and then switch off. An S pole is formed at the end where the current appears clockwise (Fig. 28.9a).

An *electromagnet* is a temporary magnet. A horse-shoe type can be made by winding solenoids in opposite directions round two soft iron cores A and B (Fig. 28.9b). The lower end of A then acts as an S pole and the lower end of B as an N pole, as shown. Large electromagnets, used for lifting heavy iron at metal goods yards, need a strong current of about 10 amperes.

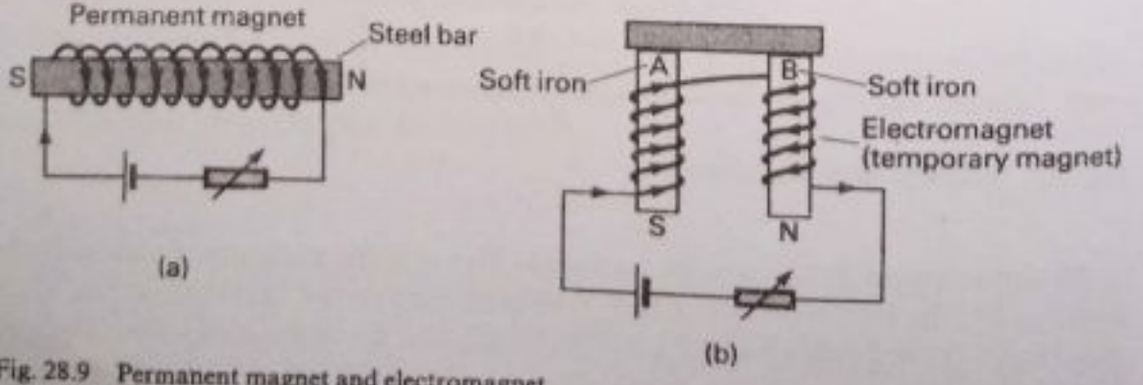


Fig. 28.9 Permanent magnet and electromagnet

Scanned with CamScanner



Plate 28A A large electromagnet, 1·65 m in diameter, being used to move steel turnings. The cable supplying the electric power can be seen. The load is released simply by turning off the current. (*GEC Ltd*)

Electric bell circuit

The electric bell works with the help of an electromagnet. It rings only when the electromagnet is energised by the current.

When a bell-push is pressed, the electric circuit in Fig. 28.10 is completed through the contact between a soft iron spring X and the point of a screw N on which it rests. The iron bar T is now attracted to the electromagnet. The contact between X and N is now broken and so T is pulled back to make contact again with the screw N.

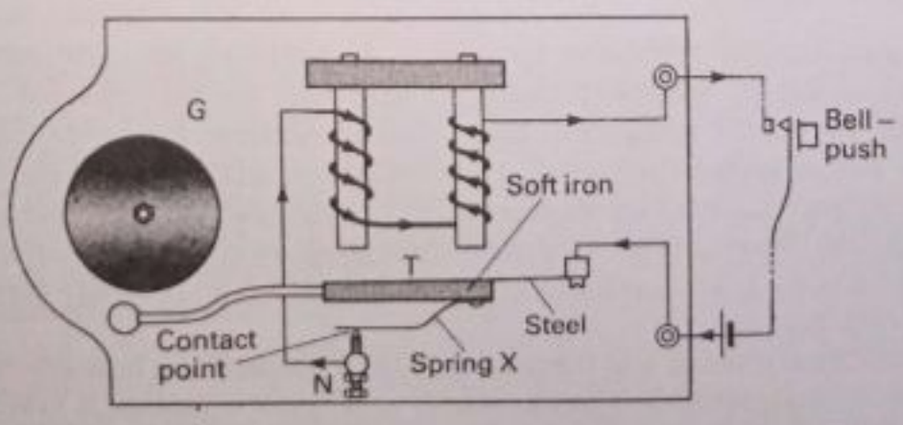


Fig. 28.10 Electric bell

So the iron T is attracted repeatedly by using a make-and-break device. Each time it moves towards the electromagnet, the clapper attached to T strikes a gong

Scanned with CamScanner

G. The screw N is tipped with tungsten or other metal of high melting-point. This is not affected by the spark which passes across the gap when the contact is broken.

Magnetic relay

A magnetic relay helps to pass messages from one part of an electric circuit to another; it is an example of an 'electromagnetic switch'.

The relay consists of a solenoid with a soft-iron core A inside it (Fig. 28.11). A bar of soft iron B helps to complete an 'iron circuit'. Near C a bent piece of soft iron called the armature (or moving part) can rock about a sharp edge at one end of B. A screw S controls the amount of armature movement.

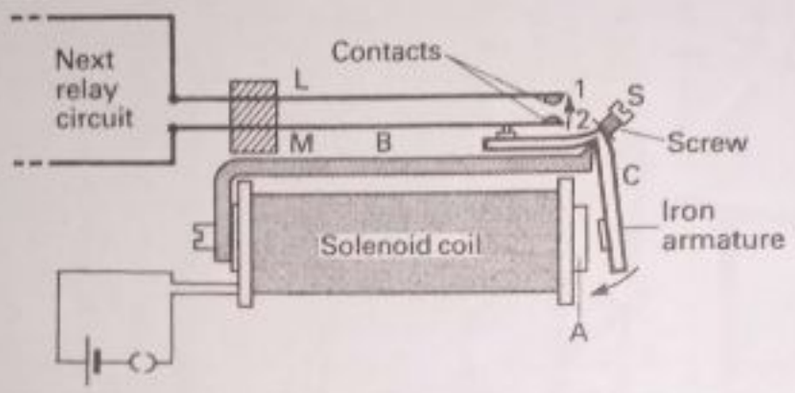


Fig. 28.11 Magnetic relay

When a telephone subscriber lifts the receiver a current flows along telephone wires to the solenoid of a relay. The magnetised iron core then attracts the armature and the contacts 1 and 2 close. This completes the circuit connected to the springs L and M, so that current may now flow in this circuit. In this way circuits with other relays can be made to close until finally a signal arrives at a switchboard at a distant exchange, for example. A complex system of telephone circuits throughout a country requires millions of relays for passing on messages.

Telephone handset

Carbon microphone

Communication by telephone requires a microphone, an instrument which changes sound to electrical energy. There are several different types of microphone, but the earliest was the *carbon microphone*. It was invented about 1878 by Bell and is still used today. Plate 28B shows a Post Office handset.

This microphone has carbon particles G between two carbon blocks X and Y (Fig. 28.12b). When a caller lifts the telephone to make a call, a battery from a central exchange is automatically connected to X and Y. A steady current then flows in the circuit.

The electrical resistance of the carbon particles depends on how much they are compressed together. If X is pushed closer to Y, their resistance is lowered; if X moves away from Y, the resistance is increased. When sound waves from a speaker pass through the plate P, the cone K vibrates. This makes X move to-and-fro slightly, and there is a corresponding variation in the resistance of the carbon particles. The current along the telephone line, or speech current, now

Scanned with CamScanner

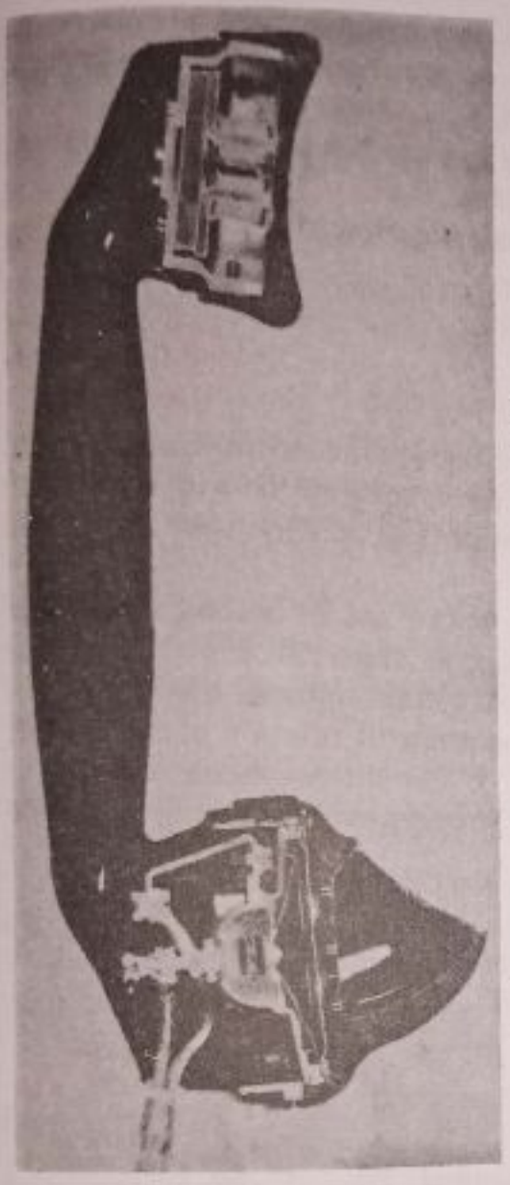
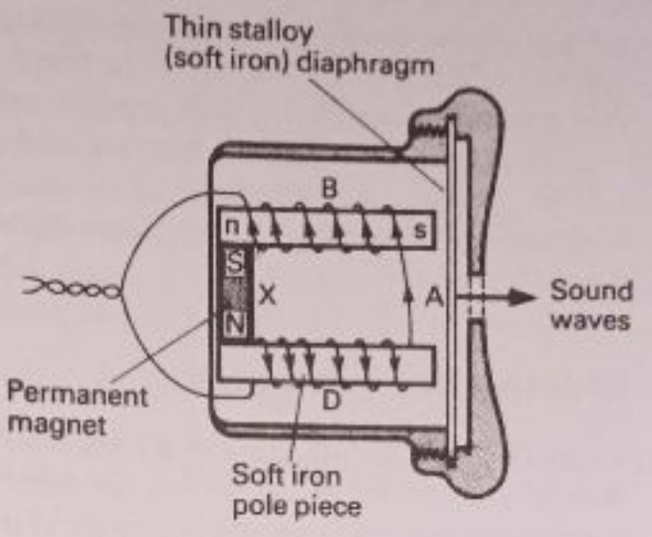
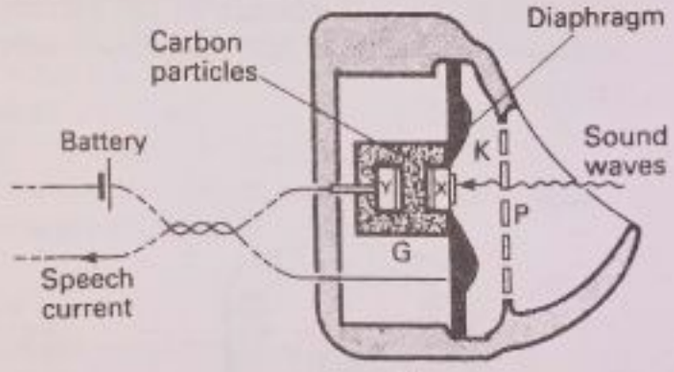


Plate 28B Earpiece (top) and carbon microphone (bottom) of a typical telephone handset



(a)



(b)

Fig. 28.12 (a) Telephone earpiece (b) Carbon microphone

varies at the same frequency. So the sound energy has been changed to electrical energy by the microphone.

Telephone earpiece

The telephone earpiece has an opposite purpose to the microphone. It changes electrical energy to sound energy, or the speech current along the telephone line to sound waves in the air.

The earpiece has soft iron pieces B and D at the S and N ends of a permanent strong magnet X (Fig. 28.12a). A flexible thin soft iron plate A is fixed near the free ends of B and D. Speech coils are placed round B and D. They are made of many hundreds of turns of very thin insulated wire, oppositely wound round B and D, and connected together. For clarity, the speech coils are shown diagrammatically in Fig. 28.12a with only a few turns.

The magnet X induces south (s) and north (n) poles at the free ends of B and D. These poles attract the iron plate A, keeping it under tension. When the varying speech current flows in the speech coils, the strength of the induced poles near A rises and falls at the same frequency as the current. So A vibrates. The vibrating

Scanned with CamScanner

plate produces sound waves of the same frequency in air. So the listener hears the speaker at the microphone end of the telephone line. As shown in Plate 28B, the microphone and earpiece are at opposite ends of the same hand unit.

If the permanent magnet X were omitted, the plate A would be attracted both on the positive *and* negative half of the same cycle of speech current. The frequency of vibration of A would then be *twice* the speech frequency. This is not desirable.

Moving iron ammeter

The magnetic effect of a current in a solenoid is used in the moving iron ammeter. Basically, as shown in Fig. 28.13a the meter has a solenoid D, with a fixed soft iron plate F and a moving soft iron plate M inside. Fig. 28.13b shows a simplified section across the solenoid and F and M.

When a current is passed into the solenoid, both F and M become magnetised with *like* poles at the ends facing each other, as shown in Fig. 28.13a. The movable iron M is then repelled from F. A spring controls the amount of movement of M, or the deflection of M. The angle of rotation of the pointer attached to M is a measure of the current. If the current in the solenoid is increased, F and M become magnetised more strongly and the pointer deflection increases.

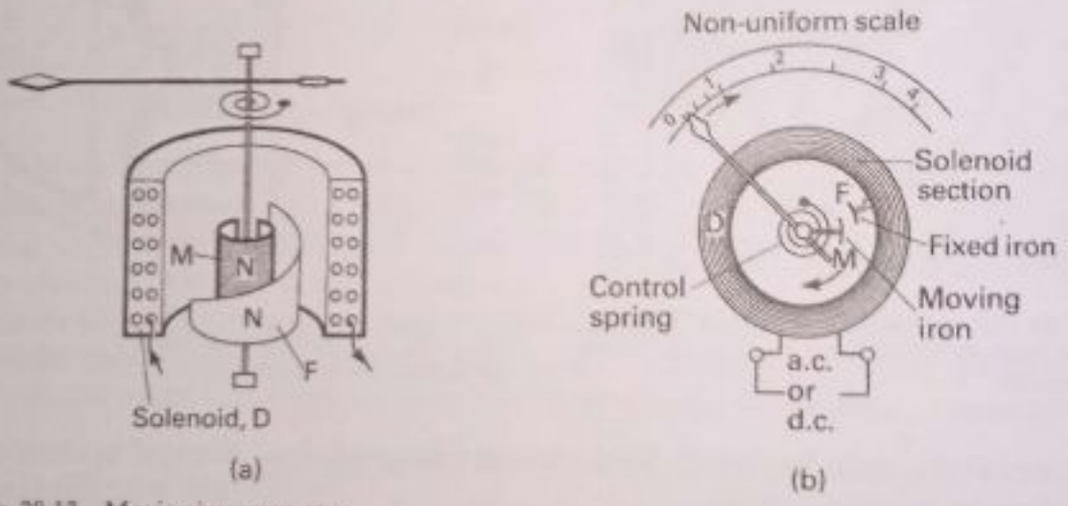


Fig. 28.13 Moving iron ammeter

If the current flows in the opposite direction in the solenoid, both poles facing each other are still like poles. So M is again repelled by F and the pointer is deflected. So this ammeter can measure both direct current (d.c.) and alternating current (a.c.). This is an advantage of the instrument. The disadvantage is that the scale is not a regular or uniform one; that is, equal intervals along the scale do not represent equal changes of current. By special shaping of the iron pieces, part of the scale can be made uniform.

Summary

1. A magnetic field is produced at right angles to the plane of a current-carrying conductor.
2. Maxwell's corkscrew rule: if a right-handed corkscrew is turned so that the point moves along the direction of the current in a *straight conductor*, the

Scanned with CamScanner

direction of rotation of the corkscrew is the direction of the flux (lines of force) round the conductor.

For a *circular coil*, a clockwise current round one face creates south polarity on that face; an anticlockwise current creates north polarity.

3. The field pattern round a *straight* current-carrying conductor is circular; with a *narrow circular coil* the field pattern is straight and perpendicular to the coil at the centre but becomes circular farther away; with a *solenoid* the field pattern outside is curved similarly to the curve created by a bar magnet but it is straight in the middle of the solenoid.

4. *Electromagnets* are usually solenoids wound round soft iron; when the current is switched off the magnetism in the iron practically disappears.

5. The *magnetic relay* has a solenoid and soft iron core. When a current flows a pivoted piece of soft iron (armature) is attracted and contacts are then made or broken in another circuit. An *electric bell circuit* has an electromagnet and a make-and-break device.

6. The *carbon microphone* contains carbon granules whose electrical resistance varies when the pressure varies. Sound wave pressure is thus changed into current variations of the same frequency.

7. When the varying current from a microphone flows through the speech coils of a *telephone receiver*, a thin iron plate or diaphragm is attracted with a varying force of the same frequency. Sound waves are then produced.

8. The moving iron meter has a solenoid, a fixed and a movable piece of iron, and a spring to control the movement of the pointer. When a current, alternating or direct (steady), flows in the solenoid the irons are always magnetised with similar poles facing, and therefore repel. The scale is non-uniform.

Exercise 28 (Answers, page 643)

1. Describe how you would proceed to magnetise an iron nail (a) electrically and (b) by making use of the earth's magnetic field. Show clearly how the polarity in case (a) is related to the direction in which the current flows and in case (b) to the way in which the nail is positioned.

Give a labelled diagram to show the construction of the telephone receiver (earpiece) and describe its mode of action. (L)

2. Give a diagram of the lines of force of the magnetic field in the immediate vicinity of (a) a straight conductor, (b) a circular conductor when each carries an electric current. Show clearly the directions of the current and the lines of force in each instance.

Why may the lines of force at a distance differ in shape from those in the immediate vicinity of the straight conductor when the field is plotted in the laboratory? (L)

3. Describe, with the aid of a diagram, the structure of an electromagnet and state clearly those features of its design and use which affect its attractive force.

Explain, with the aid of a diagram, the action of an electric bell. (L)

4. (a) With the aid of a labelled circuit diagram describe how you would magnetise a steel rod so that a marked end would be an N pole. State two ways of increasing the strength of the magnet.

(b) A wire is supported horizontally in a north–south direction above a compass needle. Describe and explain what happens when a current flows along the wire towards the north. (N)

5. Describe how an electric current may be used: (a) to magnetise a steel bar; (b) to demagnetise it.

Draw a diagram of the magnetic field produced by a current flowing in a long straight wire in a plane at right angles to the wire. State a rule which gives the relation between the direction of the current and that of the field.

A long vertical wire carrying a current passes through a horizontal bench. Give a diagram of the resultant magnetic field on the surface of the bench around the wire

Scanned with CamScanner

due to the current and the earth. Mark the positions of any neutral points formed in this field. (L)

6. (a) Describe the structure of a carbon microphone and explain its action. (b) Give an account of the structure and mode of action of a telephone earpiece. (L)

7. Explain what is meant by *magnetic field* and by magnetic induction.

Describe an experiment to plot the magnetic field inside and outside of a solenoid carrying a direct current, in a plane containing its axis. Draw a diagram of the field, marking the directions of the lines of force and the direction of the current.

If you were provided with bars of iron and steel of the same dimensions, and any other necessary apparatus, how would you demonstrate the main differences in the magnetic properties of iron and steel?

State these differences and indicate, giving reasons, which of these metals you would use for (a) a compass needle; (b) the core of an electromagnet. (L)

8. Explain carefully *two* of the following phenomena: (a) a suitably-placed bar of soft iron is drawn into a solenoid that is carrying an electric current; (b) each of two parallel wires carrying currents in the same direction experiences a force tending to draw it towards the other; (c) when a current is passed through a vertical plane circular coil set in the magnetic meridian, a small magnet placed at the centre of the coil is deflected from the meridian. (O)

9. Draw a labelled diagram of a telephone earpiece. Describe and explain the sounds heard in the earpiece (a) when it is connected to a dry cell for a few seconds and then the contact is broken; (b) when it is connected to a suitable low-voltage alternating current supply of frequency 50 Hz. (C)

10. Draw diagrams to show the magnetic field due to (a) a current in a long, straight wire; (b) a current in a single circular turn. Indicate clearly the direction of current and magnetic field.

An insulated vertical wire passes through a tank of water. A single turn of wire with its plane vertical and carrying a steady current floats on a light raft on the water. Describe and account for the behaviour of the raft when a steady current is passed upwards through the straight wire. Neglect the earth's magnetic field. (O and C)

11. Draw diagrams, with the direction of current and field clearly indicated, to show the pattern of the magnetic lines of force due to an electric current flowing in (a) a long straight wire; (b) a plane circular coil; (c) a solenoid.

Describe and explain what happens to a piece of soft iron when it is placed inside a solenoid in which a current is flowing.

Draw a labelled diagram of an electric bell, and explain its action briefly. (O)

12. Draw a labelled diagram showing the essential features of an electrical relay and describe how the relay works.

Name one situation in which such a relay could be used and explain why an ordinary type of switch is not suitable. (N)

Multiple-Choice Questions

13. When a steady current flows in a long solenoid P

A both ends of P are south poles B both ends of P are north poles
C there is no magnetic field inside P
D P settles in a north-south direction if freely suspended
E there is only a magnetic field at the ends of P

14. In Fig. 28.14, X and Y are two parallel soft iron rods inside a solenoid S. When a high current flows in S, then X and Y

A stay still B attract each other C move to the right
D move to the left E repel each other

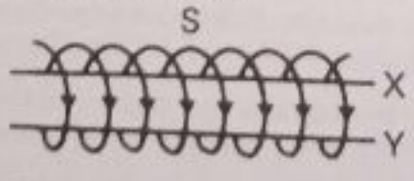


Fig. 28.14

15. Which change in A to E would NOT alter the strength of the magnetic field between the poles of an electromagnet?

A reversing the current in the coils
B halving the number of turns
C doubling the current
D changing the iron material of the core
E moving the poles nearer each other

Scanned with CamScanner

29. Forces on Conductors · The Electric Motor

In 1821 Ampere demonstrated that a current-carrying conductor experiences a force when placed in a magnetic field. Many useful machines and instruments, for example the electric motor, moving-coil meters, loudspeakers and microphones, depend on this force for their action.

Force on a straight conductor

Experiment

The force on a current-carrying conductor in a magnetic field can be shown with the apparatus in Fig. 29.1a. A smooth brass cylindrical rod AL is placed across two horizontal brass rails, PQ and RT, and a battery C and a small rheostat D are connected in series to the ends of the rails. A powerful horse-shoe magnet is then placed so that the rod AL is between its poles and the magnetic field is vertical. The magnetic field is then perpendicular to the length of the rod.

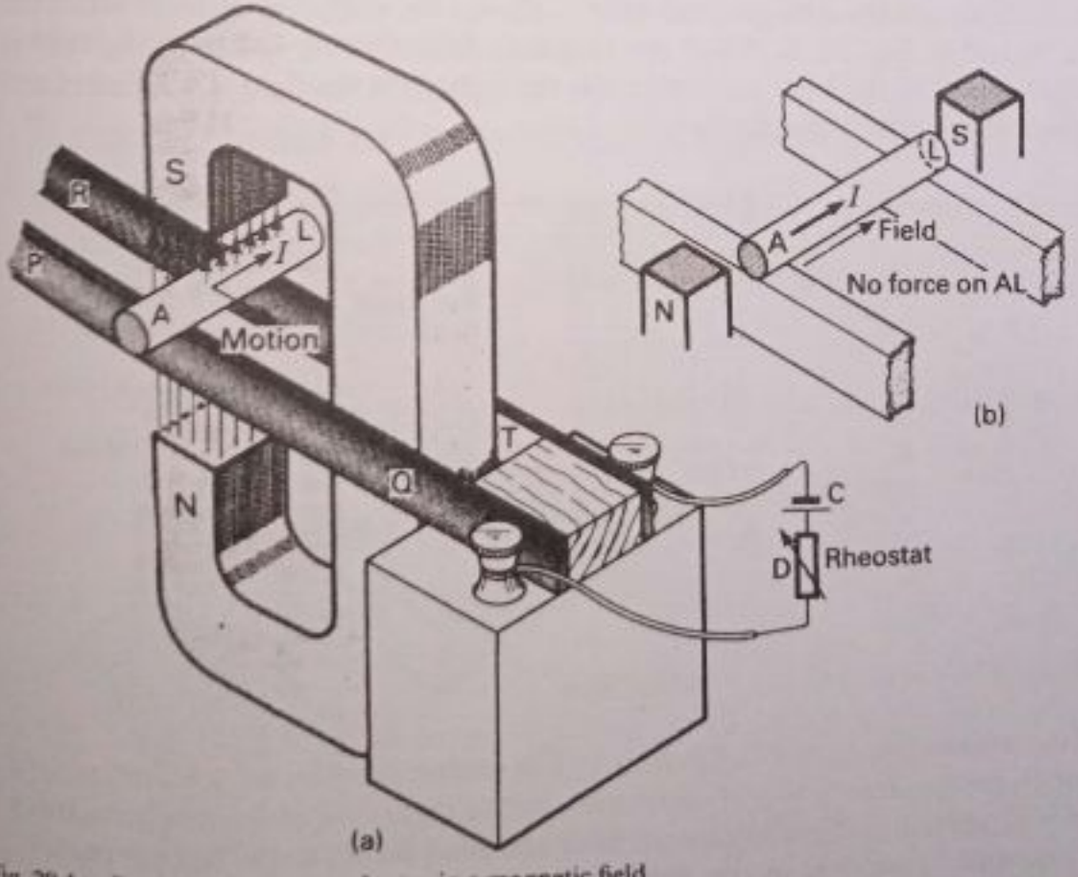


Fig. 29.1 Current-carrying conductor in a magnetic field

Scanned with CamScanner

1. When the circuit is made and a current I flows along AL the rod rolls along the rails. This shows that a force acts at right angles to the field and to the length AL of the conductor.
2. When the current is increased by altering the rheostat, the rod moves faster. So the force on the rod increases when the current I increases.
3. When the magnet is moved slightly sideways so that the strength of the magnetic field at the rod AL decreases, the rod moves less quickly. So the force decreases when the magnetic field strength decreases. If the field strength increases, the force increases.
4. When the magnet is turned round so that the field is parallel to the length AL of the rod, and the circuit is switched on, the current-carrying rod remains practically still (Fig. 29.1b). There is no force now on the rod.

Conclusions: We now see that

(a) a force acts on a current-carrying conductor in a magnetic field when it is *at right angles* to the field direction;

(b) the direction of the force is perpendicular both to the conductor and to the field;

(c) the force increases when the current increases and when the magnetic field strength increases;

(d) the force decreases when the angle between the conductor and the field is less than 90° and becomes *zero* when the conductor is *parallel* to the field.

Interaction between fields

A useful way of explaining the force is to consider the interaction when the magnetic field due to the current is combined with the magnetic field in which it is placed.

Fig. 29.2a shows the circular lines of force round the end A of the current-carrying rod AL in Fig. 29.1. The current flows downwards as if into the paper. Fig. 29.2b shows the straight lines near A due to the magnetic field between the poles N and S. Fig. 29.2c shows the resultant field. On the left of A the field is strengthened and the lines are denser. On the right of A the field is weakened and the lines are relatively less dense.

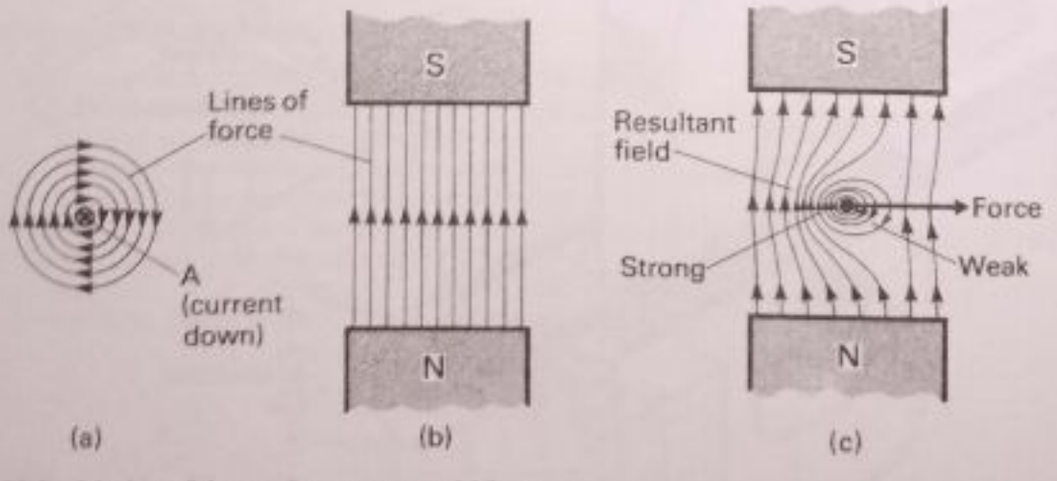


Fig. 29.2 Motion of the conductor–'catapult' force

If the lines of force are imagined to be like elastic threads, we see that, as in a catapult, the conductor A will move from left to right–this is the direction from the strong part of the field, where the lines are most dense, to the weaker part.

If the field is *parallel* to the current, however, as shown in Fig. 29.1b, the density of the lines on both sides of the conductor is practically unchanged. In

Scanned with CamScanner

fact, Fig. 29.2b is roughly the field appearance. So no catapult force acts this time on the conductor.

Direction of force

The direction of the force when a current-carrying wire is in a perpendicular magnetic field is given by *Fleming's left-hand rule*. It states *if the left hand is held with the first three fingers mutually at right angles to each other* (as shown in Fig. 29.3a), *with the forefinger in the direction of the field B and the middle finger in the direction of the (conventional) current* I, *then the motion or force on the conductor is in the direction of the thumb M*.

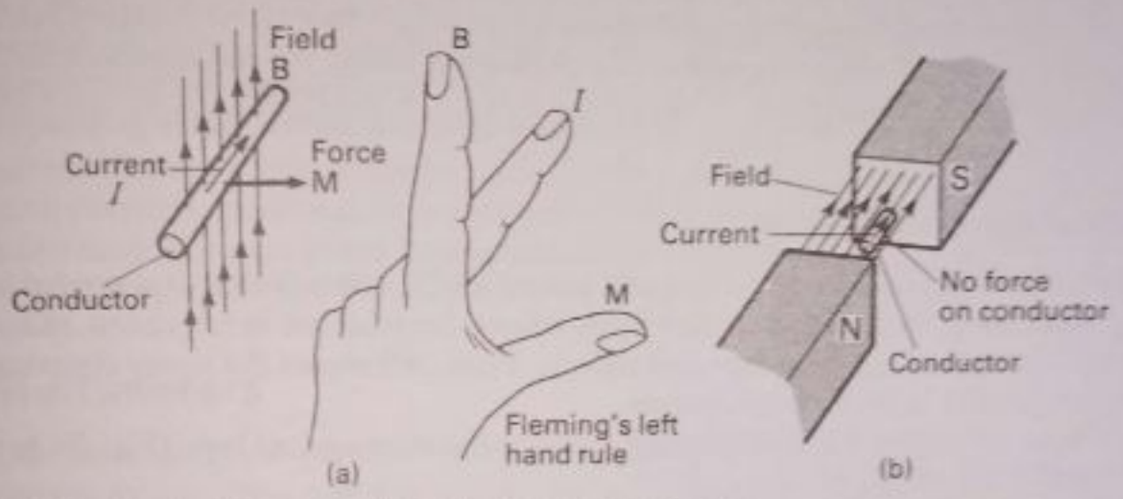


Fig. 29.3 Fleming's left hand rule for the force on a conductor

The rule can be used only if the magnetic field and current are perpendicular, or inclined, to each other. In Fig. 29.3b, the field and current are parallel to each other. No force acts on the conductor.

Turning forces on a rectangular coil

So far we have investigated the force on a straight wire or conductor carrying a current. If the wire is bent so that it forms a rectangular coil it can be suspended between two points OP by flexible strips of foil in the magnetic field between two poles N and S (Fig. 29.4).

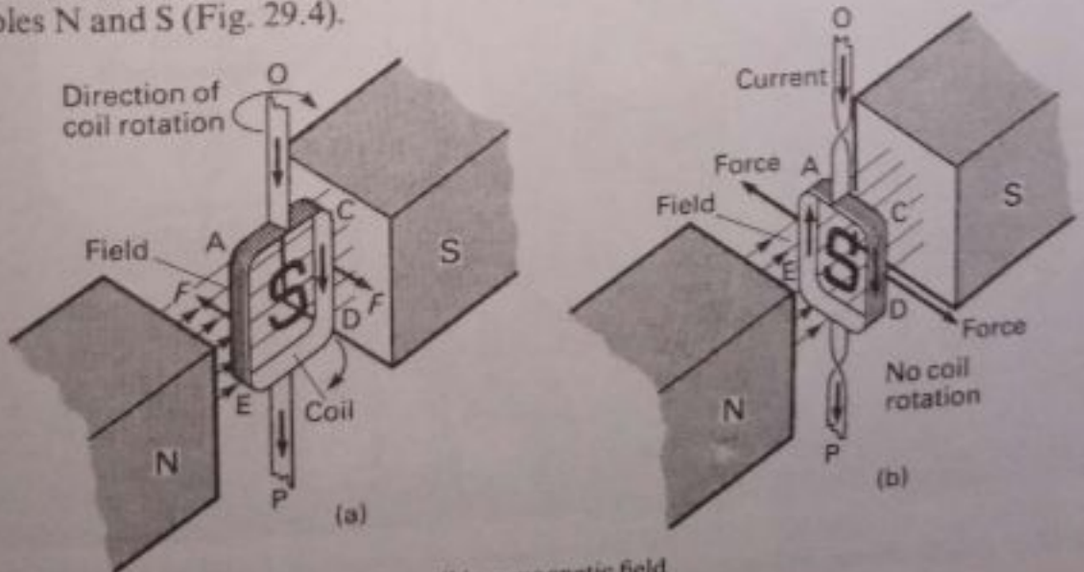


Fig. 29.4 Rotation of a rectangular coil in a magnetic field

Scanned with CamScanner

Experiment

1. When the plane of the coil ACDE is in the plane of the magnetic field (Fig. 29.4a) and a current is passed into the coil, the coil *rotates* about the axis OP. It comes to rest facing the poles (Fig. 29.4b).
2. If the circuit is broken, the coil returns to its original position. When the current is *reversed* in the coil, the coil rotates in the opposite direction.

We can explain the turning of the coil using Fleming's left hand rule. From this rule, a force F acts on the side AE and on the side CD. But these forces are in opposite directions. So they turn the coil to the position shown in Fig. 29.4b. In this position the two forces are in line with each other. So the coil no longer turns.

A simpler way of explaining the rotation is to see that the face ACDE of the coil acts like an S pole because the current flows clockwise in it (page 456). The coil then turns round to face the N pole of the magnet, to which it is attracted (Fig. 29.4b). This attraction stops the coil turning round any more.

Moving-coil meter

Accurate measurement of current and potential difference (voltage) is needed in all branches of electricity and their applications, for example in television, radio, telecommunications, dynamos and motors. Plate 29B shows the many electrical meters needed in modern aeroplanes.

The most widely used commercial meter is the moving-coil type (Fig. 29.5a). Basically, it consists of

(a) a rectangular coil ABCD with many turns,

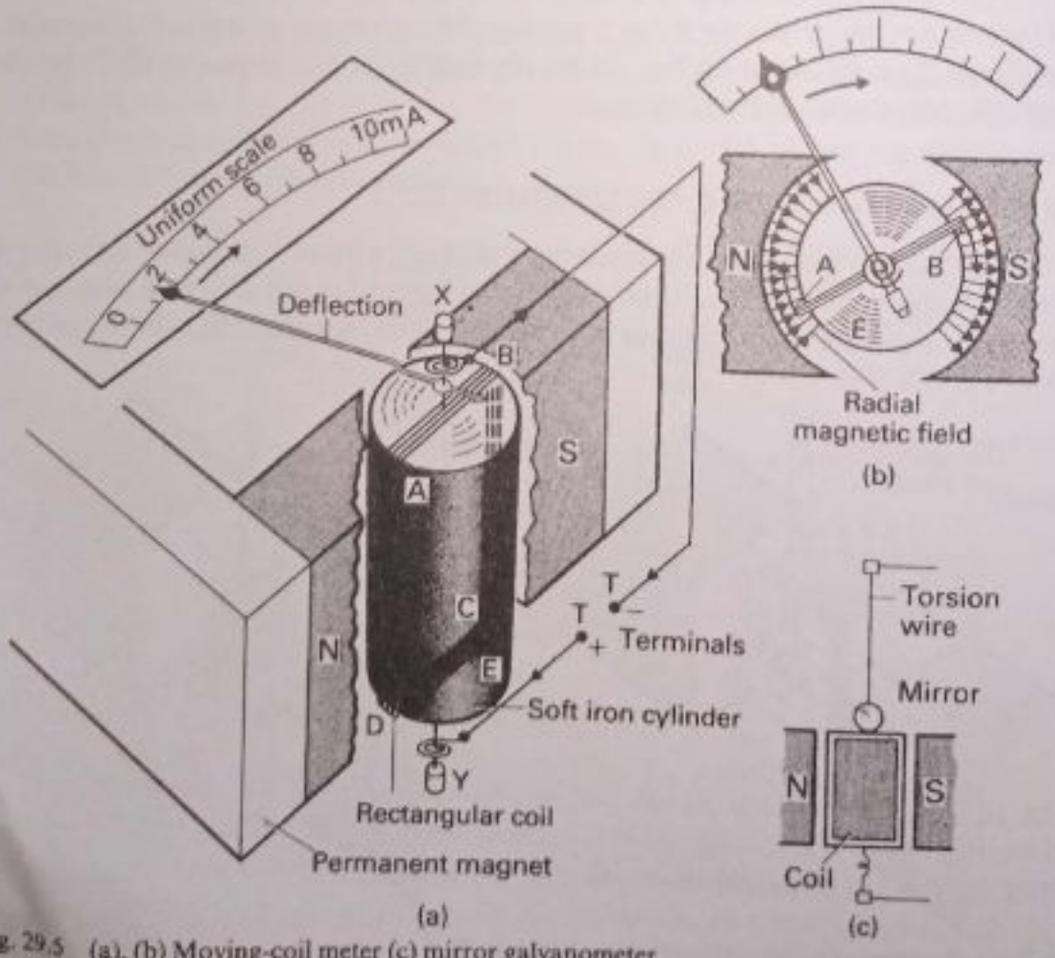


Fig. 29.5 (a), (b) Moving-coil meter (c) mirror galvanometer

Scanned with CamScanner

(b) A powerful radial magnetic field between curved pole-pieces N and S and a soft iron cylinder E (see also Fig. 29.5b),

(c) springs to control the angle of rotation of the coil,

(d) a light pointer attached to the axis XY about which the coil rotates,

(e) a uniform (linear) scale for measuring the current.

When a current flows, it enters the meter from a terminal T connected to the metal spring, goes round the rectangular coil, and leaves through the second spring at the top and the other terminal T.

The coil now rotates about the vertical axis XY between jewelled bearings. This time the angle of rotation is controlled by the springs. *The angle rotated is proportional to the current strength*, that is if a current of 2 milliamperes rotates the coil through 10°, a current of 4 milliamperes will rotate it through 20°. For this reason the scale is a uniform one as shown; equal divisions on it represent equal changes in current.

A strong spring will allow the coil to turn only through a small angle for a large current. A mirror galvanometer is also a moving-coil meter but it measures very small currents, much less than one millionth of 1 ampere. It therefore uses a so-called torsion wire as a very weak control in place of springs, and a light beam as a pointer (page 263). See Fig. 29.5c.

Multimeters

The moving-coil meter can easily be adapted to measure different ranges of current and to read potential differences. It can also measure resistance. The meter is then called a *multimeter*. Plate 29A shows the scale of a multimeter; it can read amps, volts and ohms (AVO), depending on the measurement.

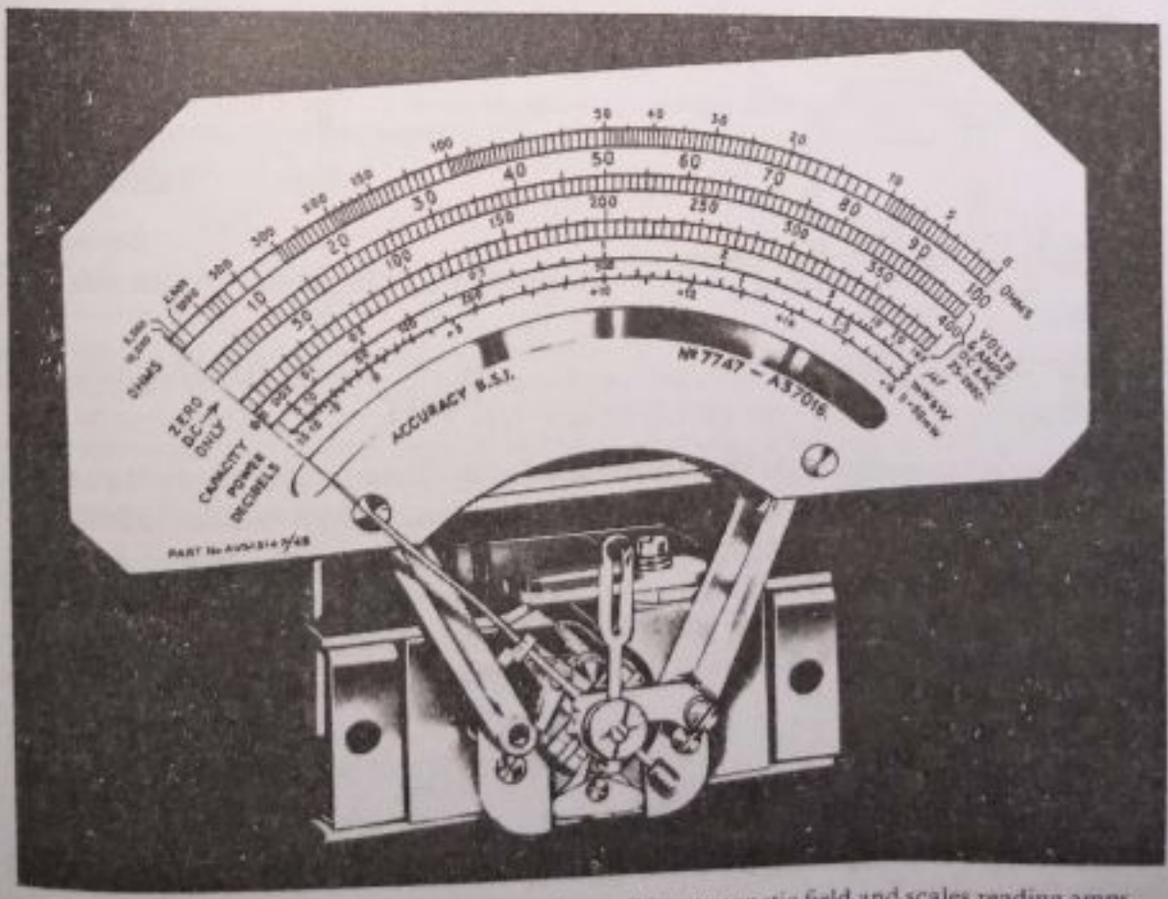


Plate 29A AVO meter, showing the rectangular coil in a magnetic field and scales reading amps, volts, ohms. A plane mirror helps to reduce parallax between the pointer and the scale.

Scanned with CamScanner

Converting to an ammeter Fig. 29.6 shows how the moving-coil meter can easily be changed to read larger currents. The basic meter can read 0–10 mA (milliampere) and the coil has a resistance of 5 Ω. If a larger current range, 0–1 A, is required, a suitable resistor R must be placed *in parallel* with the coil between the terminals A and B. Suppose the maximum current of 1·00 A flows towards A. The pointer must then be deflected full scale to the 10 mA division to use its new scale, as shown. This means that a current of 10 mA or 0·01 A flows in the 5 Ω coil. So R must divert or 'shunt' a current of 1 − 0·01 A, or 0·99 A, through itself. Now 0·99 A is 99 times as big as 0·01 A, which passes through the 5 Ω coil. So R must be 99 times *smaller* than 5 Ω, that is

$$R = \frac{1}{99} \times 5\,\Omega = \frac{5}{99}\,\Omega = 0{\cdot}0505\,\Omega$$

On switching to the 0–1 A range, then, a resistance of 0·0505 Ω is connected in parallel across the coil. A larger current range will need a smaller resistance R than 0·0505 Ω in parallel with the coil. This is calculated in the same way as before.

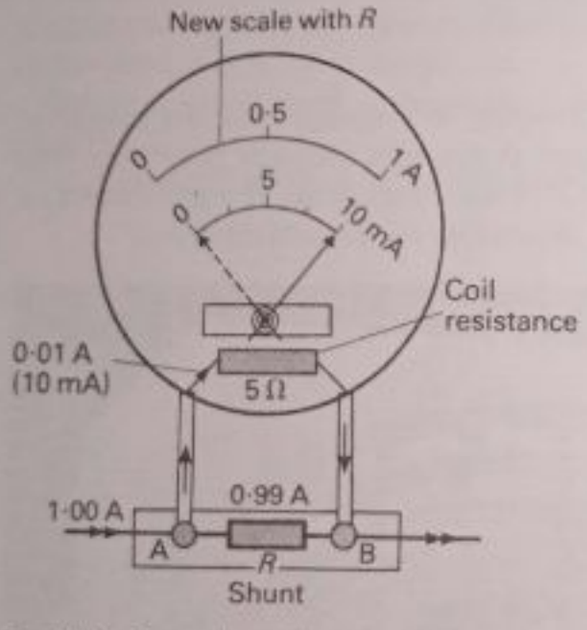


Fig. 29.6 Converting milliammeter to ammeter

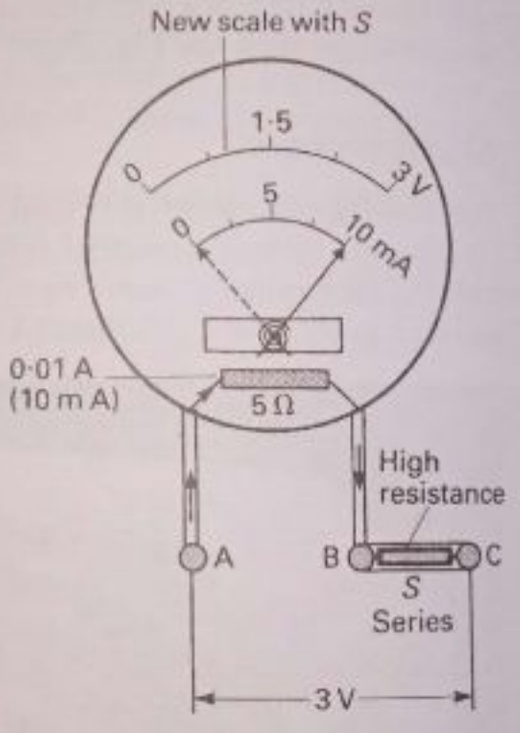


Fig. 29.7 Converting milliammeter to voltmeter

Converting to a voltmeter Fig. 29.7 shows how the same milliammeter can be converted to a voltmeter. Suppose we wish to measure a potential difference of 0–3 V. This time a suitable resistor S is connected *in series* with the coil. Suppose the maximum p.d. of 3 V is joined to the new terminals, A and C, as shown. The pointer must then deflect full scale to the 10 mA division. This means that a current of 10 mA or 0·01 A flows in the coil or along ABC. The resistance between A and C is

$$\frac{V}{I} = \frac{3\,\text{V}}{0{\cdot}01\,\text{A}} = \frac{300}{1} = 300\,\Omega$$

The coil resistance provides 5 Ω. So S must be 295 Ω. On switching to the 0–3 V range, then, a resistance of 295 Ω is connected in series. The new p.d. scale is shown in Fig. 29.7. Other p.d. ranges need different values of S.

Scanned with CamScanner

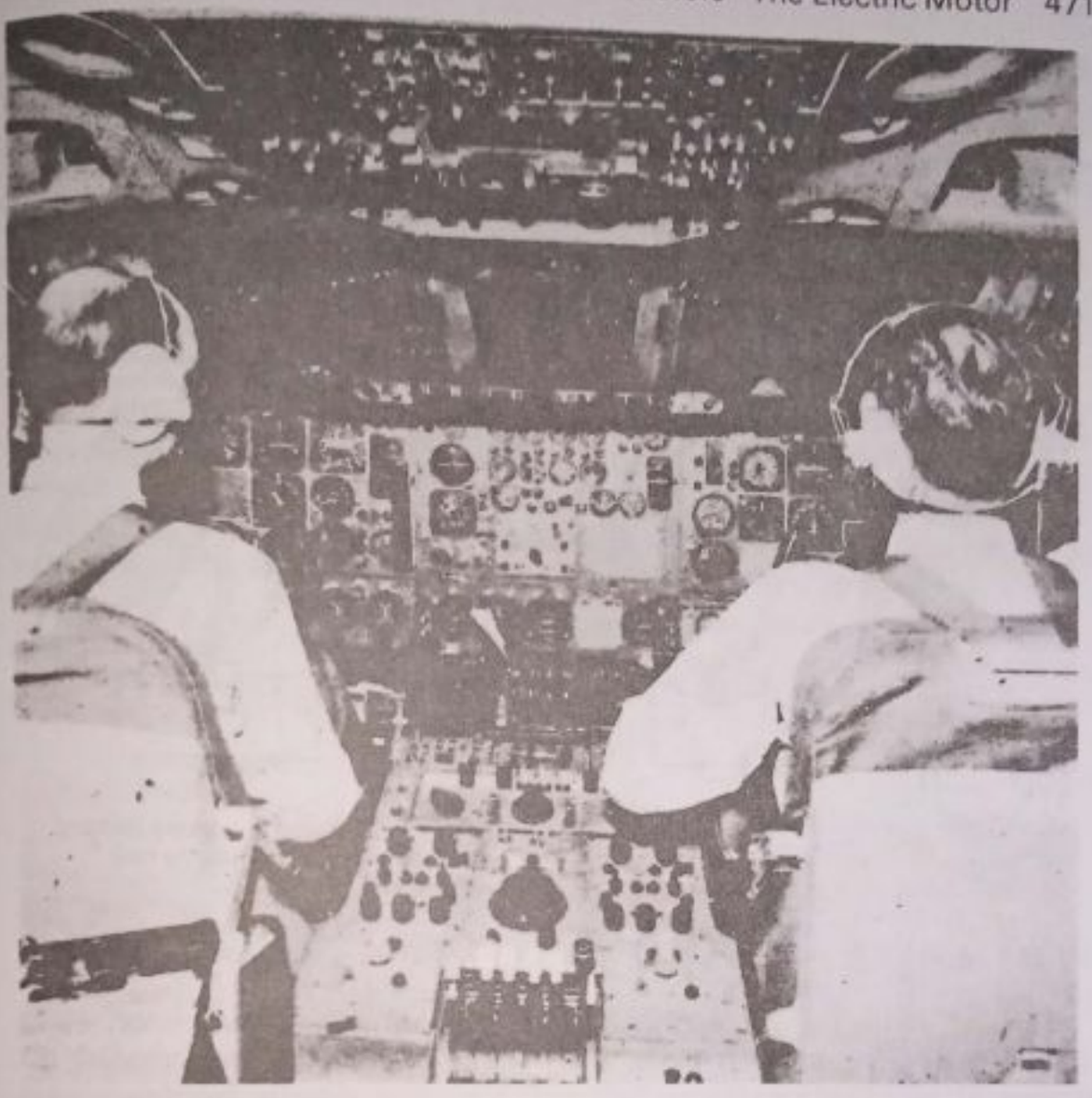


Plate 29B Flight deck of a VC 10 aeroplane, showing the many electrical instruments and automation devices (*Elliott Automation Ltd*)

Example on the ammeter and voltmeter

Describe the structure and explain the action of an instrument used for measuring or indicating an electric current. Illustrate with a clearly labelled diagram.

An accumulator of e.m.f. 2·0 V and negligible internal resistance is connected in series with a resistor of resistance 50 Ω and an ammeter of resistance 5 Ω. A voltmeter of resistance 450 Ω is connected in parallel with the 50 Ω resistor. Calculate the readings of (a) the ammeter, (b) the voltmeter. (L)

The combined resistance R of voltmeter and resistor of 50 Ω is given by

$$\frac{1}{R} = \frac{1}{450} + \frac{1}{50} = \frac{10}{450} = \frac{1}{45}$$

$$\therefore \quad R = 45\,\Omega$$

$$\therefore \text{ total circuit resistance} = 45 + 5 = 50\,\Omega$$

$$\therefore \text{ current} = \frac{E}{50} = \frac{2}{50} = 0{\cdot}04\text{ A} = \text{ammeter reading}$$

$$\text{Also voltmeter reading} = \text{p.d. across combined resistance } R$$

$$= IR = 0{\cdot}04 \times 45 = 1{\cdot}8\text{ V}$$

Scanned with CamScanner

Electric motor

The electric motor is widely used for driving electric trains and many machines in industry. A simple electric motor is shown in Fig. 29.8. It consists of

(a) a coil of wire, abcd (armature), which can turn about a fixed axis O;

(b) a powerful magnetic field in which the coil turns;

(c) a commutator, which, in its simplest form, is a split copper ring whose two halves A and B are insulated from each other.

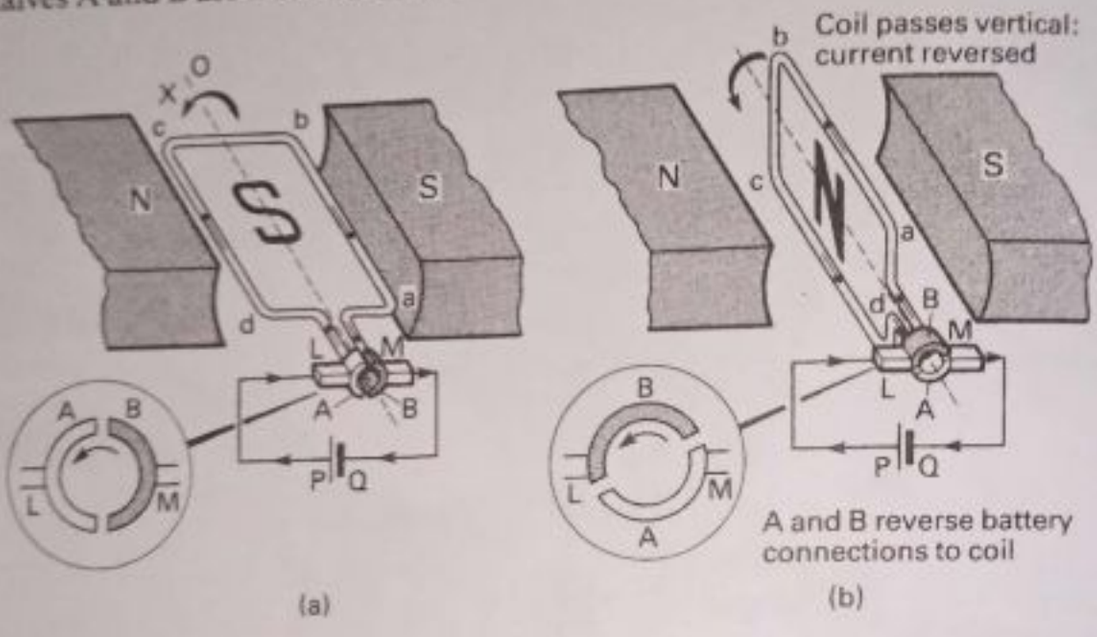


Fig. 29.8 Action of the electric motor

The ends of the coil are each soldered to one half of the commutator, which rotates with the coil. As it turns, fixed brushes L and M press against the commutator so a current from a battery PQ, connected to the brushes, always flows through the commutator into the coil.

Principle of action

Suppose the coil abcd is horizontal at first (Fig. 29.8a). The current from the battery PQ flows in a clockwise direction round the coil and therefore produces a south pole S on top as shown. This is attracted to the N pole of the permanent magnet. At the same time the north pole of the coil is attracted to the S pole of the permanent magnet. So the coil turns round its axle O in the direction X of the arrow to the position shown in Fig.29.8b.

While the coil turns, the two halves of the commutator A and B rotate with it. A keeps in contact with the brush L and B with the brush M. When the coil just passes the vertical, however, the two halves of the commutator switch over. A now makes contact with the brush M and B with the brush L, as shown inset in Fig. 29.8b. So the battery connections to the coil are reversed. The current therefore reverses in the coil. This produces an N pole, in place of the S pole, on the face of the coil abcd, as shown in Fig. 29.8b. The coil is therefore repelled away from the N pole of the magnet and so continues to turn in the same direction. The commutator is necessary as the coil would stop turning in one direction if it were not used.

Commercial electric motors in industry, which may be used for driving lathes or heavy rollers as in printing presses, are very powerful. They have strong

Scanned with CamScanner

magnetic fields, carry high currents, and use many coils of wire. This produces a powerful rotation, and machines may be coupled by belts or gear wheels to the motor.

Moving-coil loudspeaker

The moving-coil loudspeaker is used in radio receivers and in tape recorders and gramophones. It has the opposite effect to the microphone–it converts audio-frequency electric currents to sound.

Fig. 29.9a shows the principal features of the loudspeaker. A powerful magnet, with a circular S pole surrounded by a circular N pole, produces a radial magnetic field (Fig. 29.9b). A speech coil D is positioned in the narrow gap between the poles. This coil is wound on a light cylindrical former to which a large paper cone is attached. The cone itself is suspended loosely from an opening in a surrounding large board, called a baffle board.

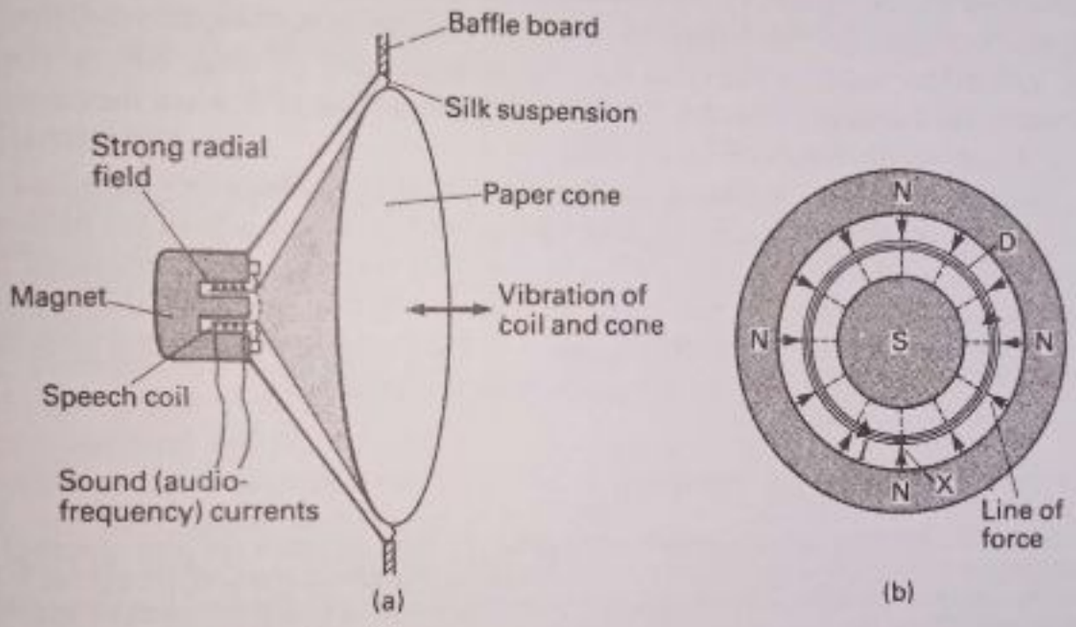


Fig. 29.9 Moving-coil loudspeaker

As in the case of a telephone earpiece, varying currents of audio-frequencies flow in the speech coil D when the loudspeaker is working. From Fleming's left hand rule (page 467), a force in the direction of the axis of the coil acts on every part of the coil (Fig. 29.9b). This force varies at the same frequency as the current frequency.

The coil is wound tightly round the former to which the cone is attached. Thus the cone vibrates at exactly the same frequency as the current in the speech coil. The large mass of air in contact with the cone is set into vibration and a loud sound is produced. The baffle board stops the sound wave behind the cone interfering with that in front.

Force between currents · Ampere balance

The ampere, the unit of current, is now defined using the force between *parallel current-carrying conductors.*

Fig. 29.10 shows the attractive and repelling forces between two parallel straight current-carrying conductors A and B. The current in B produces circular

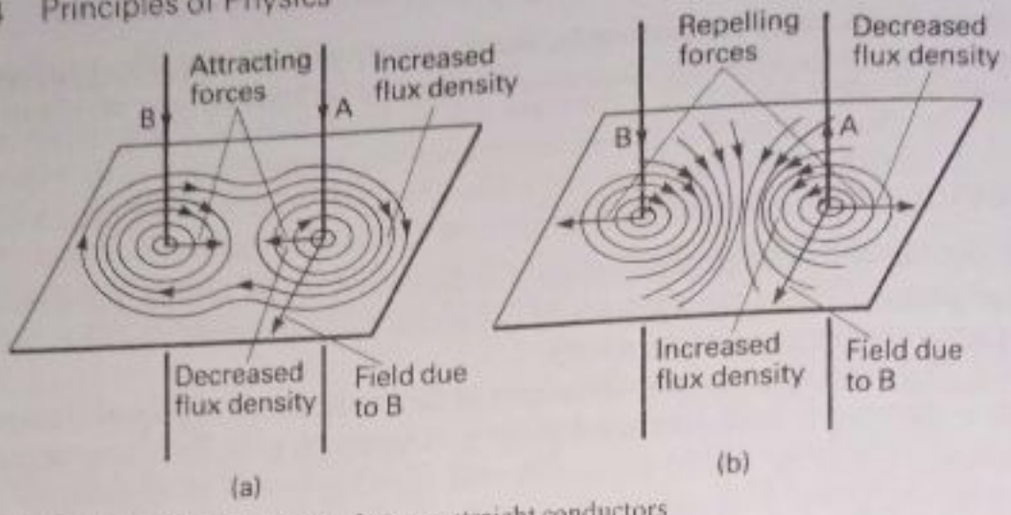


Fig. 29.10 Attraction and repulsion between straight conductors

field lines, which are perpendicular to the current at A. Applying Fleming's left-hand rule to Fig. 29.10a, A then moves towards B. The combined field round A due to both currents shows that the density of the lines is increased on the left side of A and decreased on the right side. So, as explained on page 466, A moves towards B. A similar explanation holds for the repulsion of A when the currents are in opposite directions (Fig. 29.10b).

The *ampere balance* is used at the National Physical Laboratory in Britain to measure current very accurately. It has parallel coils on one side of the balance, which carry the current, and the force between them is counterbalanced by weights on the other side. The current is calculated from the size of the force. Electrical instrument manufacturers use a 'master' ammeter, obtained from the National Physical Laboratory, to calibrate their own meters.

Example on the moving-coil meter

(a) Fig. 29.11 shows part of a moving coil galvanometer. N and S are pole-pieces of a permanent magnet, C is a fixed soft-iron cylinder about which the coil, pivoted at X, is free to rotate. Explain the advantages of this arrangement of coil, pole-pieces and soft-iron core.

(b) When a current is passed through the coil of a moving-coil galvanometer, the coil rotates through a certain angle for a given current and comes to rest. Explain, briefly, why the coil rotates and how equilibrium is achieved.

(c) Why is the coil of a moving-coil galvanometer often wound round on a frame of copper or aluminium?

(d) A moving-coil galvanometer has a resistance of 3·0 Ω and gives a full scale deflection for a current of 3·0 mA. How can it be converted into a voltmeter reading to 15 V? (O and C)

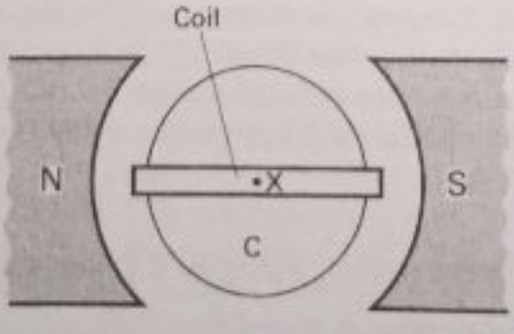


Fig. 29.11 Example

(a) The soft iron cylinder C increases the magnetic flux density and so the turning force on the coil. This makes the galvanometer more sensitive to small currents. It also produces

Scanned with CamScanner

a uniform *radial* magnetic field between the N, S poles and C, in which case the deflection of the coil is directly proportional to the current flowing. The scale of the galvanometer is then a linear one and the current can be read accurately.

(b) The coil rotates because the vertical current-carrying sides of the coil each have a horizontal force on them due to the magnetic field. Since the currents are in the opposite directions (see Fig. 29.5), the forces act in opposite directions and turn the coil. The springs round the axis of the coil oppose the turning effect of the forces. The opposing forces in the springs are proportional to the angle through which the spring is turned and so equilibrium occurs when the coil turning forces are balanced by those in the spring when it is turned through a certain angle.

(c) The coil of the galvanometer is often wound round a metal frame in order to bring the coil to rest quickly in its deflected position. The eddy currents in the frame oppose the oscillations of the coil, from Lenz's law (page 484), and bring it to rest fairly quickly.

(d) To convert the galvanometer so that it indicates 15 V for a full scale deflection, 15 V must produce a current of exactly 3·0 mA or 0·003 A through it. Using $V = IR$, the total resistance R required is given by

$$R = \frac{V}{I} = \frac{15\,\text{V}}{0{\cdot}003\,\text{A}} = \frac{15000}{3} = 5000\,\Omega$$

Since the meter has a resistance of 3·0 Ω, the additional resistance *in series* needed $= 5000 - 3 = 4997\,\Omega$.

Summary

1. A force acts on a current-carrying conductor when it is *perpendicular* to a magnetic field or inclined to the field. The direction is perpendicular to both the current and the field direction. No force acts on the conductor if it is *parallel* to the field.
2. Fleming's left-hand rule: if the first three fingers of the left hand are mutually at right angles to each other, and the forefinger points in the direction of the magnetic field and the middle finger in the direction of the current, then the thumb points in the direction of the force on the conductor.
3. The moving-coil instrument has a rectangular coil, a permanent magnet, a soft-iron core to provide a radial field and two springs. The current is proportional to the deflection, that is, the scale is uniform; this is due to the radial field. The sensitivity increases if the spring is weak or the magnetic field is strong.
4. To convert the moving-coil milliammeter to an ammeter, connect a suitable *low* resistance *in parallel* with the coil. To convert it to a voltmeter, connect a suitable *high* resistance *in series* with the coil.
5. The simple electric motor has (a) a coil of insulated wire; (b) a magnetic field; (c) a commutator which reverses the current in the coil after half a revolution so that the coil keeps turning round.
6. The ampere balance measures current by means of the force between parallel current-carrying conductors.

Exercise 29 (Answers, page 643)

1. Fig. 29.12 represents the section of a wire between the opposite poles of two bar magnets. Draw the lines of force between the poles when a current flows down the wire (into the paper). (N)
2. (a) Describe an experiment to show that a straight conducting wire carrying a current experiences a mechanical force when placed in, and perpendicular to, a magnetic field. State a rule to determine the direction of the force.
(b) A galvanometer has a resistance of 5 ohms and gives a full-scale deflection with a potential difference of 50 millivolts between its terminals. How can it be adapted to measure currents up to 5 amperes? (N)

Scanned with CamScanner

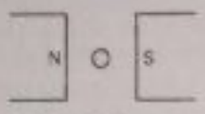


Fig. 29.12

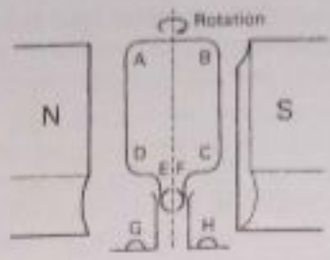


Fig. 29.13

3. (i) Fig. 29.13 is a simplified diagram to show the structure and action of a simple d.c. motor. Explain, briefly:

(a) why the coil, ABCD, will rotate when the current flows in it and why it will turn in the clockwise direction shown;

(b) why the split ring, EF, and the brushes, GH, are included in the structure;

(c) why such a motor will occasionally not start to rotate when the current is switched on but will give continuous rotation after a slight push.

(ii) An electric motor taking a current of 5 A at a p.d. of 240 V is connected by cable to a generator some distance away. If the p.d. at the terminals of the generator is 250 V determine (d) the resistance of the cable, (e) the power supplied by the generator, (f) the loss of power in the cable. (L)

4. (a) Draw a labelled diagram of the pointer type of moving-coil galvanometer and explain its action.

(b) What single resistance will have the same effect in a circuit as three resistances of 2, 4, 5Ω connected in parallel? If the total current flowing in the circuit is 1·52 A what will be (i) the current in the 5Ω resistor; (ii) the electrical energy converted into heat in the 5Ω resistor in 2 min? (N)

5. In both parts of Fig. 29.14, G represents a galvanometer of resistance 40Ω which gives a full-scale deflection (reading '2') when a current of 2 mA passes through it, and *S* represents a shunt resistor connected between the terminals A and B so that it is in parallel with the meter. In (a), $S = 40\Omega$; in (b), $S = 40/999\Omega$.

(i) In (a), what is the current through *S* when the meter reads '2'?

(ii) In (a), what is the circuit current *I* when the meter reads '2'?

(iii) In (b), when the meter reads '2', what is the current passing through the shunt *S*?

(iv) In (b), when the meter reads '2', what is the circuit current *I*? (O)

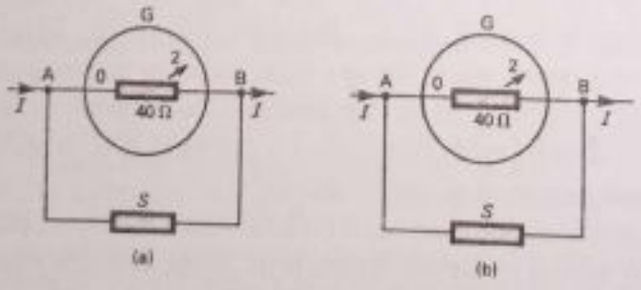


Fig. 29.14

6. Describe the structure and mode of action of a *moving-coil* loudspeaker. (L)

7. Two wires hang side by side but not touching. State what happens when electric currents flow through the wires in the same direction. Sketch the lines of magnetic force surrounding the wires. (N)

8. A moving-coil galvanometer has a resistance of 40Ω and gives a full-scale deflection of 2 mA.

(a) What is the potential difference across its terminals when this current is flowing?

(b) How can the galvanometer be converted into a voltmeter? (N)

9. Describe the structure of a voltmeter and explain how it works.

A 12·0 V battery of negligible internal resistance and resistances of 30 and 70Ω respectively are connected in series. Calculate the potential difference between the ends of the 70Ω resistance. Draw a circuit diagram showing how a voltmeter could be used to record this potential difference.

If a resistance of 60Ω is now connected in parallel with the one of 30Ω, find the new value of the potential difference between the ends of the 70Ω resistance. (L)

Scanned with CamScanner

10. Draw a diagram to show the magnetic field set up by a current flowing down a vertical wire.

Describe an experiment to show that a straight conductor carrying a current and placed in a magnetic field experiences a mechanical force. State a rule to determine the direction of the force.

Name *two* practical applications of this principle. (N)

11. A rectangular coil, carrying an electric current, is pivoted about an axis AB which is parallel to one pair of its sides. The coil is in a uniform magnetic field whose lines of force are perpendicular to AB, and is held so that its plane is inclined to the field direction.

Draw a diagram showing the relative directions of the current, the magnetic field, and the mechanical forces experienced, as a result of the interaction of the current and the magnetic field, by the two sides of the coil which are parallel to AB.

Explain how use is made of this effect in the design of the moving-coil galvanometer.

A galvanometer of resistance 5 Ω gives a full-scale deflection with a current of 15 mA. How would you convert it into a voltmeter reading up to 1·5 V? (O)

12. Describe an experiment which shows that a wire inclined at an angle to a magnetic field and carrying an electric current experiences a force. Draw a diagram showing how the directions of current, field and force are related.

Explain in detail how this effect is applied to measure current in the moving-coil galvanometer and discuss the features which determine (a) the sensitivity of the instrument; (b) the uniformity of its scale.

Illustrate your answer with a diagram. (O and C)

13. Two long vertical wires carry electric currents (a) in the same direction, (b) in opposite directions. In each case sketch a plan of the lines of magnetic flux around the wires, ignoring the effect of the earth's magnetic field. Show on your diagrams the directions of the currents, the flux lines and the forces acting on the wires. (N)

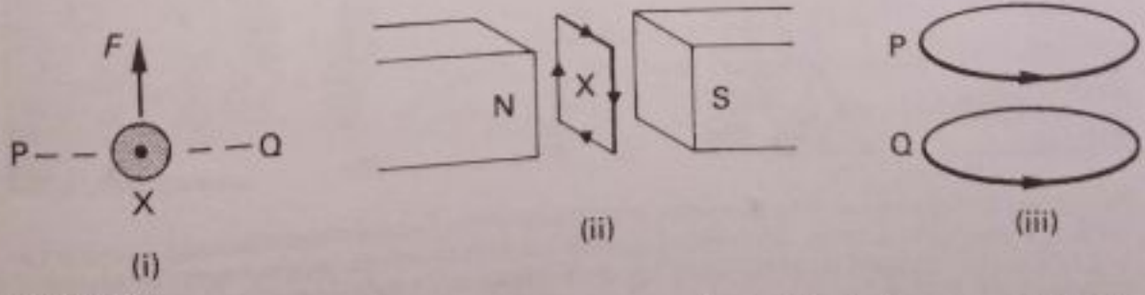
Multiple-Choice Questions

14. In a simple electric motor, the commutator

A connects the brushes together
B reverses the magnetic field
C changes the current strength in the coil
D changes the current direction in the coil
E reverses the battery poles

15. In Fig. 29.15(i), X is a section of a long straight conductor at right angles to the paper. It carries a current whose direction is vertically upwards. A force *F* acts as shown on the wire when it is in a magnetic field. The field direction is then

A PQ B parallel to *F* C QP D vertically upwards
E vertically downwards



(i) (ii) (iii)

Fig. 29.15

16. In Fig. 29.15(ii), a rectangular coil X is suspended so that its face is parallel to the N- and S-poles of a strong magnet. When a current flows in X as shown, the coil

A moves towards N B moves towards S C stays still
D turns clockwise E turns anticlockwise

17. In Fig. 29.15(iii), P and Q are two parallel circular coils each carrying current as shown. Then

A P and Q repel each other B P moves, Q stays still
C P and Q attract each other D Q moves, P stays still
E P and Q both turn round

Scanned with CamScanner

30. Electromagnetic Induction

Michael Faraday, born in 1791, is considered one of the greatest of English experimental scientists. In 1832 he discovered that electricity could be produced without any batteries by using magnets or magnetic fields. This phenomenon is called *electromagnetic induction*. His original apparatus is preserved (See Plate 30A.)



Plate 30A The Queen and Prince Philip at the opening of the Faraday Museum and Laboratory at the Royal Institution, London. Her Majesty is about to start an induced current with the original iron ring and primary and secondary coils used by Faraday in his discovery of electromagnetic induction. (*Press Association*)

Magnet and coil experiments

Fig. 30.1 shows simple apparatus for investigating electromagnetic induction. C is a solenoid or coil with many turns connected to a centre-scale zero galvanometer G. So a current in either direction in C can be detected by G.

Scanned with CamScanner

Experiment

1. *Magnet moving, coil at rest*

(a) Push the N pole of a strong magnet towards one end of C. The needle in G deflects to the right (Fig. 30.1a), so an electric current flows in the circuit. It is called an *induced current* and we say it is due to *electromagnetic induction*.

(b) Now take two, and then three, similar magnets together to increase the strength of the magnet and push them into the coil at the same speed. The induced current is seen to increase as the magnet becomes stronger.

(c) Hold a magnet at rest close to one end of the coil C or right inside it. The induced current is seen to be zero (Fig. 30.1b).

(d) Take the N pole away quickly from the coil. The current now flows in the opposite direction to when it was pushed in forward (Fig. 30.1c).

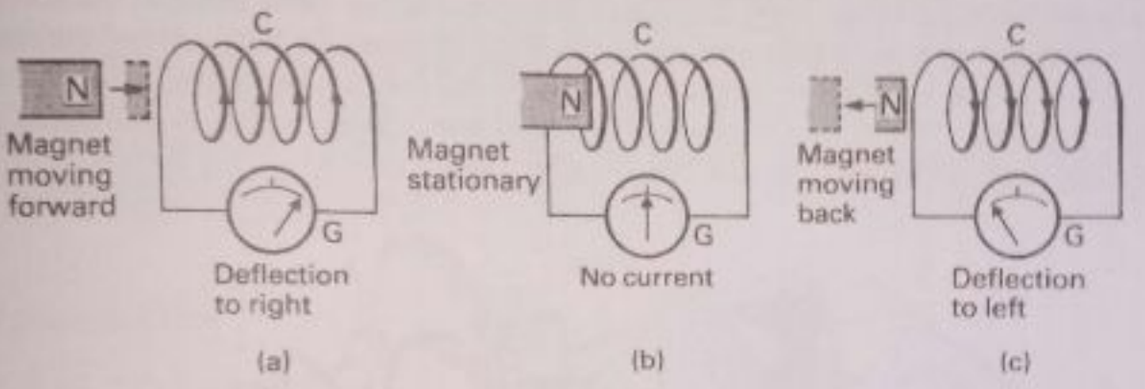


Fig. 30.1 Induced current–moving magnet

(e) Move the magnet slowly towards one end of the coil and observe the induced current. Now move the magnet faster to the end of the coil and observe that the induced current is increased.

(f) Place some soft iron, such as mu-metal, inside the coil as a core and push the magnet towards the end of the coil. A much greater induced current now flows compared to the case when there was air inside the coil.

Conclusions: An induced current flows only while the magnet is moving. The magnitude of the induced current increases when the strength of the magnet and the speed of the magnet are increased, and particularly when a soft iron core is inside the coil.

2. *Coil moving, magnet at rest*

(a) Keeping the N pole stationary, move the coil C quickly towards N. An induced current is obtained in C (Fig. 30.2a).

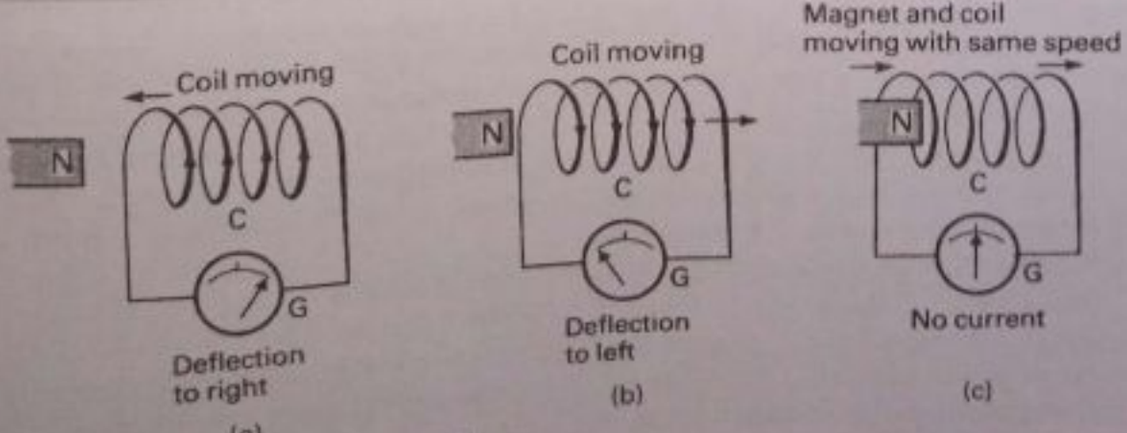


Fig. 30.2 Induced current–moving coil

Scanned with CamScanner

(b) Repeat, moving the coil C quickly back from N. An induced current is now obtained in the opposite direction (Fig. 30.2b).

(c) Move the coil and magnet together at the same speed. No induced current is obtained (Fig. 30.2c).

Conclusion: Using also the conclusion in experiment 1, we can say that an induced current flows when there is *relative motion* between the coil and the magnet.

Bicycle dynamo

A bicycle dynamo is a useful generator of current. A demonstration model is shown in Fig. 30.3. When the wheel A is turned, the disc magnet B is spun round quickly by the gearing arrangement shown. An induced current then flows in a stationary coil C in front of B. This has a large number of turns wound round soft iron and is connected to the terminals of a lamp filament L. The induced current in C makes the lamp glow. A slow rotation of the magnet produces a dim light, showing that only a small current is generated. Faster rotation produces a brighter light. A greater current is now generated.

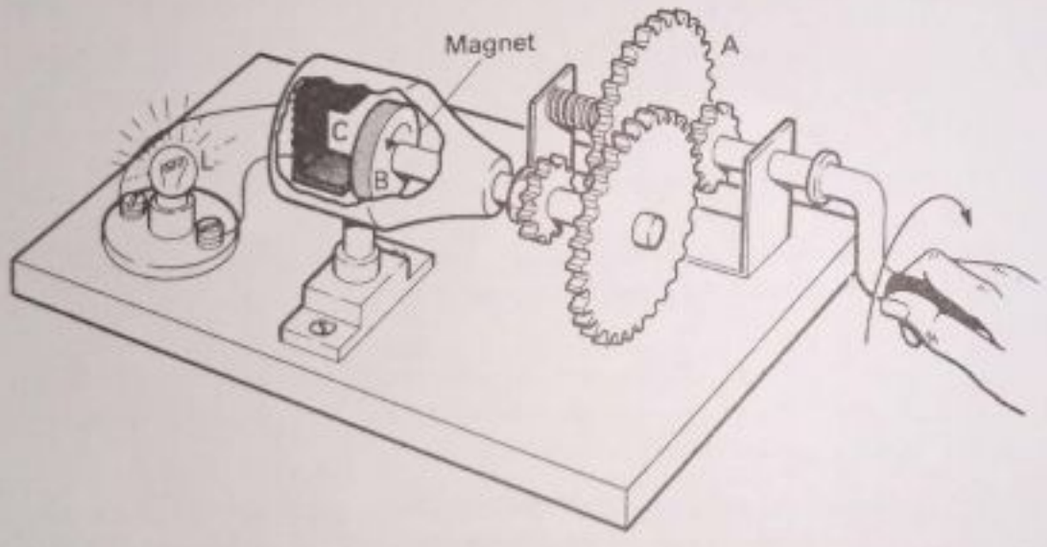


Fig. 30.3 Demonstration model of a bicycle dynamo

Primary and secondary coils

Experiment

Instead of a magnet, a solenoid A can be placed near the coil C (Fig. 30.4a). A battery L and a key K are connected to A, so when the key is pressed a current flows in A. The coil A then acts like a magnet (page 457).

(a) When A is close to C and K is depressed, a slight deflection is obtained in G (Fig. 30.4a). With a soft iron inside both A and C, as shown in Fig. 30.4b, a very much larger deflection is obtained when K is depressed.

(b) With the key K held down firmly, no deflection is observed on G. So the induced current flows only while a current *change* is made in the solenoid A.

(c) When K is released so that the solenoid circuit is broken a deflection in G is obtained in the opposite direction to that in (a). So the induced current has reversed (Fig. 30.4c). The deflection dies down quickly to zero.

Scanned with CamScanner

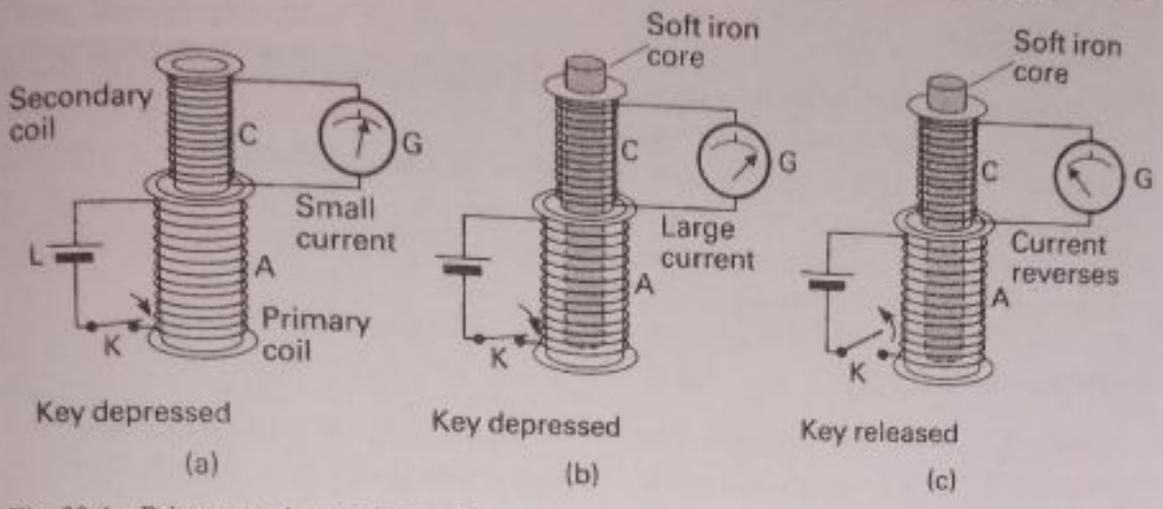


Fig. 30.4 Primary and secondary coils

Conclusion: When a current-carrying coil A is near a coil C, an induced current is produced in C whenever the circuit of A is made or broken. A soft iron core, or more turns in C, increases the induced current. Coil A is called the 'primary' coil and coil C the 'secondary' coil.

Induction coil

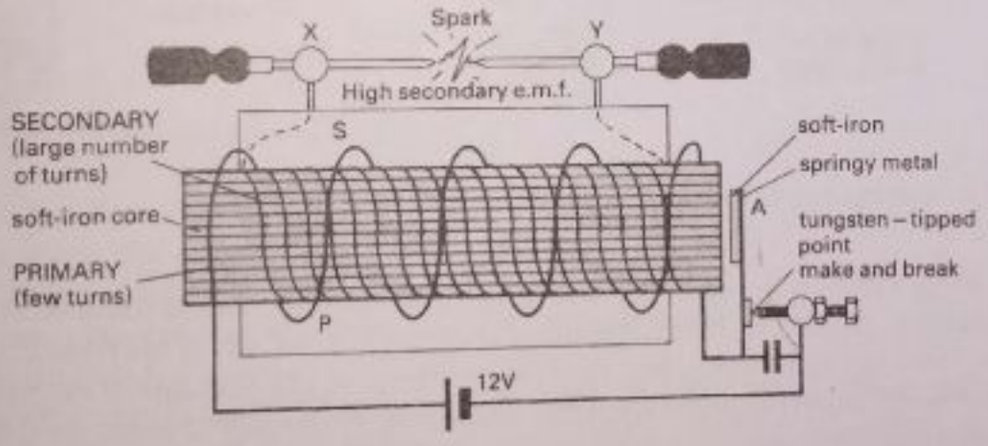


Fig. 30.5 Induction coil

The *induction coil* produces a high voltage by induction, using only a low voltage battery such as 12 V, for example. Used as the ignition coil in cars, it produces a spark to fire the petrol–air mixture for the engine.

Fig. 30.5 shows an induction coil. It has a primary coil P of a few turns and a secondary coil S of a large number of turns. When the primary circuit is made, a current flows in P. A is then attracted, so breaking the circuit at the contact point. This makes the large magnetic flux in the secondary S fall very quickly. A high voltage is then produced at its terminals X and Y, and a spark appears across the air-gap as the circuit is made and broken repeatedly.

Induced e.m.f. in a straight conductor · Fleming's rule

So far the induced e.m.f. and current has been in circular coils such as solenoids. *Straight conductors* are used in commercial generators producing high voltages for the electrical Grid system (page 491).

Scanned with CamScanner

Experiment

1. Place a cylindrical metal rod AB across two brass horizontal rails P and Q, and complete the circuit by joining a sensitive galvanometer G to the ends of P and Q (Fig. 30.6a). Arrange a powerful horse-shoe magnet round AB so that the field is perpendicular to AB. Now roll AB along the rails. An induced current is seen in G.

Note that the direction of the field, the length AB of the conductor and the direction of motion of AB are all perpendicular to each other.

2. Now turn the magnet round so that the field is parallel to the direction of movement of AB as it rolls (Fig. 30.6b). No induced current is observed in G.

Conclusion: An induced current flows in a straight wire when it is perpendicular to a magnetic field and is moving perpendicular to its length and to the field.

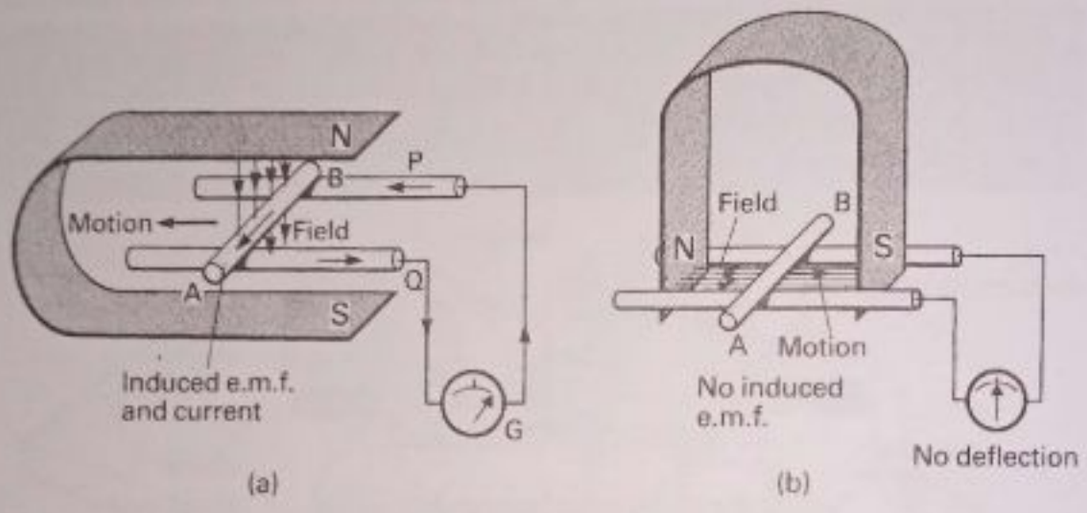


Fig. 30.6 Moving straight conductor in a magnetic field

Fleming's right hand rule helps us to find the direction of the induced current in a straight conductor. The rule states *if the first three fingers of the right hand are held at right angles to each other, with the forefinger in the direction of the field and the thumb in the direction of movement, the middle finger points in the direction of the induced current* (see Fig. 30.7).

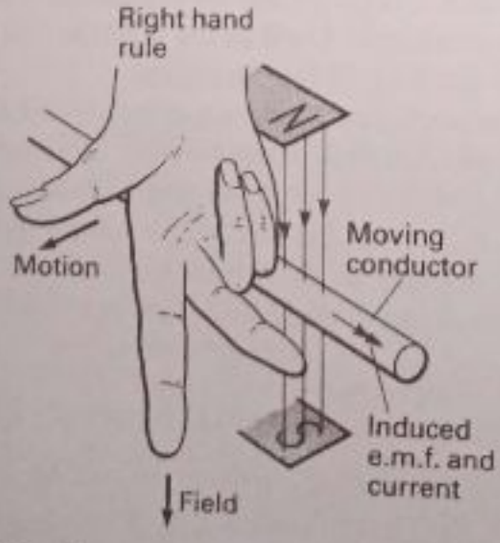


Fig. 30.7 Fleming's right hand rule

Scanned with CamScanner

You should also now apply the right hand rule to show that the induced current is in the direction BA in Fig. 30.6.

Faraday's law of electromagnetic induction

Faraday imagined lines of magnetic force (magnetic flux) everywhere in the space round a magnet or round a current-carrying coil which acts like a magnet.

When a magnet is placed near a coil, flux lines link or 'thread' the turns of the coil (Fig. 30.8a). If the magnet is moved nearer more lines link the turns. An induced e.m.f., which produces an induced current, is therefore always obtained when a *change* is made in the number of lines linking the coil. A slight movement of the magnet produces a small change; so the induced e.m.f. is small. A bigger movement of the magnet produces a greater change and a greater induced e.m.f.

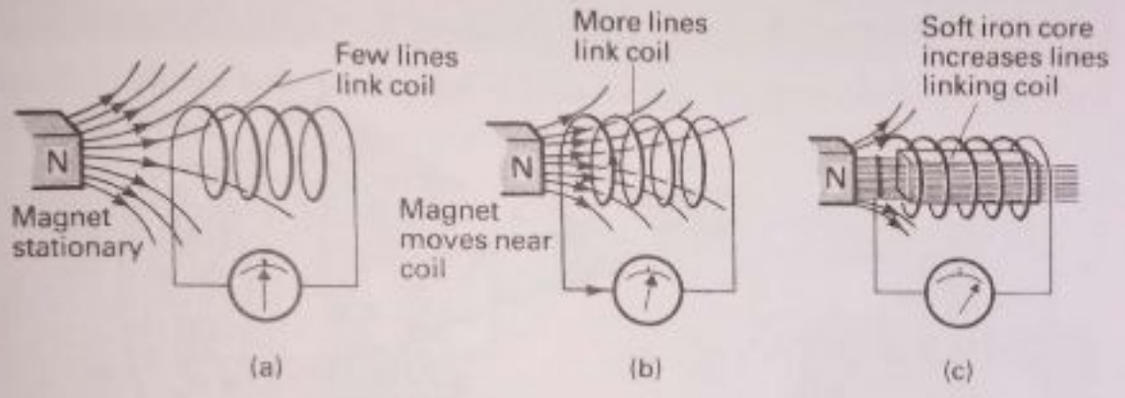


Fig. 30.8 Flux change and induced e.m.f.

As we saw from the experiments on page 479, the induced current and e.m.f. increase when the magnet moves faster towards the coil, so making a quicker change in the number of lines linking the coil. Thus the size of the e.m.f. depends on the *time* taken to make the change. The more quickly the change is made, the higher is the e.m.f.

Experiments with a bicycle dynamo rotated at different speeds show that *the induced e.m.f. in a circuit is directly proportional to the rate of change of the magnetic flux, or field lines, linking the circuit.*

This is often called *Faraday's law of electromagnetic induction.*

The number of lines or magnetic flux linking a coil depends on its number of turns, the area of its turns and the strength of the magnetic field used. The induced e.m.f. can reach several thousand volts using a very quick change of flux, a coil of many turns and a powerful magnetic field.

With a *straight conductor* moving perpendicularly to a magnetic field, as shown in Fig. 30.6a, the induced e.m.f. is proportional to the rate of 'cutting' of the lines (flux) in the field. The faster the speed of the conductor, the greater is the induced e.m.f.

Lenz's law · Eddy currents

Faraday's law refers to the magnitude or size of the induced e.m.f. A law for the direction of the induced current was first given by Lenz. *The induced current flows in such a direction as to oppose the motion or change producing it.*

We can show this 'opposition' by a simple demonstration.

Scanned with CamScanner

Experiment

A metal disc D, which can turn about a horizontal axis, is placed between the poles N and S of a strong magnet (Fig. 30.9a). D is then turned so that it starts to spin on its own but it soon slows down and stops turning. The same result is obtained even when it is pushed hard to make it spin faster.

If the magnet is moved further away, however, the metal disc spins for a long time, before eventually coming to rest because of friction.

When the disc cuts the magnetic flux between the N and S poles as it turns, induced currents flow in the metal. When the magnet is taken away, no induced currents flow in the disc when it spins.

Conclusion: Since the metal disc stops turning in the magnet's field, the induced currents *oppose* the motion of the disc or the motion producing the currents.

The induced currents in the metal are called *eddy currents* because they flow in circular paths like eddy currents in water.

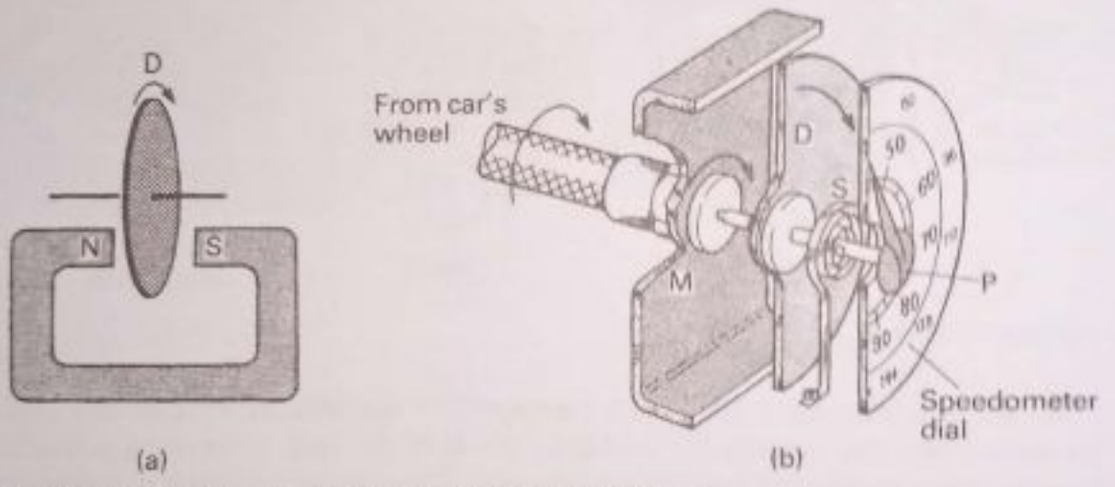


Fig. 30.9 (a) Demonstration of Lenz's law (b) Car speedometer

Lenz's law is used in the car speedometer. The principle is shown in Fig. 30.9b. A magnet M, connected to one of the car's wheels, rotates at a speed proportional to the car's speed. Induced (eddy) currents then circulate round an aluminium disc D pivoted in front of M. To 'oppose' the motion of M, D starts to rotate in the same direction. The degree of rotation is controlled by a spring S. When M turns faster, D turns through a larger angle. A pointer P attached to D can thus register the speed of the car on a dial, which is calibrated by running the car at known speeds.

Moving-coil meter damping

In a moving-coil ammeter (page 468) eddy currents are used to bring the pointer to rest quickly after it is deflected.

The copper coil is wound on a rectangular aluminium (metal) frame. Fig. 30.10 shows the top of the frame and coil viewed from above, together with the soft iron cylindrical core and surrounding magnet N, S. When a current flows into the coil, the coil oscillates in its deflected position, as shown. So the metal frame cuts the magnetic flux in the radial field and induced or eddy currents then circulate round the metal. From Lenz's law, the induced currents oppose the motion. So the

Scanned with CamScanner

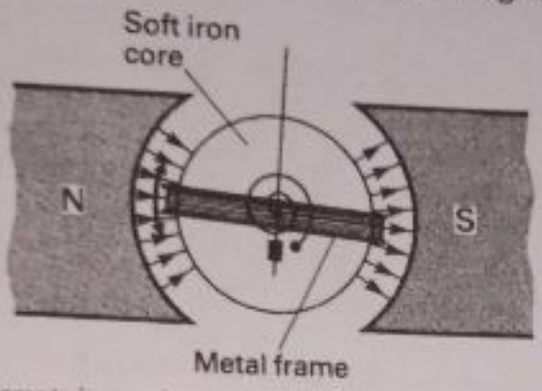


Fig. 30.10 Use of eddy currents in moving-coil meter

oscillation soon stops. We say that the oscillations of the coil and attached pointer are *damped* by using a metal frame.

Applications of Lenz's law

Fig. 30.11 shows how Lenz's law is used to find the direction of the induced current in coils.

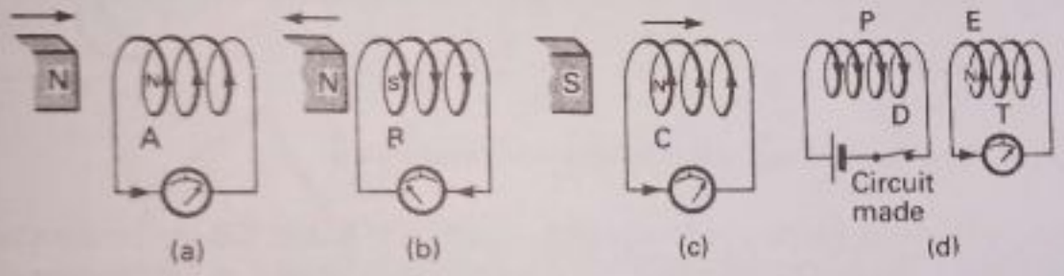


Fig. 30.11 Directions of induced current

(a) In Fig. 30.11a an N pole moves towards the face of a coil whose ends are joined. The coil opposes the movement of N. Since like poles repel, an N pole is induced in the coil and so an anticlockwise current flows in A as shown.

(b) In Fig. 30.11b an N pole is withdrawn from the coil. An S pole is therefore induced in the face B. So the induced current flows clockwise.

(c) In Fig. 30.11c a coil is moved away from a stationary S pole. An N pole is therefore induced at the face C to oppose the movement. The current in the coil flows anticlockwise, as shown.

(d) In Fig. 30.11d the circuit is made in the primary coil P, so that an N pole is suddenly obtained at the end D. This is similar to moving the N pole of a magnet quickly to the end E of the secondary coil T. To oppose the motion, an induced N pole is obtained at E, as shown. So the current flows anticlockwise. When the current in P is switched off, the current flows clockwise to produce an S pole and so oppose the removal of the N pole at D.

Principle of the simple dynamo

The dynamo is a machine which transforms mechanical energy to electrical energy. It produces an e.m.f. at its terminals by electromagnetic induction. A simple dynamo consists of a coil rotating at a steady speed about a fixed axis in a magnetic field.

Fig. 30.12 shows three positions of the rotating coil. In Fig. 30.12a, the plane of the coil is *parallel* to the field. Here the sides DA and CB are moving *at right*

Scanned with CamScanner

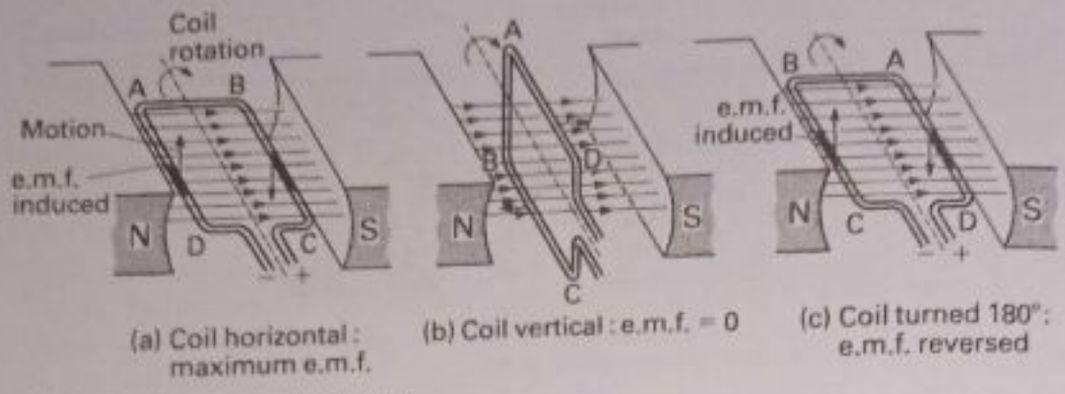


Fig. 30.12 Action of a simple dynamo

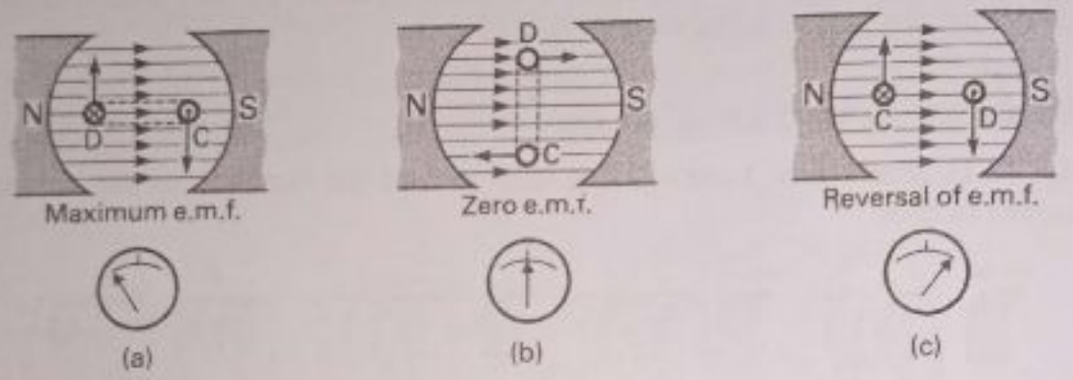


Fig. 30.13 Direction and magnitude of induced e.m.f. in a dynamo

angles to the field. So an e.m.f. is produced along DA and CB in the directions shown, from Fleming's right hand rule. There is no induced e.m.f. along the sides AB or DC because these do not 'cut' the field lines as they rotate.

In Fig. 30.12b, the coil has turned through 90°. The sides DA and CB are both moving parallel to the field at this instant. So no induced e.m.f. is obtained.

In Fig. 30.12c, the coil has turned another 90° and its plane is now horizontal or parallel to the field. But compared to Fig. 30.12a, DA and CB are moving in opposite directions. So the e.m.f. along DA and CB are in opposite directions, from Fleming's right hand rule. This means that the e.m.f. in the whole coil *reverses* each time the coil passes the vertical, or twice in one revolution.

Fig. 30.13 shows the changes in e.m.f. produced in the coil when we look at the dynamo end-on. The maximum e.m.f. is when the coil is horizontal; there is zero e.m.f. when the coil is vertical.

A.c. voltage

When the variation of the e.m.f. is plotted against the time, the graph shown in Fig. 30.14 is obtained. The e.m.f. is plotted negatively when it reverses in direction. This is called an *alternating e.m.f.* or *a.c. voltage*, that is it reverses after a time.

Generators at power stations work on a similar principle to dynamos; the mains voltage in Britain is usually standardised at 240 V a.c. So alternating current flows through lamps and electric heaters. The frequency of the mains voltage in Britain is 50 Hz (50 cycles per second); that is the voltage goes through one complete cycle of changes in 1/50 s, after which it begins all over again. In the United States of America, the frequency is 60 Hz.

Scanned with CamScanner

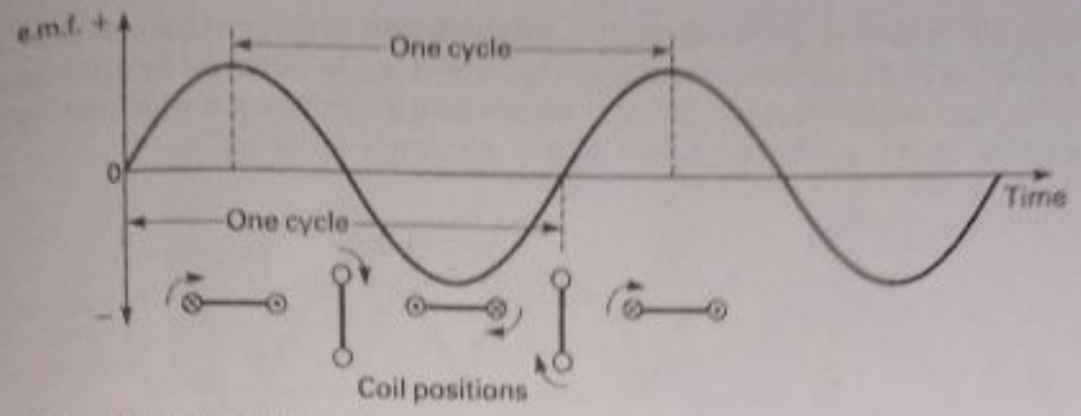


Fig. 30.14 Alternating voltage

Using the induced current

1. *Alternating current–slip rings*

Fig. 30.15a shows how the alternating current (a.c.) can be led from the rotating coil of a simple dynamo to an external circuit. Two brushes G_1, G_2, often made of carbon, are fixed so that they rub against slip-rings P and Q, which rotate with the coil. The brushes can be connected to an outside circuit such as a lamp R as shown.

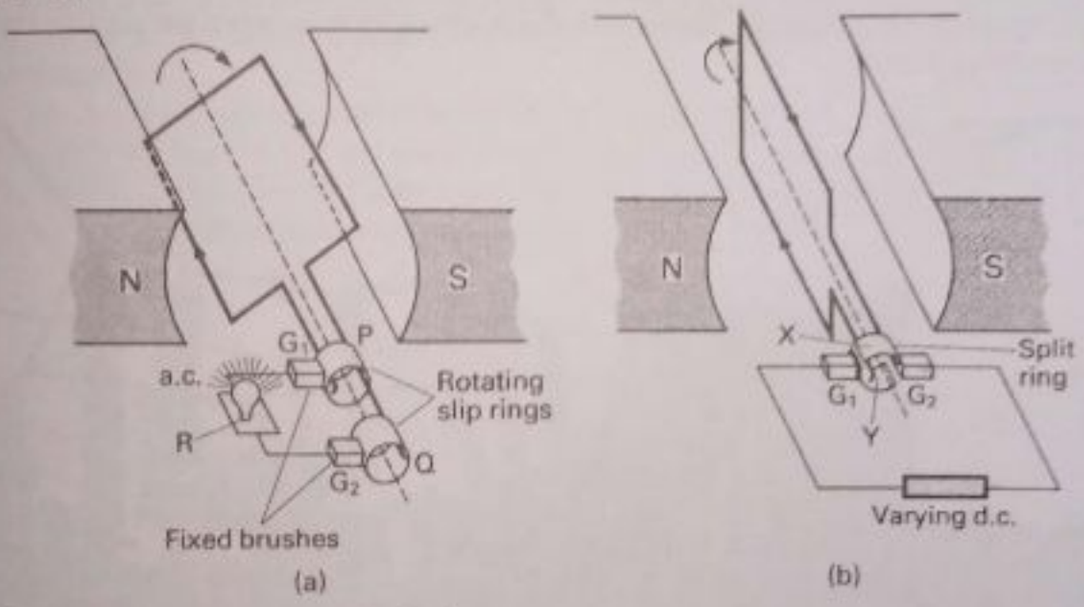


Fig. 30.15 a.c. and d.c. from a simple dynamo

2. *Direct current–commutator*

Current which flows in one direction only, even though it may vary in value, is called 'direct current' (d.c.). A simple dynamo can supply direct current by using a split-ring *commutator*, represented by G_1, G_2 in Fig. 30.15b. The ends of the coil are joined to G_1 and G_2 respectively.

Just *before* the instant shown in Fig. 30.15b, the brush G_2 was touching one half Y of a split ring and the brush G_1 was touching the other half X. As the coil rotates past the vertical to the position shown in Fig. 30.15b, Y moves round to touch G_1 and X moves round to touch G_2, as shown in the diagram. This action of the commutator reverses the connections to the external resistance connected to G_1, G_2. The reversal happens at the moment when the current is just zero (coil vertical). So instead of the current reversing in the external resistance, it continues to flow in the same direction.

Scanned with CamScanner

Fig. 30.16 shows how the current I varies with time in the external resistance. It is a varying direct current. A more steady current can be obtained by using many coils at equal angles to one another, together with a commutator which has many segments instead of the two halves used in the simple dynamo.

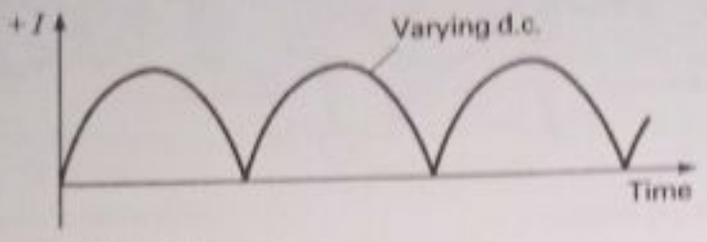


Fig. 30.16 Direct current from a dynamo

Commercial transformer

To reduce power losses, and so to transmit power economically over long distances, high a.c. voltages are needed (see page 432). Further, the a.c. voltage needs to be changed to a much lower voltage when brought into the home or factory, for safety reasons. A *transformer* changes the size of an a.c. voltage. As we see soon, it transfers electrical energy from one circuit to another by electromagnetic induction. There is no direct connection between the circuits and no moving parts.

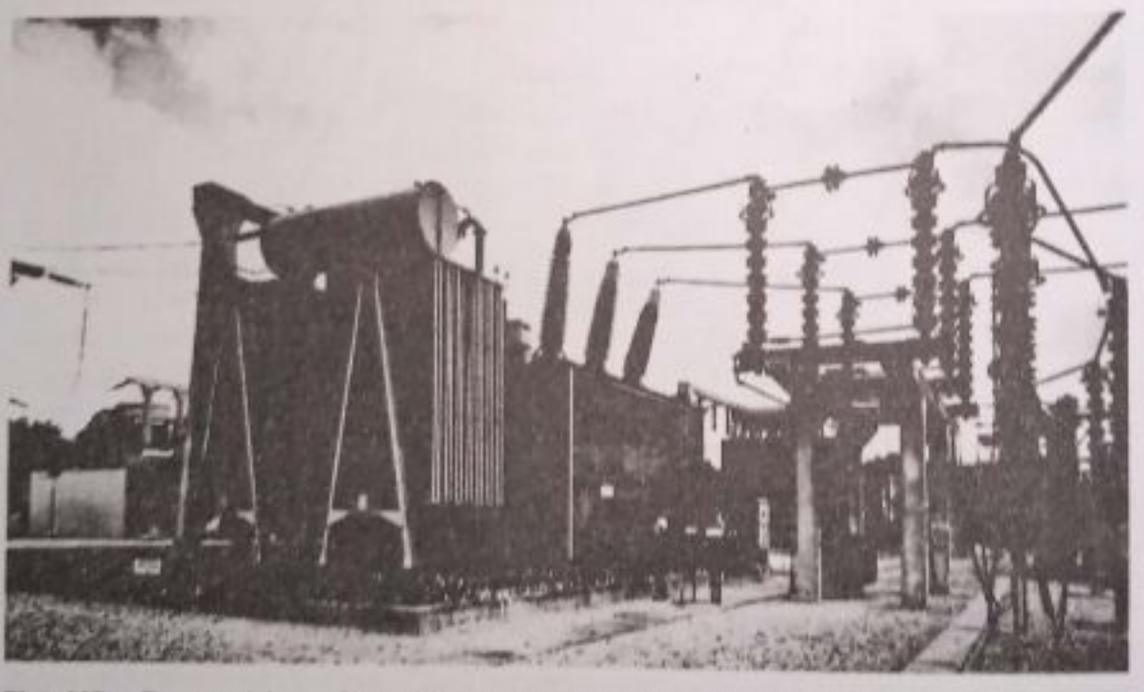


Plate 30B Commercial transformers housed in a noise-prevention building at a substation in Worcestershire. The bushes and insulators above the building have projecting wires which safeguard against damage by lightning. The transformers are cooled by oil circulating through pipes connected to the radiators and cylinder in front. (*Central Electricity Generating Board*)

The commercial transformer consists of two insulated coils P and S called primary and secondary coils, wound round a soft-iron core (Fig. 30.17). The core is made of laminations or sheets of soft iron insulated from each other and not a solid block of iron. The induced (eddy) currents in the core cannot flow across the insulation from one sheet to the next and so the loss of energy as heat is considerably reduced. This increases the efficiency of the transformer because a

Scanned with CamScanner

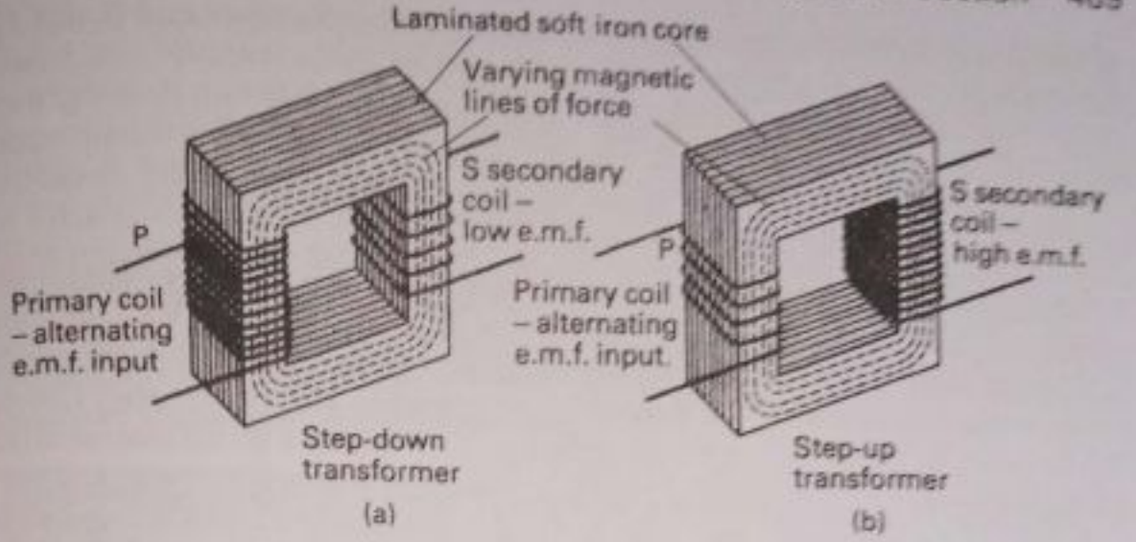


Fig. 30.17 Transformers

greater fraction of the power supplied to the primary coil is now obtained in the secondary circuit.

When an alternating e.m.f. or a.c. voltage E_p is connected to the primary coil P, an alternating current flows. This produces corresponding *variation* of magnetic flux in the iron core, which links the secondary coil S as shown in Fig. 30.17a or b. An induced e.m.f., E_s, of the same frequency as E_p, is therefore obtained in S.

The flux linking each turn of the primary and secondary coils is the same. The *total* flux linking the two coils is therefore proportional to their *number of turns*, N_p and N_s. Now from Faraday's law, the e.m.f. induced in a coil is proportional to the rate of change of the magnetic flux linking the coil. So

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

This is the law of the commercial or soft iron transformer. In one type of transformer, N_s is greater than N_p. Suppose $N_s = 1000$ turns, $N_p = 100$ turns, and $E_p = 240$ V a.c. Then

$$\frac{E_s}{240} = \frac{1000}{100} = 10$$

So
$$E_s = 10 \times 240 = 2400 \text{ V a.c.}$$

Since the 240 V a.c. at the primary coil has been increased ten times, this is called a *step-up* transformer.

A *step-down* transformer reduces the a.c. voltage. In this case the secondary coil has less turns than the primary. A ray-box transformer which produces 12 V a.c. from the 240 V a.c. mains has 20 times fewer turns in the secondary coil than in the primary.

Current and efficiency in transformer coils

Like any other machine, the transformer obeys the Principle of the Conservation of Energy. So the energy per second, or power, obtained in the secondary coil at any instant is equal to the power supplied to the primary, if the efficiency is 100%. This is practically the case for the large commercial iron core transformers. So if

Scanned with CamScanner

I_s and I_p are the secondary and primary currents when the e.m.f.s are E_s and E_p respectively

$$I_s \times E_s = I_p \times E_p$$

$$\therefore \frac{I_s}{I_p} = \frac{E_p}{E_s} = \frac{N_p}{N_s} \text{ or } N_p I_p = N_s I_s$$

So if the transformer is a step-up one with a turns ratio 60:1, the currents are stepped-down in the ratio 1:60. Suppose the primary e.m.f. is 4 V a.c. and the secondary e.m.f. is 40 V a.c. If the primary current at an instant is 1 A, the power in the primary is then 4 W. The current in the secondary is approximately 1/10 A. So the power in the secondary is approximately $1/10 \times 40$ or 4 W, the same as in the primary coil.

In the same way, the a.c. voltage in the secondary coil of a step-down transformer is less than in the primary, so the current in the secondary is *greater* than in the primary.

In practice, there is a loss of energy or power in a transformer; for example, some energy is wasted as heat in the coils, and as heat in the iron core owing to eddy currents. So the efficiency of the transformer is less than 100%. As an illustration, consider a 240 V step-down mains transformer, used for lighting eight 12 V, 24 W ray-box lamps, which then draws a current of 1 A in the primary. In this case

$$\text{power supplied to primary} = 240\text{ V} \times 1\text{ A} = 240\text{ W}$$

and $$\text{power developed in secondary} = 8 \times 24\text{ W} = 192\text{ W}$$

So $$\text{efficiency} = \frac{192}{240} \times 100\% = 80\%$$

High voltage transmission

As explained on page 432, less energy is lost in cables when power is transmitted at high voltage and low current. As the cables needed to carry a low current can be relatively thin, the cost of the metal used is also reduced.

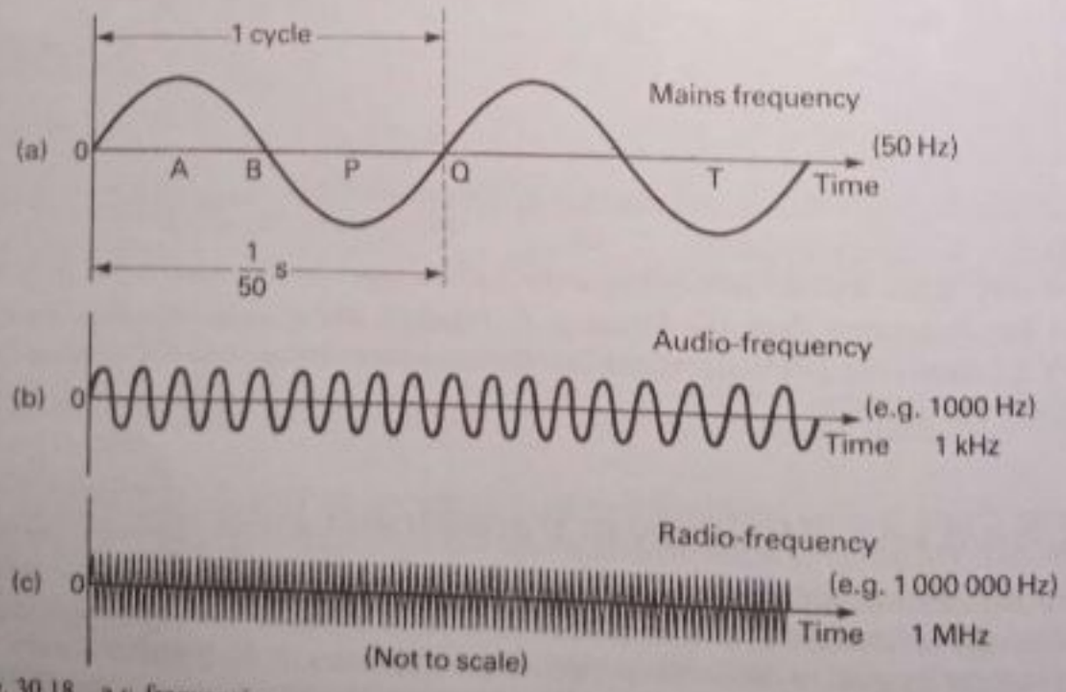


Fig. 30.18 a.c. frequencies

Scanned with CamScanner

This is why in Britain the national system for distributing electrical power, called the National Grid System, uses high 'tension' or high voltage. The generators at national power stations produce 11 000 V. This is stepped-up by large transformers to nearly 400 000 V or more for distribution over long distances. Pylons and a network of cables are used to transmit the power.

Finally, the reader should understand that alternating currents can be produced which have different frequencies from 50 Hz, the a.c. mains frequency. Fig. 30.18 illustrates (not to scale) an *audio-frequency* (a.f.) current of 1000 Hz produced in microphones and loudspeakers by speech or music, and a *radio-frequency* (r.f.) current of very high frequency of one million Hz or 10^6 Hz, produced at commercial broadcasting transmitters.

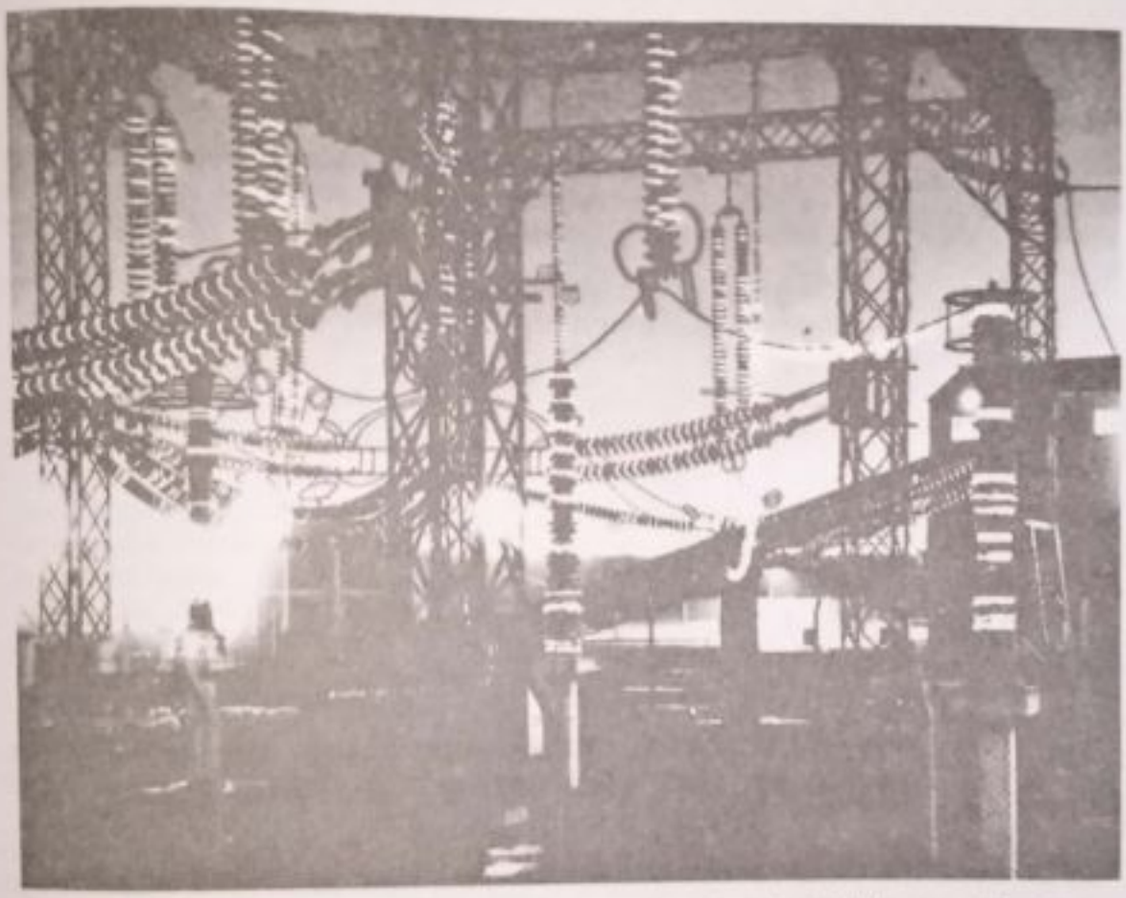


Plate 30C A night-time view of a large insulator testing station where the behaviour of insulator assemblies, such as those used in the National Grid, is observed (*Central Electricity Research Laboratories*)

Comparison of a.c. and d.c. effects

As we have seen, a direct current (d.c.) produces a heating and a magnetic effect. In the next chapter we shall also see that it produces a chemical effect; for example, copper is deposited on a metal plate inside copper sulphate solution when a direct current flows through this liquid.

When a ray-box light bulb is connected to the 12 V a.c. supply from a transformer the filament lights up. An alternating current in a wire therefore produces a heating effect. It does so because the drifting electrons in the wire collide with the stationary ions in the crystal lattice and give up their energy to the ions, whichever direction they flow along the wire. An alternating current can thus be measured by a hot-wire ammeter (page 430).

If an alternating current is passed into copper sulphate solution in a copper voltameter (page 496), the plates at the electrodes become alternately positive

Scanned with CamScanner

and negative. So any copper deposited on one plate while it is negative is removed when it is positive. Hence there is no overall chemical change with alternating current.

The magnetic field produced by an alternating current also alternates in direction as the current flows. If a moving-coil is used with low frequency alternating current, the pointer vibrates to-and-fro very rapidly; if the frequency is high, the reversals are too fast to be followed by the coil and the needle will not be deflected. Thus by itself the moving-coil meter cannot be used to measure a.c.

An electromagnet, however, will work when an alternating current is used. Iron or steel is attracted by it, whichever pole of the electromagnet becomes north when the current reverses. For this reason a moving-iron meter, described on page 462, will operate on a.c.

Examples on induced currents and transformers

1. (a) Two aluminium plates of similar mass, pivoted as shown in Fig. 30.19, are set swinging in turn between the pole pieces of a powerful horse-shoe magnet, so that the magnetic field acts perpendicularly to the plane of the plates. Describe and explain what happens in each case.

(b) The core of a transformer is made of thin layers of metal (laminations) insulated from one another. Explain why this is done. (L)

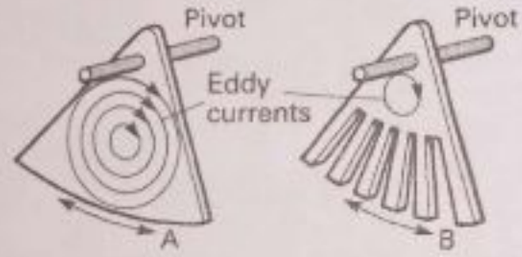


Fig. 30.19 . Example

(a) In plate A, induced or eddy currents circulate throughout the whole metal, as shown. These induced currents oppose the motion producing them, from Lenz's law. So the swings decrease rapidly and A soon comes to rest.

In plate B, however, induced currents cannot flow across the air between the slots cut into the metal. So the eddy currents produced are much less than in A. B therefore swings for a fairly long time compared with A.

(b) The core of a transformer is made of thin iron sheets insulated from one another to reduce the eddy currents flowing in the metal. (This is similar to the explanation given in (a) for B.) These eddy currents produce heat. So the power lost as heat in the core by the eddy currents is reduced.

2. Fig. 30.20a represents a transformer with a primary coil of 400 turns and a secondary coil of 200 turns.

(a) If the primary coil is connected to the 240 V a.c. mains what will be the secondary voltage?

(b) Explain carefully how the transformer works.

(c) Calculate the efficiency of the transformer if the primary current is 3 A and the secondary current 5 A.

(d) Give reasons why you would expect this efficiency to be less than 100%.

(e) The secondary coil is removed and a small coil connected to a low voltage lamp, as shown (Fig. 30.20b). Explain the following observations: (i) the lamp lights, (ii) if the coil is moved upwards, the lamp gets dimmer, (iii) if a soft iron rod is now placed through the coil, the lamp brightens again, (iv) the lamp will not light if a d.c. supply is used instead of an a.c. one. (L)

(a) Let E_s represent the voltage in the secondary coil and N_s the number of turns in that coil.

Scanned with CamScanner

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = \frac{200}{400} = \frac{1}{2}$$

$$\therefore\ E_s = \frac{1}{2} \times E_p = \frac{1}{2} \times 240 = 120\ \text{V a.c.}$$

(b) The transformer works by electromagnetic induction; the magnetic flux from the primary coil in the iron core links the secondary coil. See page 489.

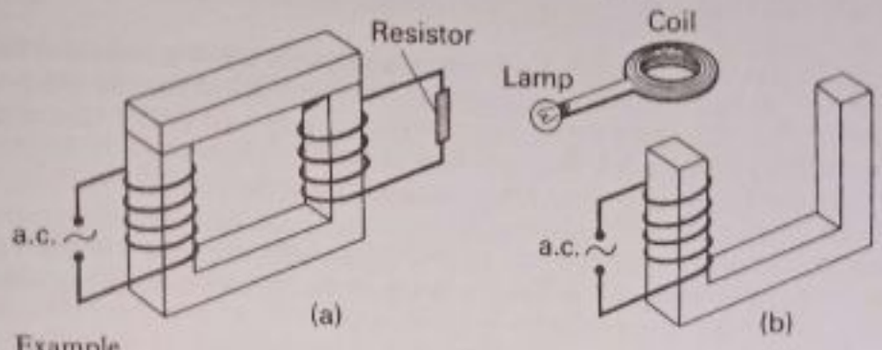


Fig. 30.20 Example

(c) Power output in secondary = current × voltage = 5 A × 120 V = 600 W

Power input to primary = 3 A × 240 V = 720 W

$$\therefore\ \text{efficiency} = \frac{\text{power output}}{\text{power input}} = \frac{600\ \text{W}}{720\ \text{W}} \times 100\,\% = 83\,\%$$

(d) The efficiency is less than 100 % because some power from the primary is lost as heat in the coils' resistance or as heat in the iron core due to eddy currents.

(e) (i) The lamp lights because some of the magnetic flux from the soft iron core links or passes through the coil above it. The flux changes continuously since the coil round the core carries alternating current and so an alternating e.m.f. is induced in the coil and the lamp then lights.

(ii) If the coil is moved upwards, the amount of flux linking the coil gets less. The lamp becomes dimmer because the induced e.m.f. is now less.

(iii) If a soft iron rod is placed through the coil, more flux now links the coil as the iron becomes magnetised and provides added flux. So the induced e.m.f. increases and the lamp becomes brighter.

(iv) If a d.c. supply is used, the current in the primary coil is now constant. The flux through the coil connected to the lamp is therefore constant, and as e.m.f. is only induced when there is a change in magnetic flux, there is no e.m.f. to provide current in the coil and lamp.

Summary

1. Induced e.m.f. is produced in a circuit whenever there is a change in the magnetic flux linking the circuit. Induced e.m.f. is produced in a straight conductor when it 'cuts' magnetic flux.
2. The magnitude of the induced e.m.f. is proportional to the rate of change of flux (Faraday's law). The direction of the induced current is such as to oppose the motion producing it (Lenz's law).
3. To produce alternating current (a.c.), a coil is rotated steadily in a magnetic field and slip-rings and brushes are used. To obtain direct current (d.c.), a split-ring commutator is substituted for the slip-rings.
4. In a commercial transformer, the alternating voltage (p.d.) to be changed is applied to the primary coil, wound round the soft iron laminated core. The output voltage is obtained from the secondary coil wound round the core.
5. Alternating current can be measured by a hot-wire or moving-iron meter but not by a moving-coil meter on its own. Alternating current produces a heating and an alternating magnetic effect but no electrolysis.

Scanned with CamScanner

Exercise 30 (Answers, page 643)

1. (a) A flat circular coil of wire, joined to a galvanometer, is placed between the N and S poles of a strong magnet so that its plane is parallel to the faces of the poles.
(i) Why is there a momentary galvanometer deflection when this is done? (ii) If the coil is held stationary in this position, what deflection, if any, occurs in the galvanometer? (iii) When the coil is turned over, why is a deflection obtained?

(b) A transformer has 800 turns in the primary winding and 40 turns in the secondary winding. The alternating e.m.f. connected to the primary is 240 V and the current flowing is 0·2 A.

Find (i) the secondary e.m.f. (ii) the current flowing in the secondary if there are no power losses (iii) the power in the secondary if this is 90% of that in the primary (iv) the e.m.f. across the secondary when the alternating e.m.f. of 240 V connected to the primary is replaced by a steady e.m.f. of 240 V.

2. Describe an electrical method of magnetising a steel bar.

The north pole of a long bar magnet is pushed into a solenoid which is connected in series with a centre-zero galvanometer. Briefly describe and explain in each case what is observed on the galvanometer when the magnet (a) *slowly* enters the solenoid (b) remains stationary inside (c) is *quickly* withdrawn.

State the two laws on which your explanation is based. (N)

3. Explain why alternating current cannot be used for electroplating by electrolysis and why it is capable of being used for transforming voltages by means of a transformer. (L)

4. A coil is rotated between the poles of a powerful horse-shoe magnet so as to produce an *alternating current* in an external resistor.

Draw a diagram in each case to show how (a) the coil is positioned in relation to the poles of the magnet, (b) electrical contact is made between the ends of the coil and the external resistor, (c) the current varies with time as the coil rotates.

Describe, or show on a diagram, the position of the coil in relation to the magnetic field when the current is at a maximum and explain why this is so. (L)

5. (a) Draw a labelled diagram of a simple alternating current generator. The output of an alternating current generator is connected to the primary coil of a simple transformer. Briefly describe the transformations which occur to the mechanical energy which is provided to drive the generator when a electric light bulb is connected across the secondary coil of the transformer (a detailed description of the action of each component in the circuit is not required).

(b) What is the power wasted when 10 kilowatts at 200 V is supplied through a cable of total resistance 0·5 ohm?

What would be the effect on the amount of power wasted if the applied voltage was increased and the same power was transmitted through the same cable? (N)

6. State *Lenz's law*.

The ends of a solenoid are connected together by a wire. Give diagrams showing the direction of the current (if any) induced in the solenoid when the N pole of a magnet is (a) thrust into the solenoid; (b) at rest in the solenoid; (c) rapidly withdrawn.

A copper disc is rotated by an electric motor. Explain why the disc comes to rest more quickly when the current is cut off if the disc rotates between the poles of a horse-shoe magnet. (L)

7. (a) Describe with a diagram, an experiment to induce a current in a coil of wire and indicate the direction of the current you would expect to obtain.

(b) With the aid of a labelled diagram describe and explain the action of a transformer. (N)

8. A rectangular coil of wire, attached to a commutator, rotates between the poles of a permanent magnet to form a dynamo.

(a) What is the position of the coil when the e.m.f. is greatest?

(b) What is the function of the commutator? (N)

9. State the essential condition for the production of an induced e.m.f. in a conductor. What determines its magnitude?

Give an account, with a diagram, of the structure and mode of action of a transformer which supplies 12 V when connected to 240 V mains. If this transformer takes 0·55 A from the mains when used to light five 12 V, 24 W lamps in parallel, find (a) its efficiency; (b) the cost of using it for 10 h, at 6p per kW h. (L)

10. Describe the structure and mode of action of a simple d.c. generator.

What alterations would you make in the simple generator in order to convert it into one of higher e.m.f.?

Explain the source of the electrical energy supplied by the generator. (N)

Scanned with CamScanner

11. Draw a labelled diagram of a simple step-up transformer.
What would be the effect of passing alternating current through acidulated water in a water voltameter? (N)

12. Draw a labelled diagram of an ammeter which could be used to measure alternating current. (N)

13. State the two laws of electromagnetic induction.
A transformer is designed to work from 240 V a.c. mains and to give a supply at 8 V to ring house bells. The primary coil has 4800 turns.
(a) Would you expect the secondary coil to be of thicker or of thinner wire than the primary?
(b) About how many turns would you expect it to have?
(c) Why is the iron core made of laminations (or sheets) of iron instead of being in one solid piece?
(d) What would happen if the transformer were connected to 240 V d.c. mains?
(e) Do you think the primary current will increase or decrease when a bell is being rung?
Give reasons for your answers. (O and C)

14. An X-ray tube is supplied with current from the secondary of a transformer working off 240 V a.c. mains. The voltage required across the tube is 96 kV. Assuming that the transformer is 100% efficient and that the X-ray tube behaves as a simple resistor, what ratio of number of turns in the secondary/number of turns in the primary is needed in its windings?
When the current in the X-ray tube is 100 mA, what current will be passing through the primary windings of the transformer? (O and C)

Multiple-Choice Questions

15. In Fig. 30.21 (i), a coil X is moved quickly away from the end Y of a stationary metal bar and a current then flows in X as shown. Then
A the bar is not a magnet
B Y is a north pole
C the current in X is constant
D Y is a south pole
E Y repels the coil X

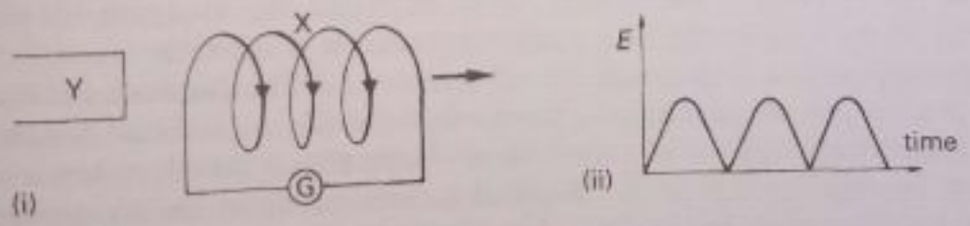


Fig. 30.21

16. When a coil rotates in a magnetic field, the induced e.m.f. E at the brushes varies as shown in Fig. 30.21 (ii). Then
A the ends of the coil are each joined to a ring
B the speed of rotation varies
C the magnetic field has changed steadily
D the magnetic field becomes zero twice per revolution
E the ends of the coil are connected to a split-ring commutator

17. A dry battery of 6 V is joined to the primary coil of a commercial transformer. The secondary has twice the number of turns of the primary. The primary resistance is 2 Ω.
The voltage obtained at the secondary is
A 0 V B 1·5 V C 3 V D 6 V E 12 V

Scanned with CamScanner

31. Chemical Effect of a Current

The flow of electricity through liquids was first studied by Faraday in 1834. By using a small light bulb in series with the liquid and a suitable battery, we find that dilute sulphuric acid solution or a dilute solution of copper sulphate or common salt are good conductors. Liquids such as solutions of acids, bases and salts are generally good conductors of electricity. Pure water is a very poor conductor of electricity. Tap water conducts better owing to the dissolved salts in it. Liquids such as paraffin or benzene are insulators.

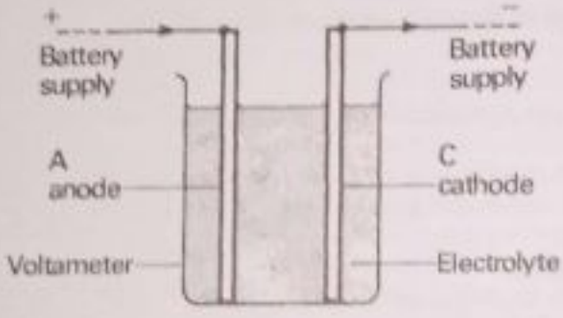


Fig. 31.1 A voltameter

Fig. 31.1 shows a voltameter, an arrangement for studying the flow of electricity through liquids. The conducting solution is called the *electrolyte*. The plates dipping into the solution are called *electrodes*. The electrode connected to the positive pole of the battery is known as the *anode*, A, and that connected to the negative pole is known as the *cathode*, C. As we see shortly, when a current flows through the electrolyte chemical actions occur at the electrodes. The chemical change in a liquid due to flow of current is called *electrolysis*.

Examples of electrolysis

1. *Electrolysis of copper sulphate solution with copper electrodes* If an electric current of about $\frac{1}{2}$ A is passed through a dilute solution of copper sulphate with copper electrodes for about 30 min, a bright fresh deposit of copper is formed on the cathode. When it is weighed, the mass of the cathode is found to have increased by about 0·3 g. But the anode looks dull in appearance after removal from the solution and on weighing is found to have decreased by about 0·3 g in mass. The density of the copper sulphate solution is found to be unchanged however long the current flows.

2. *Electrolysis of acidified water* Pure water is a poor conductor of electricity. If a little sulphuric acid is added it conducts much better. To investigate the electrolysis of water, the acidified water can be poured into a *Hoffmann voltameter*. This is a glass apparatus consisting of inverted graduated burettes

Scanned with CamScanner

with a platinum electrode at the bottom (Fig. 31.2). When a battery is connected and the circuit switched on, gases are formed above the anode A and cathode C. On testing, oxygen is found above A and hydrogen above C. The ratio of the volumes is exactly 1:2. The solution is found to be slightly more concentrated, showing that water has been lost by electrolysis and hydrogen and oxygen formed from it.

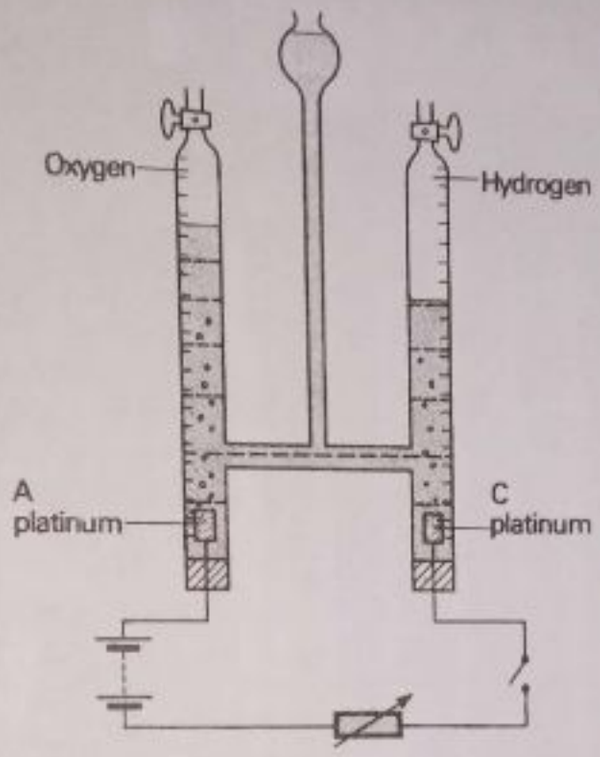


Fig. 31.2 Electrolysis of water in a Hoffmann voltameter

From the electrolysis of copper sulphate solution and acidified water, we see that *the metal or hydrogen is always deposited at the cathode.*

This general rule applies to all cases of electrolysis and should be memorised. So if silver nitrate solution is electrolysed, silver appears at the cathode. If dilute hydrochloric acid is electrolysed, hydrogen appears at the cathode.

The electrode materials affect the final products at the electrodes. For example, if copper sulphate solution is electrolysed using platinum electrodes, copper is again deposited at the cathode, but oxygen is produced at the anode.

Applications · Electrolytic capacitors

Electrolysis has many different applications.

Pure copper, which must be used in the manufacture of cables, is produced commercially by electrolysis of solutions of copper compounds, which are in large tanks. Cathode plates, dipping into the electrolyte, are raised after a time, and the pure copper deposited is stripped from the cathode. Sodium and aluminium are manufactured commercially by electrolysis.

Silver plating of objects such as spoons is carried out by making them the cathode of a voltameter containing solutions of silver compounds and passing a current through the solution. Steel objects, such as the bumpers of cars or the parts of a bicycle, are often protected by chromium-plating, which gives them a bright appearance and prevents rusting. The handlebars of a bicycle, for example, are made the cathode of a voltameter containing a solution of suitable

Scanned with CamScanner

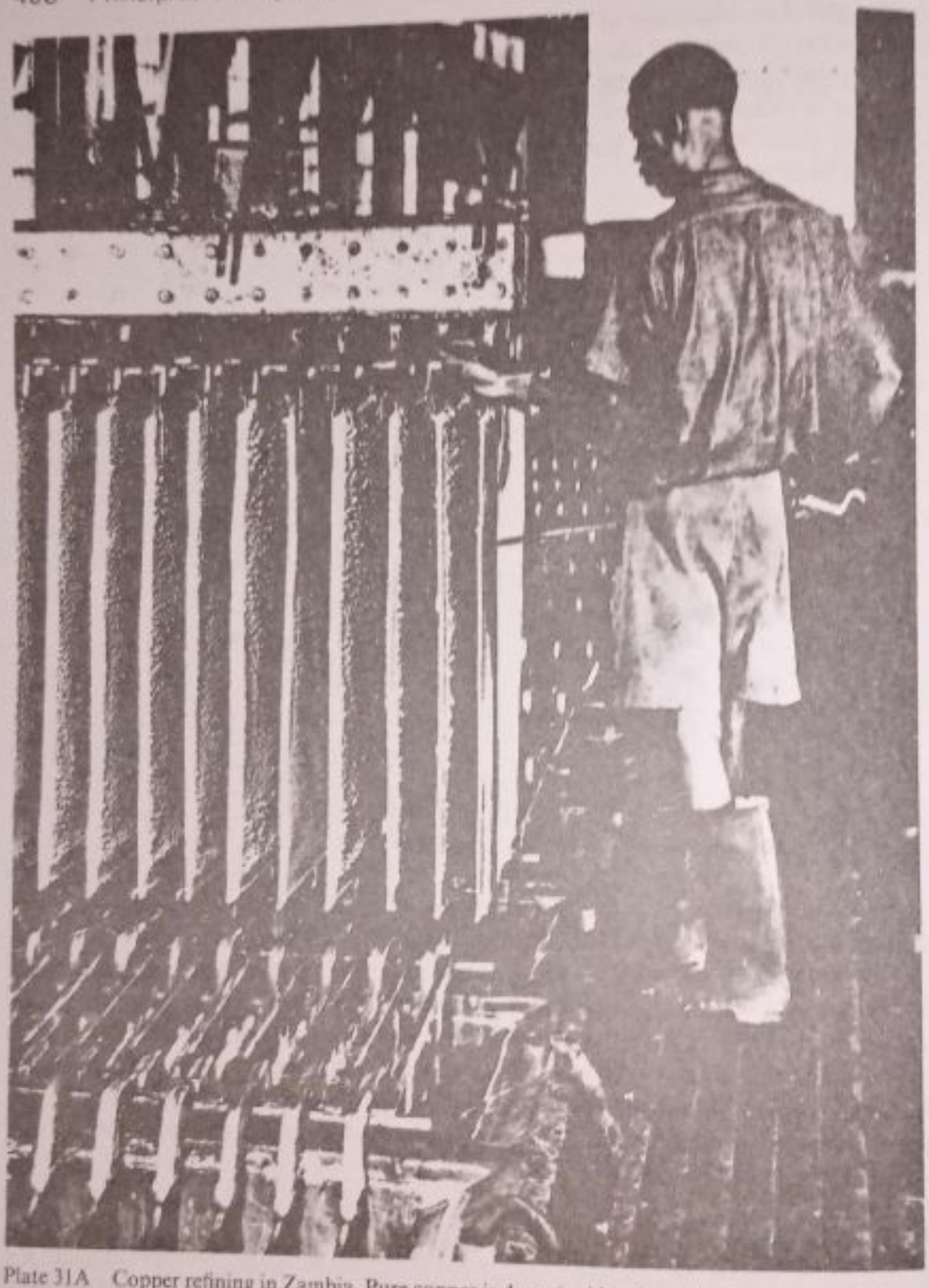


Plate 31A Copper refining in Zambia. Pure copper is deposited by electrolysis on large cathode plates, shown lifted out of the solutions of copper compounds. The pure copper is stripped from the cathodes. (*Copper Development Association*)

compounds and a current is passed through the solution until the required chromium deposit is obtained on the frame.

Electrolytic capacitors are widely used in radio receivers. This type of capacitor is made by the electrolysis of ammonium borate, for example, between aluminium electrodes (Fig. 31.3). A very thin film of aluminium oxide is formed on the anode plate. The oxide is an insulating medium or dielectric, and is so thin that the capacitance of the capacitor is extremely high (see page 526). When the capacitor is used in a transistor circuit, for example, care must be taken to

Scanned with CamScanner

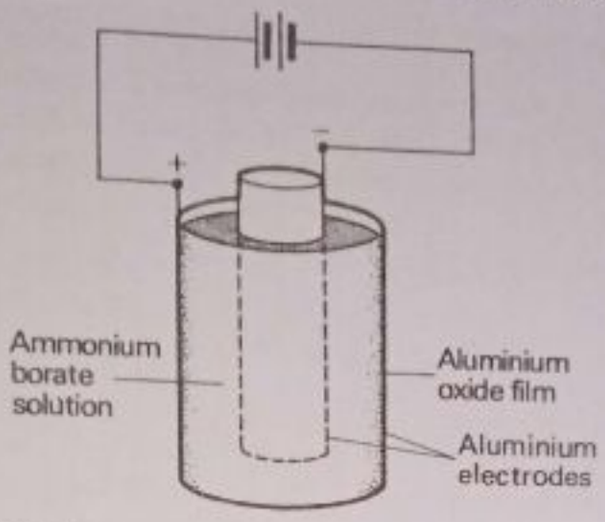


Fig. 31.3 Electrolytic capacitor

connect the terminals from the anode aluminium plate to the positive side of the circuit; otherwise the oxide film on the cathode will break down by reversal of the electrolysis action. The positive terminal of the capacitor is therefore clearly marked on it by the manufacturers. See Fig. 32.27.

Investigation of electrolysis

We have already seen that a metal or hydrogen is deposited on the cathode during electrolysis. Faraday carried out many experiments to find how the mass of the element deposited varied with the size of the current and the time.

Experiment

A quick though approximate investigation can be made with a Hoffmann voltameter filled with acidified water as shown in Fig. 31.2. A rheostat, ammeter and a suitable battery are included in the circuit. The element collected at the cathode is hydrogen, whose density varies with pressure. The pressure change during the experiment, shown by the difference in water levels, is small. So we assume that, approximately, the density of hydrogen is constant. In this case the *mass* of hydrogen collected is proportional to the volume, which we can read off.

1. *Constant current* The current is kept constant at a convenient value such 0·5 A by means of the rheostat and ammeter. The volumes V of hydrogen produced at say five suitable equal intervals of time t are measured. The ratio V/t is found to be practically constant; or if a graph of V against t is plotted, it is found to be a straight line passing through the origin.

So we conclude that, for a constant current in a voltameter

$$\text{mass deposited} \propto \text{time}$$

2. *Constant time* This time different currents are passed into the voltameter for the same time on each occasion, for example for 2 min. The volume of hydrogen produced is recorded. The results show that the ratio volume/current is practically constant. Assuming the volume of hydrogen is practically proportional to its mass, we conclude that, for a constant time

$$\text{mass deposited} \propto \text{current}$$

Scanned with CamScanner

Faraday's first law

The two results of the experiment outlined can now be combined into one general law. It is known as *Faraday's first law of electrolysis. The mass of an element deposited by electrolysis in a given voltameter is directly proportional to the current and to the time.*

So if m is the mass, I the current and t the time,

$$m \propto It$$

or

$$m = zIt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where z is a constant (discussed soon).

The product '$I \times t$' or 'current × time' is the quantity of electricity Q which has passed through the voltameter, since 1 coulomb is the quantity which has passed in 1 second when the current is 1 ampere (see page 390). So the result in (1) is better stated as

$$m = zQ \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Measurement of mass deposited per coulomb

From $m = zQ$, it follows that $z = m/Q$, or z is

the mass of an element deposited by 1 coulomb,

or

the mass deposited by 1 ampere in 1 second.

The unit of z is kilogram per coulomb (kg/C).

Experiment

The mass deposited per coulomb of an element such as copper can be measured by using two copper plates as electrodes with copper sulphate solution in a beaker as the electrolyte (Fig. 31.4). An ammeter, a rheostat, accumulators and a key K are connected in series in the circuit.

The current must not be too high, otherwise copper will not be deposited firmly on the cathode and will flake off, nor too low, otherwise the mass of copper deposited in a reasonable time will be too small to be measured accurately. Another plate, a 'dummy' cathode, is therefore placed in the beaker first and the rheostat then varied until a suitable current, 0·8 A, for example, is obtained.

The actual copper cathode is cleaned with emery paper, washed and dried, and

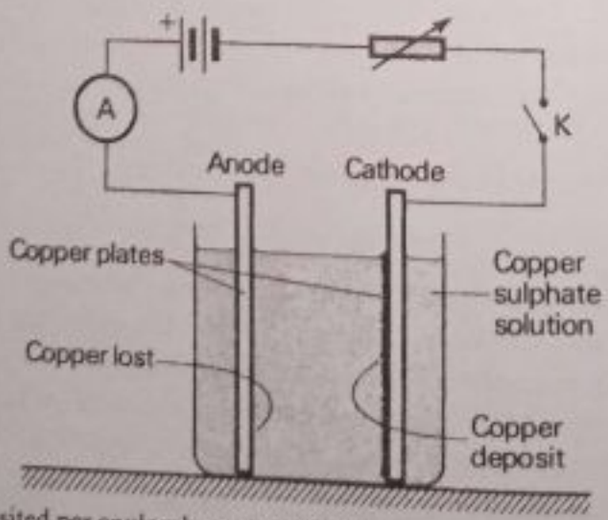


Fig. 31.4 Mass deposited per coulomb–copper voltameter

Scanned with CamScanner

weighed. It is then placed in the beaker and the current is switched on simultaneously. A constant current is now passed for a suitable time such as half an hour. The cathode is removed, washed gently with distilled water and carefully dried by holding it above a low flame. The plate must not be placed too near the flame, otherwise the copper will become oxidised. The cathode is now reweighed.

Measurements: Typical measurements are
Mass of cathode initially = 46·342 g
Mass of cathode + copper deposit = 46·822 g
Current = 0·80 A
Time = 30 min = 1800 s

Calculation: Mass of copper deposited, $m = 0{\cdot}480\,\text{g} = 0{\cdot}480 \times 10^{-3}\,\text{kg}$

From
$$m = zQ = zIt,$$
$$z = \frac{m}{I \times t} = \frac{0{\cdot}480 \times 10^{-3}}{0{\cdot}8 \times 1800}$$
$$= 3{\cdot}3 \times 10^{-7}\,\text{kg/C}$$

Similar experiments show that the mass deposited per coulomb for hydrogen, the lightest element, is $1{\cdot}04 \times 10^{-8}$ kg/C, and that for silver, a heavy element, is $1{\cdot}118 \times 10^{-6}$ kg/C. The hydrogen ion carries the same quantity of electricity or charge as the silver ion, but it is very much lighter. So the mass of hydrogen deposited per coulomb is much smaller than that of silver.

Calibration of an ammeter

Experiment

The experiment we have just described can be used to check a particular reading on the scale of an ammeter A (Fig. 31.5). The meter is used in place of the ammeter in the circuit of Fig. 31.4. The beaker is filled with copper sulphate solution, and using a dummy cathode the reading on A is first set to the required current value by means of the rheostat. After a suitable measured time the mass deposited by the electrolysis is found as before.

Calculation: As an illustration, suppose the instrument reading was kept constant at 1·20 A, and 0·982 g or $9{\cdot}82 \times 10^{-4}$ kg of copper was deposited in 40 min. If $z = 3{\cdot}3 \times 10^{-7}$ kg/C for copper, the true current I is then given by

$$m = zQ = zIt$$

So
$$I = \frac{m}{zt} = \frac{9{\cdot}82 \times 10^{-4}}{3{\cdot}3 \times 10^{-7} \times 40 \times 60}$$
$$= 1{\cdot}24\,\text{A}$$

Results: So the correction to be applied to the ammeter reading is 1·24 − 1·20 or 0·04 A.

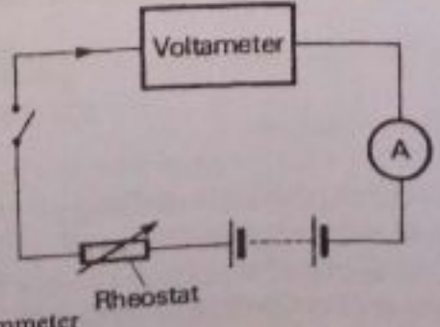


Fig. 31.5 Calibration of an ammeter

Scanned with CamScanner

The measurement of current by electrolysis can be carried out so accurately that this was used in the past to define the ampere. Nowadays, however, the ampere is defined and measured by the force between current-carrying wires (page 473).

Example on electrolysis

A steady current of 1·5 A flows through a copper voltameter. Find the thickness of the layer of copper deposited uniformly on the electrode of surface area of 50 cm^2 in 20 min. (Density of copper = 9000 kg/m^3; mass of copper deposited per coulomb = $3{\cdot}3 \times 10^{-7}$ kg/C.)

If m is the mass of copper deposited, then

$$m = zQ = zIt$$
$$= 3{\cdot}3 \times 10^{-7} \times 1{\cdot}5 \times 20 \times 60 = 5{\cdot}94 \times 10^{-4}\ \text{kg}$$

From $$\text{volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{volume of copper deposited} = \frac{5{\cdot}94 \times 10^{-4}}{9000} = 6{\cdot}6 \times 10^{-8}\ \text{m}^3$$

$$\therefore\ \text{thickness} = \frac{\text{volume}}{\text{area}} = \frac{6{\cdot}6 \times 10^{-8}\ \text{m}^3}{50 \times 10^{-4}\ \text{m}^2} = 1{\cdot}3 \times 10^{-5}\ \text{m}$$

Faraday's second law of electrolysis

Faraday's first law of electrolysis refers to the mass of an element deposited in a given voltameter. If an investigation is required into the masses of different elements deposited in *different* voltameters, a circuit can be set up containing a copper voltameter, a silver voltameter and a water voltameter, a rheostat and an ammeter in series with a battery (Fig. 31.6). A suitable current can now be passed for a convenient time, after which the masses deposited can be measured.

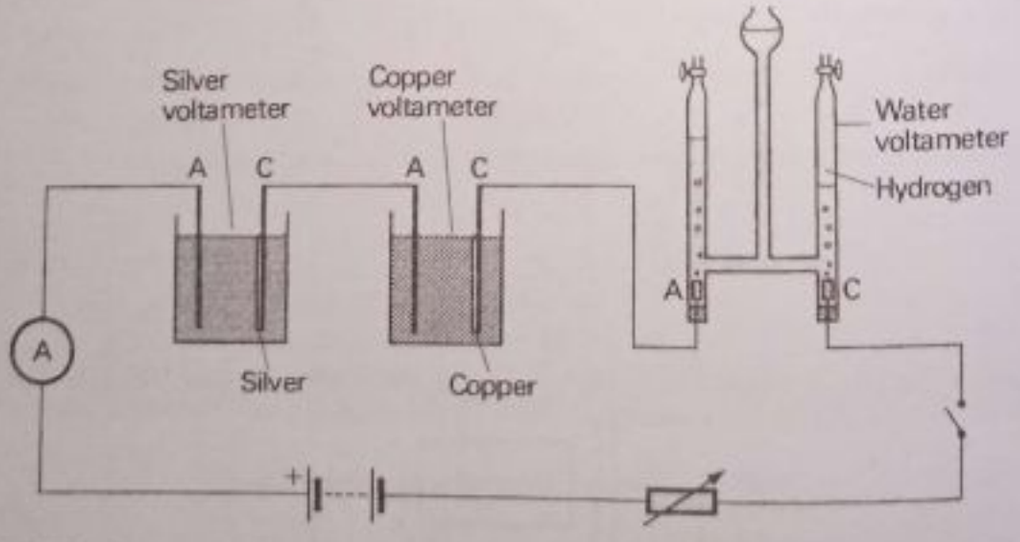


Fig. 31.6 Faraday's second law of electrolysis

Results: Mass m_1 of hydrogen collected = 0·020 g
Mass m_2 of copper deposited = 0·64 g
Mass m_3 of silver deposited = 2·18 g

Calculation: $m_1 : m_2 : m_3 = 0{\cdot}02 : 0{\cdot}64 : 2{\cdot}18 = 1 : 32 : 109$

Scanned with CamScanner

Conclusion: Now the ratio relative atomic mass/valency for hydrogen, copper and silver = 1:31·5:108. So we conclude that the masses deposited are proportional to the ratio relative atomic mass/valency.

Note carefully that because the same current flowed through each voltameter for the same time, the same quantity of charge passed through each voltameter. Faraday's second law of electrolysis can therefore be expressed as follows: *when the same quantity of charge passes through different electrolytes the masses of the elements deposited are proportional to their ratios of relative atomic mass/valency.*

Example on Faraday's second law

A copper and water voltameter are in series, and at the end of a period of time 3·0 g of copper is deposited. Calculate the mass of oxygen deposited.

Since the same quantity of electricity passes through each voltameter

$$\frac{m}{3{\cdot}0} = \frac{\text{Relative atomic mass/valency for oxygen}}{\text{Relative atomic mass/valency for copper}} \quad \text{where } m \text{ is the mass of oxygen.}$$

$$\therefore \frac{m}{3{\cdot}0} = \frac{16/2}{63/2} = \frac{8}{31{\cdot}5}$$

$$\therefore m = \frac{8 \times 3}{31{\cdot}5} = 0{\cdot}8 \text{ g (approx.)}$$

Faraday constant

From Faraday's second law, it follows that the same quantity of electricity or charge passing through different voltameters liberates the mass of one mole for a monovalent element such as silver and the mass of half a mole for a divalent element such as copper, Cu II.

Now on page 501 we saw that 0·001 118 g of silver is deposited by 1 coulomb. The mass of one mole of silver is 108 g; so this is liberated by 108/0·001 118 or 96 500 coulombs approximately. Also 31·5 g is about the mass of half a mole of copper, Cu II. Since the mass of copper deposited per coulomb is 33×10^{-5} g, 31·5 g of copper is deposited by $31{\cdot}5/(33 \times 10^{-5})$ or about 96 500 coulombs.

The quantity of charge which liberates one mole of a monovalent element is called the Faraday constant, symbol F.

$$\text{Faraday constant, } F = 96\,500 \text{ C/mol (approx.)}$$

From the value F we can easily calculate, for example, the mass of silver deposited when a constant current of 2 A flows for 30 min through a silver voltameter. Here the quantity of charge which flows $= I \times t$ $= 2(\text{A}) \times 30 \times 60(\text{s}) = 3600$ C. Now 96 500 C liberates 108 g of silver.

$$\text{So mass of silver deposited} = \frac{3600}{96\,500} \times 108 = 4 \text{ g (approx.)}$$

Ions and ionic crystals

An explanation of electrolysis was suggested by a Swedish chemist Arrhenius about 1890. He considered that a dilute solution of sodium chloride, for example, contained particles carrying charges, which he called *ions*. We now know that an ion is an atom or group of atoms which have lost or gained one or more electrons, so that ions carry positive (+ve) or negative (−ve) charges.

Scanned with CamScanner

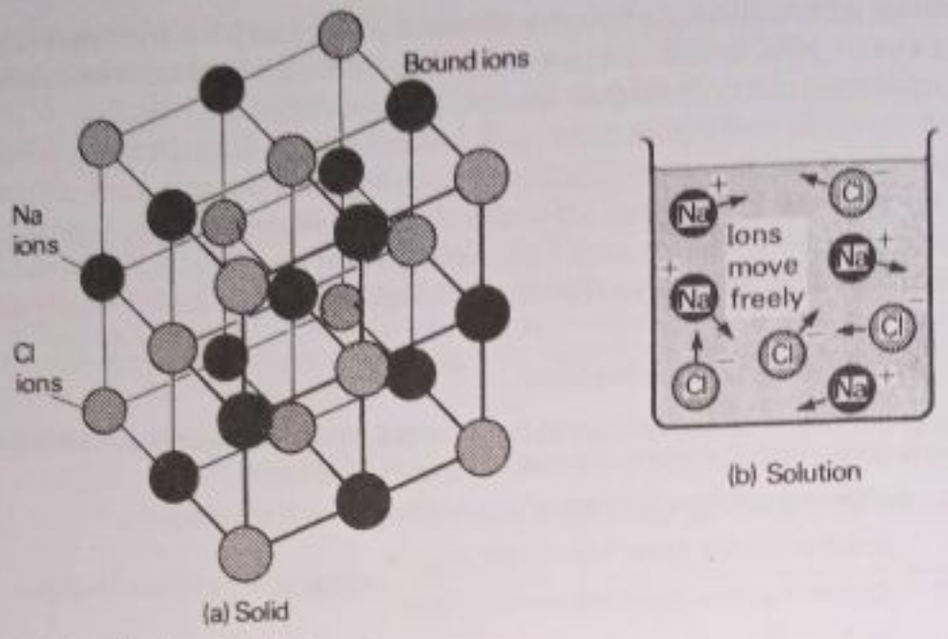


Fig. 31.7 Ions of sodium chloride

Later researches into the structure of solids such as common salt show they have a crystal structure, with ions (not atoms) at the corners of a regular three-dimensional pattern or lattice (Fig. 31.7a). In sodium chloride (NaCl), for example, +ve sodium ions (Na^+) and −ve chloride ions (Cl^-) are at the corners of a cubic structure. They are held together strongly to form a solid by the attractive force between positive and negative charges (page 510). The sodium atom 'donates' an electron to the chlorine atom when both come together to form sodium chloride, so that a chloride ion has a charge $-e$ and a sodium ion has a charge $+e$, where e is the numerical value of the electronic charge.

So $$NaCl \rightarrow Na^+ + Cl^-$$

Solid copper (II) sulphate ($CuSO_4$) crystals have +ve copper ions and −ve sulphate ions at the corners of a three-dimensional pattern, held strongly together to form a solid. This time, however, two electrons are donated by the copper atom to the sulphate group of atoms. So the +ve copper ion has a charge of $+2e$, and this is written Cu^{2+}. The sulphate ion has a charge $-2e$ and this is written $SO_4{}^{2-}$. Sulphuric acid (H_2SO_4) has two +ve hydrogen ions ($2H^+$) and a sulphate ion ($SO_4{}^{2-}$).

Water (H_2O) is a poor conductor. One molecule in ten million is dissociated into a +ve hydrogen ion (H^+) and a −ve hydroxide ion (OH^-).

So $$H_2O \rightleftharpoons H^+ + OH^-$$

When water is added to solid common salt or solid copper (II) sulphate to form a solution, the water diminishes the force of attraction between the ions to about an eightieth of that when only air was present. So the ions drift apart (Fig. 31.7b), and the solid structure disappears.

Movement of ions

In an electrolyte, the +ve and −ve ions in solution have random movements. There is no drift of ions in any particular direction until a battery is connected to the electrodes of a voltameter. The +ve ions are then attracted towards the cathode and the −ve ions towards the anode. So the +ve and −ve ions drift

Scanned with CamScanner

through the solution in opposite directions. This movement of charges is the electric current flowing through the electrolyte. When the battery is disconnected from the electrodes there is no ion drift and the movement of the ions becomes random as before.

Explanation of electrolysis

The ion drift just discussed explains what happens when a current passes through copper sulphate solution (page 496).

1. *Copper electrodes* Inside the solution are +ve copper (Cu^{2+}) ions, +ve hydrogen (H^+) ions, −ve hydroxide (OH^-) ions (from the water) and −ve sulphate (SO_4^{2-}) ions. When the current flows, the +ve ions drift towards the cathode and the −ve ions towards the anode, because of the attraction of unlike charges. At the cathode, the +ve copper ions are discharged more easily than the hydrogen ions. So *copper* is deposited at the cathode. The copper ion is discharged by taking two electrons from the cathode.

$$Cu^{2+} + 2e^- \rightarrow Cu$$

At the anode, copper is released into the solution from this metal and so the OH^- and SO_4^{2-} ions are not discharged.

$$Cu - 2e^- \rightarrow Cu^{2+}$$

The overall result is the flow of copper from anode to cathode and a flow of electrons round the external circuit. So there is no total gain or loss of copper from the solution.

2. *Platinum electrodes* In this case, still with copper (II) sulphate solution, OH^- ions are discharged more easily at the anode than the SO_4^{2-} ions. Platinum is not released into solution. On discharge, the OH^- ions react to form oxygen and electrons flow round the external circuit.

$$4OH^- \rightarrow 2H_2O + O_2 + 4e^-$$

The net result is a deposit of copper on the cathode, oxygen is produced at the anode and the concentration of the copper (II) sulphate solution is reduced.

Electrolysis of water

When a battery is connected to platinum electrodes in acidified water, H^+ ions are discharged at the cathode.

$$2H^+ + 2e^- \rightarrow H_2$$

At the anode OH^- ions are discharged more easily than SO_4^{2-} ions, resulting again in the formation of oxygen.

$$4OH^- \rightarrow 2H_2O + O_2 + 4e^-$$

The net result is the decomposition of water into its elements hydrogen and oxygen (page 497).

Faraday's laws from ionic theory

Faraday's laws of electrolysis can now be explained.

The ions which carry the current have both charge and mass, and the mass of an ion is practically the same as the mass of its atom (page 503). So if 50 ions of

Scanned with CamScanner

copper, for example, reach the cathode at the end of a given time, 50 atoms of copper are deposited and 50 packets of charge have passed through the electrolyte. If 80 ions reach the cathode, 80 atoms are deposited and 80 packets of charge have passed through the electrolyte. So Faraday's first law, mass $\propto$ charge Q (page 500) follows from the ionic structure of crystals.

The second law, like the first one, is also explained by the fact that the carriers of the current are ions. Each ion carries charge proportional to its *valency*, which is the number of atoms of hydrogen it will combine with or replace in a compound. So if e is the numerical value of the charge on an electron the sodium ion carries a charge $+e$. A copper ion in copper sulphate solution carries a charge $+2e$. A hydrogen ion and a silver ion each carry a charge $+e$.

Suppose a charge of $4e$ flows through a water and copper sulphate voltameter in series. Then 4 hydrogen ions and 2 copper ions are deposited at the respective cathodes, and form 4 hydrogen and 2 copper atoms. If the mass of 4 hydrogen atoms is, say, 4 units, the mass of copper atoms is 2×63, or 126 units, since the relative atomic mass of copper, compared to hydrogen, is 63. So

$$\text{mass of hydrogen deposited : mass of copper deposited} = 4:126 = 1:31{\cdot}5$$

But the ratio relative atomic mass/valency is 1 for hydrogen and 31·5 for copper. So the mass deposited by a given quantity of electricity or charge is proportional to the ratio relative atomic mass/valency of the element concerned (page 503).

Example on electro-plating

(a) You have a solution of silver nitrate. Draw a circuit diagram showing how you would silver plate a piece of metal, using the silver nitrate and any other apparatus which you need.

(b) The piece of metal has an area of 20 cm^2 and it takes 62·5 minutes to deposit a layer of silver 0·1 mm thick. What current is needed to do this? (Assume that 1 cm^3 of silver has a mass of 10·5 g and that 1 coulomb of electricity deposits 0·001 12 g of silver.)

(c) Why is it better to use a small current for a long time rather than a large current for a short time to plate the metal with a given thickness of silver? (L)

(a) The circuit diagram is shown in Fig. 31.4. The metal to be silver plated is used as the *cathode* in the silver nitrate solution.

(b) Since 0·1 mm = 0·01 cm,

$$\text{volume of silver deposited} = 0{\cdot}01 \times 20 = 0{\cdot}2\ cm^3$$

$$\therefore \text{ mass of silver, } m = 0{\cdot}2 \times 10{\cdot}5 = 2{\cdot}1\ g$$

If z is the mass per coulomb of silver deposited, then

$$m = zIt$$

$$\therefore I = \frac{m}{zt} = \frac{2{\cdot}1}{0{\cdot}001\,12 \times 62{\cdot}5 \times 60}$$

$$= 0{\cdot}5\ A$$

(c) It is better to use a small current for a long time because then the silver layer is built up by adding small amounts at a time and these hold firmly together. With a larger current, larger masses are added quickly and these can flake off too easily.

Summary

1. Liquids containing +ve (e.g. metal or hydrogen) ions and −ve ions conduct electric current and are known as electrolytes.
2. In the electrolysis of copper sulphate solution with copper electrodes, copper

Scanned with CamScanner

is transferred from the anode to the cathode. With the platinum electrodes, copper is deposited at the cathode and oxygen is formed at the anode.

3. *Faraday's laws of electrolysis* (a) The mass of an element deposited or liberated in electrolysis is proportional to the quantity of charge. (b) When the same current passes through different electrolytes for the same time the mass of any element deposited is proportional to its ratio *relative atomic mass/valency*.

4. The mass m of an element deposited in a time t by a current I is zIt, where z is the mass deposited per coulomb.

Exercise 31 (Answers, page 643)

1. Which of the following are electrolytes: petrol, mercury, common salt solution, molten copper, dilute sulphuric acid? (N)

2. What is the part played by the *electrolyte* in electrolysis?

It is required to cover the surface of an object of total area 600 cm^2 with nickel of thickness 0·10 mm using a constant current of 5·0 A. Determine the time for which the current must flow if the mass of nickel deposited per coulomb is 0·00030 g/C (3×10^{-7} kg/C) and its density is 9·0 g/cm^3.

If a p.d. of 40 V is employed, determine the cost at 2p per kilowatt-hour. (L)

3. State Faraday's laws of electrolysis.

Describe an experiment to determine the mass of copper deposited per coulomb. State what precautions you would take to ensure that a reliable value is obtained.

A potential difference of 6 V is maintained between the plates of a copper voltameter and 0·66 g of copper is deposited. Calculate the electrical energy consumed by the voltameter. (Mass of copper deposited per coulomb = 0·00033 g/C = $3{\cdot}3 \times 10^{-7}$ kg/C.) (N)

4. Explain the terms *anode, cathode*.

Describe an experiment to check the accuracy of the 1 A reading of an ammeter, assuming that a value for the mass deposited per coulomb of copper is available.

What is the cost of depositing a layer of silver 0·2 mm thick on an object of total surface area 150 cm^2 if a current of 1 A used for an hour costs 2p? (Density of silver is 10·5 g per cm^3; mass of silver deposited per coulomb is 0·00112 g/C = $1{\cdot}12 \times 10^{-6}$ kg/C.) (L)

5. State the *laws of electrolysis* and define the terms *anode* and *cathode*.

Describe an experiment to determine how the mass of copper deposited in a copper voltameter varies with the quantity of electricity passing through it. Give a circuit diagram and full experimental details.

An electric current passes through two voltameters in series, containing copper sulphate and silver nitrate respectively. What is the mass of silver deposited in a given time if the mass of copper deposited in that time is 1 g? (Relative atomic mass of copper is 63, of silver is 108, valency of copper is 2, valency of silver is 1.) (L)

6. State Faraday's laws of electrolysis.

Describe an experiment to decompose water by electrolysis.

Calculate the current required to liberate 10 cm^3 of hydrogen per min in electrolysis. (1 litre of hydrogen weighs 0·09 g at the temperature and pressure at which the volume is measured in this case. The mass of hydrogen deposited per coulomb is 0·0000105 g/C = $1{\cdot}05 \times 10^{-8}$ kg/C.) (N)

7. Explain the terms *electrolysis, ionisation, electrochemical equivalent*.

State Faraday's laws of electrolysis and describe how you would attempt to test the second law experimentally.

Calculate the time for which a steady current of 0·2 A must pass through a water voltameter in order to liberate a quantity of hydrogen which would occupy a volume of 50 cm^3 at s.t.p.

(Take the mass of hydrogen deposited per coulomb to be 0·00001045 g/C and its density at s.t.p. to be 0·00009 g/cm^3.) (O)

8. Explain how the ionic theory accounts for Faraday's laws of electrolysis.

Give an account of the processes occurring when dilute sulphuric acid is electrolysed between platinum electrodes.

A cell with copper electrodes and containing copper sulphate solution is in series with one containing dilute sulphuric acid and platinum electrodes. When 3·175 g of copper have been liberated, what volume of hydrogen, measured at s.t.p. is released?

Scanned with CamScanner

(Relative atomic masses: copper, 63·5; hydrogen, 1·008. Valency of copper, 2. Density of hydrogen at s.t.p., 0·0899 g per litre.) (O and C)

9. State the laws of electrolysis and describe *two* experiments to illustrate these laws.

What are the chief differences between the passage of electricity through metals and through ionised solutions?

A steady current passes through a silver voltameter and a 10 Ω resistance arranged in series. A high-resistance voltmeter placed across the terminals of the 10 Ω coil reads 6 V. Given that 0·8048 g of silver is deposited on the cathode of the voltameter in 20 min, calculate the mass of silver deposited per coulomb. (O and C)

10. State the relationship between the mass of a substance liberated during electrolysis and the charge passed. Describe an experiment by which this relationship can be established for a metal such as copper.

An ammeter and a copper voltameter are placed in series in a circuit and after 20 minutes the mass of copper deposited is 0·36 g. The ammeter reads 0·8 A. What can you deduce about the ammeter reading? (Assume mass of copper deposited per coulomb = 0·0003 g/C = 3×10^{-7} kg/C.) (L)

Multiple-Choice Questions

11. When a metal X is copper-plated in electrolysis,
- A X is the cathode and alternating current is used
- B X is the anode and direct (steady) current is used
- C X is the cathode and direct current is used
- D X is the anode and should first be cleaned
- E X is the cathode and a very high current is used

12. In electrolysis, a current *I* flows for 20 minutes and deposits a mass *m* of silver on the cathode. A current 2*I* flowing for 5 minutes will deposit a mass of silver of

A $8m$ B $4m$ C $2m$ D m E $m/2$

13. Which statement in A to E is correct?
- A Current is carried by electrons through acids
- B Current is carried by ions through metals
- C In salt solutions, positive and negative ions move in opposite directions when current flows
- D No ions move through a lead–acid accumulator when it is connected to a wire
- E Ions are not present in sea-water

Scanned with CamScanner

32. Static Electricity · Capacitors

About 300 B.C. the Greeks knew that particles of wheat were attracted to amber necklaces. They called this effect 'electricity' after the word 'electron', which meant amber. Many substances were found to attract light particles after they were rubbed and they were then said to be *electrified* or *charged*, or to have a charge. Today you can see this power of attraction if you rub vigorously a plastic pen or rubber balloon on your sleeve, and bring it near to small pieces of paper. The paper is immediately attracted to the rubbed object.

Positive and negative charges

Experiment

Charge a small ebonite rod A by rubbing it with fur, and suspend it by insulating thread in a wire stirrup (Fig. 32.1a). Charge another ebonite rod B by rubbing it with fur and bring it near one end of the rod A. You will see that A is *repelled* by B.

Now charge a glass rod C by rubbing it with silk and bring it near one end of the charged rod A (Fig. 32.1b). A is *attracted* by the charge on C.

Conclusion: This simple experiment shows that the charge on the ebonite rod B is different from that on the glass rod C.

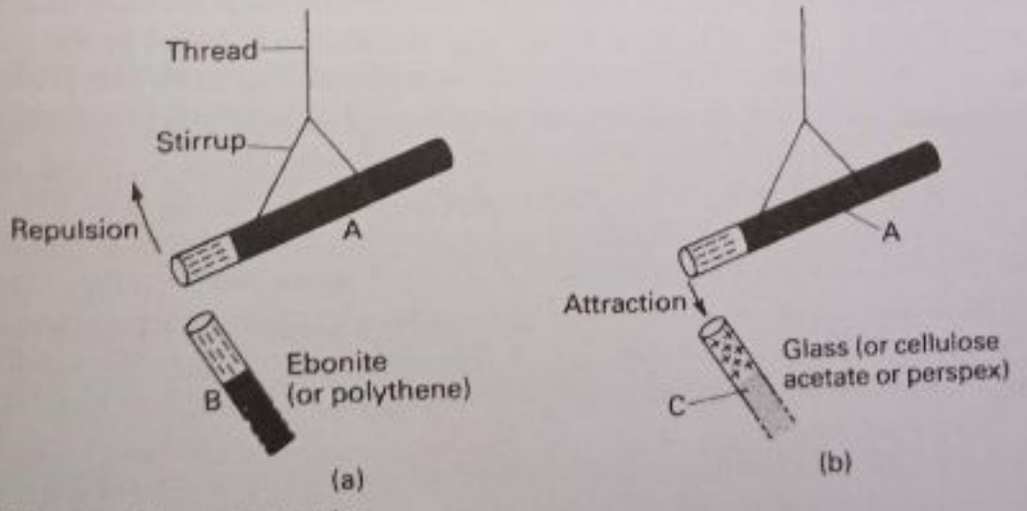


Fig. 32.1 Repulsion and attraction

Benjamin Franklin called the charge on ebonite rubbed with fur *negative electricity* or *negative charge*, and that on the glass rod rubbed with silk *positive electricity* or *positive charge*. All charges fall into one of these two classes. The kind of charge on a rubbed substance depends on the rubbing material.

Scanned with CamScanner

Today, it is convenient to use a polythene rod in place of ebonite, and a cellulose acetate strip in place of glass, as we soon discuss.

From the above experiment a general rule can be stated:

Like (similar) charges repel, unlike (opposite) charges attract.

Thus two positive charges repel each other and two negative charges repel each other. But a positive and a negative charge attract each other.

Testing charges

Since a rubbed polythene rod is repelled by a negatively-charged ebonite rod, the polythene rod must have a negative charge. A rubbed cellulose acetate or perspex rod is repelled by a glass rod rubbed with silk. So the acetate and perspex rods must have positive charges.

An uncharged object, such as the hand, will attract a suspended charged rod. So *repulsion is the only sure test of a charge.*

Conductors and insulators

Substances can be classed as conductors or insulators (non-conductors). Examples of conductors are all metals, the earth and the human body. Examples of insulators are plastic, bakelite, ebonite, paper, glass and sulphur. A metal rod held in a person's hand cannot be charged permanently by rubbing it–the charge will leak away along the metal through the body to earth. If, however, the metal rod is attached to an insulator, the metal becomes charged. The insulator stops the charge leaking away to earth.

Electrons and the atom

Later in the book we discuss atomic structure (pages 568 to 570). Here we need to know that atoms of all substances contain particles called *electrons*, which are negatively charged, and an equal number of particles called *protons*, which are positively charged. The total amount of negative charge is equal to the total amount of positive charge. So a normal atom is electrically neutral. The protons are buried deep inside the central core or nucleus of the atom. But the electrons are outside the nucleus and some are on the surfaces of the atoms.

When a polythene rod is rubbed with a duster, some of the electrons in the atoms on the outside of the duster move to the rod. Since electrons carry a negative charge, the rod now has a negative charge. Having lost negative charge, the duster has a surplus of positive charge because the atoms were electrically neutral to start with. So the rubbing material also has a charge. We see that rubbing *transfers* charge from one substance to another. Rubbing does not *create* charge (see page 512).

In a similar way, electrons are rubbed off the atoms of a cellulose acetate rod. The rod then becomes positively charged because it loses negative charge, and the duster becomes negatively charged.

Atomic theory can also explain why a metal is usually a good conductor of electricity. As we see later, atoms consist of electrons orbiting round a central nucleus which is positively charged. The silver atom has one electron furthest away from the nucleus. This is weakly bound to the atom because it is shielded from the attraction of the nucleus by other electrons nearer to the nucleus. So the

Scanned with CamScanner

lone electron, carrying its negative charge with it, can wander from one atom to the next along the metal. It is called a 'free electron'. Other metals also have free electrons which can wander along the solid. This is why metals are good conductors. However, the electrons in the atoms of insulators are firmly bound to their particular atoms and do not wander from the atom.

Gold-leaf electroscope

An electroscope is an instrument which can detect electric charges. The simplest type is the gold-leaf electroscope.

The electroscope has a metal cap C and stem S, with a gold leaf (or leaves) attached to the bottom of the stem (Fig. 32.2). The leaf (a very thin, light piece of gold foil) is surrounded by a draught-proof metal case with two plane glass sides so that it is clearly visible, and is carefully insulated from the case as shown. The cap C is one terminal of the electroscope; the case, usually earthed, screens the leaf from outside electrical influences other than that at the cap C, and is the other terminal. The leaf is the sensitive or moving part of the instrument.

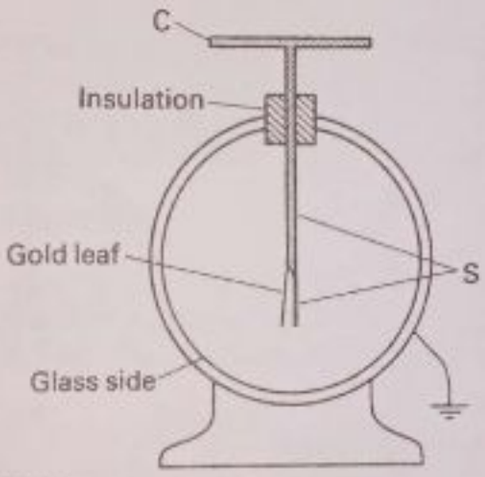


Fig. 32.2 Gold-leaf electroscope

Experiment to test conductors and insulators

1. Stroke the cap C of an electroscope with a charged strip of polythene or a charged rod such as ebonite. The leaf opens (moves away from the stem at the bottom) and stays open. The metal cap and stem have conducted some of the charge to the bottom, so the leaf is repelled from the metal to which it is attached. The electroscope is now negatively charged.
2. Touch the cap of the electroscope with your finger. The leaf now collapses. The charge on the leaf, stem and cap passes away through the body to the earth.
3. Test the conducting and insulating properties of materials, such as rods or strips of copper, brass and other metals, and wood, glass, paper, polythene and perspex. Hold one end of the material and touch the cap of a charged electroscope with the other end. Observe the speed at which the leaf falls. A rapid fall means that the charge escapes easily and so the material used is a good conductor. A slow fall means that it is a poor conductor. No fall means that the material is an insulator.

Some substances classed as insulators or poor conductors conduct much better when they are damp because water conducts electricity.

Scanned with CamScanner

Testing charges by electroscope

The gold-leaf electroscope can be used to find out whether the charge on an object is positive or negative.

Experiment

The charge on the leaf of the electroscope is first made negative by touching the cap with a rubbed polythene strip so that the leaf opens. The unknown charge on a rod X, for example, is then brought near the cap (Fig. 32.3a). If the leaf opens more from A to B, the charge on X must be negative.

When a positively charged rod Y is brought near to the cap the leaf falls a little. But this also happens when an uncharged object such as the hand is brought near the cap. So the only sure way of testing for a positive charge is to charge the electroscope positively, and then bring the charge Y near to the cap. If the leaf opens more, from C to D, Fig. 32.3b, the charge on Y is positive.

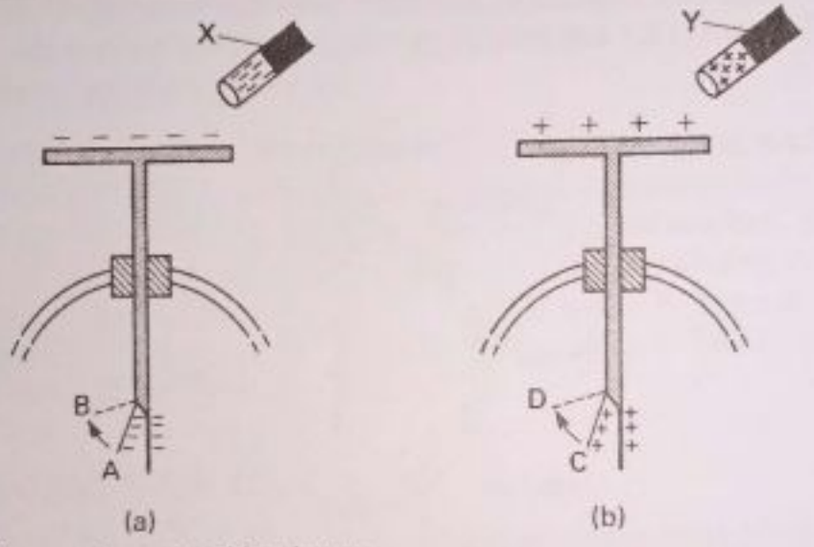


Fig. 32.3 Testing negative and positive charges

Equal and opposite charges by friction

Faraday showed with an electroscope that equal and opposite charges are produced when a body is charged by rubbing. A simple demonstration is illustrated in Fig. 32.4a.

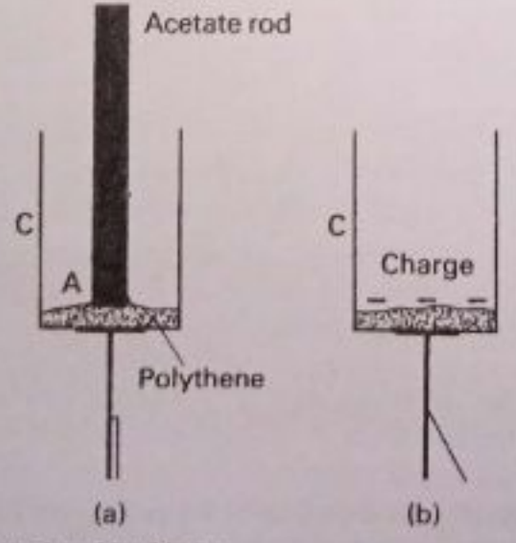


Fig. 32.4 Testing charges produced by friction

Scanned with CamScanner

Experiment

Place a deep can C on the top of an electroscope. Rub the end A of an acetate (or perspex) rod vigorously against a polythene sheet or bag at the bottom of the can (Fig. 32.4a).

When the rod is removed the leaf opens, showing that the polythene has a charge (Fig. 32.4b). When the rod is returned to the bottom of the can and touches the polythene as before, the leaf closes fully. So the charge on the rod is equal and opposite to that on the polythene when this was rubbed. Equal and opposite charges are always produced by rubbing.

Demonstration with mains electricity

Join the live lead of the 240 V a.c. mains (*care!*) to the cap of a sensitive gold-leaf electroscope and the other (neutral) lead to the case. Switch on the voltage supply. The leaf opens. Remove the live lead with an insulating rod (*care!*). Test the sign of the charge. On some occasions the charge is positive and on others it is negative.

So the electricity from the mains is the same kind of electricity as that on a charged rod. The charge on the leaf may be positive or negative, depending on whether, at the moment of disconnection, the mains alternating voltage was on the positive or negative half of the cycle.

When the lead is disconnected from the cap, the open leaf may rise or fall. This is because, at the moment of disconnecting the a.c. voltage, its value may be higher or lower than its average value (page 486).

Electrostatic induction

On page 447 we saw that magnetism could be induced in soft iron without actually touching the metal. In the same way electric charges can be obtained on an object without touching it. This is called *electrostatic induction*.

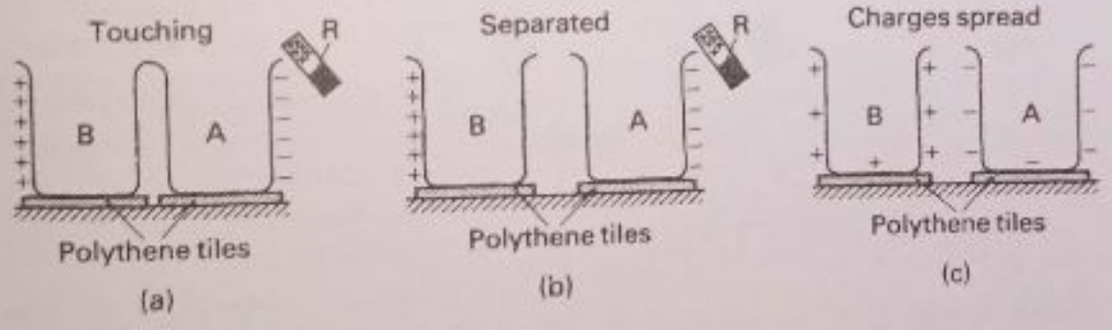


Fig. 32.5 Induced charges

Experiment

1. Place two metal cans A and B on polythene insulating tiles as shown in Fig. 32.5a, so that the cans touch one another. Place a positively charged rod R near to, but not touching, one can A, then separate the cans by moving the polythene tiles *while keeping the rod in position* (Fig. 32.5b). Finally, remove the rod R (Fig. 32.5c).

By using a charged electroscope, we find that the cans have opposite charges. The charges produced without touching by R are called *induced charges*; the charge on R is called the *inducing charge*.

2. Repeat the charging of the cans by induction. Before bringing the cans near an electroscope, however, allow them to touch. Neither can now affects the electroscope, showing that the two charges on A and B have cancelled out each other. So the induced charges were equal in size and opposite in sign.

Scanned with CamScanner

Explanation of induction

Stage 1 When the positively charged rod R is placed near to the cans A and B while they are touching, the electrons (negative charges) in the metal cans are attracted. The electrons move from B to A, where they remain as close to R as possible (Fig. 32.5a). Since B has lost negative charge from one end, it is left with an equal positive charge, as shown.
Stage 2 The cans are now separated, keeping the rod R in position (Fig. 32.5b). The positive charge on B and the negative charge on A are now separated. If R were not kept in position at this stage the electrons would move back from A to B.
Stage 3 With R taken away, the negative and positive charges now spread all over the respective metal cans (Fig. 32.5c).

Charging an electroscope by induction

An electroscope may be charged by induction, as shown in Fig. 32.6.

When a negatively-charged rod R is brought near the cap, the free electrons in the metal of the cap are repelled to the leaf and the end of the stem. So the leaf opens and a positive charge is left on the cap (Fig. 32.6a). When the cap is touched, the electrons are repelled through the body to earth. So the leaf closes (Fig. 32.6b). The finger is now removed and a positive charge is left on the cap (Fig. 32.6c). When the rod R is removed, the positive charge spreads over the cap and leaf, and this makes the leaf open (Fig. 32.6d). The charge on the leaf is an *induced charge*. To show that the electroscope is charged positively, a positively charged rod is brought near the cap. The leaf opens more.

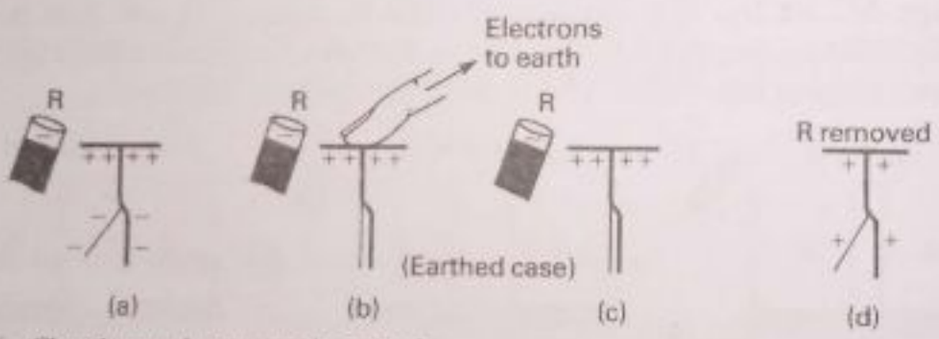


Fig. 32.6 Charging an electroscope by induction

In charging the electroscope by induction, the rod R must be taken away only in the final stage, shown in Fig. 32.6d. If R is removed before the finger is removed, the electrons return to the electroscope through the body and no induced charge is obtained.

Distribution of charge on a conductor

Experiment

Charge a can by induction, as explained above. Examine the distribution of the charge over the surface of the can. This is done by touching the can at different places with a small metal disc B with an insulating handle A, called a proof-plane (Fig. 32.7a). The size of the charge removed by the proof-plane depends on the *surface density* of the charge at the place it touched. If you lower the proof-plane into a deep can placed on top of an

Scanned with CamScanner

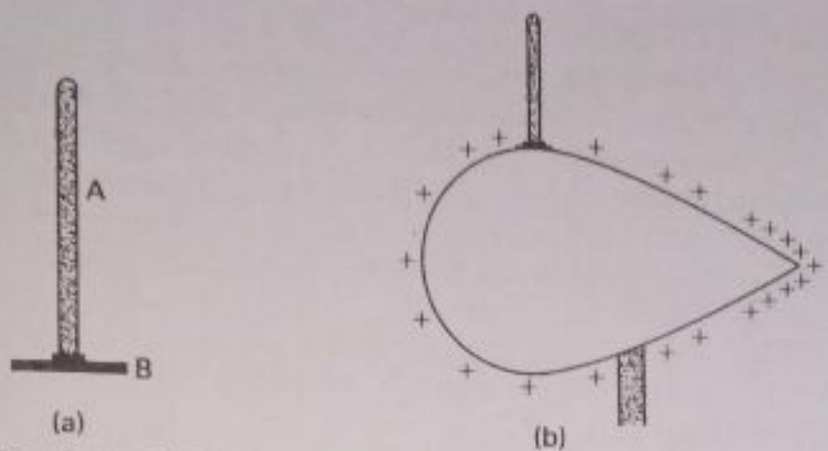


Fig. 32.7 (a) Proof-plane (b) Surface density of charge

electroscope cap, without touching the inside, the amount the leaf opens shows roughly how much charge the proof-plane has removed from the can.

Results. The proof-plane collects no charge from the inside of the can. So there is no charge inside the can. The charge on the can is only on the outside surface.

Outside the can, the proof-plane collects a larger charge from its edges and a smaller one from other parts. The density of the charge on a part of a surface is greater if that part is more sharply pointed. Fig. 32.7b shows roughly how the density of charge varies over the surface of a pear-shaped conductor; the maximum density is at the pointed end.

Electric charge at points

Demonstration

1. A demonstration of the high density of charge at a pointed conductor is shown in Fig. 32.8a. A metal 'windmill' W, with sharp points, is balanced on a metal pivot O on an insulating stand. A connecting wire OA is then joined to the terminal of a van de Graaff high-voltage generator (page 518). When the generator is started up the windmill becomes charged and spins about the pivot in the opposite direction to the points, as shown. If the experiment is carried out in a dark room a circular glow can be observed as the points rotate, showing the presence of a discharge in the air near the points. The

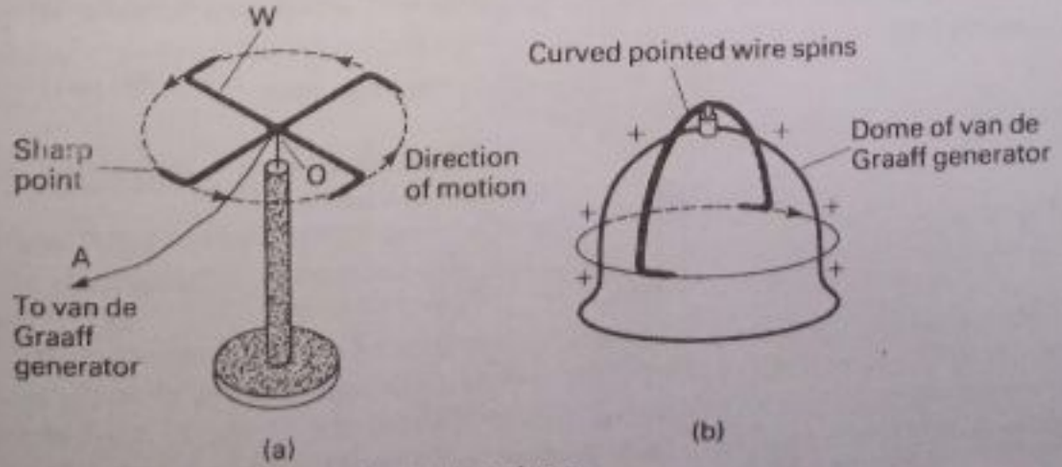


Fig. 32.8 Movement of a charged pointed conductor

Scanned with CamScanner

spinning effect due to action at points can also be seen by pivoting a thick metal with sharp points at each end on the top of the high voltage terminal of a van de Graaff machine, as shown in Fig. 32.8b. The metal spins when the machine is working.

2. The discharge or electrical flow can also be shown by pressing down the centre of the windmill with an insulating rod A so that the windmill does not move (Fig. 32.9). When a candle-flame is brought close to one point B, the flame is blown away from B. So there is a 'wind' or air movement away from B.

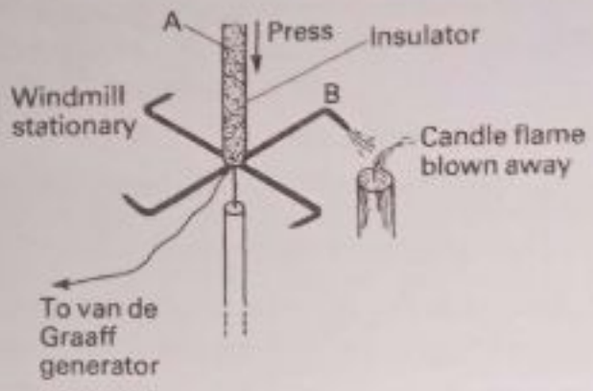


Fig. 32.9 Electric discharge at a point

Explanation Suppose the windmill is positively charged. The very high charge-density at a point attracts the outer electrons from the neutral molecules of air round it. This leaves the molecules with a positive charge. These charged particles or ions are repelled rapidly from the point because it has a similar charge. The ionised air movement produces the 'wind' which blows the candle-flame when the windmill cannot move. By Newton's law of action and reaction, the metal windmill turns in the opposite direction when it is free to move, as shown in Fig. 32.8.

The lightning conductor

Lightning conductors are used to prevent tall buildings being damaged if they are struck by lightning. As shown diagrammatically in Fig. 32.10 a long conductor, with a sharp point at the top, is connected to earth by a thick cable. The point gains an induced charge opposite to that in the thunder cloud (see Fig. 32.6). The electric field due to the very high density of charge ionises the air nearby and the charged air molecules flow upwards from the point. This discharges the cloud before a lightning flash occurs.

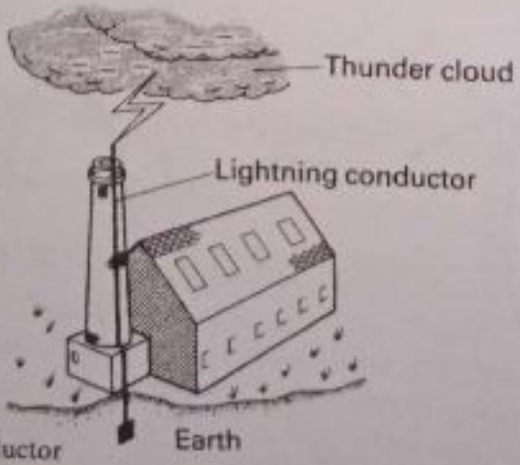


Fig. 32.10 Lightning conductor

Scanned with CamScanner



Plate 32A The Post Office Tower, London. The lightning conductor at the top is marked by an arrow. (*Post Office*)

Electrostatic generators

Various types of electrostatic generators have been used in the past. The simplest is the *electrophorus*. It consists of a metal plate fixed to an insulating handle A (Fig. 32.11).

The electrophorus plate M is placed on a negatively charged polythene tile P. There are only a few points of contact between M and P so no charge flows directly from P, but M has induced charges as shown in Fig. 32.11a. When the plate M is touched by a finger (Fig. 32.11b), electrons are repelled through the

Scanned with CamScanner

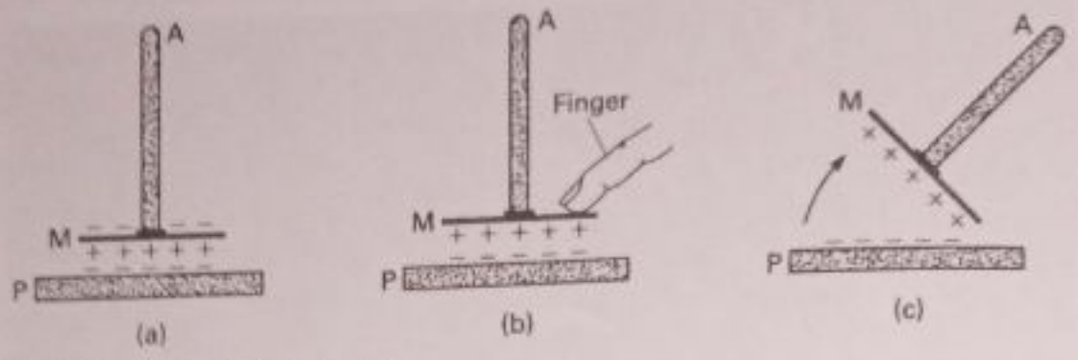


Fig. 32.11 Action of an electrophorus

body to earth. The finger is removed and after this M is removed from P, as shown in Fig. 32.11c. The plate M now has an induced positive charge. When M is brought close to one's knuckle, a small spark is heard to pass from M to the finger. The energy in a spark drawn from M comes from the mechanical work done in moving it away from P against the force of attraction (Fig. 32.11c).

Van de Graaff generator

In 1930, a scientist called van de Graaff made a new type of electrostatic generator. He used an idea from Kelvin's work on electrostatics.

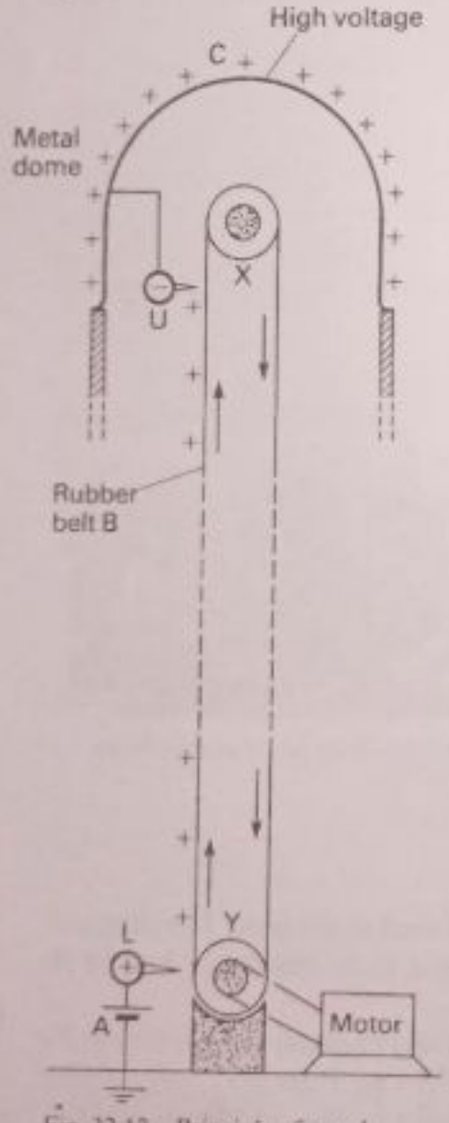


Fig. 32.12 Principle of van de Graaff generator



Plate 32B Van de Graaff electrostatic generator. It produces 6 million volts at the metal dome and is used in nuclear investigations.

Scanned with CamScanner

Fig. 32.12 shows the principle of a model of the generator. Plate 32B shows an actual generator which can produce 6 million volts. In Fig. 32.12, a rubber belt B moves round two insulating rollers X and Y, the roller Y being driven by a motor. A set of metal points, represented by L, faces the belt at the bottom. Another set of points, represented by U, faces the belt at the top. U is connected to the inside of a large metal dome C at the top. C is the high-voltage terminal of the generator when it is working.

The generator is self-exciting. When the motor is switched on, the belt at the bottom rises with a +ve charge produced by friction with a metal surface of the roller Y. When the charge arrives opposite the points U, it induces a −ve charge on U and therefore a +ve charge on C. By the action of points, −ve charged or ionised molecules of air are repelled by U to the +ve charge on the belt opposite U. So the belt is discharged and the metal dome C gets a +ve charge. This builds up steadily as the belt rotates. A high-voltage battery A may be connected with its positive pole to L to assist the action.

The amount of charge the dome C can store depends on its size, the efficiency of the insulation and the air humidity. A voltage of say 10000 V can be generated with a small model generator of the type shown in Fig. 32.12.

Large van de Graaff generators are used nowadays in the study of atomic structure and atomic energy. A high potential difference of several million volts can be obtained from them to accelerate electrical particles to high energies and then to disrupt atoms.

Electric fields

Intensity of electric field

When a small +ve charge on a light metal sphere is placed at a point S near to the +ve charge X on a van de Graaff machine, the sphere is repelled with a large force (Fig. 32.13). At a point A much farther away, the force on the charge is

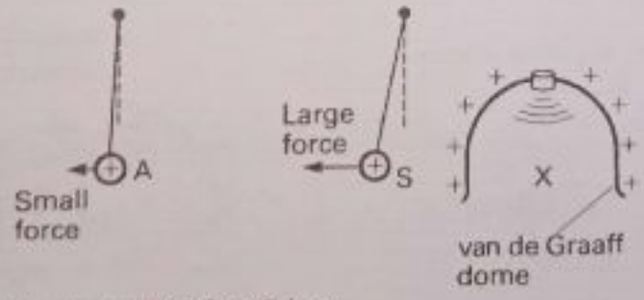


Fig. 32.13 Electric field round van de Graaff dome

small. The space round X, where a force is exerted on a charge placed in it, is called an *electric field*. If the force is large, the field intensity or strength is high at this point. The intensity is high at S and weak at A.

Electric field patterns

As we saw earlier, a magnetic field exists round bar magnets and current-carrying conductors. Each field has a particular pattern of magnetic lines of force, which can be shown by using iron filings.

Scanned with CamScanner

Demonstration

In a similar way, electric field patterns can be obtained. In Fig. 32.14a, two parallel metal strips A and B are placed inside a Petri dish containing a light oil and semolina particles are sprinkled lightly between A and B on the oil. When the high-voltage terminal of a van de Graaff generator is connected to A, and B is earthed, the semolina particles move and tend to form straight parallel lines between A and B.

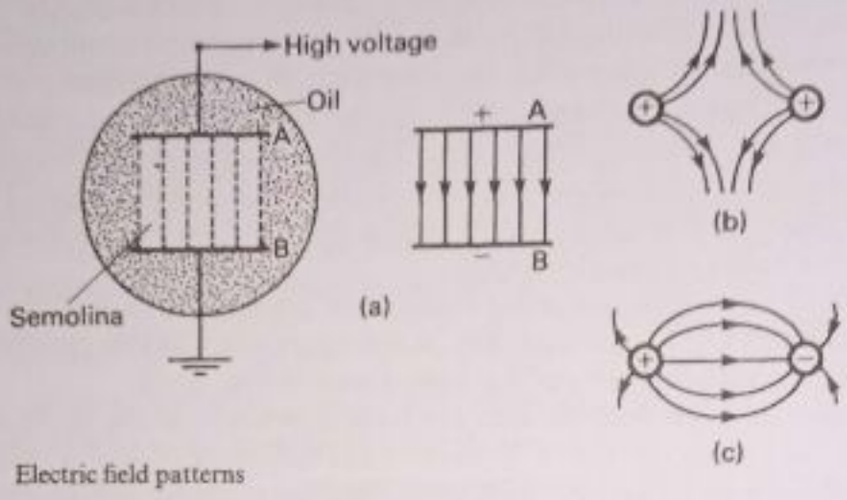


Fig. 32.14 Electric field patterns

This is the pattern of the electric lines of force between two parallel charged plates A and B, when A has a +ve charge and B has an induced equal −ve charge. The electric field pattern between two +ve charges is shown in Fig. 32.14b and that between a +ve and a −ve charge is shown in Fig. 32.14c.

Electrostatic screening

When a leaf electroscope L is placed near the high-voltage terminal D of a van de Graaff machine, the leaf opens (Fig. 32.15a). When a metal gauze G is held and

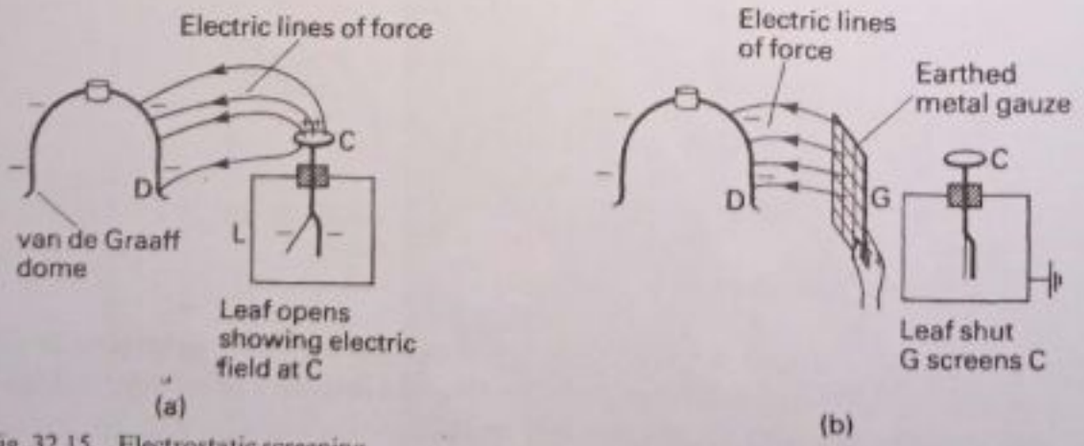


Fig. 32.15 Electrostatic screening

placed between the electroscope and D, the leaf closes (Fig. 32.15b). The electroscope is now screened from the charge on D. Fig. 32.15a, b shows why screening takes place. The electric lines of force from D end on G, the earthed metal gauze, in Fig. 32.15b, whereas they were able to reach the electroscope cap C in Fig. 32.15a. So no electric field exists beyond the earthed metal gauze.

Scanned with CamScanner

Electric potential

An object in the earth's gravitational field, for example, at a height above the ground, has gravitational potential energy. In a similar way, a charge in an electric field has electrical potential energy. We say that all points in an electric field have an *electric potential*. Near a +ve charge, the electric potential in the field is high. Further away, the electric potential is smaller. A very long way from the the charge the electric potential is practically zero.

In practice, sea level is usually the zero of gravitational potential. In electric fields, the electric potential of the earth is taken as the zero. This is because the earth is an enormously big conductor and so its potential is practically unchanged by any charge it may collect or lose. As we have seen earlier, one cable in the domestic mains electricity supply is at earth potential.

Energy changes occur when an object moves from one place to another in the earth's gravitational field. In a similar way, electrons (negative charges) move from the negative to the positive pole of a battery through a lamp filament joined to the battery (Fig. 32.16). The negative pole has a lower potential than the

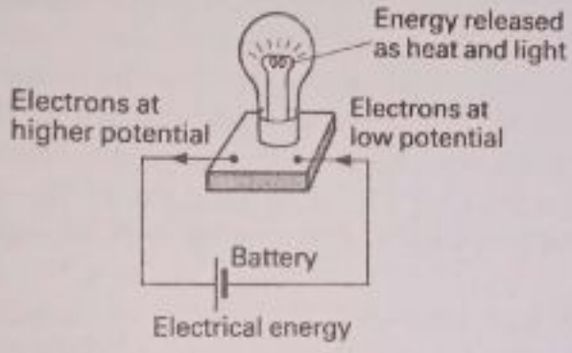


Fig. 32.16 Potential difference and energy change

positive pole and the potential difference produces the energy change of the electrons which is transferred to heat in the filament. In a television tube, the electrons gain energy from a high potential difference inside the tube. This is transferred to kinetic energy, and then to luminous energy and heat when the electrons strike the screen and produce a picture.

Electroscope and potential difference

We can show by simple experiments that the leaf of an electroscope opens only when there is a *potential difference* between the cap and the case.

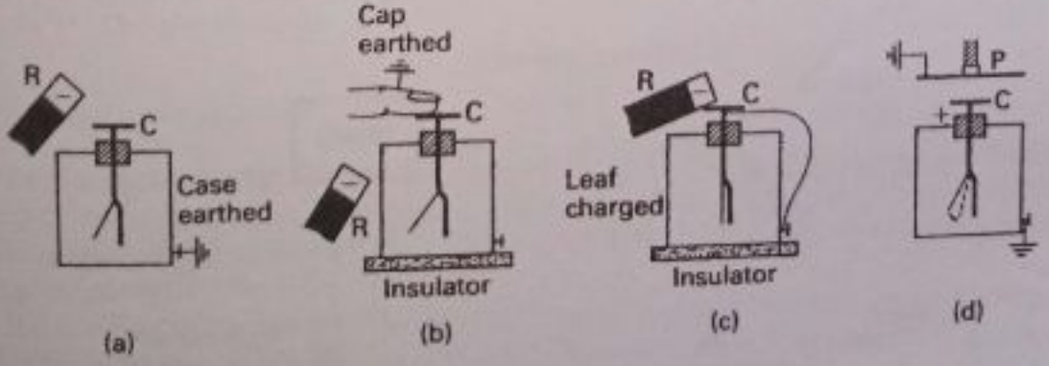


Fig. 32.17 Electroscope and potential difference

Scanned with CamScanner

Experiment

1. A charged rod R is brought (a) near the cap C of an electroscope when the case is earthed (Fig. 32.17a); (b) near the insulated case when the cap C is earthed (Fig. 32.17b). In both cases the leaf is seen to open. Now in Fig. 32.17a, C has a −ve potential because it is in the electric field of R and the case has earth ('zero') potential, so there is a potential difference between the cap and case. Similarly, there is a p.d. between the cap and case in Fig. 32.17b. So we conclude that the leaf opens when there is a p.d. between the cap and case.
2. The charged rod R is now brought near to the cap and electroscope when the cap is connected to the insulated case (Fig. 32.17c). The leaf remains closed, even when the charged rod touches the cap or the case and gives them a charge. This confirms our result in 1. The cap and case have no p.d. because they are connected and so the leaf does not open.
3. Finally, in Fig. 32.17d, an earthed metal plate P is brought near the cap C of a positively charged electroscope. We see that the leaf closes a little, although the charge on the electroscope is unaltered. When the earthed metal plate P is brought closer to the cap, the leaf closes more.

Explanation When P is brought near to the +ve charge on C, an induced −ve charge is obtained on the side of P near the cap, as shown. The *potential* of C is now less than before because of the −ve charge near it. The closer P is to C, the more the potential is reduced. The experiment shows that the amount the leaf opens is *a measure of the potential difference* between the cap and the case of the electroscope.

We can therefore get a measure of the potential of the surface of a conductor by connecting one end of a wire to the cap of an electroscope whose case is earthed. Move the other end of the wire all over the surface using an insulator to hold the wire. We then find that the leaf remains open the same amount. So every part of the surface has the same potential. This is true whatever the shape of the charged conductor. If it were not true, electric currents would flow between points which had different potentials; no current can be detected, however.

Static and current electricity

As we saw in earlier chapters, current electricity produces heating, magnetic and chemical effects. An electric current is due to moving charges. When it is made to move, static electricity produces the same effects as current electricity.

For example, a spark can be obtained from an electrostatic generator; that is, heat is produced. A 'discharge' of static electricity through the coil of a moving-

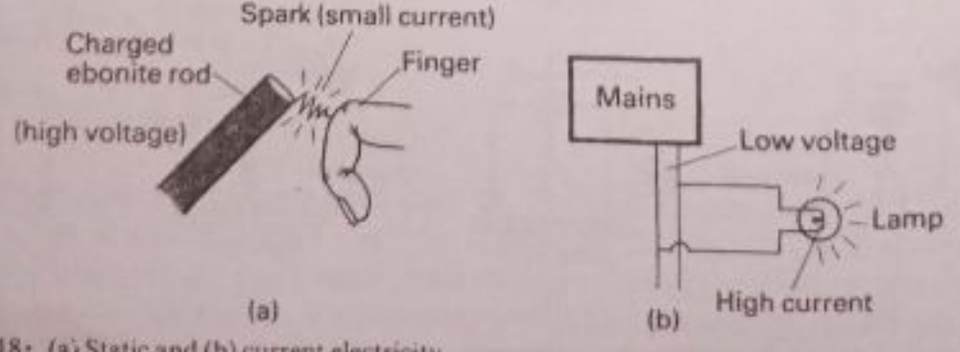


Fig. 32.18 (a) Static and (b) current electricity

Scanned with CamScanner

coil galvanometer makes the coil deflect, showing there is a magnetic effect. And a red colour change is produced on discharging an electrostatic generator through blue litmus paper dipped in sodium sulphate solution, showing a chemical effect.

So static electricity is the same kind of electricity as current electricity. In static electricity, however, we deal with high voltages and small currents. In current electricity we deal with relatively low voltages and high currents (Fig. 32.18).

Capacitors

Charging a capacitor

A capacitor is a device which can store charge. Basically, it has two metal plates separated by an insulator. Capacitors are widely used in all types of electronic equipment.

When a capacitor is connected to a battery B (Fig. 32.19), the positive pole of the battery attracts electrons from a plate X and they are pushed round the circuit on to plate Y. So plate X has a +ve charge and plate Y an equal −ve charge. The capacitor is now said to be *charged*. The electron flow or current in the circuit stops when the electrons on Y repel other electrons and stop them from reaching Y

The process of charging shown in Fig. 32.19 is finished in a very short time. If large capacitors are used and a large resistor is placed in the circuit, the charging takes longer to complete.

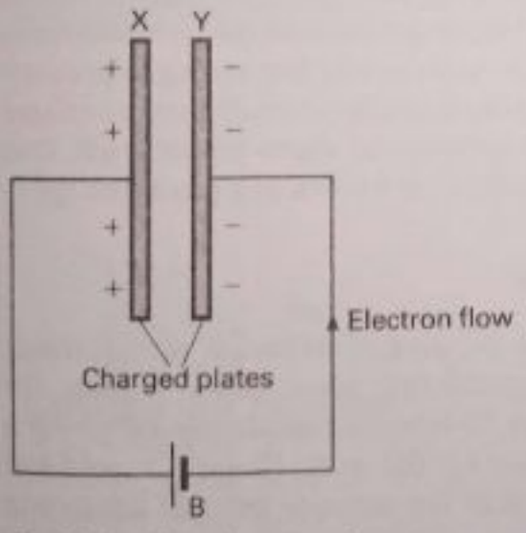


Fig. 32.19 Charging a capacitor

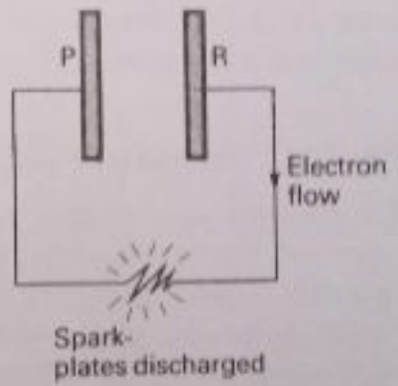


Fig. 32.20 Discharging a capacitor

Discharging a capacitor

If the plates of a charged capacitor are joined, a current will flow for a short time. Electrons from the negatively charged plate flow round the circuit to the positively charged plate and the charges neutralise each other. The capacitor is then 'discharged'.

If a wire is used to connect the charged capacitor plates P and R in Fig. 32.20, a spark may be seen as the plates are joined. When the current has stopped flowing, the capacitor is discharged.

Scanned with CamScanner

Capacitance

Suppose a capacitor such as the pair of plates X and Y in Fig. 32.19 store a charge Q of 2 units when a p.d. V of 10 V is connected to them. Experiment shows that the charge stored is 4 units when the p.d. is increased to 20 V and 6 units when the p.d. is increased to 30 V. We can see that more charge should be stored on the capacitor when the p.d. is increased, because more electrons are driven to the plate Y when the battery p.d. is increased.

We see that for a given capacitor

$$\frac{Q}{V} = C, \text{ a constant}$$

where Q is the charge stored when a p.d. V is applied. The constant C is called the *capacitance*. Using the same battery, a large pair of plates stores more charge than a small pair of plates, other factors being equal. So it has a larger capacitance.

Values of capacitance

The unit of capacitance is the *farad*, symbol F. By definition, 1 F is the capacitance of a capacitor when 1 V p.d. provides it with a charge of 1 C (coulomb). A 1 F capacitor has an enormous size, so the practical unit of capacitance is the microfarad (μF). A capacitor has a capacitance of 1 μF if it stores a charge of 1 microcoulomb (one-millionth coulomb) when a p.d. of 1 V is applied. In radio receivers capacitors of the order of 1 μF are considered relatively large. Very large capacitors, such as 100 μF or more, are needed in some parts of radio circuits.

At the other end of the scale, the capacitor used for tuning to different stations in radio receivers is small. Its maximum capacitance is 0·0005 μF, or 2000 times smaller than 1 μF. Still smaller capacitors are used in very fine tuning to produce a change of pitch. A small change of treble or bass in a good quality record-player is obtained by turning a small capacitor of the order of micro-microfarads. One micro-microfarad, one-millionth of a microfarad, is known as a picofarad (pF).

Factors affecting capacitance

The capacitance of a capacitor depends on the area of its plates, their distance apart and the nature of the insulating material (air, glass, paper or mica, for example) between them. These factors can be investigated simply by giving a charge Q to the plates and then measuring the p.d. V with a gold-leaf electroscope. On page 521 it was shown that the amount the leaf opens is a measure of the p.d. between the cap and case. The capacitance C can then be found from the ratio Q/V.

Experiment

1. *Distance between plates* To investigate this factor, two large insulated plates A and B (Fig. 32.21) are connected to the cap X and case Y of a gold-leaf electroscope. Using a rubbed polythene rod, A is given a charge Q. The leaf opens. The plate B is now moved nearer to A. The leaf closes a little. Since V is smaller and Q is unchanged, we see from $C = Q/V$ that the capacitance C is increased when the distance between the plates is decreased.

Scanned with CamScanner

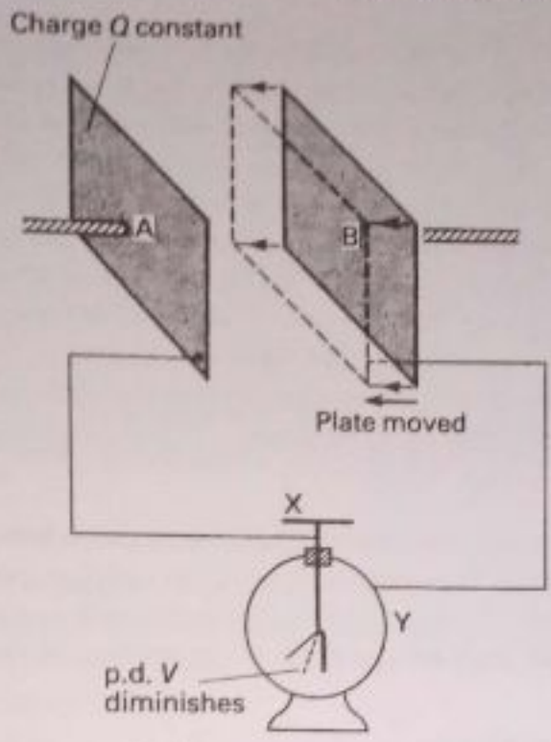


Fig. 32.21 Effect of distance apart of plates

2. *Medium between plates* As before, the plate A is charged and the leaf opens (Fig. 32.22). A thick book is now carefully placed between the plates so that paper replaces part of the air between the plates. The leaf closes a little. The same effect is produced when polythene or perspex is used in place of the peper. Since V diminishes and Q is unchanged, from $C = Q/V$, the capacitance C increases when paper, polythene or perspex is used between the plates instead of air alone.

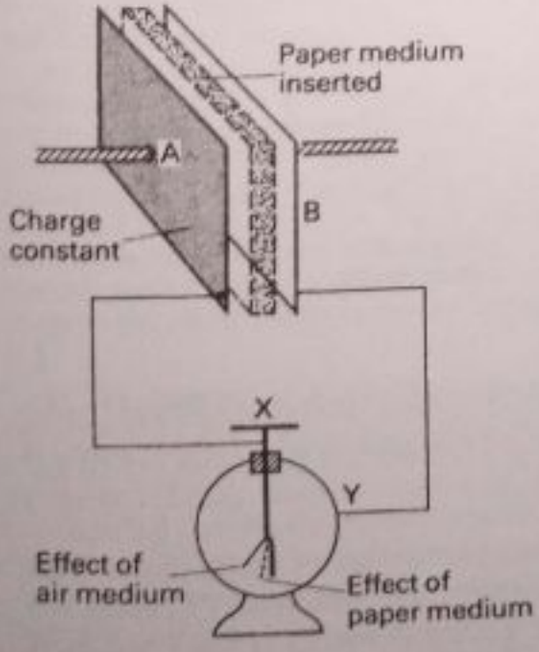


Fig. 32.22 Effect of dielectric

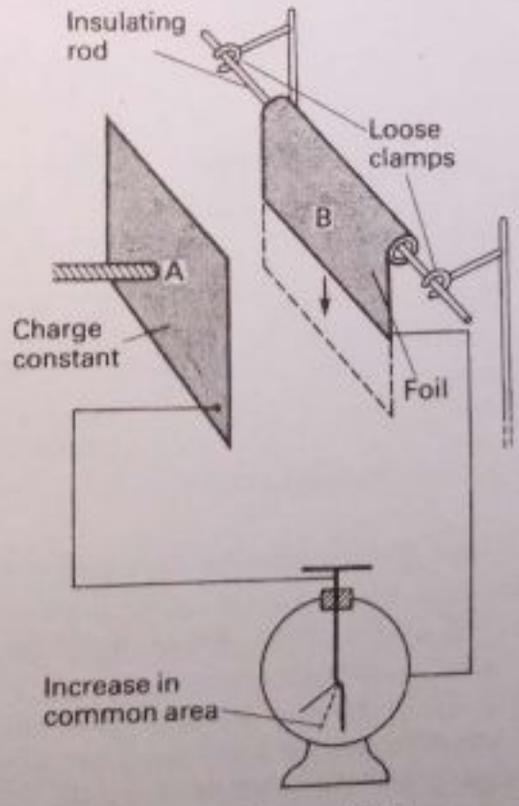


Fig. 32.23 Effect of area of plate

Scanned with CamScanner

The material or medium between the plates of a capacitor is called the *dielectric*. When a dielectric such as waxed paper completely fills the space between the plates, the capacitance is increased about 3 times compared with air as the dielectric. The *dielectric constant* of waxed paper is said to be 3. Mica has a higher dielectric constant of about 7.

3. *Common area of plates* The plate B is now replaced by a roll of aluminium foil (Fig. 32.23). With an appreciable area of foil facing the plate A, A is charged and the leaf opens. The foil is now wound up to decrease the area facing A. We see that the leaf then opens more. So, from $C = Q/V$, the capacitance C decreases when the common area between the capacitor plates decreases. When the foil is wound back so that the common area increases, the leaf closes a little. So V decreases. Hence the capacitance C increases when the common area between the plates increases.

Summarising The capacitance of a capacitor increases when the common area of its plates *increases*, when the distance apart of the plates *decreases* and when the insulating medium between the plates is a material such as paper or glass in place of air. These facts help the manufacturer to design practical capacitors.

Practical capacitors

1. *Variable air capacitor*

This capacitor has a set of metal plates M on a spindle which can be turned round so that they move between another set of fixed metal plates, F (Fig. 32.24). The two sets of plates are insulated from each other. In this way the common area, and therefore the capacitance, can be varied easily.

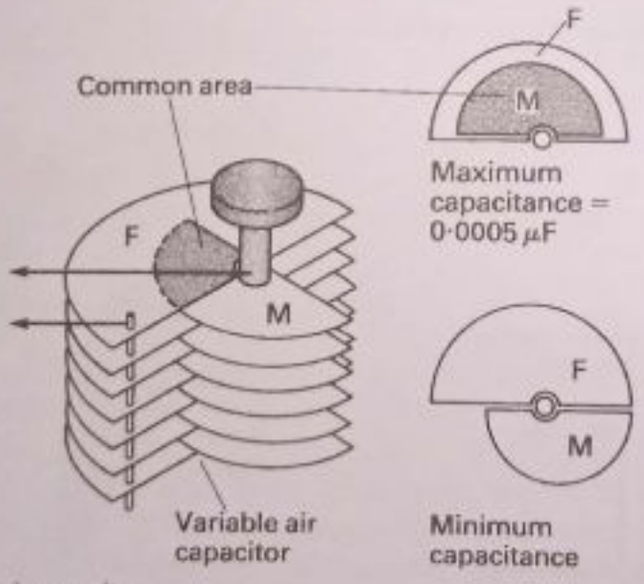


Fig. 32.24 Variable air capacitor

Variable air capacitors are used in radio receivers for tuning to different radio stations. Changing the capacitance alters the wavelength to which the receiver is tuned, and so different stations can be heard.

2. *Mica capacitor*

Fig. 32.25 shows a capacitor with mica between a number of metal plates. This form of capacitor has a fixed capacitance between the terminals TT. Mica is a 'high-grade' dielectric in the sense that little electrical energy is absorbed by it when the capacitor is used.

Scanned with CamScanner

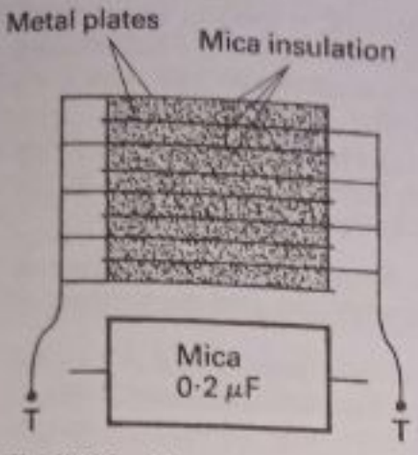


Fig. 32.25 Mica capacitor

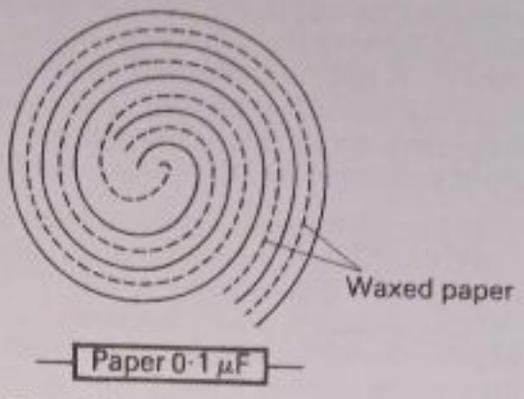


Fig. 32.26 Paper capacitor

3. *Paper capacitor*

Dielectrics other than mica are often used in capacitors. Polystyrene is used, for example, and Fig. 32.26 shows a paraffin-waxed paper capacitor. This has two long strips of metal foil with the paper dielectric rolled up tightly together and enclosed in a plastic insulating case.

4. *Electrolytic capacitor*

If a very large capacitance is required, an electrolytic capacitor is used. To produce this capacitor an electric current is passed through a solution of ammonium borate into which two aluminium electrodes dip (see Fig. 31.3). As the capacitor may be ruined by reversing the current through it, the manufacturers show clearly the +ve terminal for correct connection in circuits, as in Fig. 32.27.

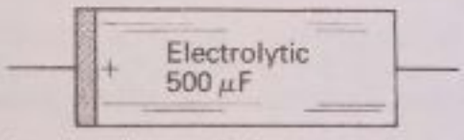


Fig. 32.27 Electrolytic capacitor showing terminals

Examples on electrostatics

1. (a) Draw a diagram of a leaf electroscope, labelling clearly the parts. What does the leaf electroscope measure?

(b) (i) A copper calorimeter containing a piece of dry flannel is placed on top of an electroscope and a dry polythene rod is pushed in. There is no deflection of the electroscope, but when the rod is pulled out the leaves diverge. Explain these observations. (ii) Why do the rod and the flannel have to be dry? (iii) If, after the rod is pulled out, it is quickly placed in a similar empty calorimeter on a similar electroscope, what will happen? Explain. (iv) If instead the rod is brought near to a metal-coated pith ball suspended by a nylon thread, the pith ball is first attracted to the rod, touches it and is then repelled. Explain. (v) If instead, the rod is placed over a small light piece of paper the paper sticks to the rod. Explain why the paper is not immediately repelled. (L)

(a) The diagram of a leaf electroscope is shown in Fig. 32.2. The leaf electroscope measures the potential difference between the cap and the case.

(b) (i) The polythene rod obtains a negative charge, and the flannel an equal positive charge by rubbing. Since the *total* charge is zero, the leaves remain closed. But when the rod is pulled out, the flannel in the calorimeter is left with its +ve charge. So the leaves diverge.

(ii) The rod and flannel have to be dry as water is a conductor; otherwise any charge on them would be conducted back or leak away to the metal calorimeter.

(iii) If the rod is also placed in an empty calorimeter, the leaves would diverge. The charged rod induces a charge on the calorimeter and electroscope cap.

Scanned with CamScanner

(iv) When the rod is brought near the metal-coated pith ball, it induces an opposite charge on the side near it and this causes attraction to take place. When the ball touches the rod, some of the rod's charge is transferred quickly by contact to the ball, which is a conductor, and the like charges on the rod and ball then repel.

(v) The paper also obtains a similar charge to that on the rod by contact but it does this much more slowly than the metal-coated pith ball because paper is an insulator. So the paper sticks to the rod for a little time and is not immediately repelled.

2. In Fig. 32.28, A, B and C are insulated metal spheres, A and B being the same size and C much larger. A is negatively charged. B and C are uncharged, and are connected to identical electroscopes E and F.

Explain clearly, giving reasons in each case, (a) how you would use A to give *positive* charges to both B and C, (b) whether the electroscope deflections will differ if the charges given to B and C are equal, (c) why if A and B are now joined by a length of resistance wire, heat will be dissipated in the wire.

If in (c) a charge of 10^{-6} coulomb flows in a time of 10^{-6} second through a wire of $2\,\Omega$, how much heat is dissipated in the wire? (L)

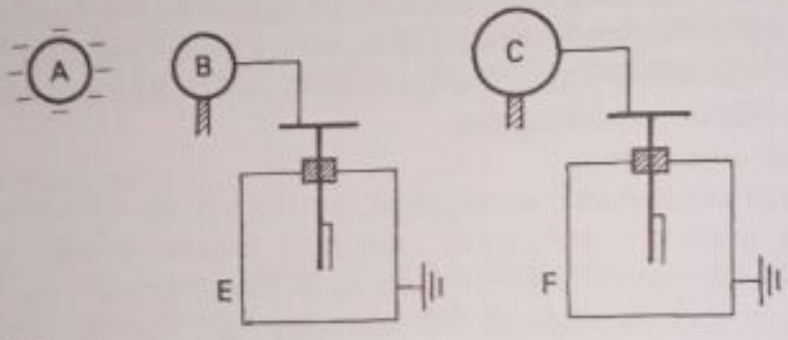


Fig. 32.28 Example

(a) To give a +ve charge to B, bring A near to B, then touch B with a finger, remove the finger, and finally remove A.

Reason The −ve charge on A repels electrons on the side of B nearer to it, leaving this side with a +ve charge. On touching B with a finger the electrons are conducted through the body to earth. When the finger is removed, the excess +ve charge remains on B and spreads over it.

To give a +ve charge to C, repeat the procedure as for B.

(b) The electroscope deflection is a measure of the potential difference V between the cap and the case. Since B has a smaller area than C, its capacitance C will be smaller. So from $V = Q/C$, where Q is the equal charge on B and C, it follows that V is greater for B than for C. So the deflection of the electroscope is greater for B.

(c) Since the potential of A is −ve and the potential of B is +ve, a current will flow along a resistance wire connecting A to B. The current produces heat in the wire.

Current flowing, $I = Q/t = 10^{-6}\text{C}/10^{-6}\text{s} = 1\text{ A}$

$\therefore$ p.d. across wire, $V = IR = 1 \times 2 = 2\text{ V}$

$\therefore$ heat produced $= IVt = 1 \times 2 \times 10^{-6} = 2 \times 10^{-6}\text{ J}$

(*Alternatively*, heat produced = energy provided $= QV = 10^{-6} \times 2 = 2 \times 10^{-6}\text{ J}$.)

Summary

1. Like electrical charges repel, unlike charges attract.
2. Negative charge is the charge on an ebonite rod rubbed with fur or on a rubbed polythene strip. Positive charge is the charge on a glass rod rubbed with silk or on a rubbed cellulose acetate strip. Negative charges are carried by electrons.
3. Conductors (metals, the earth, the human body) allow electric charge to flow through them. Insulators (ebonite, mica, plastics) do not normally allow charge to flow through them.

Scanned with CamScanner

4. If an unknown charge is brought near to a charged electroscope and the leaf opens more, the unknown charge is similar to the charge on the electroscope.
5. To charge an electroscope by induction, bring a charged object near, touch the cap, remove the finger from the cap, and finally remove the charged object.
6. A pointed charged conductor has a high density of charge at the point. This is shown by the rotation of a metal windmill. A lightning conductor has a pointed end.
7. The van de Graaff machine has an insulating belt which is driven continuously, and a metal dome or terminal on which charges continue to be induced. Pointed conductors help to transfer charges to the belt and to the dome.
8. Electric fields exist all round metals which carry electric charges. Objects can be 'screened' by earthed metal plates or cages.
9. Positive charges in an electric field move from high to low potential points. Electrons (negative charges) would move in the opposite direction. 'Zero' potential is earth potential.
10. For a capacitor, $C = Q/V$. C is larger if (a) the plates are closer together, (b) the overlapping area of the plates is larger, (c) a material of higher dielectric constant is between the two plates.

Exercise 32 (Answers, page 643)

1. A piece of dry silk when rubbed against a polythene rod and then held near a leaf electroscope produces a small deflection. A second piece of moist silk when used and tested similarly does not produce a deflection. Explain both of these results. (N)
2. Fig. 32.29 represents two insulated metal spheres. A is positively charged and B is uncharged. Show on the diagram the distribution of charge on the spheres. (N)



Fig. 32.29

3. Assuming you have an insulated positively-charged conductor, describe how you would use it to charge a leaf electroscope (a) negatively, (b) positively.
Describe and explain the effect on the deflection of a negatively charged electroscope when an insulated positively-charged conductor is brought near to the cap without making contact with it. (L)
4. (a) Describe an experiment you could perform to show that equal and opposite charges are produced when two bodies are electrified by rubbing them together. Explain how the sign of each of these charges would be identified.
(b) An insulated spherical conductor has a short steel needle projecting from its surface. The conductor is then rapidly charged. Describe and explain the effect of the needle on the charge on the conductor. (N)
5. Describe and explain how you would charge a leaf electroscope negatively using a positively charged insulating rod.
The negatively charged electroscope is then connected by a wire to an uncharged metal can which stands on an insulating support. Describe what you would see and explain your observations in terms of the charge, potential and capacitance of the system. (N)
6. Describe how a leaf electroscope may be used to (a) test the insulating properties of a given material, and (b) identify the sign of the charge on a charged insulated conductor.
Explain in each case how your conclusions are derived from your observations. (L)
7. (a) Describe and explain how the whole of the charge on a small insulated conductor can be transferred to a deep metal can, open at the top, standing on a non-conducting base.
(b) A positively charged rod is brought near to the cap of a gold-leaf electroscope. Describe and explain the effect it has when (i) the electroscope is positively charged.

Scanned with CamScanner

(ii) the electroscope is uncharged and the cap is momentarily touched with a finger and the rod is then removed. (N)

8. Explain, briefly, the following:

(a) When an uncharged metal plate held by an insulating handle is brought into contact with the cap of a negatively charged leaf electroscope the divergence of the leaf is decreased.

(b) As the same metal plate is then brought near the cap again the leaf begins to diverge.

(c) When the plate again makes contact with the cap the divergence is of the same value as in the final stage of (a), above. (L)

9. When a polythene rod which has been rubbed with a dry cloth is held high above small pieces of paper on a table, the pieces of paper jump up and down repeatedly between the table and the rod. Account for this. (C)

10. Describe and explain how you would use a negatively charged leaf electroscope to show that the charge acquired by a dry glass rod when it is rubbed with silk is positive. You must state what you would observe during the test and draw diagrams to explain how your observations are linked with the movement of charge (or electrons) on the cap and leaf of the electroscope. (L)

11. Draw a fully labelled diagram of a gold leaf electroscope.

Explain the nature of the charge on a gold leaf electroscope if the divergence of the leaf is observed to *decrease* when an ebonite rod which has been rubbed with fur is brought towards the cap. Explain, also, why, if the rod is brought still nearer to the cap, but without touching it, the divergence will first decrease to zero and then increase again.

Describe and explain an experiment to demonstrate that the capacitance of a conductor increases as its area is increased. (L)

12. Describe an electrophorus and state how it is used to give an insulated conductor a positive charge. Give diagrams to illustrate the distribution and nature of the charge in each stage of the operation.

Why is it possible to obtain a series of approximately equal charges from this instrument without recharging it and what is the source of energy of these charges? (L)

13. Describe experiments, one in each instance, to show that: (a) equal and opposite charges are produced when ebonite is rubbed on fur; (b) the inside surface of a charged hollow can has the same potential as the outside surface but has no charge; (c) silk thread is a better insulator than cotton thread. (L)

14. (a) Describe and explain what happens when a charged rod is brought near a pith ball suspended by a dry silk thread.

(b) Why has one end of a lightning conductor a sharp point? Describe a laboratory experiment to support your explanation. (N)

Multiple-Choice Questions

15. Which statement of A to E is correct?

A capacitor

A need not have an insulator between its plates
B must have metal plates to store charge
C has no voltage across it when working in a radio receiver
D has a very high capacitance when air is between its plates
E has a capacitance defined by $C = V/Q$

16. In Fig. 32.30, two vertical plates P and Q have a large positive and negative charge respectively. When a small conducting sphere S with a positive charge is suspended between P and Q, it swings to Q and then keeps oscillating from Q to P and back. This is because

A P has a greater charge than Q
B S is always repelled from P and attracted by Q
C S is always attracted by P and repelled by Q
D Q has a greater charge than P
E S receives a similar charge from P and Q each time it touches the plates

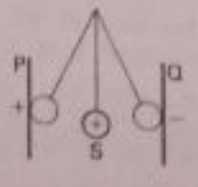
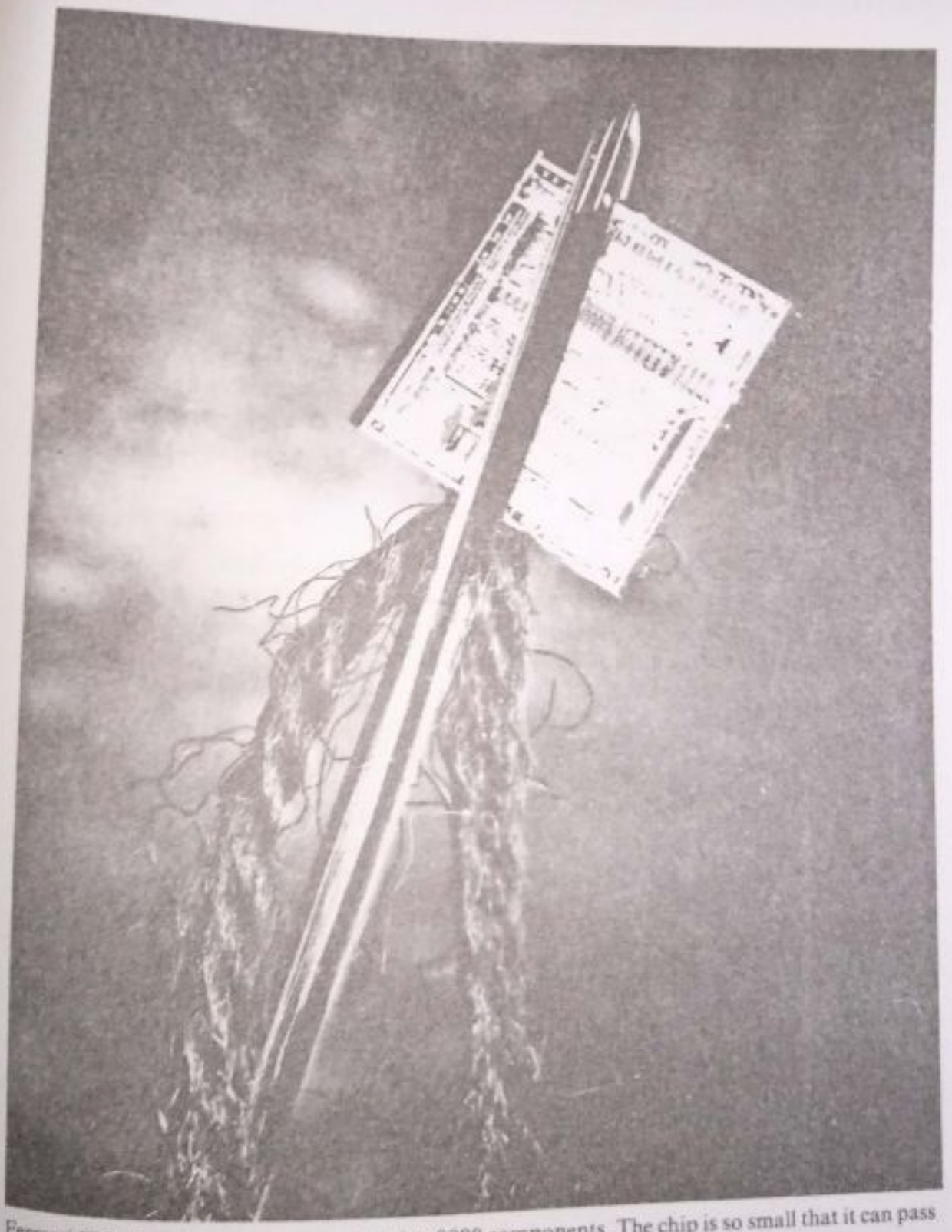


Fig. 32.30

Scanned with CamScanner

ATOMIC PHYSICS



Ferranti F100–L microprocessor, containing 9000 components. The chip is so small that it can pass through the eye of a needle. (*Ferranti Ltd*)

Scanned with CamScanner

33. Electrons and Electronics

Conduction in gases

Towards the end of the 19th century, scientists began to investigate the conduction of electricity through gases. A long glass tube called a *discharge tube* was used and a high voltage of several thousand volts was connected across two electrodes C and A at the ends (Fig. 33.1). As in electrolysis, C is called the cathode because it is connected to the − ve terminal of the high voltage and A is the anode since it is connected to the + ve terminal. A vacuum pump is connected to a side tube.

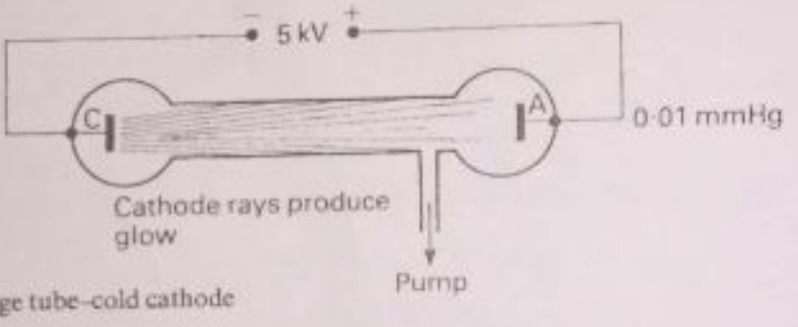


Fig. 33.1 Discharge tube–cold cathode

As most of the air is pumped out, the air left begins to conduct. Long blue streaks of light appear between C and A. At a low pressure of 0·5 mmHg the air glows near the cathode. See plate 33A. At the very low pressure of 0·01 mmHg there is no gas glow inside the tube but the glass of the tube itself glows green.

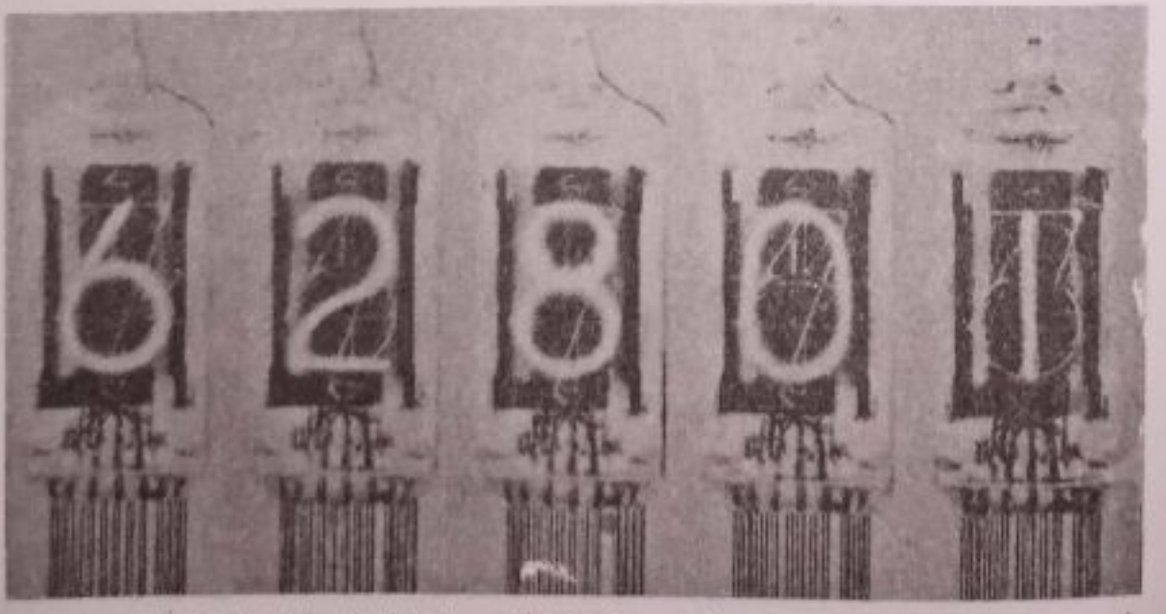


Plate 33A Electrical (cold cathode) tubes which display numbers. They are used in computers and in cash registers. (*Mullard Ltd*)

Scanned with CamScanner

Discovery of electrons

The green glow is due to particles or rays which appear to come from the cathode. They were called *cathode rays* about 1890 when the phenomenon was first observed. For many years scientists were puzzled by the nature of cathode rays. The glow on the glass moved when a strong magnet was brought up at right angles to the tube, so the cathode rays were easily deflected by a magnetic field. This meant they could not be electromagnetic waves like light waves. They were probably very light particles carrying electric charges, and they were later called *electrons*.

In 1896 Sir J J Thomson showed by experiment that, approximately

$$\text{mass of electron } (m_e) = \frac{1}{1840} \text{ mass of hydrogen atom } (m_H)$$

So, in round figures, the mass of an electron is only 1/2000th of the mass of a hydrogen atom, the lightest atom. We now know that electrons are particles present in all atoms. In the discharge tube, electrons come from the atoms of the metal cathode; others are produced after collisions of electrons with molecules of air. This is called production of electrons by a 'cold cathode'.

Hot cathodes · Electron deflection by electric field

In 1902 Richardson found that electrons could escape from some metals such as tungsten when a fine wire of the metal was heated to a high temperature. This is called *thermionic emission* and the metal is called a *hot cathode*.

Demonstration

Note: *Care must be taken in this and other demonstrations where high voltages are used.*

Fig. 33.2a shows an evacuated 'Teltron' tube with a 6 V supply connected to a hot cathode C at one end to produce electrons. A metal anode cylinder A in front of C has a high +ve potential of about 4 kV relative to C so that the electrons are accelerated towards A. Together, C and A form an 'electron gun'. After passing through a hole in A the electron beam traces a luminous straight horizontal line XY on a fluorescent screen S held between two plates P and Q.

When a high p.d. battery of 2 kV is connected between P and Q, with its +ve terminal to P and its −ve terminal to Q, the electron beam is deflected upwards to P. When the +ve

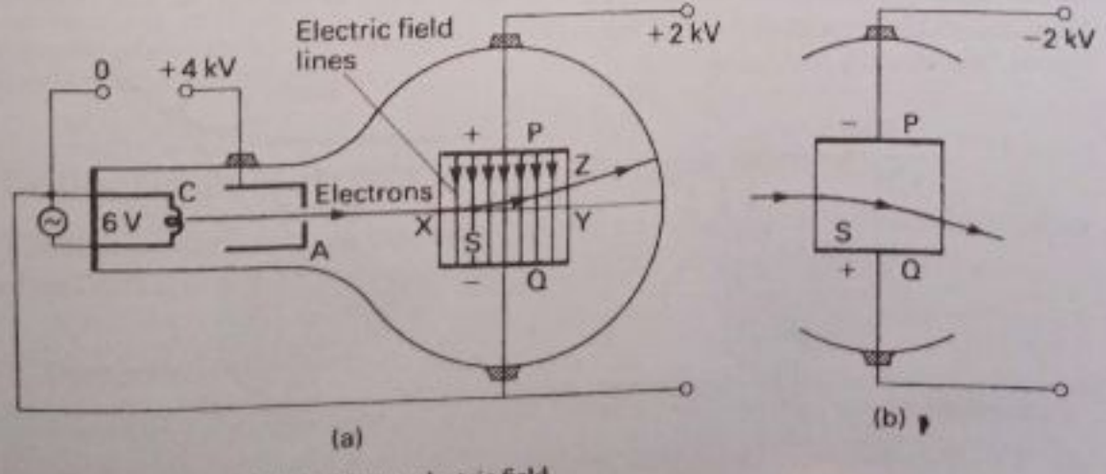


Fig. 33.2 Electrons deflected by an electric field

and −ve terminals are reversed, so that Q is now +ve and P is −ve, the electron beam is deflected downwards to Q (Fig. 33.2b).

Conclusion: Since the charge on the electron is attracted towards the plate of opposite sign to itself, we see from Fig. 33.2 that the charge on an electron is negative.

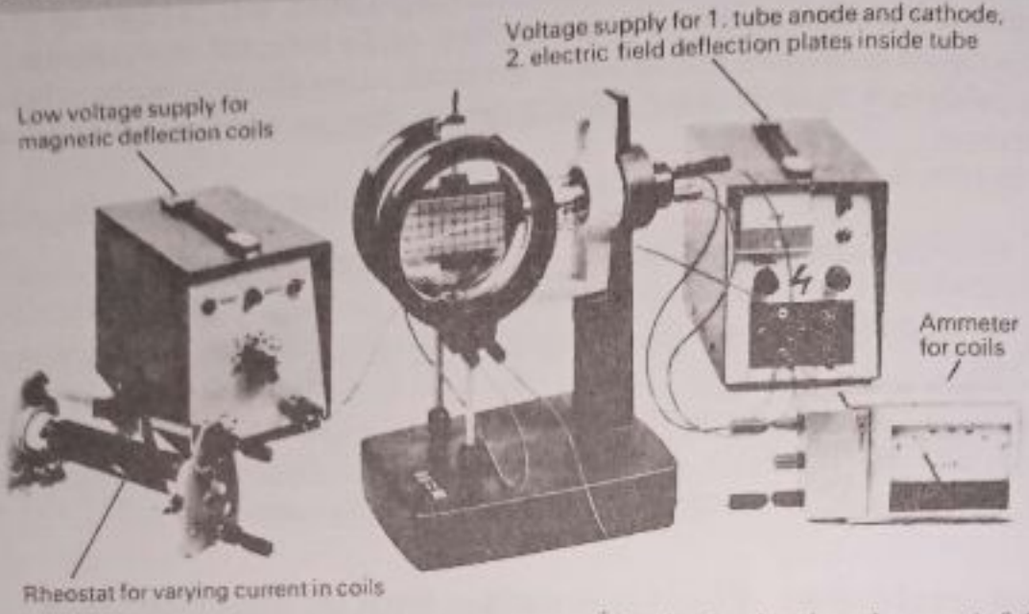


Plate 33B Teltron tube, used for experiments in electron deflection by electric and magnetic fields (*Philip Harris Ltd*)

The p.d. of 2 kV connected between P and Q produces an electric field between the two plates. Fig. 33.2a shows some typical field lines, which pass from P to Q. When they enter this field, the electrons are deflected along the curved path XZ, which is part of a parabola. In Fig. 33.2b, the path of the electrons is like the path of a ball falling in air, because the electron is accelerated vertically with constant acceleration in the electric field while travelling with a uniform velocity in a horizontal direction (see page 31).

Deflection of electrons in magnetic field

With the battery disconnected from P and Q, we can examine the effect on the electron beam of a *magnetic field*.

Fig. 33.3a shows the path of the beam when the N pole of a strong magnet is placed perpendicular to the beam. The electron beam is deflected along BD in a circular path. Fig. 33.3b shows the electron beam deflected in the opposite direction in the magnetic field of the S pole.

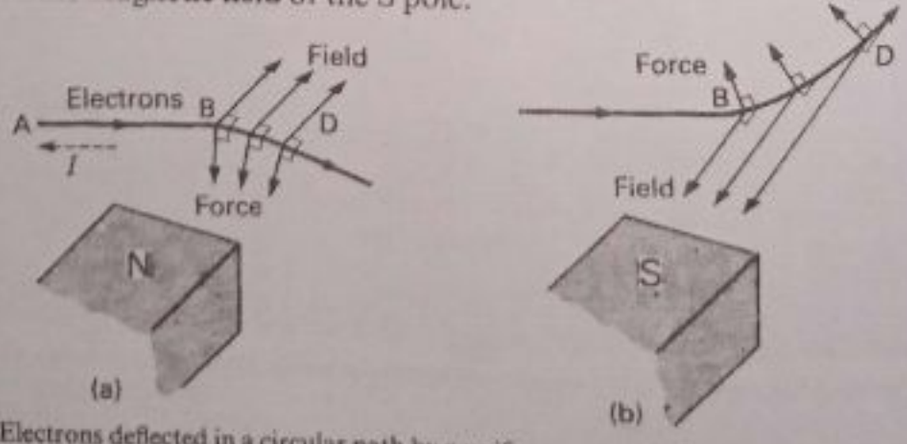


Fig. 33.3 Electrons deflected in a circular path by a uniform magnetic field

Scanned with CamScanner

Explanation We can explain this effect from our previous study of the force on a current-carrying conductor. The electron beam is an electric current. The magnetic field is perpendicular to this current. The beam is therefore deflected by a force *perpendicular* to both the beam and the field, as shown at B in Fig. 33.3a. If we apply Fleming's left-hand rule (page 467) in this case, we find that the current that gives a downward force flows in the direction BA. Fleming's rule applies to the 'conventional' current I, the movement of positive charge. Since the movement of positive charge from B to A is equivalent to the movement of negative charge from A to B, we conclude again that the electrons moving from A to B carry a *negative* charge.

At every point in the electron beam, such as at B or D in Fig. 33.3a, the force is perpendicular to the direction of the beam (since this is the direction of the current). So the force is a centripetal force (page 43). The path is circular as long as the magnetic field is uniform.

Plate 33C shows the effect on an electron beam when the magnetic field makes a small angle with the beam direction. This time the beam moves in a spiral path. The glow in the sky at the North pole, called the Northern Light or *aurora borealis*, is caused by electrons spiralling and bunching together on entering the earth's magnetic field. The concentrated electron beam produces a glow on hitting atoms of air. 'Van Allen radiation belts' are layers of spiralling ions and electrons which are trapped high in the atmosphere by forces of the earth's magnetic field.



Plate 33C Spiral electron path in Leybold fine-beam tube, produced by a magnetic field applied at a small angle to the electron beam. (*Leybold-Hiraeus GMB and Co*)



Plate 33D The Aurora Borealis seen from a forest in Alaska. (*U.S. Department of Energy*)

Collecting electrons · Properties of electrons

The charges carried by electrons can be collected by a Teltron Perrin tube, named after Perrin, a French scientist who devised the method about 1895.

Demonstration

The circuit is arranged as shown in Fig. 33.4. The 6 V supply is used to heat the filament F and produce electrons, and the high voltage (E.H.T.) supply of 2–5 kV between the anode

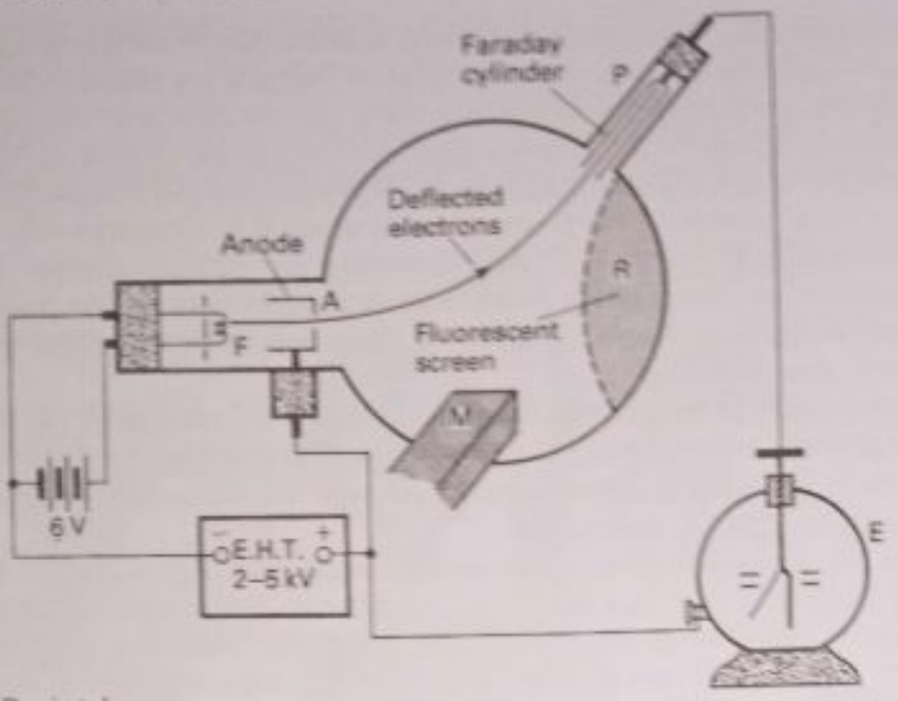


Fig. 33.4 Perrin tube

A and F makes the electrons pass at high speed through the hole in A. A leaf electroscope E is joined to a metal cylinder P called a Faraday cylinder and to the +ve terminal of the high voltage supply.

A strong magnet M is now brought up slowly *perpendicular* to the electron beam. This deflects the beam and at some stage the electrons enter P as shown in Fig. 33.4. The electroscope leaf is then seen to open steadily.

To find the sign of the charge on the leaf, the circuit between P and the electroscope is broken. A rubbed polythene rod, which has a −ve charge, is then brought near the cap of the electroscope. The leaf opens more, showing that the charge on the electrons was −ve.

Other properties of electrons can be shown by Teltron tubes.

1. Electrons travel in straight lines: a sharp shadow of a metal Maltese cross is seen on the fluorescent screen in the tube shown in Fig. 33.5.
2. Electrons have momentum: they exert a force when incident on light metal

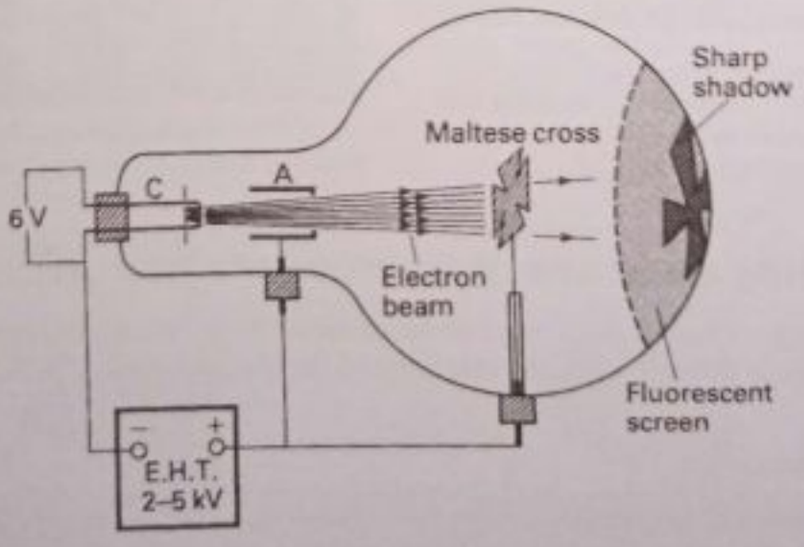


Fig. 33.5 Maltese cross shadow showing electrons travel in straight lines

Scanned with CamScanner

vanes which makes the vanes turn round. The particles have energy: a small piece of metal glows when electrons strike them, showing heat has been produced.
3. Electrons make certain materials fluoresce.

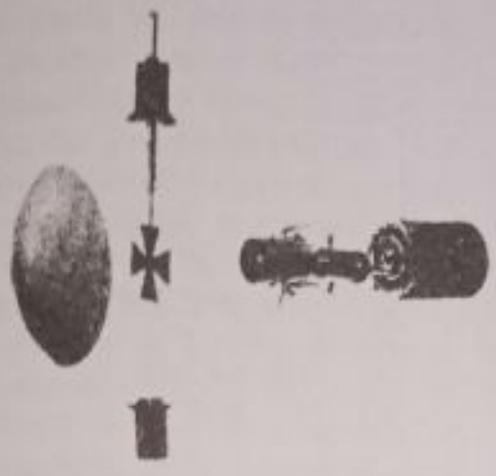


Plate 33E Teltron Maltese cross tube.
(*Philip Harris Ltd*)

Diode valves

The diode valve was invented by Sir J A Fleming in 1902 for use in radio communication.

Fleming's valve contained (a) an emitter of electrons, which was a cathode C, (b) a nickel plate or anode A, which collects electrons from the cathode. This is called a *diode* (two-electrode) valve. The electrodes are inside a small glass container from which all the air is pumped out, so that the electrons emitted from C are not stopped by air molecules from reaching A.

In early radio valves, the cathode was a hot fine tungsten wire. This is called a *directly heated* valve. Modern valves also contain a tungsten wire, but it is used only as a source of heat. It warms a small cylinder near it whose surface is coated with a mixture of barium and strontium oxides, which readily emits electrons (Fig. 33.6a). The small cylinder is therefore the cathode in this case and the wire is the heater. This is an *indirectly heated* cathode. The voltage needed for the heater is low, such as 4 V or 6·3 V; it can be obtained from an a.c. or d.c. supply.

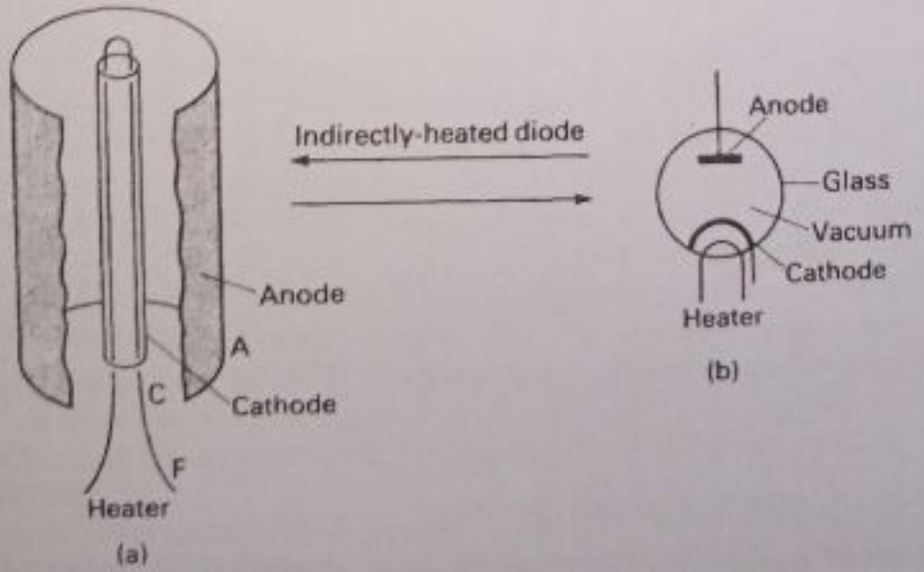


Fig. 33.6 Diode valve, indirectly-heated cathode

Scanned with CamScanner

Diode characteristic

Like any other type of conductor, the diode characteristic (I–V curve) can be found by varying the voltage V applied to it and measuring the current I.

Experiment

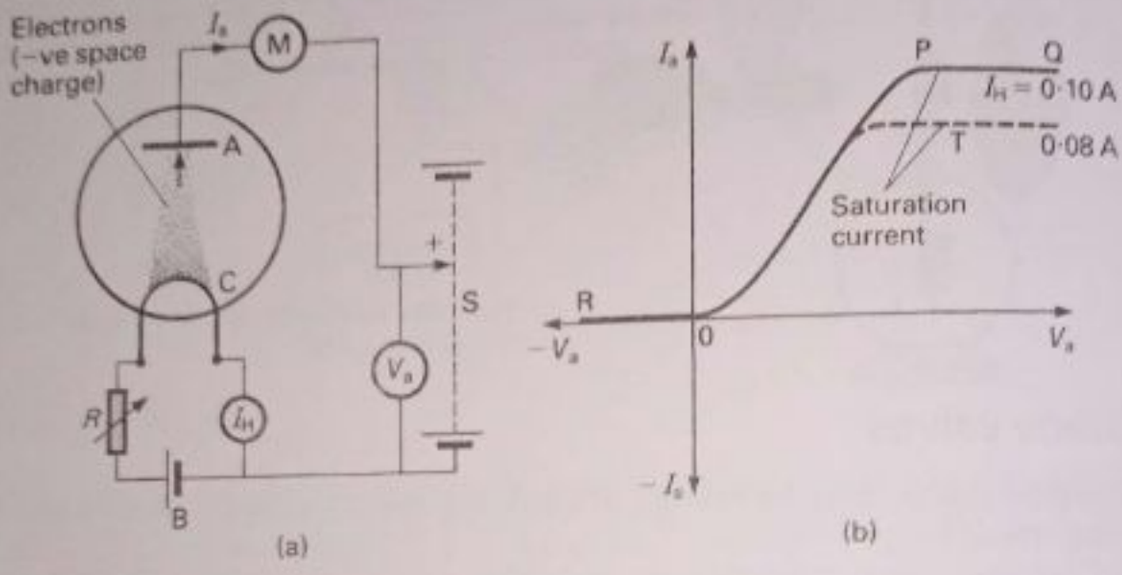


Fig. 33.7 Diode valve characteristic

The cathode C is heated directly by connecting a low voltage battery B such as 6 V to it, with a rheostat R included to vary the heater current I_H (Fig. 33.7a). A second battery supply S enables the voltage V_a between the anode A and cathode C to be varied. A is +ve in potential relative to C. The current I_a in the anode circuit is measured by a meter M.

Starting from zero, V_a is increased in suitable steps. At each stage, V_a and I_a are measured. The rheostat R in the heater circuit is then altered to decrease the heater current I_H, and the experiment is repeated.

The battery supply terminals of S are now reversed, so that A has a −ve potential relative to the cathode C. The anode voltage V_a is increased but this time no anode current I_a flows.

Results: Fig. 33.7b shows the results obtained when I_a is plotted against V_a. After an initial bend, a straight line is obtained. As the voltage is increased, however, the current reaches a maximum value at P and remains constant as the voltage is increased further. So PQ is a horizontal line. This is the *saturation current*, explained shortly.

When the *heater current* is decreased, a graph of similar shape is obtained but the saturation current is less, as shown at T.

When the anode voltage is reversed so that A is −ve in potential relative to C, no anode current is obtained. So for −ve voltage V_a, the graph is a line OR along the voltage axis. A typical diode characteristic, then, is a graph similar to ROQ.

Explanation of the diode characteristic

We can now explain the shape of the diode characteristic. When the anode voltage V_a is small and +ve, electrons from C are attracted to the anode A. These

Scanned with CamScanner

electrons flow through the meter M round the circuit and reach the cathode C again. The current I_a is a measure of the electron flow.

The number of electrons emitted per second by the cathode depends only on its temperature. The number reaching A is less than this number when the voltage V_a is small, because the electrons moving in the space between C and A at any instant repel some of the electrons back to C. We say that the 'cloud' of electrons between C and A form a negative 'space charge'.

As the +ve anode voltage V_a is increased, the attractive effect on electrons emitted from the cathode increases. The repulsion effect of the space charge is then diminished, and so the anode current I_a increases. At one stage, when the anode voltage reaches a high value corresponding to P, *all* the electrons emitted per second by the cathode at its particular temperature are collected by the anode; and as further increase in anode potential does not now increase the current, as shown by PQ in Fig. 33.7b, this is the saturation current.

When the cathode temperature is lowered by decreasing the heater current I_H, fewer electrons per second are emitted by the cathode. This is why the saturation current is less than before.

No anode current flows when the anode voltage is negative relative to the cathode. The electrons, which carry negative charges, are all repelled back to the cathode in this case. The diode is called a 'valve', because electrons flow only in one direction through it, from cathode to anode, and then only when the anode is positive in potential relative to the cathode.

Resistance of the diode

We can think of the diode as a special kind of electrical resistance component. A metal wire such as copper or tungsten at constant temperature has the same electrical resistance R whichever way the terminals of a battery are connected to it. So the current I has the same numerical value when the p.d. V connected to it is reversed (Fig. 33.8a).

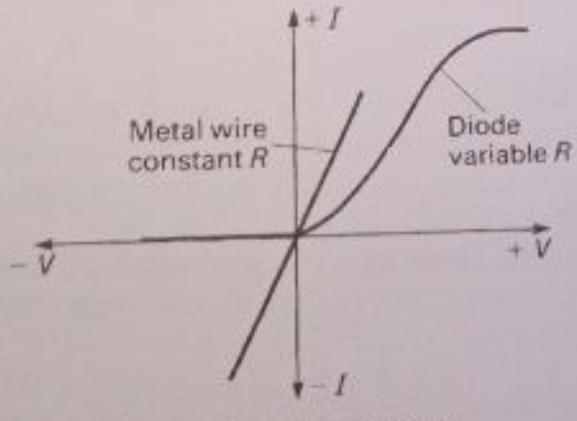


Fig. 33.8 Metal (ohmic) and diode (non-ohmic) characteristics

In contrast, the diode conducts relatively easily when the anode potential is positive relative to the cathode; its 'resistance' is then low. But when the anode potential is negative relative to the cathode no current flows. Its resistance is then infinitely high.

The metal wire obeys Ohm's law, that is, the current is always proportional to the p.d. applied (Fig. 33.8a). The diode valve does not obey Ohm's law (Fig. 33.8b); it is called a *non-ohmic conductor*.

Scanned with CamScanner

The diode as a rectifier

We now investigate a property of the diode which is used in television and radio receiver circuits.

Experiment

The circuit in Fig. 33.9a is set up. The diode is connected in series with a low *alternating voltage*, or a.c. voltage, V, of a few volts and a resistance R of say 2000 ohms. For convenience, the cathode heating supply is omitted from the diagram.

The voltage V_R across R is compared with V by connecting one end of R to the Y_2 plate (see page 542) of a double-beam oscilloscope, one end of V to the Y_1 plate and their common terminal to earth (Fig. 33.9a).

Results: From the waveforms on the screen (Fig. 33.9b) we see that only the positive half of the varying voltage V appears across R. So V_R is a varying direct voltage, or d.c. voltage, because all its values are positive. We say that the applied alternating voltage V has been rectified by the diode and the diode is called a *rectifier*.

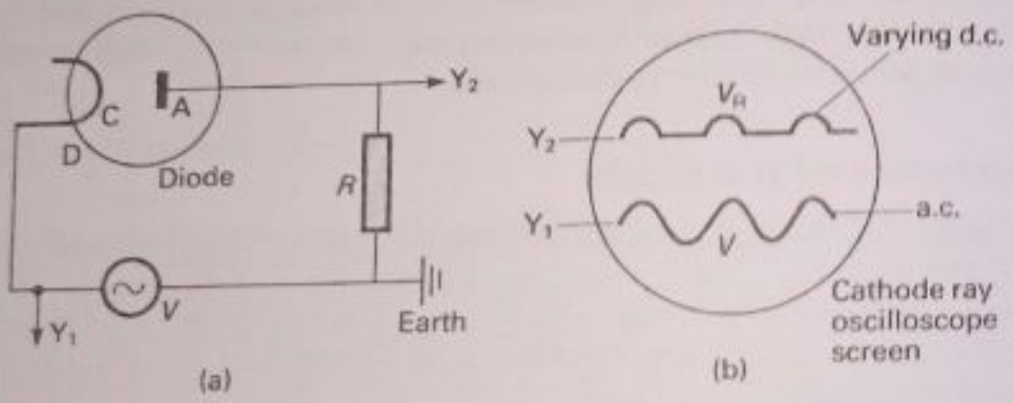


Fig. 33.9 Diode valve as a rectifier

The explanation of diode rectification is simply that on the +ve half of the cycles of V, the diode conducts. On the −ve half of the cycles, the diode does not conduct. So on the +ve halves only, electrons flow through R. The voltage V_R at any instant is IR, where I is the current then flowing.

Used as a rectifier, the diode provides the necessary direct voltage (d.c.) for components in mains television and radio receivers. No batteries are then needed.

Metal rectifier

One of the earliest rectifiers was a carborundum crystal with a steel point pressing against it. This led to the symbol now used to represent a rectifier in circuits, shown in Fig. 33.10a. The crystal–steel junction had a low resistance in the direction from crystal to steel and a high resistance in the opposite direction.

A copper oxide/copper junction, made by oxidising one face of a copper disc, acts as a rectifier (Fig. 33.10b). These *metal rectifiers* are used in battery chargers to provide direct current (d.c.) from the a.c. mains supply.

Two rectifiers can be used to obtain 'full-wave rectification'. Using a

Scanned with CamScanner

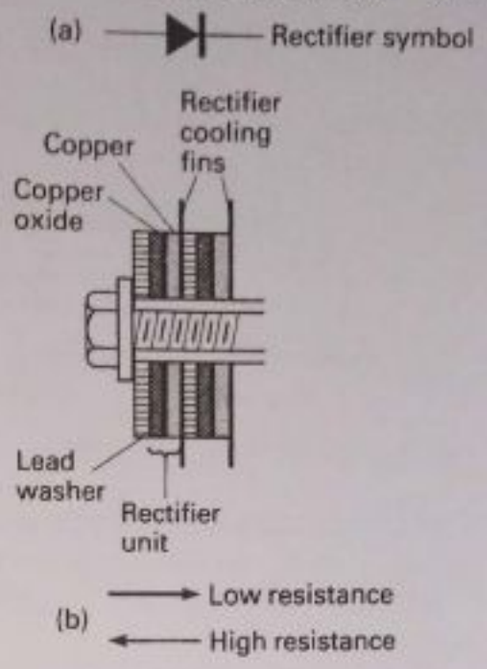


Fig. 33.10 Metal (copper oxide) rectifier

transformer with a centre tap T in the secondary coil (Fig. 33.11) the diode D1 or other rectifier conducts on one half of the a.c. voltage between A and B and the diode D2 conducts on the other half of the *same* cycle. So the voltage V_R across R is one with consecutive half-way voltages all in one direction, as shown in the figure.

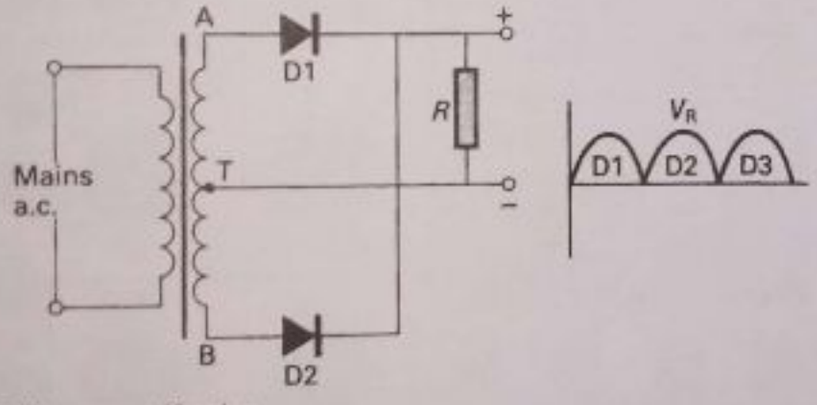


Fig. 33.11 Full-wave rectification

Cathode ray oscilloscope

The discovery of cathode rays or electrons led to a large number of applications now grouped together under Electronics. The cathode ray tube is an important application; it is used in television and radar. In the cathode ray oscilloscope (C.R.O.) it is used for investigating electronic circuits in radio, television and telecommunications.

Basically, the C.R.O. has (Fig. 33.12)

(a) a hot cathode C which emits electrons;

(b) a plate G at a −ve potential relative to C to control the number of electrons per second reaching the fluorescent screen or to control brightness of the glow seen;

(c) an accelerating anode A_1, which accelerates the electrons to high energy and forms an 'electron gun' with C;

(d) an *electron lens*, which consists of metal plates or cylinders A_1, A_2, A_3 all at high +ve potential relative to C of the order of a thousand volts. The electron

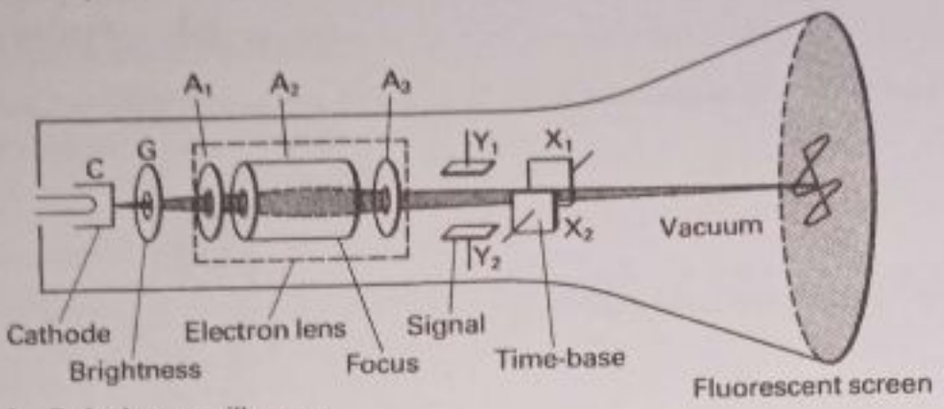


Fig. 33.12 Cathode ray oscilloscope

beam, which passes through these anodes, is focused on the screen by varying the potential of A_2. So a *focus control* is obtained;

(e) two metal plates, Y_1, Y_2, called Y plates, to which the voltage examined is connected;

(f) two metal plates X_1 X_2, called X plates, to which a *time-base* voltage can be connected.



Plate 33F Oscilloscope used in industry to test electronic apparatus. (*J. Allan Cash Ltd*)

Scanned with CamScanner

Time-base · Brightness of trace

The electron beam passes through the Y and X plates on its way to the screen. An unknown varying voltage or signal connected to the Y plates makes the beam move up and down. So a vertical line is traced on the screen. When the time-base voltage is switched on to the X plates, the beam is now also swept horizontally and provides a time-axis. So the waveform of the unknown voltage can be seen. In this way a faulty component can be traced in a radio or television receiver.

Increasing the heater current will increase the number of electrons per second emitted from the cathode or 'electron beam current'. The number of electrons per second striking the screen will then increase and so the brightness of the spot, and waveform on the screen, will increase. The brightness will also increase if the voltage of the accelerating anode is increased, as the energy of the electrons reaching the screen will then increase.

The spot is swept across the screen with a speed controlled by the frequency of the time-base voltage. If the speed is increased, an a.c. voltage connected to the Y plates produces fewer waves on the screen. When the spot is swept across the screen it spends less time at a given place as the speed of the time-base increases. So the brightness of the waveform on the screen decreases.

Oscilloscope as d.c. and a.c. voltmeter

The oscilloscope can be used to measure d.c. and a.c. voltages either by using a spot of light on the screen or by using the time-base.

1. *Using a spot* Fig. 33.13a shows a spot on the screen, that is the time base is not connected to the X plates.

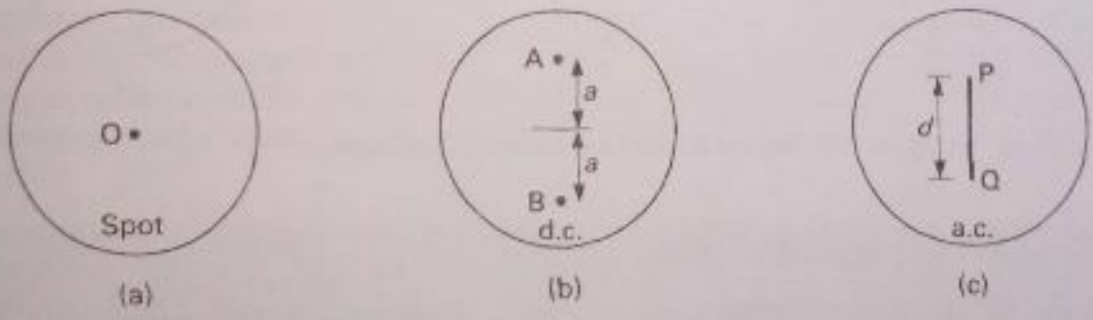


Fig. 33.13 Using a spot for d.c. and a.c. voltage measurement

(a) After centering the spot at O on the screen, a d.c. voltage is connected to the Y plates so that the top plate is + ve. The spot is then deflected upwards to A (Fig. 33.13b). With the d.c. voltage reversed, so that the bottom plate is + ve, the spot is now deflected downwards to B. In each case the displacement a of the spot is proportional to the voltage, V. So by measuring a and the displacement due to a *known* d.c. voltage, V can be found. The oscilloscope can therefore be used as a d.c. voltmeter.

(b) Using the X plates instead of the Y plates, the same d.c. voltage deflects the spot sideways or horizontally.

(c) With an a.c. voltage of very low frequency connected to the Y plates, the spot is seen moving up and down. If the mains frequency, 50 Hz, is used, the up and down movement of the spot is now so fast that a line PQ is obtained (Fig. 33.13c).

Scanned with CamScanner

The length d of PQ represents a voltage from the maximum of the a.c. voltage to the minimum. So d is proportional to twice the maximum (peak) a.c. voltage, V. To find V, the length of the line produced by a *known* a.c. voltage is measured and compared with d, the length for the unknown voltage. So the oscilloscope can be used as an a.c. voltmeter.

2. *Using the time-base* Fig. 33.14 shows some demonstrations and uses of the oscilloscope when the time-base is connected to the X plates. If the time-base is one corresponding to 2 milliseconds or 0·002 s, this means that the spot moves horizontally across the screen with a steady velocity in 0·002 s, then flies back very fast to its starting point, and repeats this movement across the screen. So a straight line D is seen on the screen (Fig. 33.14a) and the length of D represents a time of 0·002 s.

(a) *d.c. voltage* When a d.c. voltage is connected to the Y plates so that the top plate is +ve, the line is displaced to E (Fig. 33.14b). If the bottom plate is +ve, the line is deflected downwards to F. As explained before, the displacement a in either case is proportional to the d.c. voltage V, and so V is found by using a known d.c. voltage.

(b) *a.c. voltage* When an a.c. voltage is connected to the Y plates, a stationary wave can be obtained on the screen (Fig. 33.14c). The height b from maximum to minimum represents twice the maximum voltage, V. As explained before, V is found by using a known a.c. voltage.

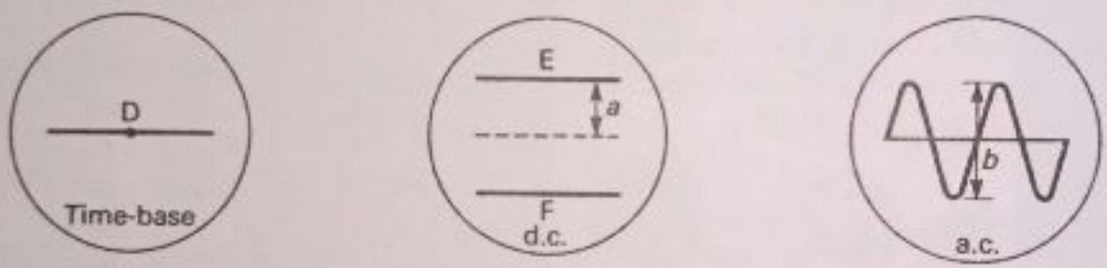


Fig. 33.14 Using the time-base for d.c. and a.c. voltage measurement

Oscilloscope as a clock

In addition to its use as a voltmeter and its use to show waveforms of voltages the oscilloscope can be used as a clock for radar and frequency measurement of a.c. voltages. This is illustrated in the following examples.

Examples on oscilloscopes

1. The length 30 mm of the horizontal line XY due to the time-base represents a time of 0·003 s (Fig. 33.15a). A radar signal is sent out at a time corresponding to A and received back after reflection from a distant aeroplane at a time corresponding to B. Calculate the distance of the aeroplane if the length AB is 10 mm. (Speed of radio waves = 300000 km/s.)

$$\text{Time to and from aeroplane} = \frac{10\text{ mm}}{30\text{ mm}} \times 0{\cdot}003\text{ s}$$
$$= 0{\cdot}001\text{ s}$$
$$\text{So distance to and from aeroplane} = 0{\cdot}001 \times 300000$$
$$= 300\text{ km}$$
$$\text{So distance of aeroplane} = \tfrac{1}{2} \times 300 = 150\text{ km}$$

Scanned with CamScanner

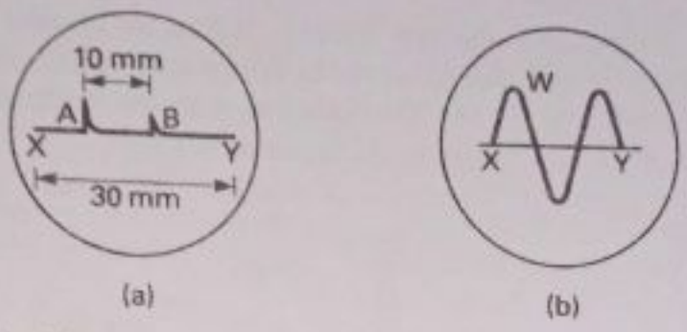


Fig. 33.15 Measuring (a) time and (b) frequency

2. The length XY of the time-base on an oscilloscope represents a time of 0·003 s. A stationary wave W is obtained on the screen as shown in Fig. 33.15b when an a.c. voltage V is connected to the Y plates. Find the frequency of V.

W represents $1\frac{1}{2}$ cycles of the a.c. voltage V. So $1\frac{1}{2}$ cycles takes a time of 0·003 s

$$\text{So 1 cycle takes a time of } \frac{1}{1\frac{1}{2}} \times 0{\cdot}003 = \frac{2}{3} \times 0{\cdot}005 = 0{\cdot}002\,\text{s}$$

$$\therefore \text{ number of cycles per second} = \frac{1}{0{\cdot}002} = \frac{1000}{2} = 500$$

$$\therefore \text{ frequency} = 500\,\text{Hz}$$

X-rays

In 1895 a German physicist, Roentgen, found that a penetrating radiation was coming from a discharge tube when cathode rays were produced inside. He covered the tube with black paper, but found that a screen near it coated with a barium salt fluoresced every time the tube was operated. As Roentgen said when reporting this discovery, 'The most striking feature of this phenomenon is the fact that an active agent here passes through a black cardboard envelope, which is opaque to the visible and ultra-violet rays of the sun ... an agent which has the power of producing active fluorescence'.



Plate 33G X-ray photograph of a crab. (*Barnaby's Picture Library*)

Scanned with CamScanner

He called the unknown radiation X-rays. It is now known that X-rays are produced when fast-moving electrons strike heavy atoms such as those of metals or even atoms of glass (Fig. 33.16). On collision, some of the kinetic energy of the electrons is transformed to energy in the form of X-rays.

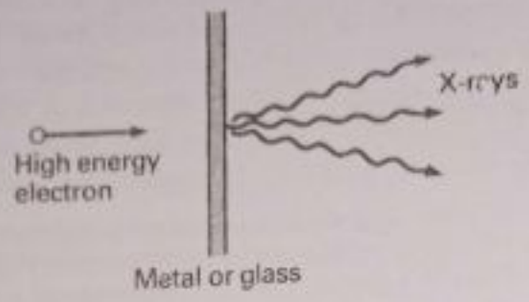


Fig. 33.16 Production of X-rays

Nature of X-rays

The nature of X-rays was not definitely known for many years after their discovery. It was thought they were of the same nature as light rays. If so, X-rays should be diffracted at very narrow openings like light rays, as explained on page 355. The diffraction gratings used for light rays, however, produced no diffraction of X-rays. So X-rays probably had a wavelength much shorter than light waves.

In 1911 Laue realised that the atoms of a crystal could form a natural diffraction grating for very short wavelengths; they were regularly spaced and about 10^{-10} m apart. When X-rays were passed through a crystal a diffraction pattern was formed on a photographic plate. We now know that *X-rays are electromagnetic waves with wavelengths about* 1000 *times shorter than light waves.*

After Laue's discovery was announced two British scientists William Bragg and his son Lawrence Bragg began researches into X-rays, for which they were awarded the Nobel Prize in 1915. They measured the wavelengths of X-rays and then began a study of the structure of crystals using X-rays. This has led to a knowledge of the structure of molecules of fibres, for example, the manufacture of new materials, and the discovery of the structure of the molecule DNA, a vital constituent in the living cell.

Some properties of X-rays are (a) they travel in straight lines, (b) they are not deflected by magnetic or electric fields because they are electromagnetic waves, (c) they cause some substances to fluoresce, (d) they are diffracted by crystals, (e) they produce ionisation in gases.

X-ray tubes

X-ray tubes are widely used in medical treatment and in scientific laboratories. Fig. 33.17 shows the essential features of the interior of a modern X-ray tube.

A hot filament or cathode emits electrons. T is a metal 'target', made of copper or tungsten, for example, which emits X-rays when it is bombarded by the electrons. The target is embedded in the copper block A at the end of a metal rod. A is the anode, and it is kept at a high voltage, such as 100 kV or higher, depending on the use of tube. Electrons then reach A with high energy.

Scanned with CamScanner

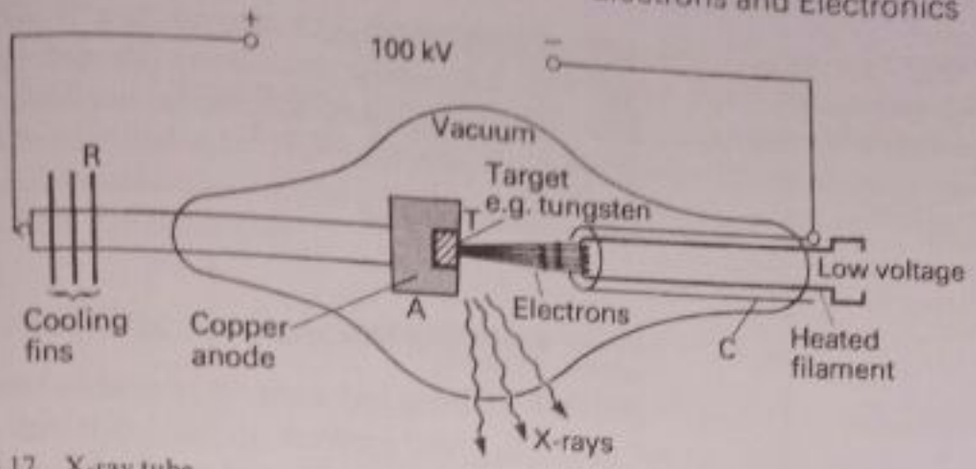


Fig. 33.17 X-ray tube

Only a very small part of this energy is converted into X-radiation. The rest is converted into heat, so that A becomes very hot. Cooling fins R, as shown, may therefore be used to transfer the heat conducted along the rod to the surrounding air by convection and radiation. In large X-ray tubes used in hospitals the anode is cooled by oil flowing behind it continuously. The X-ray tube is evacuated of air so that the electrons are not hindered in moving from the cathode to A, and a metal cylinder C is used to focus the electrons on the target area.

Photoelectric emission

As we have seen, X-rays are emitted from some metals when high energy electrons are incident on them. The opposite effect can also be obtained. When ultra-violet light (electromagnetic waves) falls on zinc, electrons are emitted from the metal surface. The metal caesium emits electrons when visible light falls on its surface.

This is called *photoelectric emission* of electrons. In contrast, thermionic emission of electrons (page 533) occurs when a metal is heated. The photoelectric effect is widely used in industry. Photoelectric cells contain light-sensitive metals and they are used to make the sound track on films work by transforming light energy to electrical energy. A photoelectric cell can be used to make doors open automatically in stores when a light beam is interrupted by someone and also to operate burglar alarms.

Semiconductors

Semiconductors are materials which have an electrical resistance between the high value of insulators such as perspex and the low value of metals such as copper. Silicon and germanium are examples of solid semiconductors. Semiconductors are widely used in the electronics industry. The silicon chip, used in microelectronics, is a tiny piece of semiconductor containing many components for use in computers or communications. See page 531.

Electron and hole carriers

When a battery is connected to a pure semiconductor S, a small current flows (Fig. 33.18). The current is carried by two types of carriers, in a similar way as current is carried through electrolytes by two carriers, negative and positive ions. One carrier in a semiconductor is a free electron, which has a negative charge.

Scanned with CamScanner

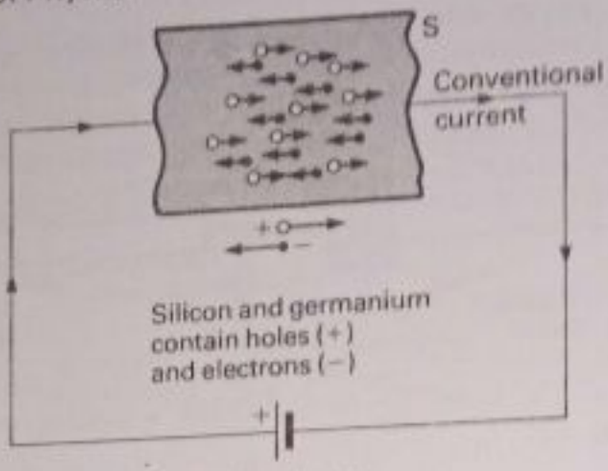


Fig. 33.18 Electron and hole carriers in semiconductors

The other has a positive charge equal numerically to the charge on an electron; this carrier is called a *hole*. Its existence can only be explained by the theory of semiconductors, which is outside the scope of this book.

So, in a semiconductor, the carriers are free electrons (negative charges) and holes (positive charges). In metals, however, the carriers of current are only free electrons. Unlike a metal, the resistance of a pure semiconductor *decreases* with temperature rise, because this sets free more electrons and holes inside the semiconductor.

N- and p-type semiconductors

Pure semiconductors have equal numbers of electrons and holes. There are not many electrons and holes. If a great number of holes or positive charges are required, the element is 'doped' by adding to it a tiny amount of a suitable foreign element. The impure semiconductor is then called a p-semiconductor (p means

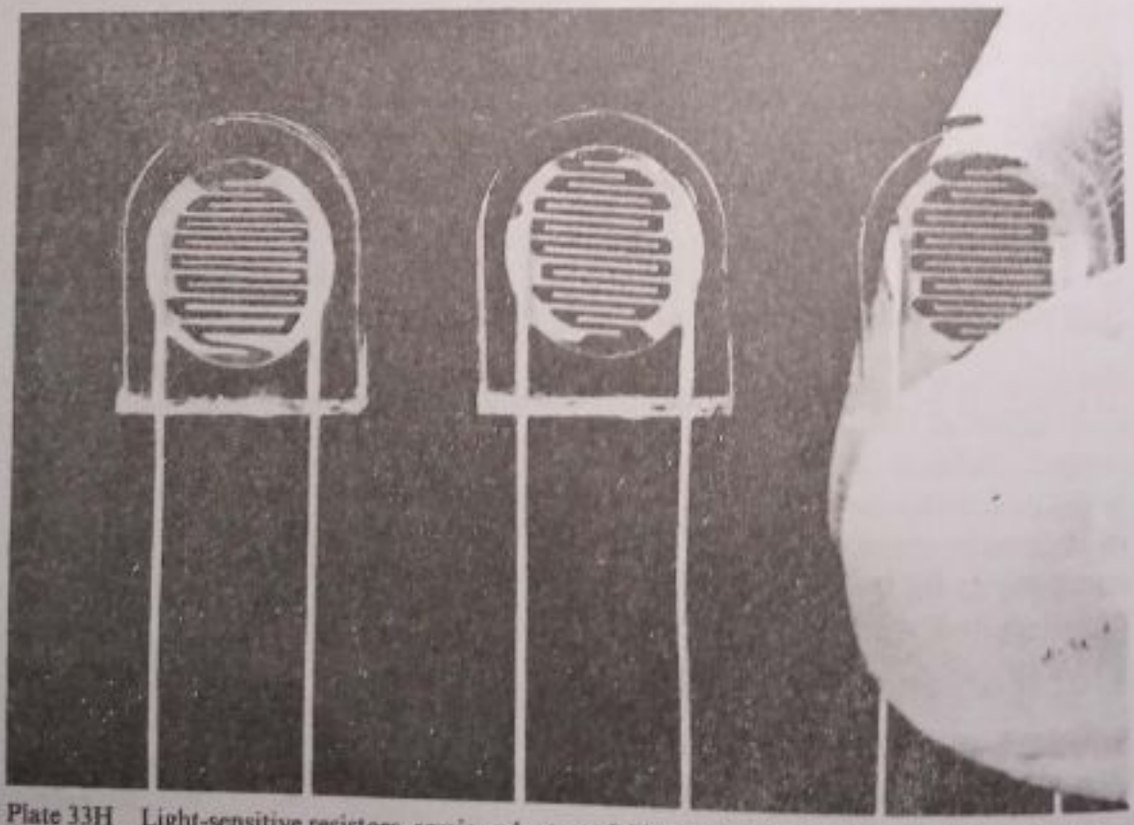


Plate 33H Light-sensitive resistors, semiconductors, falling from 10 megohms (dark) to 100 ohms (bright). They are used to trigger electric circuits in computers by using a light source. (*Mullard Ltd*)

Scanned with CamScanner

positive). If a great majority of electrons or negative charges are required, a different impurity is added to the pure semiconductor. An n-type semiconductor is now obtained (n means negative). P- and n- semiconductors are widely used in the electronics industry. *Thermistors* are heat-sensitive resistors that can be made from semiconductors. Light-sensitive resistors are also made from semiconductors. See plate 33H.

Semiconductor junction diode

A special industrial process can produce an alloy of p- and n-semiconductors with a very thin junction between them. This is called a *p–n junction* and is shown diagrammatically in Fig. 33.19. The junction has a very useful property which can be demonstrated simply.

Experiment

A low voltage dry battery is joined with its +ve pole to the p-terminal and its −ve pole to the n-terminal of the junction, with a milliammeter in series as shown in Fig. 33.19a. We see from the meter that the p–n junction diode conducts well, that is, it has a fairly low resistance.

The battery is now reversed as in Fig. 33.19b. This time there is only a very slight deflection on the meter. So the junction is conducting only very slightly; that is, it has a high resistance.

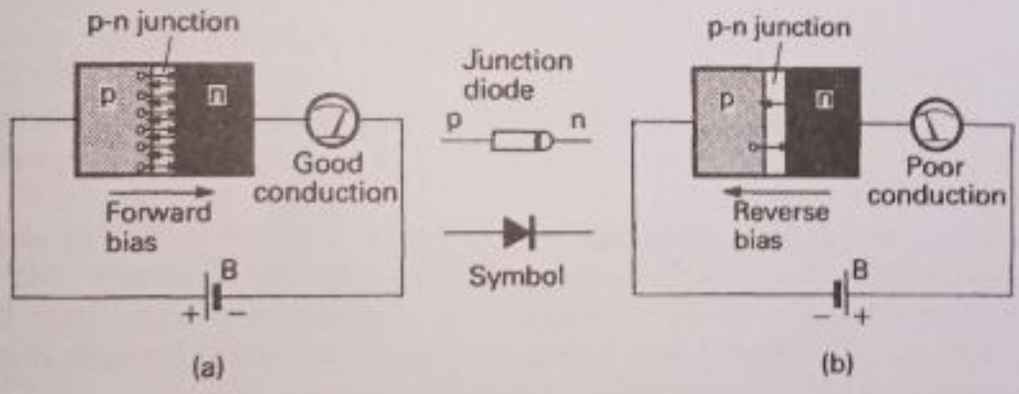


Fig. 33.19 Junction diode

Explanation In Fig. 33.19a, the +ve terminal of the battery repels the numerous p-charges or holes in the p-semiconductor because they are +ve charges. They are pulled across the junction by the attractive effect of the −ve terminal of the battery. In the same way, the n-charges or electrons (−ve charges) in the n-semiconductor are repelled by the −ve battery terminal and pulled across the junction by the +ve battery terminal. The many +ve and −ve charges moving across the junction produce an appreciable current in the circuit. So the junction diode conducts well when the p-terminal is +ve and the n-terminal is −ve.

When the battery is reversed, however, the −ve pole of B repels only relatively few n-charges in the p-semiconductor across the junction to the +ve pole. Similarly, only relatively few p-charges in the n-semiconductor are repelled across the junction to the −ve pole. So a very small current flows. We see that the junction has a low resistance in one direction but a high resistance in the reverse direction.

Scanned with CamScanner

The characteristic, or I–V graph of the junction diode as it is known, is shown in Fig. 33.20a. The junction diode is a 'non-ohmic' conductor. Like the diode valve, whose characteristic or I–V graph is similar to that of Fig. 33.20a, the junction diode can be used as a rectifier, that is it can change a.c. to d.c. voltage. Fig. 33.20b shows how this is done for an alternating voltage V. The circuit is similar to that for the diode valve on page 540.

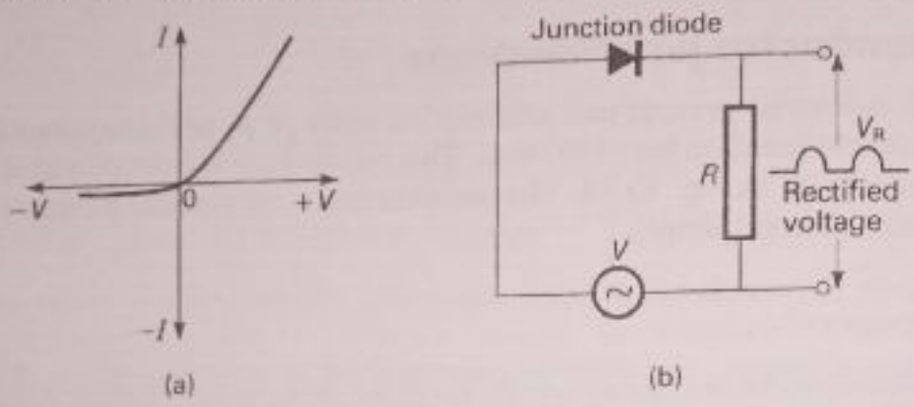


Fig. 33.20 (a) Junction diode characteristic (b) the junction diode as a rectifier

In many respects the junction diode is superior as a rectifier to the diode radio valve. For example (a) no heater is needed to produce the current carriers–they are already there; (b) it is less liable to break; (c) it is much smaller; (d) it is cheaper to manufacture in large numbers; (e) it needs only a small p.d. to work in a radio receiver.

Transistors

The junction diode can only rectify. In 1948 two American scientists, Bardeen and Brattain, found how to use a semiconductor to amplify current. They called their invention a *transistor*.

Basically, a transistor has three semiconductor layers. Fig. 33.21a shows diagrammatically two outer n-semiconductors with an extremely thin p-semiconductor sandwiched between them. This is called an n–p–n transistor. Fig. 33.21b shows an extremely thin n-semiconductor sandwiched between two p-semiconductors. It is called a p–n–p transistor.

Fig. 33.21c shows a whole transistor. The dot on the casing, or other mark, helps to identify the wire connected to the *collector* c. Fig. 33.21d shows the circuit symbol for an n–p–n transistor. The *emitter* e emits electrons which pass

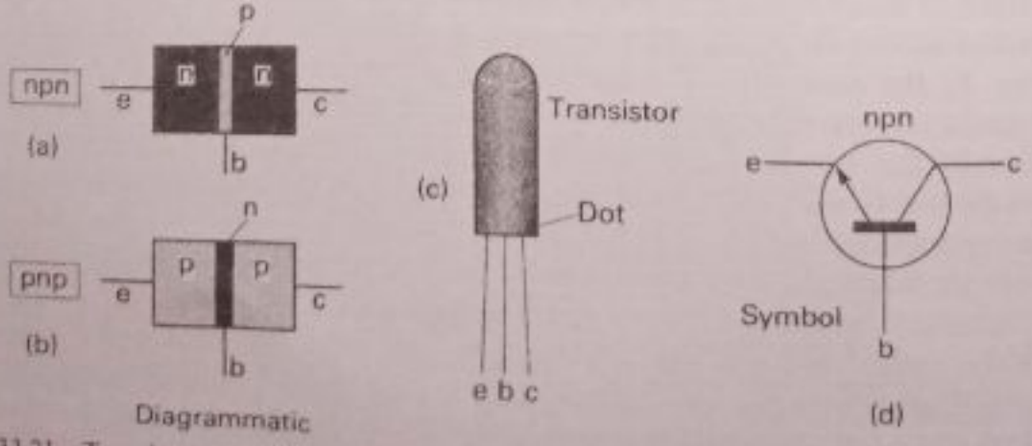


Fig. 33.21 Transistors

Scanned with CamScanner

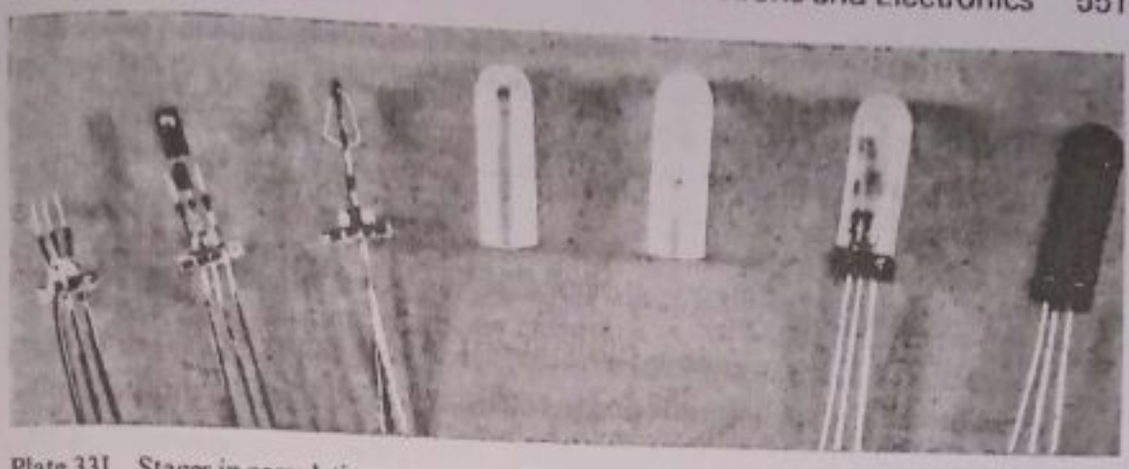


Plate 33I Stages in completing a transistor (from left to right). 1. Three leads for emitter, base and collector, 2. transistor attached to base (central) lead, 3. emitter and collector leads attached, 4. glass capsule, 5. capsule filled with protective grease, 6. transistor housed in sealed capsule, 7. capsule coated with black lacquer to exclude light, to which a transistor is sensitive (*Mullard Ltd*)

through the *base* b to the collector c. The arrow inside the transistor symbol shows the direction of conventional current flow; this is opposite to the electron flow.

Common-emitter amplifier

A simple circuit using an n–p–n transistor as a *current amplifier* is shown in Fig. 33.22. The −ve pole of a battery L is joined to the n-emitter and its +ve pole to the p-base b. Electrons in the n-emitter are then driven towards the base. The +ve pole of the battery M is joined to the n-collector and its −ve pole to the emitter. So the electrons reaching the base are now attracted towards the collector. The collector current I_c is shown on the meter Y. The base current I_b is shown on the meter X and can be varied by the rheostat R (Fig. 33.22b).

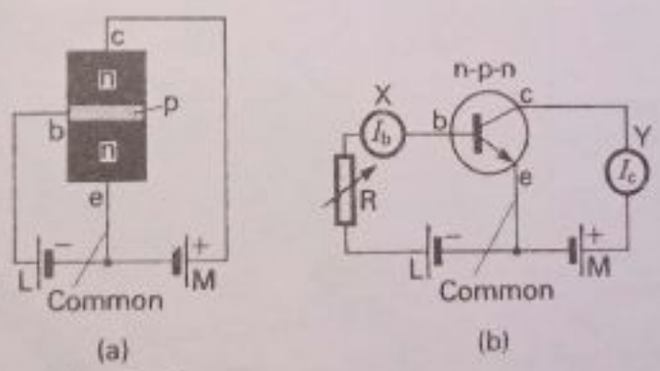


Fig. 33.22 Common-emitter circuit

Care must be taken to ensure that the collector c is +ve in the n–p–n transistor (it is −ve in the p–n–p transistor). Fig. 33.22 shows that the emitter lead is common to the base and collector circuits. This is why the circuit is called a common-emitter circuit.

If an n–p–n Mullard BC108 transistor is used with suitable batteries, experiment shows that a base current change from 60 μA to 90 μA, or 30 μA, produces a collector current change from 25 mA to 40 mA, or 15 mA. So the current change of 30 μA in the base circuit becomes a current change in the collector circuit of 15 mA, which is 15000 μA. So

$$\text{amplification} = \frac{15000}{30} = 500 \text{ times}$$

Scanned with CamScanner

This is why the common-emitter circuit is widely used as audio-frequency (a.f.) amplifier circuits in transistor radio receivers. The a.f. signal is always connected in the base circuit and the amplified signal is obtained in the collector-emitter circuit. With a suitable arrangement, *one* battery can be used to provide the two voltages of L and M in Fig. 33.22.

Simple receivers

The simple circuit shown in Fig. 33.23a can receive home or local broadcasting stations. AB is a coil of about 100 turns of thin (e.g. 36 s.w.g.) enamelled or

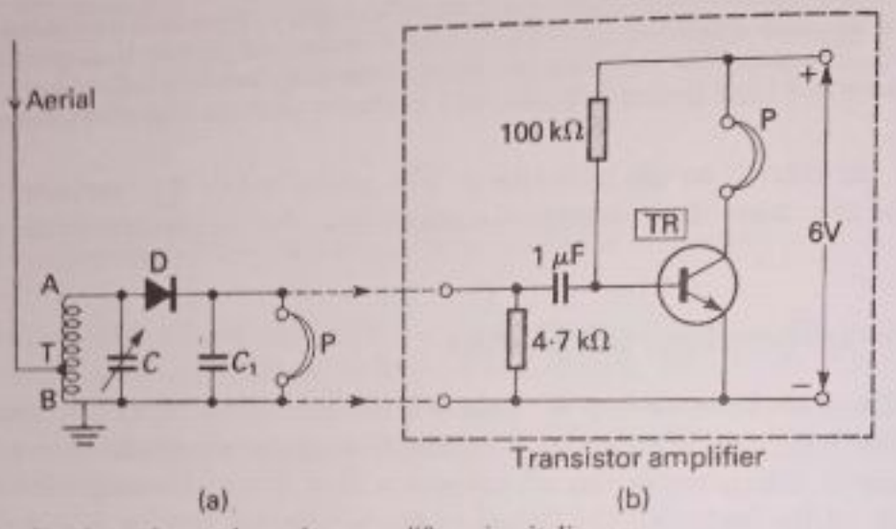


Fig. 33.23 Simple receiver and transistor amplifier: circuit diagram

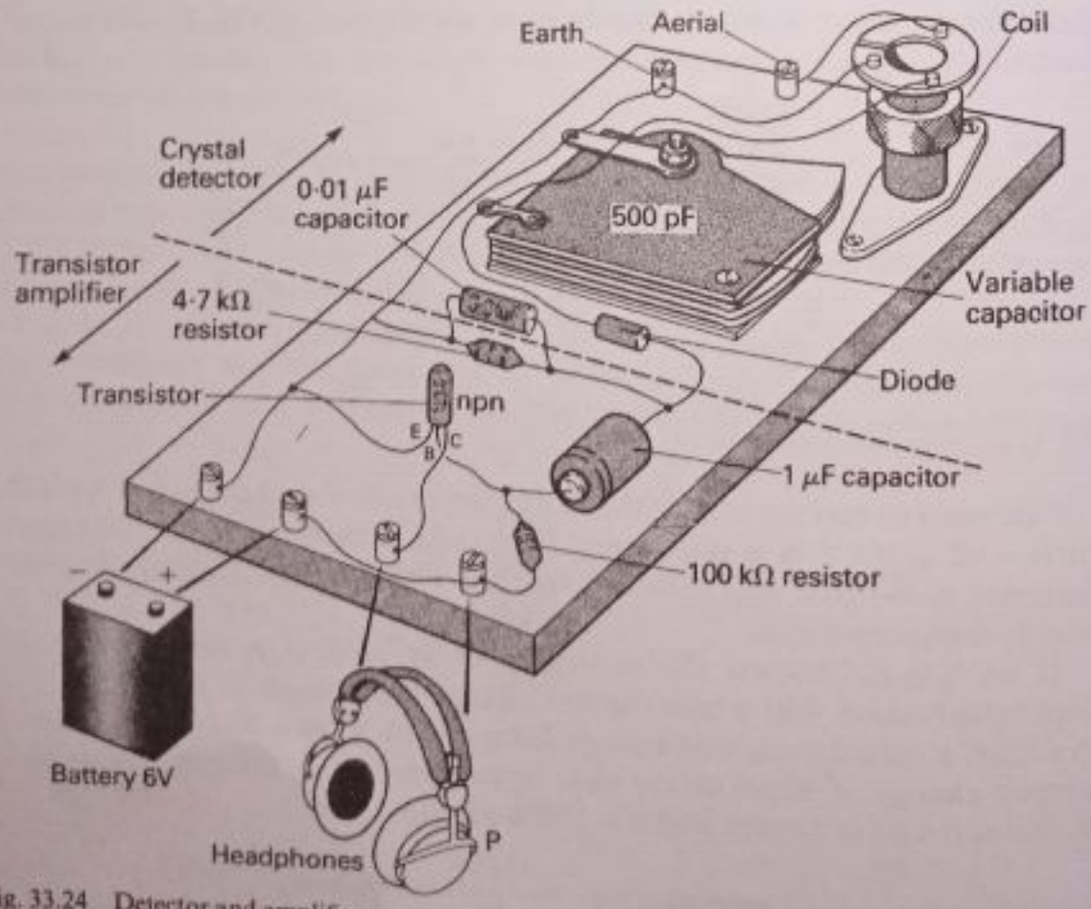


Fig. 33.24 Detector and amplifier components

Scanned with CamScanner

insulated wire on a small former (a suitable coil can be purchased); C is a small variable air capacitor, 0·0005 μF (500 pF) maximum; D is a junction diode (OA 60 or similar); C_1 is a 0·01 μF capacitor; P is a pair of high resistance headphones (e.g. 2000 ohms per earpiece). The aerial can be joined to A, or to T, about one third from B along the coil, for better reception. A good earth at B, such as a water-pipe, will increase the sensitivity. The aerial wire should be placed as high as possible. Fig. 33.23a is a semiconductor version of the old 'crystal' receiver. Fig. 33.24 shows the actual components.

To amplify the a.f. signals, an n–p–n transistor TR and the circuit shown in Fig. 33.23b can be added. The headphones P are transferred to the collector side of TR. The values of components given are not critical. Note that, for an n–p–n transistor, the negative pole of the battery must be connected to the emitter and the positive pole to the collector side, as shown. See also Fig. 33.24.

Example on cathode rays, thermionic emission, diode valves

(i) State one way in which 'cathode rays' differ from 'electromagnetic waves'? What experiment would you carry out to show this difference?

(ii) What particles are emitted in *thermionic emission*? Explain why they are emitted.

(iii) A diode valve circuit is arranged with a heater supply for its cathode and a battery is connected between the anode and cathode. Under what conditions is the anode current (a) zero, (b) saturation current?

(i) Cathode rays are particles carrying negative charges. Electromagnetic waves, such as light or radio waves, carry no charges and are not particles.

This can be shown by bringing up a strong magnet perpendicular to a beam of cathode rays and then to light rays. The cathode rays are deflected; the light rays are not affected.

(ii) The particles emitted in thermionic emission are electrons. When a cathode, for example made of tungsten wire, is heated to a high temperature, the free electrons in the metal gain sufficient energy to break through the surface of the metal.

(iii) (a) The anode current is zero when the anode voltage is zero. In this case the electrons emitted return to the cathode because the anode does not exert any attractive force on them.

(b) A saturation current is obtained when the attraction of the positive anode voltage on the electrons emitted by the cathode overcomes the repulsion on them due to the negative 'space charge'. All the electrons emitted by the cathode then reach the anode. So the saturation current is a measure of the number of electrons per second emitted by the cathode at the temperature concerned.

Summary

1. Electrons are minute particles about 1/2000th of the mass of a hydrogen atom; they carry a negative charge.
2. Electrons are obtained from hot cathodes such as a directly heated tungsten wire or an indirectly heated surface coated with a mixture of barium and strontium oxides.
3. Electrons (a) travel in straight lines, (b) are deflected in a circular path by a uniform perpendicular magnetic field, (c) are deflected by a perpendicular electric field applied between two plates.
4. A diode valve has an anode A and a cathode C inside an evacuated glass bulb. It conducts when A is positive in potential relative to C and has an infinitely high resistance when A is negative in potential relative to C. The current is space-charge limited at first, and reaches a saturation value when all the electrons per

Scanned with CamScanner

second emitted from the cathode are collected by the anode as the anode potential is increased.

5. A cathode-ray tube has a hot cathode, a brightness control, an electron lens, Y plates for connecting the voltage to be examined or received, and X plates to which a time-base voltage is usually applied.

6. X-rays are electromagnetic waves of very short wavelength, such as 10^{-8} cm or less. They are produced when high-speed electrons bombard a metal target such as a tungsten one. In an X-ray tube a hot cathode produces electrons, and a high voltage, such as 50 kV, gives the electrons high energy.

7. A pure semiconductor at room temperature has an equal number of electrons and holes. Hole movement is equivalent to movement of a positive charge numerically equal to the charge on an electron.

8. A p-type semiconductor is one which has a majority of holes and a minority of electrons. An n-type semiconductor is one which has a majority of electrons and a minority of holes.

9. A p–n junction diode is formed from p- and n- semiconductors. It is a rectifier, conducting well when the potential of the p-semiconductor is positive relative to that of the n-semiconductor; it conducts only slightly when the reverse is the case.

10. The n–p–n transistor has an n-emitter, a thin p-base and an n-collector. With correct voltages, the transistor acts as an amplifier of current.

Exercise 33 (Answers, page 643)

1. Fig. 33.25 shows a simple cathode ray tube with only one pair of deflector plates. What would be the effect on the fluorescent spot if (a) the temperature of the filament of the electron-gun were raised? (b) the deflector plate A was positively charged while B was negatively charged? (c) the anode voltage was increased while the plates remained charged? (N)

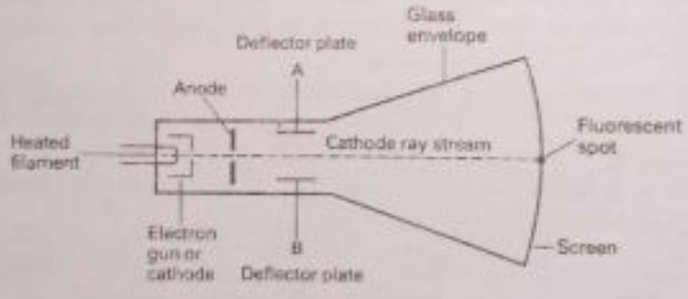


Fig. 33.25

2. Describe the structure of a diode valve and state the conditions for an electric current to pass between its electrodes.

Show by means of a diagram how the current through a constant resistance will vary with time when it is connected in series with a diode and an alternating voltage supply. Account briefly for the shape of the diagram drawn. (L)

3. Describe an experiment that can be carried out in a school laboratory which shows that *either* cathode rays *or* beta particles carry a negative charge. You need only indicate your source of rays or particles and the apparatus used for detection or measurement, but you should give as full a description as possible of what is done and observed and explain how you link the conclusion with the experimental observations. (L)

4. In one type of cathode ray oscilloscope the electron beam is deflected by electrically charged plates. Makes a labelled diagram of the side-view of the oscilloscope tube showing its main features.

If you wished to examine the waveform of the a.c. mains supply where would you connect the mains leads? (O and C)

Scanned with CamScanner

5. Fig. 33.26 shows a tube suitable for producing a narrow beam of electrons and accelerating them so that they pass between two horizontal plates, L and M, before striking the far end of the tube which is coated with a fluorescent material.

(a) How is a copious supply of electrons produced in the tube?

(b) How and why are the electrons accelerated?

(c) How can the plates L and M be used to deflect the electron beam and in what direction will it be deflected?

(d) Describe briefly some other method of deflecting the beam and then show how the direction of the deflection is related to the method used. (L)

6. 'X-rays are particles, cathode-rays are waves.' Is this statement true? If not, correct it, and explain how the nature of X-rays was demonstrated. Explain the relation between X-rays and visible rays.

7. Describe the diode valve and give details of the cathode circuit. State the essential condition, in each instance, to produce an anode current which is (a) zero, (b) a maximum, assuming that the cathode is at a constant temperature suitable for the normal working of the valve. (L)

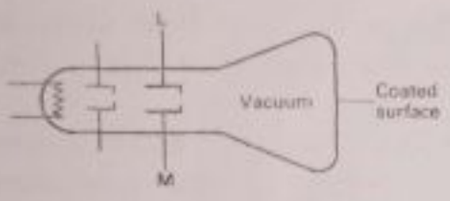


Fig. 33.26

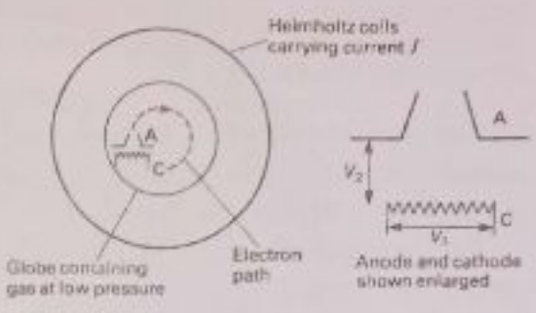


Fig. 33.27

8. Fig. 33.27 shows in a very simplified form the essentials of an apparatus for showing that electrons can describe a circular path in a magnetic field. The p.d. across the heated filament C is V_1 and that between C and the anode A is V_2; the current through the large Helmholtz coil system is I.

(a) Is it a magnetic field or an electric field which causes the electrons to follow this circular path?

(b) In which direction does this field act if the electrons follow a clockwise path as shown?

(c) Explain how this enables you to determine the direction of the current I in the Helmholz coils.

(d) Which, if any, of the three quantities V_1, V_2 and I can on its own be reversed in direction without affecting the sense in which an electron describes its path? Explain. (O)

Multiple-Choice Questions

In a cathode-ray oscilloscope, which of the changes A to E below can produce on the screen

9. a brighter trace

10. a vertical line

11. a wave pattern

12. a horizontal line

A use time-base and connect alternating voltage to Y-plates
B use time-base only
C switch off time-base and connect a battery to Y-plates
D switch off time-base and connect alternating voltage to Y-plates
E increase the anode voltage

13. Which of the statements A to E is correct?

A electrons and X-rays are both particles
B electrons can be deflected by magnetic fields but X-rays are not deflected by these fields
C electrons are deflected by magnetic fields but not by electric fields
D electrons are heavy particles
E in thermionic emission, only ions are produced

Scanned with CamScanner

34. Radioactivity · Atomic Structure · Nuclear Energy

Radioactivity

Warning *In all radioactivity demonstrations or experiments, safety regulations of the Department of Education and Science, and age regulations for experimenters, must be followed.* For example, **1.** radioactive sources must be stored in special boxes lined with thick lead, **2.** the sources must never be handled but must be lifted with long tweezers or tongs, **3.** the open ends of the sources must always point away from the body when handled.

In the previous chapter we saw that electrons can be liberated from atoms in metals. Since these particles are obtained fairly easily, it is likely that the electrons are in the outermost parts of the atoms.

The first clues to the structure of the heart or *nucleus* of the atom, and of atomic structure, were found by Becquerel in 1896. He happened to place a uranium compound near a photographic plate wrapped in black lightproof paper. When he developed the plate he found, to his surprise, that the plate was fogged. He traced this effect to some unknown radiation coming from the uranium compound. The phenomenon was called radioactivity. It was investigated in 1897 by a famous New Zealand scientist, Ernest Rutherford, who became Lord Rutherford.

Types of radiation · Ionising properties

Rutherford found that three different types of radiation were emitted by uranium. They were called α (alpha), β (beta) and γ (gamma) radiation. A radioactive source may be recognised because it gives off one or more of these three radiations. The radiations are emitted when some of its atoms break down and new atoms are produced which may also be radioactive.

It was soon discovered that α- and β-radiations are really *particles*. So radioactive atoms emit α-particles, β-particles and γ-rays. α-particles are relatively heavy and have considerable energy. So when they collide with atoms or molecules they knock out electrons from them. As α-particles move in air, then, ions and electrons are produced along their paths by collisions with air molecules. β-particles are relatively light and only a small amount of ionisation is produced in air compared with that due to α-particles. γ-rays produce much less ionisation than β-particles.

When an α radioactive source is brought close to the cap of a gold-leaf electroscope which has a +ve or −ve charge, the leaf falls slowly. Ions of opposite sign to the charge are attracted to the cap and so the charge is slowly neutralised. The experiment does not work for β- or γ-radiations because they produce much less ionisation.

Scanned with CamScanner

Photographing alpha and beta tracks

C T R Wilson was a distinguished pioneer in radioactivity. He showed that if the air inside a chamber was dust-free and contained excess water vapour, the water vapour would condense round ions in the chamber if the air was suitably cooled. The droplets of water round the ions reflect light. So if the chamber is suddenly illuminated, the water droplets can be photographed.

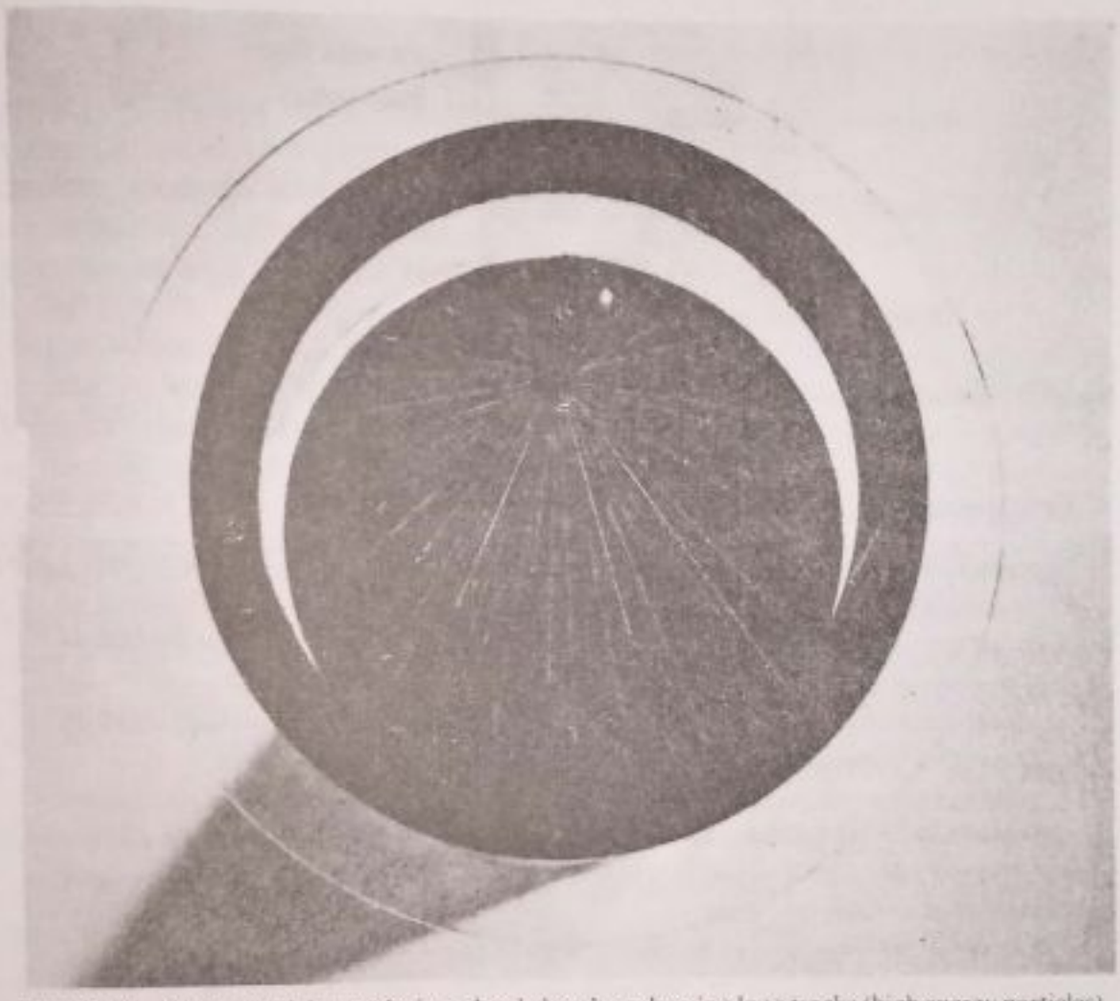


Plate 34A Alpha-particle streaks in a cloud chamber, showing long tracks (high energy particles) and short tracks (lower energy particles) (*W. B. Nicholson (Glasgow) Ltd*)

In 1911 Wilson designed a 'cloud chamber' for photographing the tracks of α- and β-particles in air and other gases. Plate 34A illustrates α-particle streaks in a type of cloud chamber to be discussed shortly. The streaks are similar to the vapour trails sometimes seen at the rear of a high-flying aeroplane, which show the path of the plane. One famous photograph, taken in 1917, shows the collision of an α-particle with a nitrogen atom. See page 572.

Diffusion cloud chamber

Fig. 34.1 shows one type of cloud chamber called a *diffusion cloud chamber*. It consists of a cylindrical perspex chamber with a metal base. The base is covered with black velvet, which helps to show up the tracks of ionising radiations when they are viewed from above.

Scanned with CamScanner

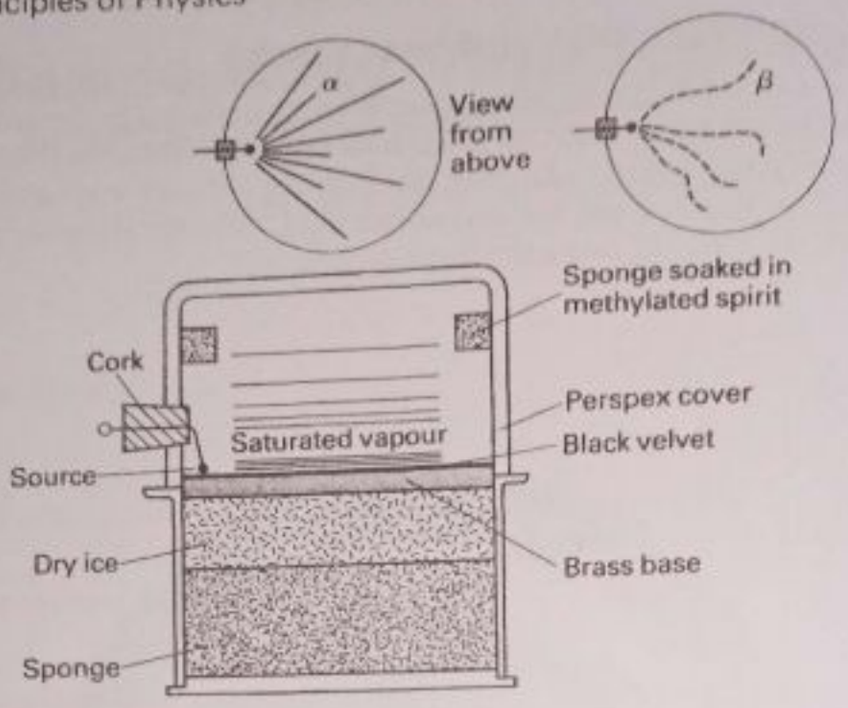


Fig. 34.1 Diffusion cloud chamber

Demonstration

To prepare the cloud chamber for seeing tracks, some methylated spirit is placed round the felt or sponge ring inside the lid. The base of the container is then unscrewed and dry ice (solid carbon dioxide) is pressed on the metal plate to cool it. The sponge at the bottom helps to keep the dry ice in contact with the plate and the base is screwed on again. Finally the chamber is levelled to reduce convection currents inside and the perspex top is rubbed to remove any ions already inside the chamber.

The spirit evaporates continuously from the ring in the lid and sinks to the region above the metal base. This is cooled to about −60 C by the dry ice. The vapour is then in a cold region where it is supersaturated, that is, its vapour pressure is higher than the saturated vapour pressure at that temperature.

The radioactive sample, such as radium, is put into the chamber by means of a wire holder passing through cork on one side. On looking down through the top of the chamber, vapour trails can be seen which start from the radium. They are due to vapour which condenses round the ions produced by the α-particles shooting out from the radium. The vapour trails show the paths of the α-particles.

As shown at the top of Fig. 34.1, the α-particle paths are straight. But the β-particle paths are straggly, that is, they wander about, because these relatively light particles are knocked off their paths by collisions with the air molecules.

Geiger–Müller (G–M) tube

We now discuss a detector of radiation often used in radioactivity experiments.

Geiger was a student of Lord Rutherford and carried out many important experiments in radioactivity. Working with Müller, he designed a detector of radiation which is widely used today. It is called a Geiger–Müller (G–M) tube (Fig. 34.2).

Basically, the G–M tube is a small closed glass tube with a thin mica end

Scanned with CamScanner

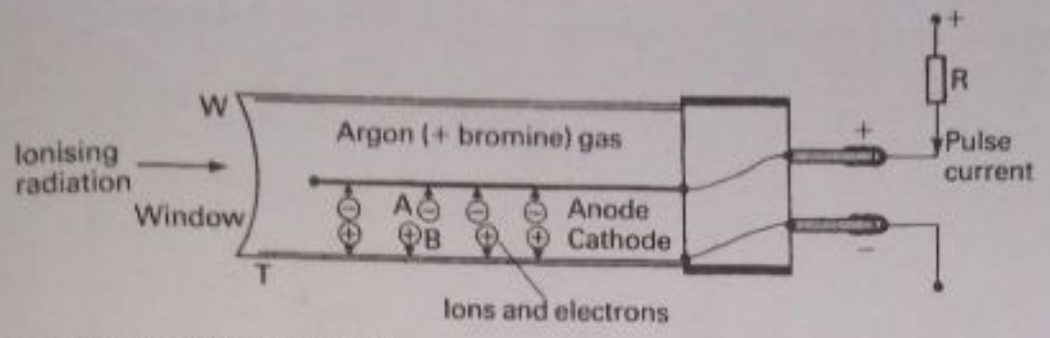


Fig. 34.2 Geiger–Müller (G–M) tube

window W. It may contain a gas such as argon at very low pressure, together with a small amount of halogen gas such as bromine vapour. A central wire A, insulated from the tube, is one electrode. The inside of the tube is coated with a conductor and forms the second electrode B. The + ve terminal of a high voltage is connected to A through a high resistance *R* and its negative terminal is joined to B.

If an ionising particle or radiation enters a G–M tube, some argon atoms are ionised by collision. The numbers of ions and electrons are increased many times by more collisions. The electrons are attracted to the wire A and the positive ions to the tube B and so a small current is obtained for a short time. This produces a voltage in *R*, which is amplified and recorded on a counting unit such as a scaler or ratemeter discussed shortly. See Fig. 34.3.

β-particles and γ-rays are easily detected by a G–M tube. The mica window of a G–M tube, although very thin, would normally absorb α-particles. So a specially thin perspex window is used for detecting α-particles by G–M tubes.

A *solid state detector*, which uses a semiconductor, can detect α-particles more easily than a G–M tube can.

Counting units

A *scaler* uses a number of magic-eye tubes, called dekatron tubes, to count the brief currents, called *pulses*, produced in the G–M tube by ionising particles or

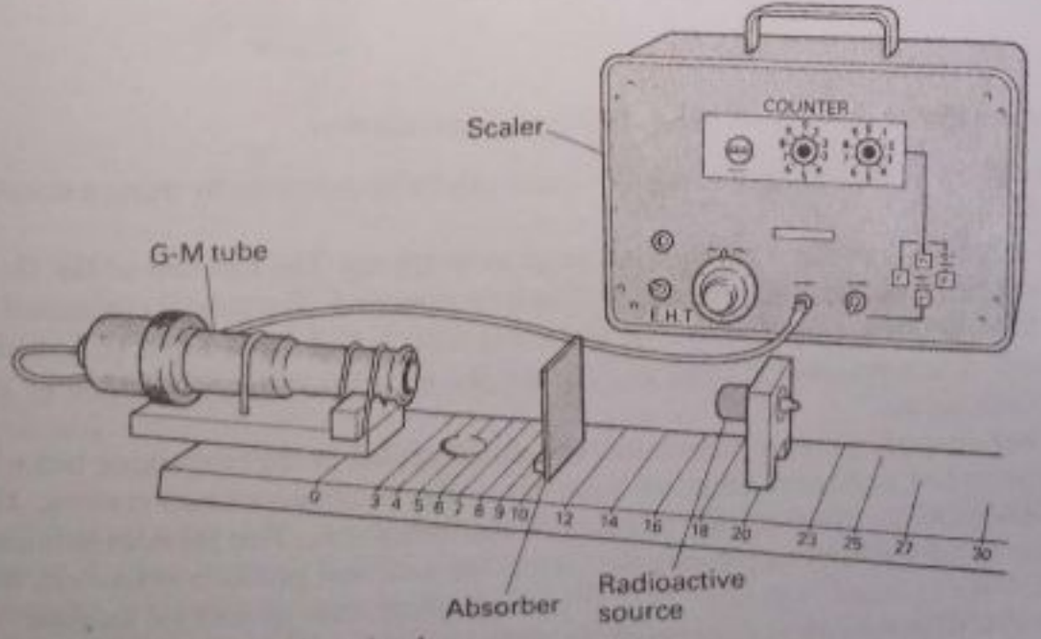


Fig. 34.3 Geiger–Müller tube and scaler

Scanned with CamScanner

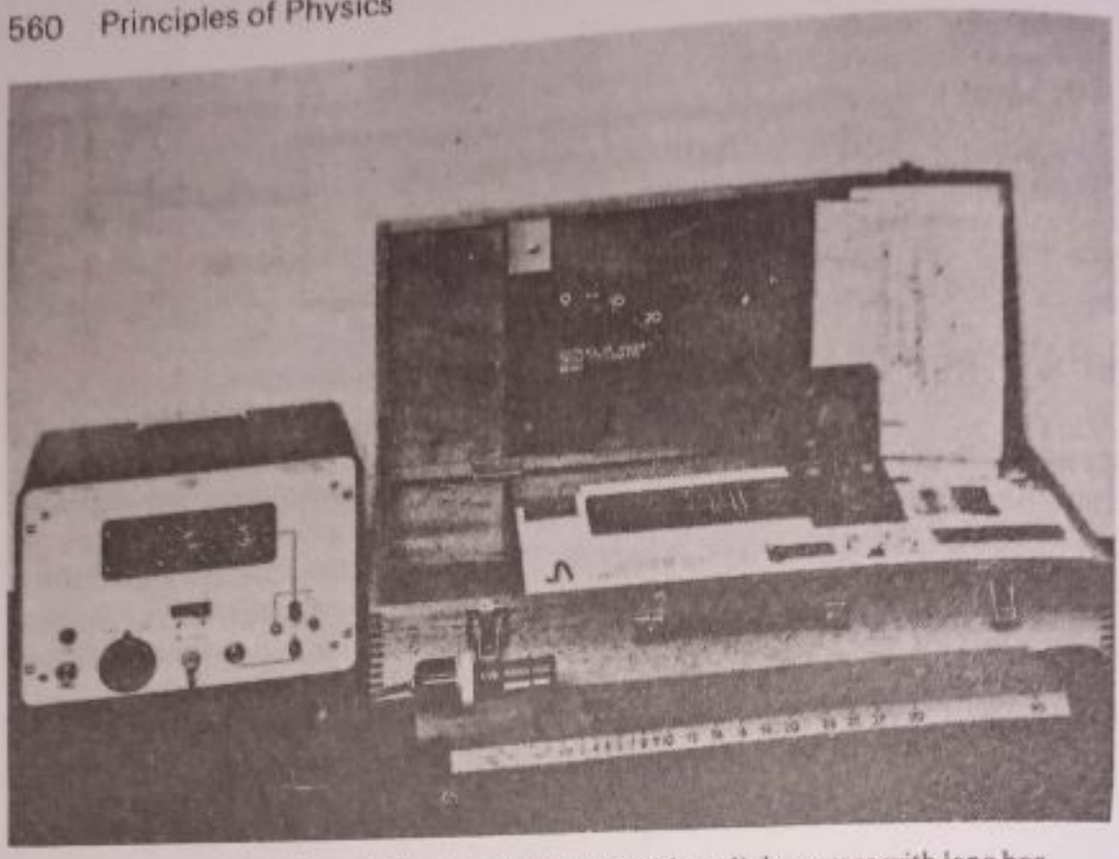


Plate 34B Scaler (left), Geiger-Muller tube (on graduated board), large case with long box containing aluminium and lead absorbers, wide open box containing weak sources of radiation, and a stronger source, in centre, covered with a lead disc

radiations. The number of pulses per second, or *count rate*, is measured with the help of a stopwatch and this gives a measure of the radiation intensity or activity.

With a *ratemeter*, the average number of pulses per minute, or the count rate, is recorded directly by a pointer moving over a scale of a sensitive current meter.

The pulse currents obtained from a G–M tube can also be passed to a loudspeaker. They are then heard as a succession of clicks. With such an arrangement, workers in radiological laboratories can hear when the radioactivity in the air reaches a dangerous level.

Investigations with G–M tube · Background radiation

The radiation from radioactive substances can be investigated by using a suitable G–M tube and scaler.

The scaler is switched on and allowed to warm up. The end-cap of the G–M tube, which protects the thin window, is then removed. The tube is connected to the high-voltage supply inside the scaler marked 'E.H.T.' and the E.H.T. dial is turned until a suitable voltage is obtained for the tube as recommended by the manufacturers.

Even though the radioactive sources are all sealed in the containing box and this is placed well away from the tube, the scaler indicates a small reading. The reading changes every minute showing its random nature. This random radiation is due to cosmic rays which pass into the G–M tube and produce ionisation. It is called *background radiation*. If it is a small count rate it may be ignored; in accurate experiments it must be taken into account.

Scanned with CamScanner

Properties of α-particles

Absorption An α-particle source R, americium-241, (Am-241) is placed in the perspex holder H (Fig. 34.4). The G–M window is placed very close to the source and the count rate is observed. A thin sheet of paper is now placed between the source and the G–M tube as shown. The count rate is seen to be greatly decreased. So α-particles are absorbed by thin paper.

If the tube window is moved back more than about 4 cm from the source, the count rate is small. So the range of α-particles in air at atmospheric pressure is only a few centimetres.

The G–M window is made of specially thin perspex so that the α-particles are not absorbed when the source is placed close to it. The solid state detector (page 549) is better for detecting α-particles than the G–M tube and can be used in its place.

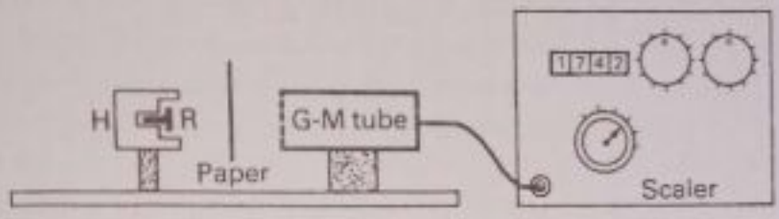


Fig. 34.4 Absorption of α-particles

Deflection by magnetic field As illustrated in Fig. 34.5a, α-particles are only very slightly deflected by a magnetic field. By applying Fleming's left-hand rule we see that α-particles carry a *positive* (+ve) charge. Further, since the deflection is very small, their mass is relatively large.

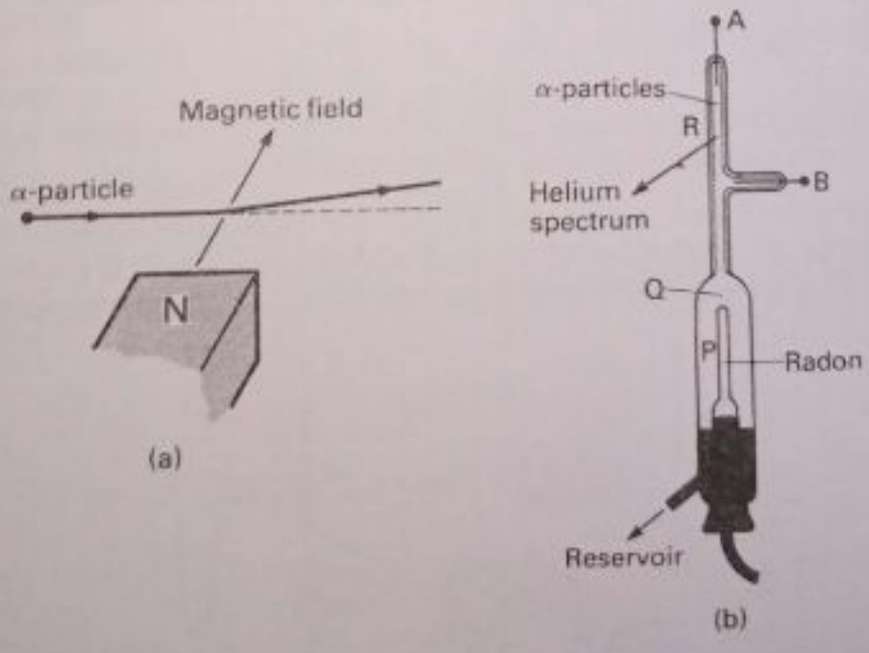


Fig. 34.5 (a) Deflection of an α-particle by magnetic field (b) Experiment to find nature of α-particles

Nature of α-particles The nature of α-particles was identified by an experiment due to Rutherford and Royds. Radon, a gas given off by radium which emits α-particles, was collected above mercury in a thin-walled tube P (Fig. 34.5b). After several days, some of the α-particles passed through P into a surrounding vacuum

Scanned with CamScanner

Q and were than pushed up into the narrow tube R by raising the mercury reservoir. The charge on the α-particles was neutralised by electrons within the tube and so gas atoms were now in R. A high voltage was then connected to the terminals A and B, which made the gas glow, and the spectrum of the glowing gas was identified to be that of *helium* gas. Helium is an element of about four times the atomic mass of hydrogen. We shall see later that when a helium atom loses the electrons surrounding its nucleus, the nucleus remaining has a +ve charge equal to that on the α-particles. So

an α-particle is a helium nucleus

Properties of β-particles

Absorption A strontium-90 source R, a β-particle emitter, is placed in a holder H, and positioned in front of a G–M tube connected to a scale (Fig. 34.6). A fast count rate is observed on the scaler. When a sheet of thin paper is placed between the source and the tube, the count rate is practically the same. But when an aluminium plate a few millimetres thick is placed between the source and tube, as shown, the count rate is appreciably reduced. Thicker aluminium reduces the count rate to a low value.

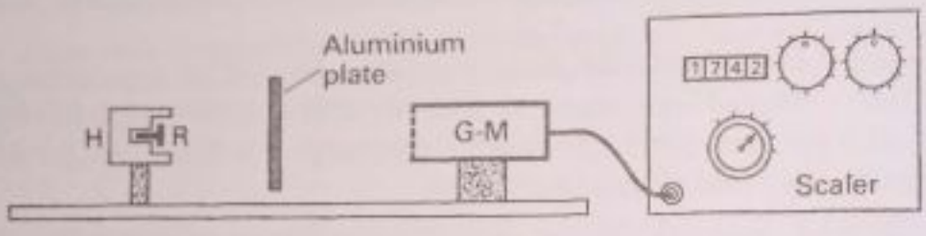


Fig. 34.6 Absorption of β-particles

So β-particles, unlike α-particles, are not absorbed by thin paper but they are absorbed by aluminium plate a few millimetres thick.

Deflection by magnetic field A narrow beam of β-particles is placed directly in front of the G–M tube shown (Fig. 34.7) and the count rate is measured on a scaler. A strong magnet is now placed near the source so that the magnetic field is

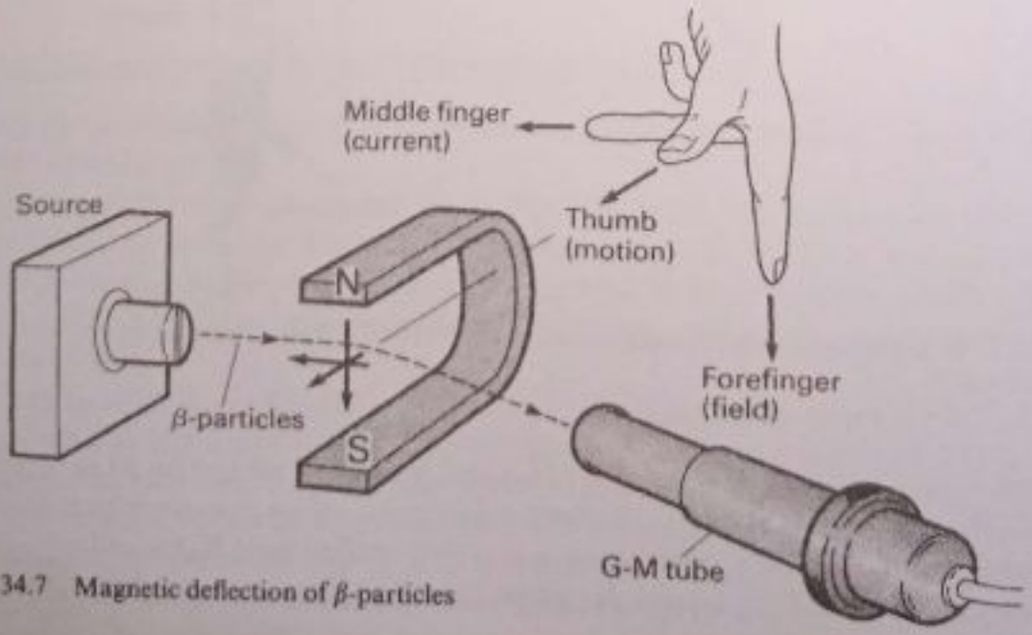


Fig. 34.7 Magnetic deflection of β-particles

Scanned with CamScanner

perpendicular to the direction of the β-particles. The count rate goes down. So β-particles are deflected by the magnetic field.

The direction the β-particles have been deflected can be found by moving the G–M tube round on either side of the original path of the β-particles. On one side, as illustrated in Fig. 34.7, the count rate increases again. So the β-particles have been deflected in this direction.

By applying Fleming's left-hand rule to the direction of deflection, we find β-particles carry $-$ve charges. Further experiments show that these relatively light particles are electrons. So

β-particles are usually electrons

Properties of γ-rays

Absorption With the apparatus shown in Fig. 34.6, a cobalt-60 source, a γ-emitter, is placed at R in front of the G–M tube. A high count rate is observed. Thin paper and then an aluminium plate are placed between the γ source and the tube. The paper has no effect on the count rate and the aluminium plate has only a small effect. When the aluminium plate is replaced by thicknesses of lead block the count diminishes appreciably using a thickness of several cm.

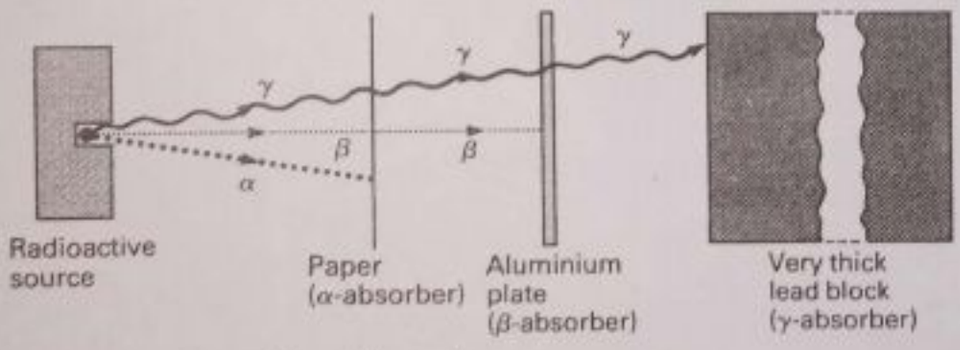


Fig. 34.8 Absorbers of α- and β-particles, and γ-rays

So γ-rays are very penetrating and can be stopped only by thick blocks of lead. See Fig. 34.8.

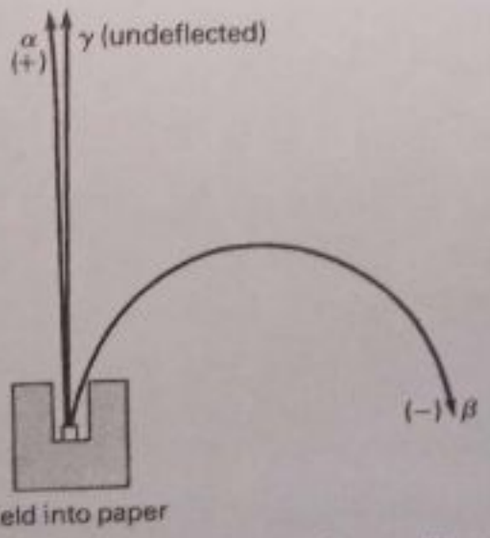


Fig. 34.9 Effect of magnetic field on α- and β-particles, and γ-rays–diagrammatic

Scanned with CamScanner

Magnetic field Using a powerful magnet, it is found that γ-rays are not deflected by magnetic fields. See Fig. 34.9. This shows they are not particles.

Inverse-square law A small γ-source is placed in front of a G–M tube. The distance of the source from the tube is varied and the count rate measured each time.

Suppose the count rates were 3592 per min at 4 cm; 912 per min at 8 cm; and 408 per min at 12 cm. To the nearest hundred, then

$$\text{ratio of count rates} = 3600:900:400$$

We see that the count rate is reduced to one quarter (3600 to 900) when the distance is doubled (4 cm to 8 cm) and to one ninth (3600 to 400) when the distance is trebled (4 cm to 12 cm).

The intensity I of the γ-radiation is inversely-proportional to the square of the distance from the source, that is $I \propto 1/d^2$. This is the same kind of law obtained for luminous radiation from a light source. We now know, in fact, that γ-rays are electromagnetic waves like light and they travel at the speed of light. But their wavelength is about ten thousand times shorter than that of visible light and shorter than X-rays (page 335).

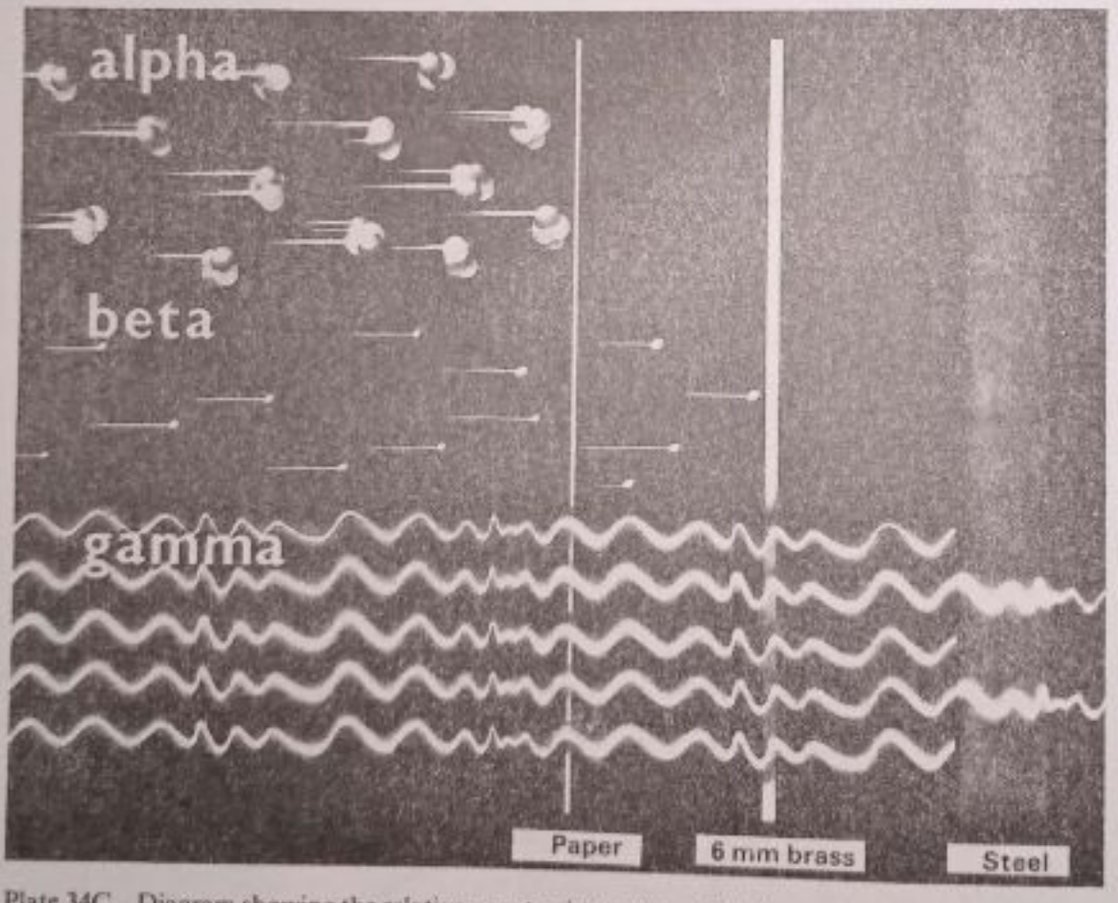


Plate 34C Diagram showing the relative penetrating powers of alpha- and beta-particles and gamma-rays. (*UKAEA*)

Summary of properties

The following table summarises some of the main points about α- and β-particles, and γ-radiation.

Scanned with CamScanner

	Nature	Electrical charge	Mass	Velocity	Relative penetration	Absorber
Alpha, α-particle	Helium nuclei	$+2e$	4 units	about $\frac{1}{10}$ velocity of light	1	Thin paper
Beta, β-particle	Electrons	$-1e$	$\frac{1}{1840}$ unit	3–99% velocity of light	100	Metal plate
Gamma, γ-ray	Electromagnetic radiation	No charge	Negligible	Velocity of light	10000	Large lead blocks

e = numerical value of charge on electron

Health hazards

α-particles are stopped by the outer layers of the skin as their penetration power is small. If they penetrate into the body, however, they are dangerous and damage organs.

β-particles are more penetrating than α-particles. Inside the body they are dangerous.

γ-rays and energetic β-particles destroy cells in tissue when the body is exposed to them, and upset the natural chemical reactions. This may lead to injury or death. X-rays have the same effect.

The body is exposed to radiation from outer space, but the amount absorbed per year is small and harmless. Luminous paint used on the dials of some watches is radioactive.

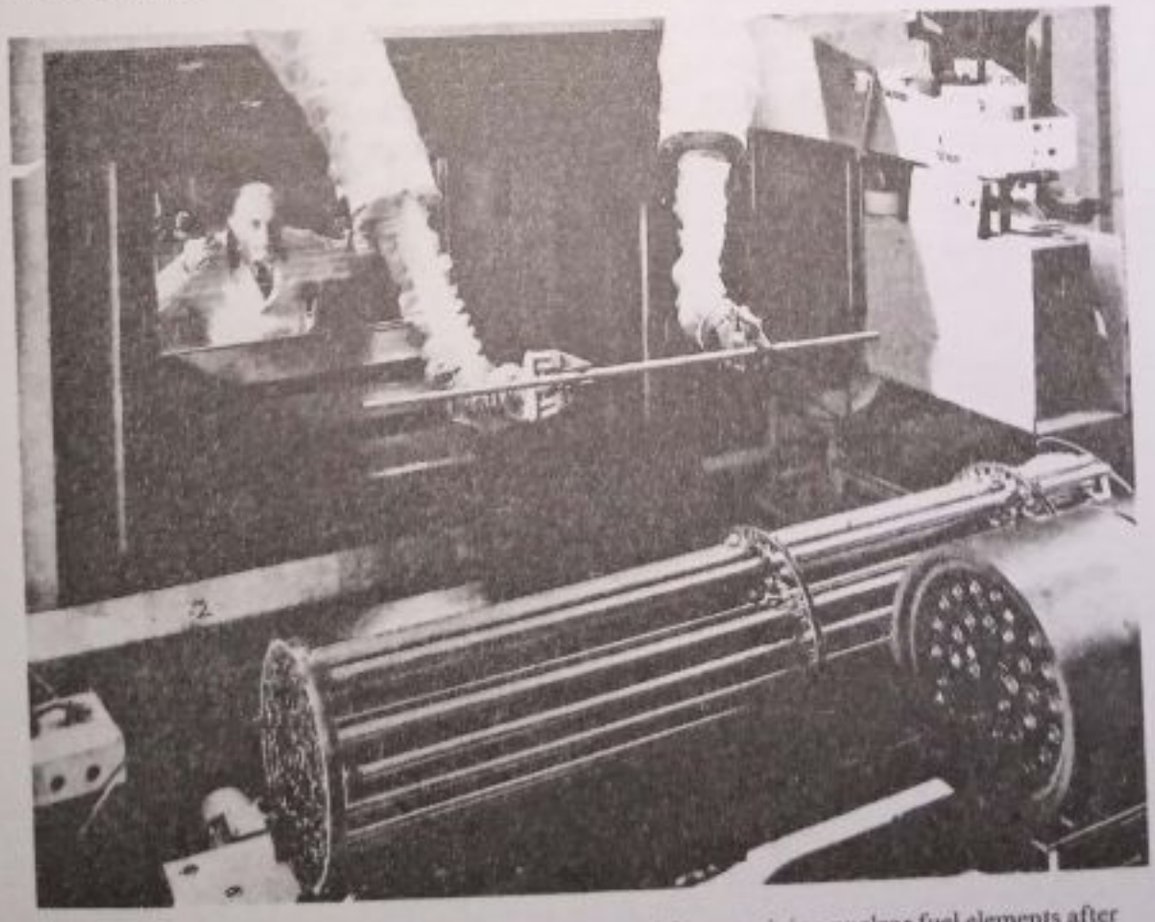


Plate 34D Active Handling Building at Winfrith, used for examining nuclear fuel elements after they have been irradiated. As shown, remote handling in large, thick-walled, concrete shielded chambers is used. (*UKAEA*)

Scanned with CamScanner

When an atomic bomb explodes, apart from the blast hazard, dangerous products are released which are swept into the atmosphere. Strontium-90, for example, may settle thousands of miles away from the centre of an explosion on grass or soil. Animals may eat the grass or the foods produced from the soil. Man may eat the animals and be affected by the radiation. Strontium-90 is absorbed by bone, and as it emits β-particles it upsets the formation of the blood cells in the body. Other dangerous products are also emitted in an atomic explosion.

The best shields against γ-radiation in the laboratory are thick blocks of lead, iron and high-density concrete. Workers in radiological laboratories are required to wear badges containing film sensitive to radiations, and these are checked at least once a month to find out whether the wearer has been exposed to a dangerous dose of radiation.

Randomness of decay

Natural radioactivity, such as the radioactivity of uranium salts, is due to the disintegration (break-up) or decay of some of the uranium atoms. The decay of the atoms is independent of chemical combination, temperature and other external conditions such as pressure. It is therefore completely different from any known chemical change. This is because the α- or β-particles and the γ-radiation are emitted from the *nucleus* of the atoms. The nucleus is the small central core of an atom which contains practically the whole mass of the atom. It is discussed on page 569. Nuclei of many heavy elements, such as uranium, contain so many particles that they tend to become unstable and break up.

Radioactive decay is a 'random' process, that is, we can never say which atom will decay at any instant. But we can say that, on average, the actual number of atoms per second which decay at any instant is proportional to the number of atoms present, although there will be small fluctuations about the average.

As time goes on there are fewer undecayed atoms. However, if one-thousandth of the atoms of a given element decay per second at the start, then one thousandth of the remaining atoms decay per second at any later time.

Half-life

The time taken for the atoms of a radioactive element to decay to *half* the original number is called the *half-life* of the element.

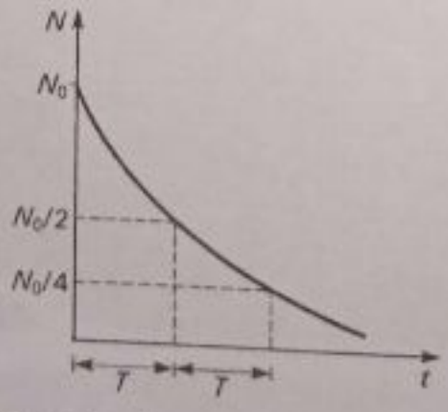


Fig. 34.10 Decay curve for a radioactive element

Scanned with CamScanner

Suppose the half-life of a particular element is 2 hours. Then if there were N atoms at one stage, $\frac{1}{2}N$ atoms are left undecayed after 2 hours. After 4 hours or two half-lives, the number of atoms remaining $= \frac{1}{2} \times \frac{1}{2}N = \frac{1}{4}N$. So $\frac{3}{4}N$ atoms have decayed. In 6 hours or three half-lives, the number of atoms left $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}N = \frac{1}{8}N$; so $\frac{7}{8}N$ have decayed.

Fig. 34.10 shows the curve obtained when the number of atoms N remaining is plotted against the time, t. The half-life is represented by T.

Different radioactive elements have different half-lives. So the elements can be identified by measuring their half-life, described shortly. Radium has a half-life of about 1600 years, radon has a half-life of about four days and polonium a half-life of 3 min. Radon-220, called 'thoron', the radioactive gas formed by the decay of thorium, has a half-life of about 1 min, which is convenient for measurement.

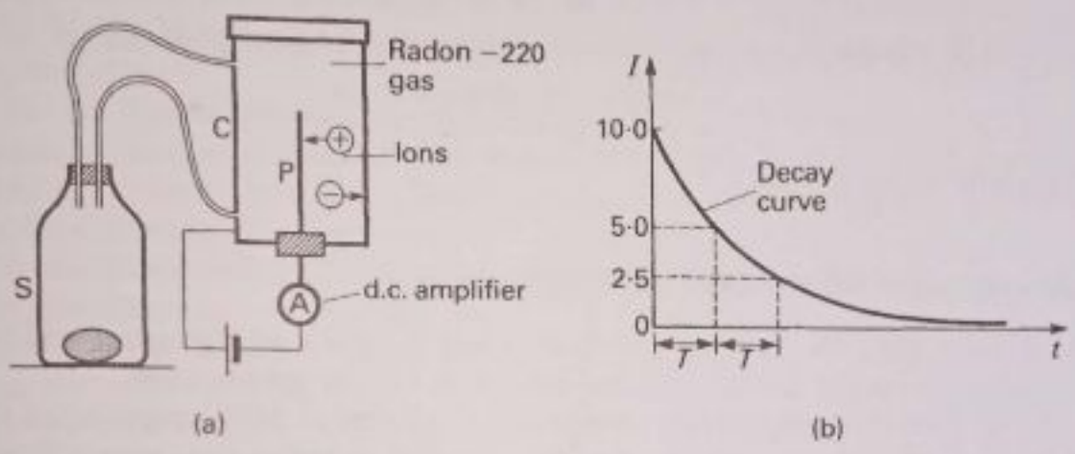


Fig. 34.11 Half-life of radon-220

Experiment

Fig. 34.11a shows how the half-life of radon-220 can be measured. A plastic bottle S containing the gas is connected to a metal case C with an insulated metal rod P inside it. When the bottle is squeezed gently, a little radioactive gas is passed into C. Here the decaying atoms emit α-particles which ionise the air molecules. If a battery is included in the circuit the ions drift between P and C and produce an *ionisation current*. This tiny current is amplified and then read on a meter A.

The current I in A is proportional to the rate of decay of the radon-220 atoms. Starting with a suitably high reading on A, the decreasing current is read by a stopwatch at equal intervals of time, such as 15 s.

Results: Fig. 34.11b shows the graph obtained when I is plotted against the time t. The number of radioactive atoms decreases with time. From the graph, the time T is found for the current I to decrease to *half* its initial value, for example, from 10·0 to 5·0 or from 5·0 to 2·5. The half-life is the average of several values of T.

A G–M tube connected to a scaler can be used to measure the rate of decay of radioactive atoms that emit β-particles. In this case the number of disintegrations per second is proportional to the 'counts per second' or *count rate* recorded on the scaler. The half-life is the time for the count rate to decrease to half its initial value.

Scanned with CamScanner

Examples on half-life

1. A radioactive element has a half-life of 3 min. If the initial count rate is 512 per min, how long does it take to reach a count rate of 64 per min? What fraction of the original number of atoms is then left?

After successive 3 min intervals, the count rate reduces from 512 to 256 (half) to 128 (half) to 64 (half). Since there are three half-lives

$$\text{time taken} = 3 \times 3\,\text{min} = 9\,\text{min}$$

$$\text{Also, fraction of atoms left} = \tfrac{1}{2} \times \tfrac{1}{2} \times \tfrac{1}{2} = \tfrac{1}{8}$$

2. If 15/16th of the radioactive atoms of an element decay in 8 min, find the half-life of the element.

Since 15/16 decay, fraction of atoms left is 1/16.

$$\text{Now} \quad \frac{1}{16} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

So in 4 half-lives, the number of atoms left is 1/16th of the original number

$$\therefore \text{ half-life} = \tfrac{1}{4} \times 8\,\text{min} = 2\,\text{min}$$

Atomic structure

Geiger and Marsden's experiment

As we have seen, the α-particle is a heavy particle moving at high speed and has considerable energy. Rutherford decided to use it to disrupt the atom.

At Rutherford's suggestion, Geiger and Marsden in 1909 investigated the effect of α-particles incident on thin films of metal. A radioactive source R was placed in an evacuated vessel V, and emitted α-particles in a narrow beam on to a very thin metal foil F of gold or other element of high atomic mass (Fig. 34.12). The path of the α-particles could be traced by the pin-points of light (scintillations) they produced on hitting a fluorescent screen S attached to a microscope.

Geiger and Marsden investigated the scattering of the beam of α-particles by the metal atoms. They saw scintillations not only in the straight-through position, $\theta = 0°$, but also in directions which were at large angles such as 45° to the incident beam. A few scintillations were even seen in a direction FB, which

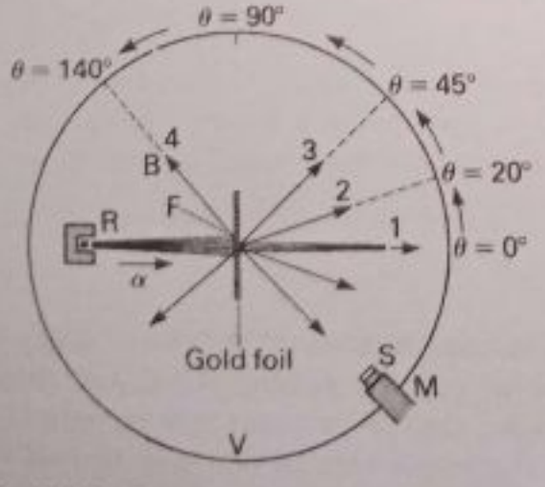


Fig. 34.12 Scattering of α-particles by gold foil

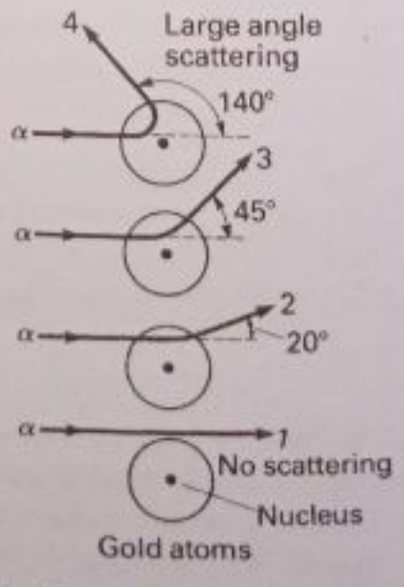


Fig. 34.13 Scattering of α-particles by nuclei

Scanned with CamScanner

was an *opposite* direction to the incident beam. So some α-particles were deflected through very large angles, as if they had reached a centre inside the metal atom which repelled them violently.

The nucleus

Rutherford recognised the significance of these observations. He said at the time that the repulsion of α-particles in an opposite direction was as surprising as if a shell fired from a gun had been repelled by tissue paper! Sir J J Thomson had thought that an atom might be pictured as a cloud of positive charge, with negative electrons spread through the cloud like currants in a pudding. Rutherford proposed, however, that there was a *concentration* of positive charge inside the atom, which repelled the positively charged α-particles. On this basis he calculated how many α-particles would be scattered through a particular angle (Fig. 34.13). The relationship was verified by Geiger and Marsden in subsequent experiments.

In 1911 Rutherford suggested the first correct structure of the atom. The atom, he said, consists of a concentration of positive charge in a very tiny volume in the heart or centre of the atom, called the nucleus, with electrons moving round the nucleus. Most of the atom is empty. If we imagine the nucleus magnified to the size of a cricket ball the farthest part of the atom would be a few miles away, and the space between the ball and the farthest part would be empty except for electrons. The nucleus of the hydrogen atom is called a proton. Since there is only one electron round the hydrogen nucleus, the proton carries a charge of $+e$, where e is the numerical value of the charge on the electron (Fig. 34.14a).

Protons and neutrons

By using α-particles as 'bullets', Rutherford and collaborators found that protons were thrown out from the nuclei of many elements when collisions occurred. So atomic nuclei contain protons. In 1932 Sir James Chadwick carried out experiments which showed that a nucleus contained another particle. This carries *no* charge and has a mass about the same as a proton. This was called a neutron, symbol n.

We now believe that the nucleus of an atom is built up of protons and neutrons. (Other particles have been discovered in the nucleus, but this will not concern us.) The particles inside a nucleus are called *nucleons*. The hydrogen nucleus, the simplest nucleus, is the proton, p. This has a positive charge $+e$, where e represents the numerical value of the negative charge on an electron, but it is nearly 2000 times as heavy as the electron. The next heavier element is helium; this has a charge of $+2e$ on its nucleus, that is, the nucleus contains 2 protons. The mass of the helium nucleus is 4, and so the nucleus must also contain 2 neutrons. The symbol for the helium nucleus is ${}^{4}_{2}\mathrm{He}$; the top number is the *mass number* or *nucleon number*, and the lower number is the *atomic number* or *proton number* of the nucleus (Fig. 34.14).

The proton itself is represented by ${}^{1}_{1}\mathrm{H}$, the neutron by ${}^{1}_{0}\mathrm{n}$. Each has a mass 1, but the neutron has no charge. An electron is represented by ${}^{0}_{-1}\mathrm{e}$, since it has negligible mass and a charge of $-1e$. The nitrogen nucleus is written ${}^{14}_{7}\mathrm{N}$; it has an atomic mass of 14 and atomic number 7, that is, there are 7 protons and 7 neutrons in the nucleus. Oxygen, ${}^{16}_{8}\mathrm{O}$, has 8 protons and 8 neutrons in its nucleus. The chemical behaviour of an element depends on the number of electrons round

Scanned with CamScanner

the nucleus. This is the same as the atomic number, since a normal atom is electrically neutral. So the elements are identified by their atomic (proton) number.

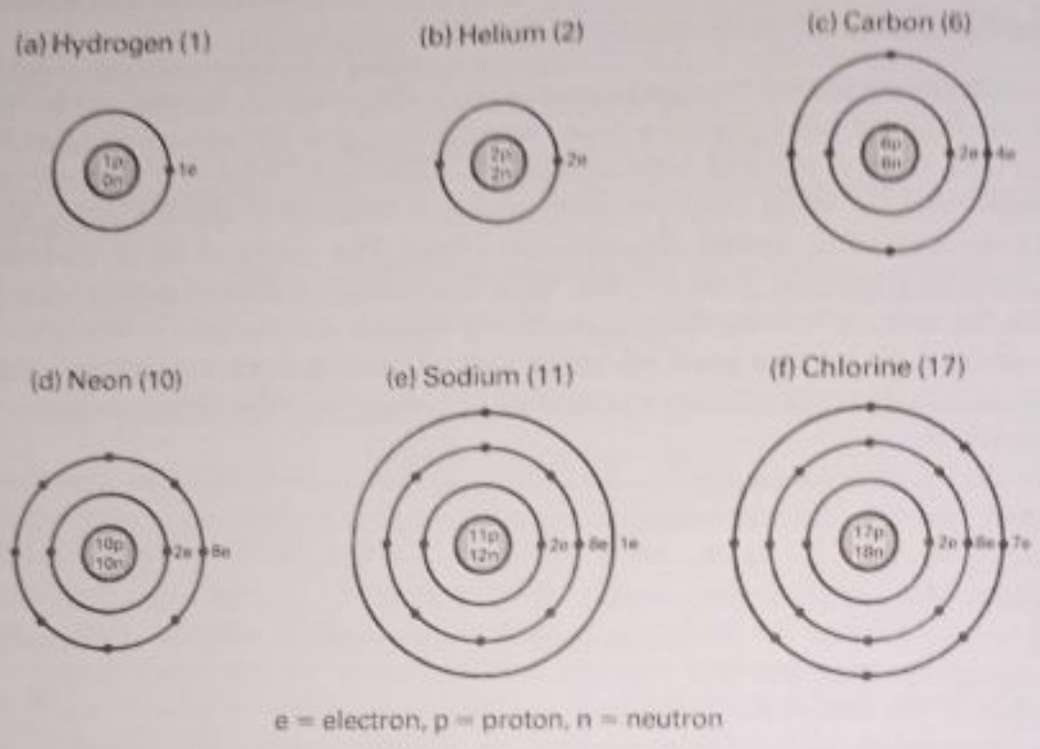


Fig. 34.14 Atomic structure and atomic (proton) number of some atoms

Radioactive transmutation · Nuclear changes

We now know that natural radioactivity is caused by the disintegration (decay) or break-up of a heavy atomic nucleus. This nucleus may throw out or emit an α-particle or a β-particle, accompanied by a γ-ray. As the new nucleus left may now have a different atomic (proton) number, *a new element is formed*. This is called transmutation of the atom. The γ-ray emission alone leaves the nucleus practically unchanged because a γ-ray has negligible mass and no charge.

In nuclear changes (or nuclear collisions), the total number of the nucleons concerned must remain constant. So the total nucleon number before the change is equal to the total nucleon number after the change. Also, the total *charge* of all the particles concerned must remain constant. So the total proton number before the change is equal to the total proton number after the change.

This is illustrated in the following examples.

Examples on nuclear changes and equations

1. The radium nucleus $^{226}_{88}$Ra decays to radon (Rn) by α-particle emission. Write down the nuclear equation for this change.

An α-particle, $^{4}_{2}$He, has 4 nucleons (top number in symbol) and 2 protons (bottom number). Using the fact that the total nucleon number and total proton number are both constant, we therefore write the nuclear equation as

$$^{226}_{88}\text{Ra} \rightarrow {}^{4}_{2}\text{He} + {}^{222}_{86}\text{Rn}$$

Scanned with CamScanner

Check Total nucleon number on right of equation = 222 + 4 = 226 = nucleon number on left

Total proton number on right of equation = 86 + 2 = 88 = proton number on left

2. Radon, $^{222}_{86}$Rn, decays to radium, $^{222}_{88}$Ra, by the emission of 2 β-particles (electrons). Write down the nuclear equation.

A β-particle, $^{0}_{-1}$e, has a nucleon number 0 and a proton number −1. So we write the nuclear equation as

$$^{222}_{86}\text{Rn} \rightarrow 2\,^{0}_{-1}\text{e} + ^{222}_{88}\text{Ra}$$

Check Total nucleon number on right of equation = 222 + 0 = 222 = left side nucleon number

Total proton number on right side = 88 − 2 = 86 = left side proton number

3. A neutron changes into a proton by the emission of a β-particle (electron) from the nucleus. Write down the nuclear change.

A neutron has the symbol $^{1}_{0}$n; the proton symbol is $^{1}_{1}$H; the β-particle symbol is $^{0}_{-1}$e. So the nuclear change is

$$^{1}_{0}\text{n} \rightarrow {}^{0}_{-1}\text{e} + {}^{1}_{1}\text{H}$$

Check Total nucleon number on right = 0 + 1 = 1 = nucleon number on left

Total proton number on right = −1 + 1 = 0 = proton number on left

4. A uranium nucleus, U-238, atomic number 92, emits two α-particles and two β-particles (electrons) and finally forms a thorium (Th) nucleus. What is the symbol of this nucleus?

For 2 α-particles, $^{4}_{2}$He, and 2 β-particles, $^{0}_{-1}$e

$$\text{total nucleon number} = (2 \times 4) + (2 \times 0) = 8$$

and

$$\text{total proton number} = (2 \times 2) - (2 \times 1) = 2$$

So the nuclear change is represented by

$$^{238}_{92}\text{U} \rightarrow 2\,^{4}_{2}\text{He} + 2\,^{0}_{-1}\text{e} + ^{230}_{90}\text{Th}$$

Check Total nucleon number on right = 230 + (2 × 4) + (2 × 0) = 238

Total proton number on right = 90 + (2 × 2) − (2 × 1) = 92

An isobar is a nucleus with the same nucleon number or mass number as another but with a different atomic number. Isobars are usually produced in β-emission, as in Example 2. Here radon and radium are isobars, each having the same mass number of 222.

Artificial transmutation

Artificial disintegration was first produced when energetic α-particles were used by Rutherford to disrupt the nitrogen nucleus, symbol $^{14}_{7}$N. He found that protons were emitted after the collision. See Plate 34E. The α-particle is a helium nucleus with 2 protons, the nitrogen nucleus has 7 protons, and so the total proton number is 9. After the disintegration, the products are one proton together with another atom X.

So proton number of X = 9 − 1 = 8

Oxygen has a proton number 8. So X is an oxygen atom, O. Also

$$\text{nucleon number of X} = 14\,(\text{N}) + 4\,(\text{He}) - 1\,(\text{H}) = 17$$

So the atom X is $^{17}_{8}$O.

We can therefore write the nuclear reaction as

$$^{4}_{2}\text{He} + {}^{14}_{7}\text{N} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$$

Scanned with CamScanner

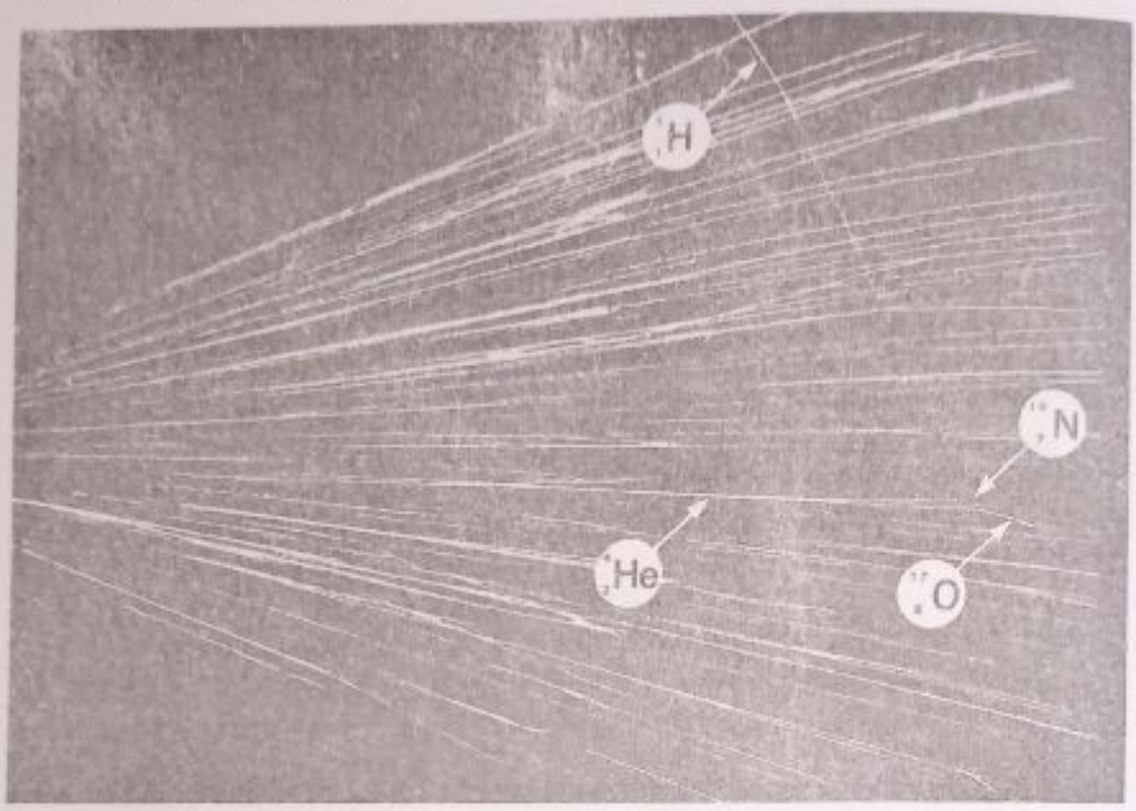


Plate 34E Nuclear collision by α-particle, which transmutes a nitrogen atom to an oxygen atom. The streaks show α-particles moving through nitrogen gas. One α-particle (right, centre) has collided and merged with a nitrogen nucleus, and this has broken into an oxygen nucleus (short track) and a proton (long thin track crossing others). This historic photograph was taken by Lord Blackett in 1925 using a Wilson cloud chamber. (*Lent to the Science Museum, London by The Royal Society*)

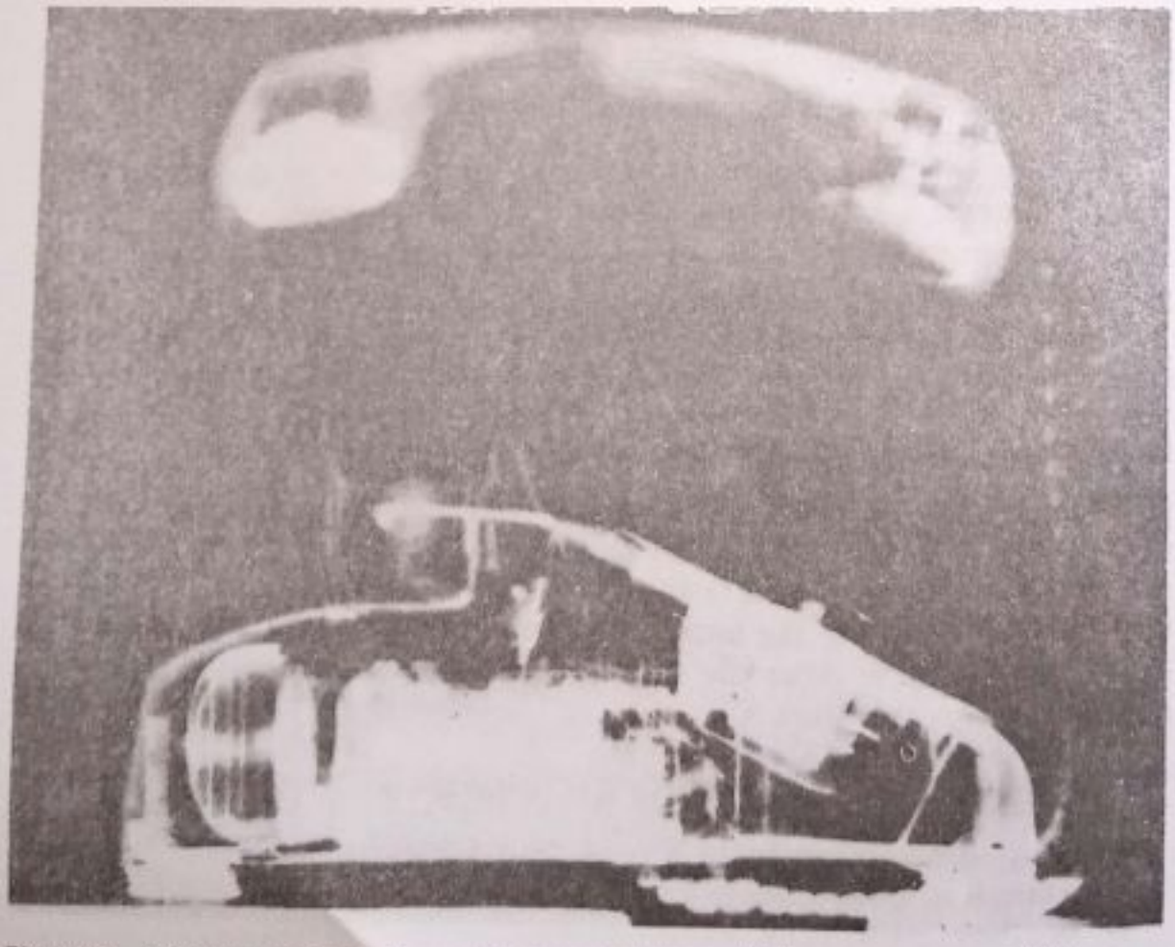


Plate 34F Photograph of a telephone using a neutron beam, which penetrates materials like an X-ray beam. (*Barnaby's Picture Library*)

Scanned with CamScanner

Isotopes

Thomson found by experiment that a sample of neon gas contained atoms of mass 20 and atoms of mass 22. Neon gas actually has about 90% of atoms of mass 20 and 9% of mass 22. The 'atomic weight' measured by chemists gives the average mass, 20·2, of the atoms in the gas; the individual atoms cannot be distinguished. In Thomson's experiment, however, the masses of individual atoms could be measured.

Atoms of the same element which have the same chemical properties but different masses are called *isotopes*. The chemical properties depend on the number of electrons round the nucleus, or the proton number, because chemical combination is due to an exchange of outer electrons between elements. So the proton number of the two isotopes of neon is the same. It is 10. The symbols for the isotopes are written

$$^{20}_{10}\text{Ne and } ^{22}_{10}\text{Ne; or } ^{20}\text{Ne and } ^{22}\text{Ne; or Neon-20 and Neon-22}$$

Since there are 10 protons in each of the neon isotopes, the remaining particles in the nucleus are 10 neutrons in neon-20 and 12 neutrons in neon-22.

Chlorine has isotopes of mass number 35 (^{35}Cl) and 37 (^{37}Cl). Each has 17 protons in the nucleus but ^{35}Cl has 18 neutrons and ^{37}Cl has 20 neutrons. The atomic weight of chlorine is about 35·5; there are about four times as many atoms of mass number 35 compared with atoms of mass number 37 in a given sample of chlorine gas.

Hydrogen has isotopes which are important in the study of nuclear energy (page 577). Most atoms in ordinary hydrogen gas have a mass number 1 but a tiny percentage have masses twice as much or a mass number 2. These are known as *heavy hydrogen* or deuterium, symbol $^{2}_{1}$H or D. So deuterium has 1 proton and 1 neutron in its nucleus, and 1 electron moving round the nucleus. A 'deuteron' is a nucleus of deuterium.

A third isotope of hydrogen is unstable. It is called tritium, symbol $^{3}_{1}$H.

Uses of radioactive isotopes

Isotopes can be made artificially by firing neutrons or protons or deuterons at elements. For example, the isotope sulphur-35 can be made by bombarding the normal atom sulphur-34 with neutrons.

$$^{34}_{10}\text{S} + ^{1}_{0}\text{n} \rightarrow ^{35}_{10}\text{S}$$

Isotopes made in this way are unstable. Their nuclei decay and emit α- or β-particles and γ-rays. They are therefore called radioisotopes. The half-life of sulphur-35 is about 87 days. The half-life of iodine-131 is about 8 days. The half-life of cobalt-60 is about 5 years.

As they are radioactive, radioisotopes can be traced by instruments such as Geiger–Müller tubes. Radioisotopes are widely used in industry and medicine. Compounds of carbon-14 have been mixed with insecticide and used to trace their action on plants. The γ-rays from cobalt-60 are used to sterilise medical supplies as viruses and bacteria are destroyed by exposure to the rays. Water-marks in printing may be made by exposing the paper to the weak β-particles from sulphur-35.

Scanned with CamScanner



Plate 34G Checking the thickness of rubber using a strontium-90 source of β-particles. A counter (out of sight) measures the reduction in intensity of the radiation. The rubber is moved down by the operator. (*UKAEA*)



Plate 34H Measuring iodine-125, an isotope which has been sublimed into the tyre surface of a car. As the tyre wears, some of the isotope is lost and the rate of wear can be found by measuring the lower count rate. (*Vauxhall Motors Ltd*)

Scanned with CamScanner

Nuclear energy

The particles in the nucleus, protons and neutrons, are called nucleons. The nucleons are kept together in the very tiny volume of the nucleus by powerful forces whose nature is not yet fully known. To take all the nucleons apart requires an enormous amount of energy. It is called the *binding energy* of the nucleus. By comparison, the energy needed to remove electrons from an atom is about a million times smaller.

Nuclear fission · Chain reaction

When the nucleus of the heavy element uranium is split into two parts of roughly the same mass, the nucleus is said to undergo *fission*. The mass of the two parts together is less than the mass of the original uranium nucleus. The difference in mass is a measure of the nuclear energy released. Einstein's mass–energy relation shows how the energy can be calculated and is the key formula in all nuclear energy calculations. It states

$$\text{energy released in joules} = \text{mass change} \times 9 \times 10^{16}\ \text{J}$$

where the mass change is in kg. Calculation shows that if all the atoms in 1 kg of uranium could undergo fission, the energy released would be as much, theoretically, as that obtained by burning about 1 million kg of coal.

Nuclear fission can be produced by bombarding a uranium-235 atom with a neutron. The neutron has no charge. It can therefore penetrate deeply into the nucleus. Fig. 34.15 illustrates the reaction. Two neutrons are produced in

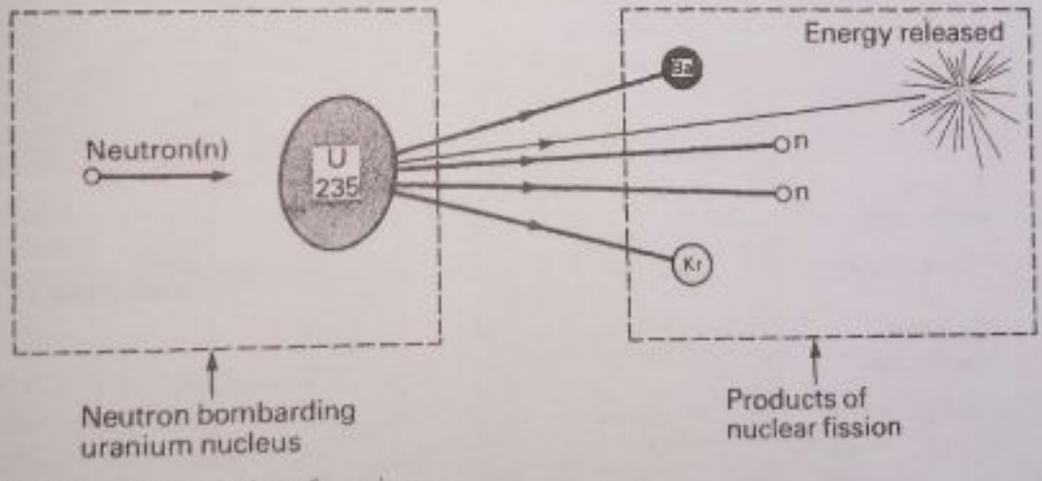


Fig. 34.15 Nuclear fission of uranium

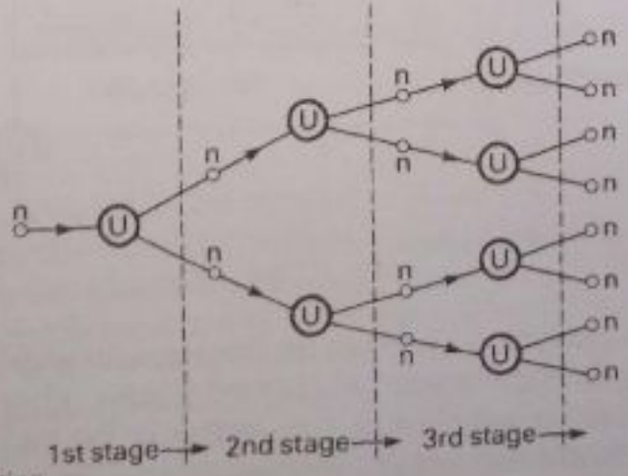


Fig. 34.16 Chain reaction

Scanned with CamScanner

addition to the two nuclei of barium (Ba) and krypton (Kr). If these neutrons in turn can be captured by other uranium nuclei, the number of atoms that undergo fission would multiply rapidly. Fig. 34.16 illustrates how one neutron in this way produces fission of 4 atoms (3rd stage). This multiplying action is called a *chain reaction* and it releases an enormous amount of energy in the atomic bomb.

Nuclear reactor principle

Natural uranium contains about 1 part by weight of atoms of atomic weight 235 and 139 parts by weight of atoms of atomic weight 238. Nuclear fission occurs with the atoms of atomic weight 235 when uranium is used in nuclear reactors.

Fission takes place only if the bombarding neutron is slow. Graphite is therefore used round the uranium to slow down the neutron, so that the chain reaction is prevented from dying out. If there are too many neutrons the reaction will be too fast and get out of hand, and this is controlled by neutron-absorbing boron steel rods. These rods are moved in and out of the reactor from the control room by small electric motors, which raise them through the floor to a suitable position. If there is an electrical failure the rods fall and shut off the reactor automatically.

The reactor is started by a charge machine. This lowers uranium rods, each about 1 m long and 2 cm in diameter, into about 1700 fuel channels in the graphite core (Fig. 34.17). The boron steel rods are then raised slowly, and in a certain position the chain reaction proceeds at the required rate. The reactor is now said to have 'gone critical', and heat is produced steadily.

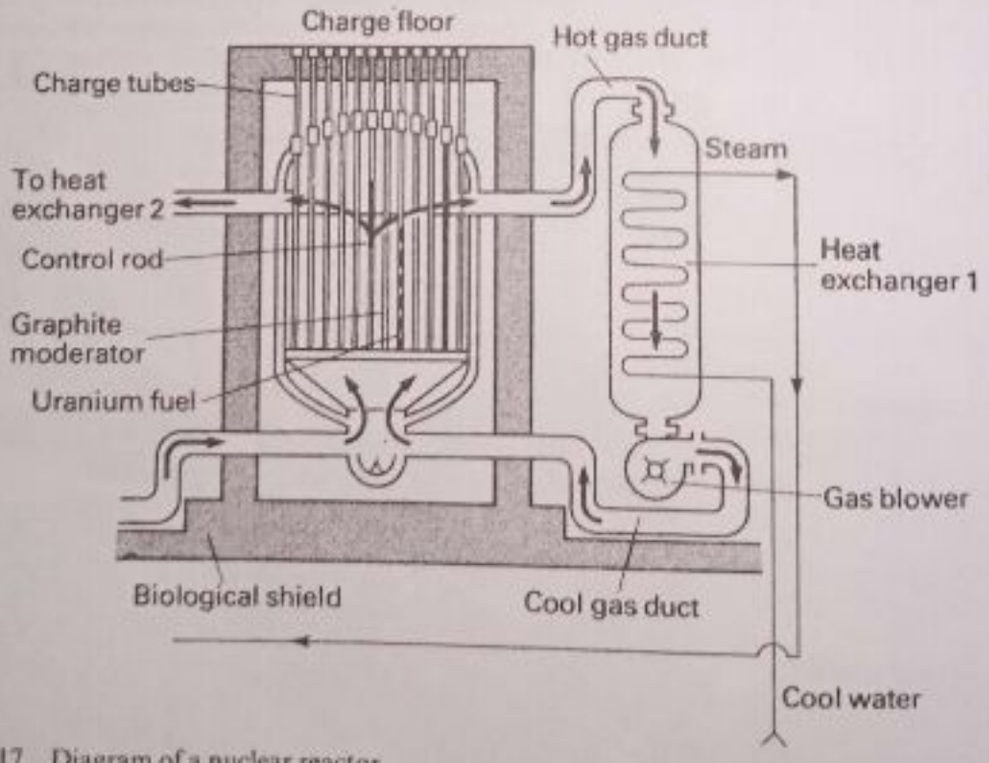


Fig. 34.17 Diagram of a nuclear reactor

Carbon dioxide gas is blown through the fuel elements under pressure, and the hot gas is led into heat exchangers outside the reactor. Here it heats up water which flows through an independent pipe system in the exchanger, and this is converted into high-pressure and low-pressure steam. The steam is used to drive

Scanned with CamScanner



Plate 34I A uranium–plutonium oxide fuel rod being lowered into the core area of a prototype fast reactor. The high speed neutrons emitted by the fission reactions are reflected by the surrounding rods and this improves the efficiency of the chain reaction. (*UKAEA*)

the turbines which turn electrical generators. In Britain, the electricity produced is fed to the National Grid System to increase the electrical energy produced by burning coal at conventional power stations. The nuclear reactor can operate day and night for a few years with the same uranium fuel. The reactor core is shielded by a welded-steel pressure vessel and a concrete biological shield, which protects operators outside from the intense radiation inside the core.

Nuclear fusion

As we have seen, a considerable amount of energy is released when a heavy nucleus such as uranium disintegrates into two large parts; this is nuclear fission. Experiments show that a considerable release of energy is also obtained when the nuclei of very light elements are fused to form a heavier nucleus, and this is known as nuclear *fusion*.

One transformation of matter into energy is the fusion of deuterons, ${}^{2}_{1}H$, the nuclei of heavy hydrogen, into helium nuclei. As the sun contains a considerable amount of hydrogen, it was suggested that the energy of the sun was basically due

Scanned with CamScanner

to nuclear fusion. It has been estimated that the energy released from the fusion of all the atoms in 1 kg of deuterium gas, heavy hydrogen, is equivalent to that released by burning nearly 3 million kg of coal.

To obtain nuclear fusion, the nuclei must come near enough to each other to overcome the repulsion of their like charges. The lightest elements have the smallest nuclear charge, and so the chance of nuclear fusion is greatest for such elements. The most practical way of getting nuclear fusion is to raise the temperature of deuterium gas to millions of degrees, which is the temperature in the heart of the sun. At these very high temperatures deuterons fuse together. This is known as *thermonuclear fusion*. In laboratory experiments, very heavy electrical discharges are sent through the gas to heat it to these enormously high temperatures.

Heavy hydrogen could be extracted from sea water. So a cheap method of producing enormous quantities of power would be available if methods for nuclear fusion were successful.

Further examples on radioactive emission and half-life

1. A radioactive nucleus is denoted by the symbol $^{288}_{97}$Y. Write down how you would expect the nucleus to be composed, and state the composition of the nucleus at the end of each of the following stages of disintegration, illustrating by means of a symbol similar to the example given, (a) the emission of an alpha-particle, (b) the *further* emission of a beta-particle, (c) the *further* emission of some gamma-radiation. (L)

The nucleus $^{288}_{92}$Y consists of 92 protons and (288 − 92) or 196 neutrons.

(a) *Alpha-particle emission* An alpha-particle is the helium nucleus $^{4}_{2}$He. Since the total number of nucleons (288) and the total number of protons (92) remain constant

$$^{288}_{92}\text{Y} \rightarrow {}^{4}_{2}\text{He} + {}^{284}_{90}\text{Z}$$

where Z is the nucleus after disintegration.

(b) *Beta-particle emission* If the beta-particle is an electron, represented by $^{0}_{-1}$e and A is the nucleus remaining after its emission, the reaction is represented by

$$^{284}_{90}\text{Z} \rightarrow {}^{0}_{-1}\text{e} + {}^{284}_{91}\text{A}$$

to keep the total number of nucleons and protons the same on both sides of the equation.

(c) *Gamma radiation emission* Gamma radiation has no charge or mass, so the nucleus remaining after its emission is still $^{284}_{91}$A.

2. The count rate recorded by a Geiger–Müller tube and counter close to an alpha-particle source is 400 per minute after allowing for the background count. If the half-life of the source is 4 days, (a) what would be the count rate 12 days later, (b) why should the rate be determined over periods of several minutes rather than over a few seconds, (c) why, if the mass of the source was determined many days later, would there be no apparent change?

What is meant by *background count*? (L)

(a) The count rate diminishes to half or 200/min in 4 days, to 100/min in another 4 days, and to 50/min in another 4 days. So the count rate is 50/min after 12 days.

(b) Disintegration or the breakdown of nuclei does not occur steadily but takes place at random. An average rate can only be estimated by taking a count over several minutes and not for just a few seconds.

(c) The only change in mass that could occur would be by the emission of alpha particles. The mass of an alpha particle is only a small fraction of the total mass of the nucleus (about 1/70th in Example 1) and many days later the mass of the source would diminish only by a very small amount as the number of emissions is by then very small.

The 'background count' is a measure of natural or other radiation always present in the neighbourhood of a detector. Some of this may be due to cosmic rays from outer space and some may be due to atmospheric pollution from man-made radioactive sources.

Scanned with CamScanner

Summary

1. α-particles are positively charged nuclei of helium (mass about four times that of the hydrogen atom and carrying a charge of $+2e$). They are stopped by thin paper. They ionise air considerably. Owing to their heavy mass, they are not deflected appreciably in a magnetic field.
2. β-particles are usually electrons and are negatively charged. They are stopped by aluminium plate several millimetres thick, and ionise air less than α-particles. They are easily deflected in a powerful magnetic field, as they are so light.
3. γ-rays are electromagnetic waves which have shorter wavelength than X-rays. They are stopped by very thick lead blocks and ionise air slightly. They are not deflected by magnetic fields.
4. Detectors of ionising particles and radiation are (a) Geiger–Müller tube or solid-state detector and scaler (or ratemeter)–the count rate is proportional to the intensity; (b) diffusion cloud chamber–tracks can be seen. The G–M tube and solid state detector detect all three radiations, but the cloud chambers detect mainly α- and β-particles.
5. The atom consists of a small central nucleus, containing protons and neutrons; a proton is a hydrogen nucleus. The number of protons is called the *proton* or *atomic number*, the total number of protons and neutrons together is called the *nucleon* or *mass number*. In nuclear reactions, the total nucleon number and total proton number on one side of the equation are the same as on the other side of the equation.
6. An *isotope* is an atom which has the same proton or atomic number, and hence the same chemical properties, as another atom with a different mass or nucleon number.
7. The *half-life* of a radioactive element is the time taken for the atoms to decay to half the original number.
8. Nuclear *fission* is the 'splitting' of a heavy atom, such as uranium, into two heavy parts; it is produced when neutrons of suitable speed are fired into a mass of uranium-235; nuclear energy is released. Nuclear *fusion* occurs in the sun; hydrogen nuclei fuse to form helium nuclei, with the release of nuclear energy.

Exercise 34 (Answers, page 643)

1. If e is the charge and m the mass of a proton, complete the following table.

Particle	Charge	Mass
proton	$+e$	m
neutron		
hydrogen nucleus		
alpha particle		

(L)

2. Name the three fundamental particles of which the atoms of an element are composed. How are these particles distributed in the atom of an element whose atomic number is 3 and mass number 7? (L)

3. The half-life of a certain radioactive substance is 57 days. Explain the meaning of this statement. (N)

4. (a) What is understood by *atomic number*? (b) What is the effect of the emission of a ß-particle from an atom of radioactive phosphorus of atomic number 15, and mass number 32? (C)

5. (a) When alpha-particles strike a thin metal foil they are scattered. Draw a sketch of a simplified experimental arrangement to observe such scattering.

State the results which have been obtained and the conclusions which have been drawn from these results.

Scanned with CamScanner

(b) A radioactive source emitting only one type of radiation produces a count rate of 1000 counts/minute when placed close to a detector. The insertion of a thin aluminium foil between the source and detector reduces the reading to 300 counts/minute. With the foil removed, increasing the separation of the source and detector reduces the count rate until beyond 30 cm the rate remains constant at 50 counts/minute.

State the most probable type of radiation emitted. Give reasons for your choice and explain the readings obtained. (N)

6. Arrange the following radioactive radiations in order of their ability to penetrate matter, placing the most penetrating radiation first; β-particles, α-particles, γ-rays. State the electric charge associated with each of these radiations. (C)

7. (a) Explain the changes in the composition of the nucleus of a radioactive atom that result from the emission of (i) an alpha-particle, (ii) a beta-particle.

(b) Describe an experiment to show the ability of beta particles to penetrate matter. In what way do the penetrating powers of alpha particles differ from those of beta particles? (N)

8. Outline one way of detecting α-particles and describe, so far as you can, how it works.

The counting rate recorded by a detector fixed in front of an α-particle emitter is 256 per sec. This figure is an average rate worked out from a count lasting for several minutes. What is the average counting rate 20 days later, for the same arrangement, if the half-life of the emitter is 5 days?

If the number of α-particles were recorded at this time for 1 sec precisely, would you expect to find the number you have just calculated? Give a reason for your answer. (O and C)

9. Draw a diagram of a hydrogen atom and label the particles of which it is composed.

If a proton is considered to have mass m, what is the mass of (a) a neutron; (b) an electron? (N)

10. The simplest model of an atom consists of electrons which are in orbit around a nucleus.

Describe the differences between the nucleus of hydrogen, mass number 1; atomic number (proton number) 1; and

(a) the nucleus of helium, mass number 4; atomic number (proton number) 2;

(b) the nucleus of deuterium, mass number 2; atomic number (proton number) 1.

Which of these three substances are isotopes of the same element? Give a reason. (L)

11. Explain the meaning of the parts in italics in the following true statement: 'Plutonium-238 is an *alpha-emitter* but strontium-90 which has a *half-life of 28 years* emits mainly *β-particles*, with a little *gamma-radiation*.'

If a certain quantity of strontium-90 was emitting a very large number of particles per second in the year 1971, how long will it take to emit the same number of particles in the year 2055?

Describe an apparatus for detecting alpha-particles and explain how it works. (O and C)

12. What is meant by (a) *radioactive decay*, (b) the *half-life* of a radioactive element?

Thoron is a radioactive gas with a half-life of about 1 minute. When some of the gas is put in a cloud chamber, the tracks of alpha-particles can be detected. How would you use this fact to determine the half-life of the gas? Illustrate your answer by reference to the types of observations you would expect to make if, on first observing tracks in the chamber, there were 14 tracks visible.

What changes take place in the atomic structure of thoron (atomic number 86, mass number 220) if it emits an alpha-particle and then, later, another alpha-particle? (C)

13. The count rate recorded by Geiger–Müller tube and counter close to an alpha-particle source is 400 per minute after allowing for the background count. If the half-life of the source is 4 days, (a) what would the count rate be 12 days later, (b) why would the rate be determined over periods of several minutes rather than over a few seconds, (c) why, if the mass of the source was determined many days later, would there be no apparent change?

What is meant by *background count*? (L)

14. When a pure radium salt is completely sealed up in a glass tube and left for some time, the tube contains elements which emit, between them, α-, β-, and γ-radiations. For medical use, such tubes were sheathed completely in a layer of platinum about 0·5 mm thick.

State, giving reasons for your answer, what radiation you would expect to find

Scanned with CamScanner

outside the platinum sheath. Describe the apparatus and method you would use to detect this radiation.

The radioactive isotope iodine-131 has a half-life of approximately 8 days. A hospital physicist received a quantity of this material for use on a certain day. What *fraction* of it would still have remained if he had not been able to use it until (a) 8 days (b) 16 days after he received it? (O and C)

Multiple-Choice Questions

15. Which particle is NOT affected by a magnetic field or an electric field?
A electron B proton C neutron
D alpha-particle E beta-particle

16. In a cloud chamber, straight-line streaks of vapour are produced by a source emitting
A beta-particles B gamma-rays C electrons
D alpha-particles E light rays

17. The radium atom $^{226}_{88}Ra$ has
A 138 protons and 88 electrons
B 138 neutrons and 88 electrons
C an atomic number 226
D a mass number 88
E 138 protons

18. When a beam of alpha-particles is fired at a thin gold film, some of the particles were considerably deflected.
This experiment shows that
A alpha-particles are very light
B a gold nucleus contains alpha-particles
C electrons in a gold nucleus attract alpha-particles
D a gold nucleus is concentrated in a small volume
E a gold nucleus emits alpha-particles

19. Which of the atoms A, B, C, D and E are isotopes?
A $^{234}_{92}P$ B $^{218}_{84}Q$ C $^{226}_{88}R$ D $^{234}_{91}S$ E $^{214}_{84}T$

20. The half-life of an element is 24 hours. What fraction of the atoms have disintegrated in 72 hours?
A 7/8 B 3/4 C 1/2 D 1/4 E 1/8

Scanned with CamScanner

35. Further Mechanics: Projectiles · Simple Harmonic Motion

Projectiles

In an earlier chapter, we showed that the horizontal and vertical motion of an object thrown forward into the air could be treated *independently* of each other (see page 31). We now deal generally with *projectiles*, as these objects are called. Examples of projectiles are a ball thrown at an angle to a wall, a football kicked into the air towards the goal or a rocket fired at an angle to the horizontal.

Time of Flight · Maximum height · Range of projectile

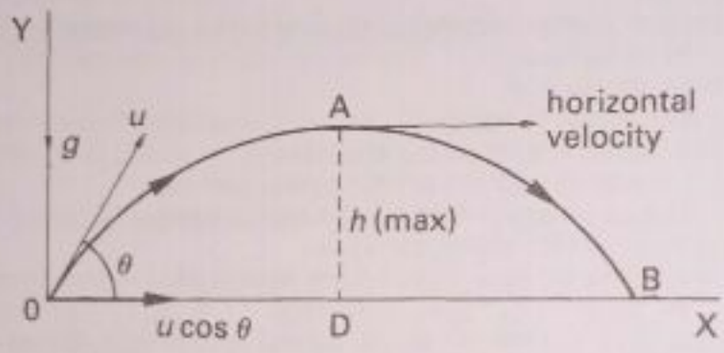


Fig. 35.1 Projectile motion

Consider a ball thrown initially with a velocity u from 0 at an angle θ to the horizontal, Fig. 35.1. At A it reaches a maximum height and then comes down to the horizontal again at B. OB is called the horizontal *range* of the ball.

We now consider separately the motion of the ball in the vertical direction and in the horizontal direction. In the vertical direction, the component of u is $u \sin \theta$; in the horizontal direction, the component of u is $u \cos \theta$ (see p. 30).

Vertical motion

1. *Time of flight.* In the vertical direction OY, the component of u is $u \sin \theta$. When the ball reaches B, the vertical distance travelled is *zero*.

Using the equation of motion $s = ut + \frac{1}{2}at^2$ (p. 18) for vertical motion, then

$$s = 0, u = \text{initial vertical velocity, } u \sin \theta, \text{ acceleration } a = -g$$

We use $-g$, where g is the *numerical* value of the acceleration due to gravity, because the ball decelerates when moving upwards along OY.

$$\therefore 0 = u \sin \theta \, . \, t + \tfrac{1}{2}(-g)t^2$$

Cancelling t, $$\therefore u \sin \theta = \tfrac{1}{2}gt$$

$$\therefore t = \frac{2u \sin \theta}{g} \qquad (1)$$

Scanned with CamScanner

This is the time of flight from O to B.

2. *Maximum height.* At A, the maximum height, the velocity of the ball is horizontal. So the vertical velocity is zero at A. We can use the equation of motion $v^2 = u^2 + 2as$ (p. 19) to calculate the maximum height h. For vertical motion,

$$\text{final velocity } v = 0, \text{ initial velocity } u = u\sin\theta, s = h, a = -g$$

$$\therefore 0 = (u\sin\theta)^2 - 2gh$$

$$\therefore h = \frac{u^2\sin^2\theta}{2g} \qquad (2)$$

Horizontal motion · Range

We now deal with the horizontal motion of the ball or projectile. Since the acceleration due to gravity, g, always acts vertically downwards, it has no component or effect in the horizontal direction.

The horizontal velocity $= u\cos\theta$. Since $u\cos\theta$ is *constant*, then if OB is reached from O in a time t,

$$\text{distance OB} = \text{velocity} \times \text{time} = u\cos\theta \times t$$

From (1)
$$t = \frac{2u\sin\theta}{g}$$

$$\therefore \text{OB} = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g}$$

since $2\sin\theta\cos\theta = \sin 2\theta$. So

$$\text{horizontal range OB} = \frac{u^2\sin 2\theta}{g} \qquad (3)$$

The *maximum range* is obtained when $\sin 2\theta = 1$ or $2\theta = 90°$. So $\theta = 45°$ for maximum range with a given velocity.

Example on projectiles

A ball is thrown with an initial velocity of 20 m/s at an angle of 30° to the horizontal. Calculate (a) the time of flight, (b) the maximum height, (c) the horizontal range of the ball. (Assume $g = 10\,\text{m/s}^2$.)

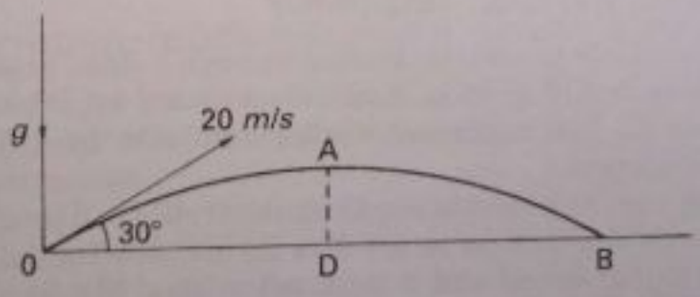


Fig. 35.2 Example

(a) *Time of flight.* When the ball reaches B, vertical distance travelled, $s = 0$, Fig. 35.2.
For vertical motion, $u = 20\sin 30° = 10\,\text{m/s}$, $a = -g = -10\,\text{m/s}^2$, $s = 0$

Scanned with CamScanner

From $s = ut + \frac{1}{2}at^2$

$$\therefore 0 = 10t + \tfrac{1}{2}(-10)t^2 = 10t - 5t^2$$

Cancelling t, $0 = 10 - 5t$

$$\therefore t = \frac{10}{5} = 2\text{ s}$$

(b) *Maximum height.* At A, vertical velocity = 0. From $v^2 = u^2 + 2as$ for vertical motion,

$$0 = (20\sin 30°)^2 + 2(-10)s$$

$$\therefore s = \frac{(20\sin 30°)^2}{20} = \frac{10^2}{20} = 5\text{ m}$$

(c) *Range.* For horizontal motion,

$$\text{range} = \text{OB} = s = \text{velocity} \times \text{time} = 20\cos 30° \times 2$$
$$= 40\cos 30° = 34{\cdot}6\text{ m}$$

$$\left(\text{alternatively, range} = \frac{u^2\sin 2\theta}{g} = \frac{20^2\sin 60°}{10} = 34{\cdot}6\text{ m}\right)$$

Exercises 35A (Answers, page 643)

(*Assume $g = 10\,m/s^2$*)

1. Choose the best answer from A, B, C, D or E:
When a projectile is fired with a velocity u at an angle θ to the horizontal,
   A the vertical velocity is constant
   B the horizontal range is a maximum when θ is 30°
   C the horizontal velocity is constant
   D the time of flight is $u\sin\theta/g$
   E the vertical velocity never reaches zero
2. A ball is thrown with an initial velocity of 40 m/s at 30° to the horizontal. Calculate (i) the time to reach the greatest height, (ii) the greatest height, (iii) the range of the ball.
3. A rocket is fired with an initial velocity of 200 m/s at an angle of 60° to the horizontal. Find (i) the maximum height reached, (ii) the time of flight.
4. When a ball is thrown with an initial velocity u at an angle of 30° to the horizontal, the time taken to reach the horizontal again is 4 s. Calculate u and the horizontal range of the ball.
5. A bullet is fired with a velocity of 50 m/s at an angle θ to the horizontal. (a) What is the value of θ for maximum range with this velocity? (b) Calculate the maximum range.
6. A rocket fired with an initial velocity u at an angle of 60° reaches a maximum height of 200 m. Calculate (i) u and (ii) the time of flight.

Simple Harmonic Motion

Acceleration, period, frequency

The bob O of a simple pendulum, moving through a small angle θ, has *simple harmonic motion* (s.h.m.) Fig. 35.3a. As it swings to-and-fro, it has an average or central position O. The maximum displacement, OA or OB, is called the *amplitude* of the motion.

From O to A, then to B and back to O again, is called one complete *cycle*. The time for one cycle is called the *period* T of the motion. The *frequency* f is the number of cycles per second and is measured in hertz, Hz. So a strip of metal vibrating at 2 Hz does 2 cycles per second. Its period $T = 1/2$ seconds. Always

$$T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T}$$

Scanned with CamScanner

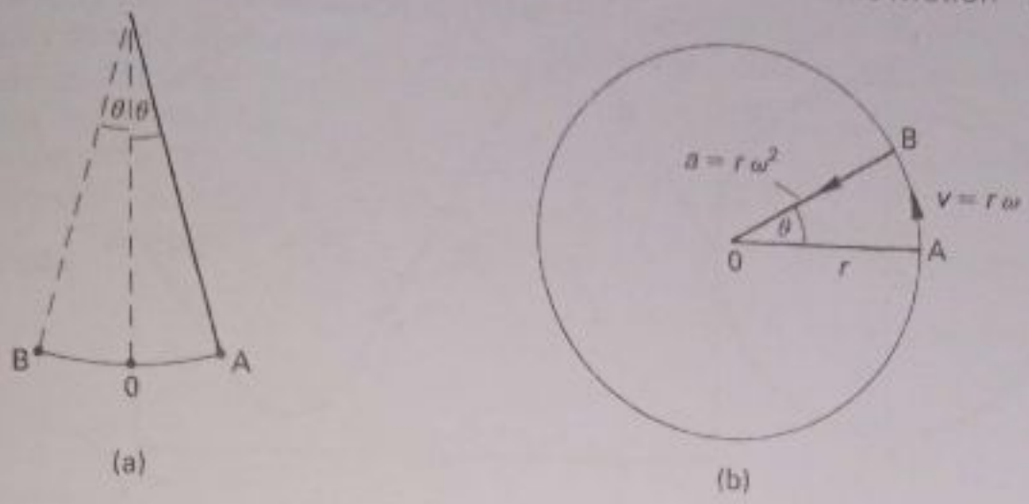


Fig. 35.3
(a) Simple pendulum
(b) Circular motion

Circular motion

Simple harmonic motion is connected with circular motion as we see shortly. So we first consider a small object moving round a circle of radius r with a constant linear speed v, Fig. 35.3b. The constant *angular speed* ω round the circle is defined as θ/t, where θ is the angle at the centre O of the circle when the object moves from A to B in a time t. θ is measured in *radians* (rad) (2π rad $= 360°$), so ω, from $\omega = \theta/t$, is measured in rad/s.

By definition, θ in radians $=$ AB$/r$, or AB $= r\theta$. So AB$/t = r\theta/t$. But AB$/t = v$ and $\theta/t = \omega$.

$$\therefore v = r\omega \qquad (1)$$

As explained on p. 43, an object moving round a circle with a constant speed v has an acceleration *a towards the centre* O. Calculation shows that

$$a = \frac{v^2}{r}$$

Using (1) for v,
$$a = \frac{(r\omega)^2}{r} = r\omega^2 \qquad 2)$$

At A, the acceleration a is directed towards O along AO. At B, the acceleration is directed towards O along BO.

Simple harmonic motion from circular motion

When a small object moves at constant angular speed round a circle, the motion of the *foot of the perpendicular from the object to a fixed diameter* of the circle is simple harmonic motion.

In Fig. 35.4, CZF is a fixed diameter. Starting at O, the foot of the perpendicular from O to CZF is Z. Moving along the circle to A, the foot of the perpendicular is now M; at B it is N; and at C it is C itself.

As the object continues to move round the circle to D and E, the foot of the perpendicular moves down CZF. When the object goes past F and returns to O, the foot of the perpendicular moves up from F to Z. This to-and-fro motion along CZF can be defined as simple harmonic motion.

Scanned with CamScanner

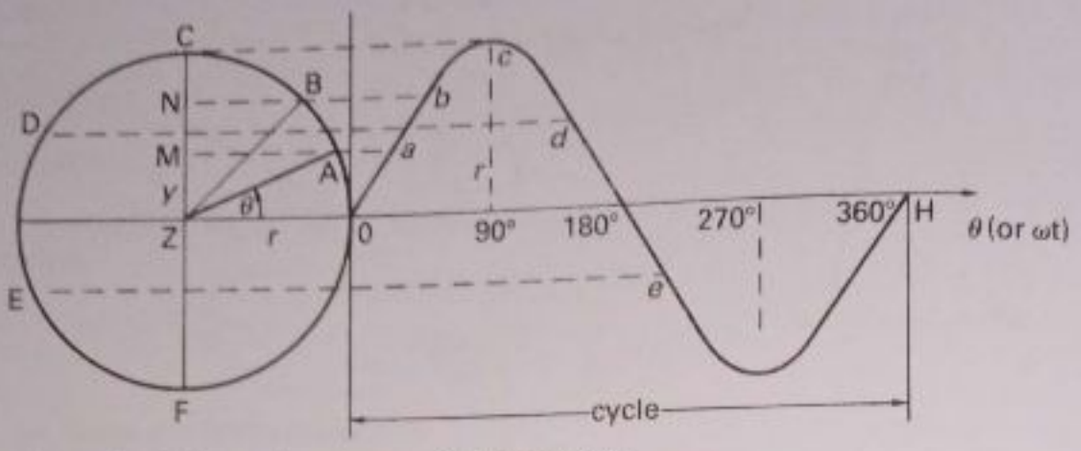


Fig. 35.4 Simple harmonic motion-graph of displacement

Then Z is the centre of the motion, MZ(y) is the *displacement* of M from Z at some time t measured from the centre Z, and CZ or r is the maximum displacement or *amplitude* of the simple harmonic motion.

Simple harmonic motion formulae

We can now obtain formulae for use in simple harmonic motion. These formulae are needed, for example, to design the springs below car bodies or to calculate the power from a loudspeaker. Waves of all kinds, such as sound, light and water waves are produced by simple harmonic motion of particles or fields.

1. *Acceleration* The acceleration of M towards Z = component of A's acceleration in direction MZ, Fig. 35.4.

Now acceleration of A along AZ = $r\omega^2$ (p. 585)

$$\therefore \text{ acceleration of M towards Z} = r\omega^2 \sin\theta$$

But $r \sin\theta$ = ZM = y say.

$$\therefore \text{ acceleration of M towards Z} = -\omega^2 y, \qquad (1)$$

the minus being essential if we measure y positive in the direction ZMC because the acceleration of M is in the opposite (negative) direction MC.

From (1), we can define simple harmonic motion as the motion of an object whose acceleration is proportional to its displacement y from a fixed point and is directed towards that fixed point.

2. *Displacement, y* From Fig. 35.4, $y/r = \sin\theta$.

$$\therefore y = r\sin\theta = r\sin\omega t \qquad (2)$$

since $\theta = \omega t$. So the graph of y against θ, or y against ωt, is a sine curve as shown. The sine curve is obtained from the circle by plotting a, b, c, d, e and so on, as shown in Fig. 35.4.

3. *Cycle · Period* When the object goes once round the circle in Fig. 35.4, y varies from O to H along the sine curve. The graph between O and H represents *one cycle* of the motion.

The time for 1 cycle is called the period T of the motion. This is the time for the object to move once round the circle, which is an angle of 2π radians (360°). Since ω is the angular speed,

$$T = \frac{2\pi}{\omega} \qquad (3)$$

Scanned with CamScanner

4. *Velocity* Fig. 35.5a shows the displacement y plotted against the time t. The velocity at any instant is the rate of change of displacement or displacement per second (p. 15), which is equal to the *gradient* of the graph of y against t at the instant considered. At O, the gradient g or velocity v is a maximum as represented by OX. At C, the gradient $g = 0$, so $v = 0$. In this way, we see that the velocity v is a cosine curve. From mathematics,

$$v = \pm r\cos\omega t = \pm\omega\sqrt{r^2 - y^2} \tag{4}$$

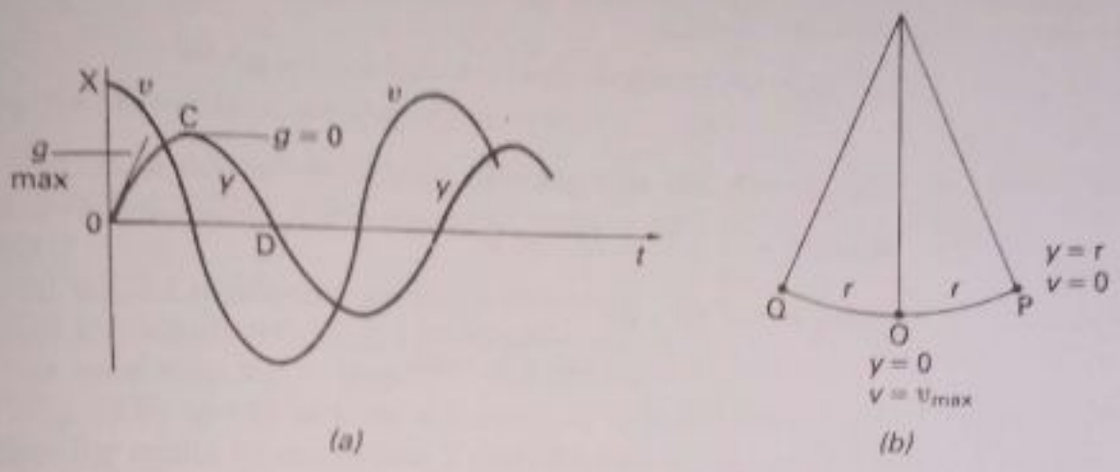


Fig. 35.5 Displacement and velocity

At the centre O of the oscillation of the bob of a pendulum, $y = 0$ and the velocity v is a maximum, Fig. 35.5b. From (4), since $y = 0$ in this case,

$$v_{max} = \omega r \tag{5}$$

At the end of the oscillation, $y = r$. So $v = 0$ from (4).

Also, since the acceleration $a = -\omega^2 y$, note that the acceleration is numerically a maximum when $y = r$. So

$$a_{max} = -\omega^2 r$$

The acceleration is zero when $y = 0$, at the centre of the oscillation.

Summary of s.h.m. formulae

$$\text{acceleration } a = -\omega^2 y \tag{1}$$

$$\text{period } T = 2\pi/\omega \tag{2}$$

$$\text{velocity } v = \pm\omega\sqrt{r^2 - y^2} \tag{3}$$

So $a_{max} = -\omega^2 r$ and $v_{max} = \pm\omega r$

Examples on s.h.m.

1. A metal strip is fixed at one end. The other end vibrates with simple harmonic motion and has an acceleration of 2 m/s² when the displacement from the centre of oscillation is 2 cm (0·02 m).

Calculate the period T of the motion.

$$\text{Acceleration } a = -\omega^2 y$$

So numerically, omitting the minus,

$$2 = \omega^2 \times (0{\cdot}02)$$

Scanned with CamScanner

$$\therefore \omega^2 = \frac{2}{0{\cdot}02} = 100$$

$$\therefore \omega = \sqrt{100} = 10$$

So $$\text{period } T = \frac{2\pi}{\omega} = \frac{2\pi}{10} = 0{\cdot}63\text{ s}$$

2. The bob of a simple pendulum oscillates with s.h.m. and has an amplitude of 10 cm (0·1 m). The acceleration $a = -4y$, where y is the displacement.
Calculate (a) the maximum acceleration, (b) the period, (c) the velocity 4 cm from the centre, (d) the maximum velocity.

(a) $$a_{max} = -4 \times \text{amplitude} = -4 \times 0{\cdot}1 = -0{\cdot}4\text{ m/s}^2$$

(b) $$a = -4y = -\omega^2 y$$

$$\therefore \omega^2 = 4 \quad \text{or} \quad \omega = \sqrt{4} = 2$$

$$\therefore \text{period } T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \omega = 3{\cdot}1\text{ s}$$

(c) $$\text{velocity } v = \omega\sqrt{r^2 - y^2}$$

$$\therefore v = 2\sqrt{0{\cdot}1^2 - 0{\cdot}04^2} = 2 \times 0{\cdot}09$$

$$= 0{\cdot}18\text{ m/s}$$

(d) $$v_{max} = \omega r = 2 \times 0{\cdot}1 = 0{\cdot}2\text{ m/s}$$

Kinetic and potential energy in s.h.m.

In simple harmonic motion, the energy of the moving object is both kinetic and potential. For the oscillating bob of a simple pendulum, for example, the kinetic energy at any instant is $\frac{1}{2}mv^2$, where m is the mass and v is the velocity. At the same instant, the bob has potential energy and this varies with the height of the bob above the lowest point of swing.

There is a constant exchange between kinetic energy (k.e.) and potential energy (p.e.) in simple harmonic motion. At the end of an oscillation, the bob of a pendulum has maximum p.e. and zero k.e. As the bob moves back towards the centre of oscillation, the p.e. becomes smaller and the k.e. becomes correspondingly bigger. The change in p.e. = the gain in k.e. At the centre, the k.e. is a maximum and the p.e. is then zero, if we measure the p.e. from the lowest point or centre of the oscillation.

Generally, neglecting friction, in s.h.m.

$$\textit{kinetic energy} + \textit{potential energy} = \textit{constant} = \textit{total energy}$$

K.e. and p.e. values in s.h.m.

Suppose the oscillating bob has a mass m and a velocity v when it is a distance y from the centre of oscillation. If r is the amplitude, then

$$\text{kinetic energy, k.e.} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2(r^2 - y^2), \qquad (1)$$

since $v = \pm\omega\sqrt{r^2 - y^2}$ (p. 587). So the maximum k.e., when $y = 0$, is $\frac{1}{2}m\omega^2 r^2$. This is the *total* energy of the bob, since the potential energy is then zero. So at a distance y from the centre,

$$\text{potential energy, p.e.} = \tfrac{1}{2}m\omega^2 r^2 - \tfrac{1}{2}m\omega^2(r^2 - y^2)$$

$$\therefore \text{p.e.} = \tfrac{1}{2}m\omega^2 y^2 \qquad (2)$$

Scanned with CamScanner

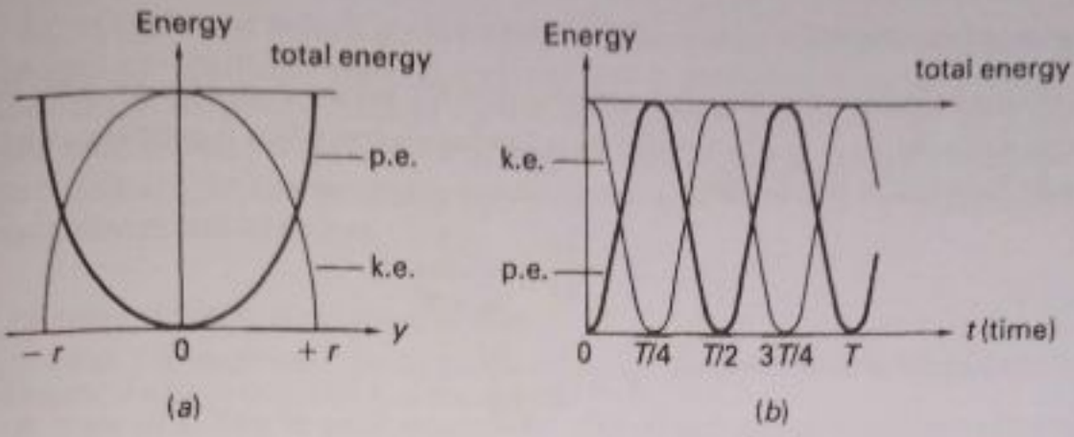


Fig. 35.6 Kinetic and potential energy variation

Fig. 35.6a shows how the k.e. and p.e. vary with the displacement y from the centre of oscillation. From the oscillation of a pendulum bob, you can remember that:

(a) k.e. is a maximum and p.e. is zero when $y = 0$
(b) k.e. is zero and p.e. is a maximum when $y = r$
(c) maximum k.e. $= \frac{1}{2}m\omega^2 r^2 =$ maximum p.e.

Fig. 35.6b shows how the k.e. and p.e. vary with time t, when t is measured from the centre of oscillation. If T is the period, then $T/4$ is the time when the object is at the end of its oscillation, $T/2$ is the time when it returns to the centre, and so on. The graphs are sine curves in shape.

Oscillating systems · Mass–spring system

We now deal with examples of oscillating systems. One important example is a mass m attached to the end of a vertical helical spring S, Fig. 35.7. When m is pulled down a little and then released, it oscillates about a centre O. The centre O is the equilibrium position of m when it is first attached to S at A and stretches the spring a distance e as shown.

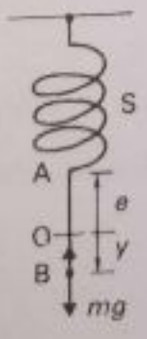


Fig. 35.7 Mass–spring oscillation

Force constant of spring

Provided its elastic limit is not exceeded, a spring stretches a length x proportional to the force F applied (see p. 152). In this case, $F = -kx$, where k is known as the 'force constant' of the spring. The minus shows that F acts in the opposite direction to the direction in which x is increased. So if a force F of 2 N extends the spring 8 mm, numerically

$$k = \frac{2\text{ N}}{8\text{ mm}} = 0{\cdot}25\text{ N/mm} = 250\text{ N/m}$$

Scanned with CamScanner

Period of oscillation

In simple harmonic motion, we have seen that the period $T = 2\pi/\omega$ (p. 586). We prove soon that $\omega = \sqrt{k/m}$ for the mass–spring system when it oscillates, where k is in N/m and m is in kg. So

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}}$$

$$\therefore T = 2\pi\sqrt{\frac{m}{k}} \qquad (1)$$

Proof

In Fig. 35.7, suppose the mass m extends the spring a distance e to O when first tied to it at A. Then numerically,

$$F = \text{weight of } m = mg = ke \qquad (2)$$

Now suppose the mass m is pulled down a little below O and released. At one instant in its oscillation, suppose m reaches a position B at a distance y below O. The resultant force F in the direction OB = mass weight − upward force in spring ($k \times$ extension). Since the spring extension is $(e + y)$,

$$F = mg - k(e + y) = mg - ke - ky$$

But $mg = ke$, from (2).

$$\therefore F = -ky$$

From force = mass × acceleration, $F = ma$.

$$\therefore ma = -ky$$

$$\therefore a = -\frac{k}{m}y = -\omega^2 y$$

where $\omega^2 = k/m$. So the motion of m is simple harmonic. Also

$$\text{period } T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} = 2\pi\sqrt{\frac{m}{k}}$$

We can write the formula for T in another useful way. Since $mg = ke$, then $m/k = e/g$. So

$$T = 2\pi\sqrt{\frac{e}{g}} \qquad (3)$$

Energy in mass and spring

When the mass–spring system oscillates, the mass m has kinetic energy of $\frac{1}{2}mv^2$ at any instant, where v is its velocity.

The spring has potential energy during the oscillation. This is molecular energy, due to changes in the displacement of its molecules when the spring is stretched or compressed. As we showed on page 152,

$$\text{potential energy of spring} = \tfrac{1}{2}kx^2 \qquad (4)$$

where x is the extension of the spring from its unstretched or natural length.

Scanned with CamScanner

As we explained before, (a) the potential plus kinetic energy of this system is constant and (b) the potential energy and kinetic energy interchange in value. So when the mass passes through the centre of oscillation O in Fig. 35.7, it has maximum kinetic energy and the spring has zero potential energy. At the end of the oscillation the kinetic energy becomes zero and the stretched spring then has maximum potential energy.

Example on mass–spring oscillation

A force of 2 N stretches a spring 40 mm (0·04 m). Assuming Hooke's law is obeyed, calculate the force constant k of the spring.

A mass of 0·5 kg is now attached to end of the spring which is suspended vertically. The mass is pulled down a distance 10 mm from its equilibrium position and then released. Calculate the period of oscillation, the maximum kinetic energy of the mass, and the kinetic energy of the mass and the potential energy of the spring when the mass is 4 mm below its centre of oscillation.

Force constant $F = 2\,\text{N}$ and $x = 0{\cdot}04\,\text{m}$.

So $$\text{force constant } k = \frac{F}{x} = \frac{2\,\text{N}}{0{\cdot}04\,\text{m}} = 50\,\text{N/m}$$

Period $$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0{\cdot}5}{50}}$$

$$= 2\pi\sqrt{\frac{1}{100}} = 2\pi \times \frac{1}{10} = 0{\cdot}63\,\text{s}$$

Maximum kinetic energy Maximum k.e. $= \frac{1}{2}m\omega^2r^2$.

Since $r = 10\,\text{mm} = 0{\cdot}01\,\text{m}$, $\omega = \sqrt{k/m} = \sqrt{50/0{\cdot}5} = 10$, $m = 0{\cdot}5\,\text{kg}$

$\therefore$ maximum k.e. $= \frac{1}{2} \times 0{\cdot}5 \times 0{\cdot}01^2 \times 10^2 = 0{\cdot}0025\,\text{J}$

Kinetic energy of mass With mass 4 mm below centre of oscillation,

$$x = 4\,\text{mm} = 0{\cdot}004\,\text{m},\ r = 0{\cdot}01\,\text{m}$$

So $$\text{k.e.} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2(r^2 - x^2)$$

$$= \tfrac{1}{2} \times 0{\cdot}5 \times 10^2 \times (0{\cdot}01^2 - 0{\cdot}004^2)$$

$$= 0{\cdot}0021\,\text{J}$$

Potential energy of spring The mass 0·5 kg, weight 5 N, extends the spring initially a distance $e = 5 \times 0{\cdot}02\,\text{m}$, using the force constant value, $= 0{\cdot}1\,\text{m}$.

With the mass 0·004 m below this extension,

$$\text{total extension of spring} = 0{\cdot}1 + 0{\cdot}004 = 0{\cdot}104\,\text{m}$$

$$\therefore \text{p.e. of spring} = \tfrac{1}{2}kx^2 = \tfrac{1}{2} \times 50 \times 0{\cdot}104^2 = 0{\cdot}27\,\text{J}$$

(The p.e. stored in the spring by the initial extension e of $0{\cdot}1\,\text{m} = \frac{1}{2}ke^2 = \frac{1}{2} \times 50 \times 0{\cdot}1^2 = 0{\cdot}25\,\text{J}$.)

Measurement of *g* by helical spring

As we saw on p. 590, the period of oscillation T of the mass at the end of a helical spring is given by $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{e/g}$, where e is the extension due to the weight mg of the mass. Squaring both sides, then

$$T^2 = 4\pi^2\frac{e}{g}$$

$$\therefore g = 4\pi^2\frac{e}{T^2} \qquad (1)$$

Scanned with CamScanner

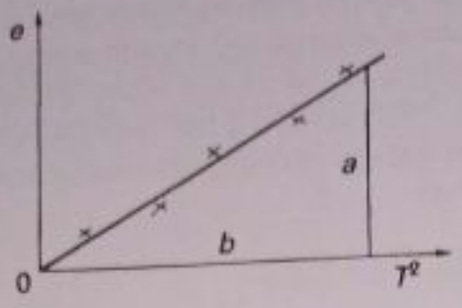


Fig. 35.8 Measurement of g

Equation (1) can be used to measure g. Using five different masses whose weights do not exceed the elastic limit, the extension e is measured each time. The period T of oscillation is also measured for each mass by taking a suitable number of oscillations.

A graph of e against T^2 is then plotted and the best straight line is drawn through all the points, Fig. 35.8. The gradient a/b of the line is then measured and g calculated from

$$g = 4\pi^2 \times \text{gradient}$$

The experiment gives only an approximate value of g as the mass of the spring has not been taken into account.

Oscillation of loaded tube in liquid

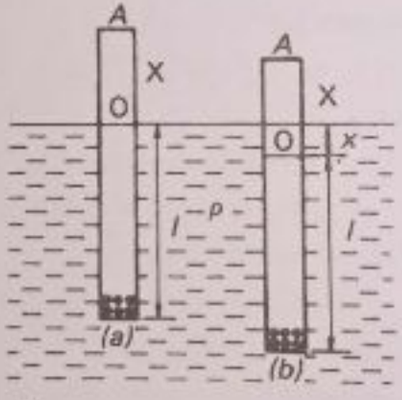


Fig. 35.9 Oscillation of loaded tube

Consider a tube X, loaded at the bottom with lead shot for stability and floating upright in a liquid of density, Fig. 35.9a. The upthrust U = weight of liquid displaced, from Archimedes' principle. See page 106. So if the depth of X below the surface is l as shown, corresponding to O on the tube,

$$U = \text{mass of liquid displaced} \times g = V\rho g,$$

since mass = volume V displaced × density ρ of liquid. Assuming X has a uniform cross-sectional area A, then $V = Al$. So $U = Al\rho g$.

If X is pushed down a small distance below the surface and released, it oscillates vertically. Suppose O is now a distance x below the water level at some instant, Fig. 35.9b. Then new upthrust U_1 = weight of liquid displaced of length $(l+x)$. So

$$U_1 = A(l+x)\rho g$$

$\therefore$ resultant upward force on X, $F = A(l+x)\rho g - mg$,

since mg is the downward weight of X. But X was in equilibrium in Fig. 35.9a, so $U = Al\rho g = mg$.

Scanned with CamScanner

$$\therefore F = A(l+x)\rho g - Al\rho g = Ax\rho g$$

So, from $F = ma$, where a is the acceleration of X at this instant,

$$ma = -A\rho gx$$

the minus showing that F is opposite in direction to x measured positively in a downward direction.

$$\therefore a = -\frac{A\rho g}{m}x = -\omega^2 x$$

where $\omega^2 = A\rho g/m$. So the period T is given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{A\rho g/m}} = 2\pi\sqrt{\frac{m}{A\rho g}}$$

Simple pendulum oscillation

Fig. 35.10 shows a simple pendulum of length l suspended from a fixed point X.

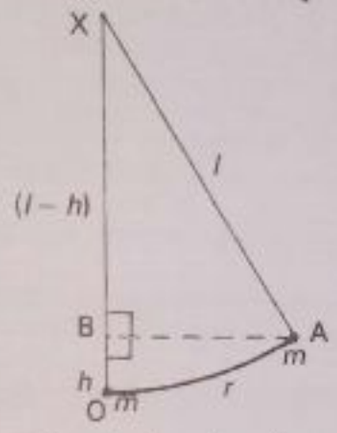


Fig. 35.10 Simple pendulum

The small mass m of the pendulum bob swings from the end A of its oscillation to its centre of oscillation O through a very small arc AO of length r. When the angle of swing θ is very small, AO is practically a straight line.

We can find the constant ω in the period formula $T = 2\pi/\omega$ from an *energy equation*. When m moves from A to O, the gain in kinetic energy at O = the loss in potential energy at A. The maximum kinetic energy at O $= \frac{1}{2}m\omega^2 r^2$ (see page 588).

$$\therefore \tfrac{1}{2}m\omega^2 r^2 = mgh \qquad (1)$$

where h is the length BO in Fig. 35.10 and AB is horizontal.

Applying Pythagoras' theorem to triangles ABO and ABX respectively,

$$r^2 = h^2 + \mathrm{BA}^2 \quad \text{and} \quad l^2 = (l-h)^2 + \mathrm{BA}^2$$

So $$\mathrm{BA}^2 = r^2 - h^2 = l^2 - (l-h)^2 = l^2 - (l^2 - 2hl + h^2) = 2hl - h^2$$

$$\therefore r^2 = 2hl$$

Substituting this value of r^2 in (1), then

$$\tfrac{1}{2}m\omega^2 . 2hl = mgh$$

Simplifying, $$\therefore \omega^2 = \frac{g}{l} \quad \text{or} \quad \omega = \sqrt{\frac{g}{l}}$$

$$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

Scanned with CamScanner

All kinds of simple harmonic systems, such as sound and electrical systems in addition to mechanical systems, can be set into *forced vibrations* and into *resonance* by outside periodic forces. This is discussed on page 346, to which the reader is referred.

Exercises 35B (Answers, page 644)

(Where necessary, assume $g = 10\,\text{m/s}^2 = 10\,\text{N/kg}$.)

In questions 1, 2 and 3, choose the answer you think is most correct.

1. In simple harmonic motion,
   A the displacement is proportional to the time
   B the velocity is proportional to the displacement
   C the maximum acceleration is at the centre of oscillation
   D the acceleration is proportional to the displacement
   E the maximum velocity is at the end of the oscillation
2. In simple harmonic motion,
   A the kinetic energy is constant in value
   B the potential energy is constant in value
   C the kinetic energy is a maximum at the end of the oscillation
   D the kinetic plus potential energy is constant
   E the potential energy is a maximum when the displacement is zero
3. A mass X at the end of a spring has a natural frequency of oscillation of 4 Hz. When a simple harmonic force frequency f is applied to X to force it to oscillate, resonance (maximum amplitude of oscillation) occurs when f is
   A $\frac{1}{2}$ Hz B 1 Hz C 2 Hz D 3 Hz E 4 Hz
4. A small mass oscillates with simple harmonic motion. Its amplitude is 0·04 m and its period is 1 s.
   Calculate (i) the constant ω in 'acceleration $= -\omega^2 x$', (ii) the maximum acceleration of the mass, (iii) its maximum velocity, (iv) its velocity when 0·02 m from the centre of oscillation.
5. An oscillating pendulum bob has a mass of 0·1 kg. Its amplitude is 0·01 m and its period 2 s.
   Calculate (i) the constant ω in the acceleration formula, (ii) the maximum acceleration, (iii) the maximum kinetic energy of the bob, (iv) the kinetic energy when the bob is 0·004 m from the centre of oscillation.
6. A helical spring is extended 8 mm (0·008 m) by a force of 1·6 N. What is the force constant of the spring?
   A mass of 0·5 kg is suspended from the vertical spring and is pulled down a small distance of 4 mm (0·004 m) and then released. Calculate (i) the period of the motion, (ii) the maximum kinetic energy of the mass, (iii) the potential energy of the spring when the mass is 2 mm (0·002 m) below the centre of oscillation.
7. A sealed tube of uniform cross-sectional area $4 \times 10^{-4}\,\text{m}^2$ is loaded with lead shot so that the total mass is 0·12 kg. The tube is then floated upright in oil of density $800\,\text{kg/m}^3$.
   (i) Calculate the depth of immersion of the tube in the oil.
   (ii) When the tube is pushed down slightly and then released, the acceleration a of the tube is given by $a = -(A\rho g/m)x$, where A is the cross-sectional area, ρ is the density of the oil, m is the total mass and x is the displacement from the centre of oscillation. From this relation for a and x, calculate the period of oscillation of the tube, using the measurements given.
8. Draw sketch graphs showing for simple harmonic motion how (a) the acceleration varies with *displacement*, (b) the kinetic and potential energy vary with *time*.
9. A simple pendulum has a length of 1·5 m and a bob of mass 0·2 kg. Calculate the period of oscillation.
   The amplitude of oscillation of the bob is 0·05 m. Calculate (i) the maximum acceleration of the bob, (ii) its velocity 0·02 m from the centre of oscillation, (iii) its maximum kinetic energy.
10. A mass of 0·4 kg is suspended from the end of a vertical helical spring of force constant 20 N/m. Calculate the extension of the spring, assuming Hooke's law.
    The mass is now pulled down a distance of 5 cm (0·05 m) and released. Find (i) the period of oscillation, (ii) the maximum potential energy of the spring, (iii) the kinetic energy of the mass when 2 cm from the centre of oscillation.

Scanned with CamScanner

36. Gravitational and Electric Fields · Capacitors · Magnetic Fields

Gravitational Fields

Gravitational forces

The pull of gravity acts on masses of all sizes. A car slipping downwards on an inclined road due to faulty brakes, for example, is pulled down by gravity or the attraction of the Earth.

The relatively large mass of the car, and the very small masses of air molecules are both attracted by the Earth. We say that the Earth has a *gravitational field* all round it. The large mass of the Moon moves round the Earth in its gravitational field. Planets such as Mars, Mercury and the Earth move round the Sun in the gravitational field of the Sun.

Newton's law of gravitation

About 1666, Newton discovered a universal *law of gravitation*. Newton's law states that the force of attraction F between two masses m_1 and m_2 separated by a distance r is given by

$$F = G\frac{m_1 m_2}{r^2} \qquad (1)$$

where G is a universal constant called the *gravitational constant*.

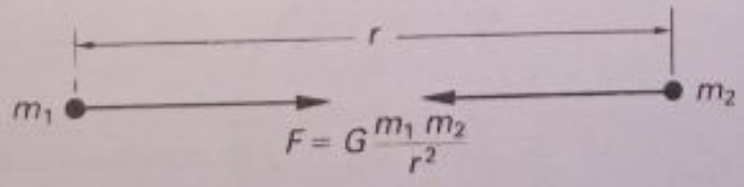


Fig. 36.1 Newton's law of gravitation

In the formula (1), m_1 and m_2 are in kilograms (kg), r is in metres (m) and F is in newtons (N). The value of G was found by experiment to be about $6{\cdot}7 \times 10^{-11}$ in units $\mathrm{N\,m^2/kg^2}$ from (1).

Force between masses

We can use Newton's law to calculate the sizes of gravitational forces between large and small masses.

Scanned with CamScanner

1. *Masses of planets* The Earth has a large mass of about 6×10^{24} kg and the Moon a mass of about 7×10^{22} kg. The Earth–Moon distance is about 4×10^{8} m. So the Earth attracts the Moon with a force F given, from (1) by

$$F = G\frac{m_1 m_2}{r^2} = \frac{6{\cdot}7 \times 10^{-11} \times 6 \times 10^{24} \times 7 \times 10^{22}}{(4 \times 10^{8})^2}$$

$$= 2 \times 10^{20}\ \text{N}$$

2. *Atomic masses* In contrast to the enormous masses of planets, let us calculate the gravitational force between two protons in a chemical compound. The proton is the nucleus of a hydrogen atom (p. 569) and its mass is about 2×10^{-27} kg. If the distance r between two protons is 10^{-10} m, then

$$F = G\frac{m_1 m_2}{r^2} = \frac{6{\cdot}7 \times 10^{-11} \times (2 \times 10^{-27})^2}{(10^{-10})^2}$$

$$= 3 \times 10^{-44}\ \text{N (in round figures)}$$

3. *Proton and electron* In the hydrogen atom, an electron of mass about 10^{-30} kg moves round the nucleus, a proton of mass about 2×10^{-27} kg (see p. 569). The distance between the electron and proton is about 5×10^{-11} m. So the gravitational attraction F between the two particles is

$$F = G\frac{m_1 m_2}{r^2} = \frac{6{\cdot}7 \times 10^{-11} \times 10^{-30} \times 2 \times 10^{-27}}{(5 \times 10^{-11})^2}$$

$$= 5 \times 10^{-45}\ \text{N}$$

We see later that the electrical force of attraction between these masses is enormously greater than the gravitational force just calculated.

Relation between G and g

Suppose the Earth is a sphere of radius R and mass M, Fig. 36.2. If we assume the Earth has a uniform density to help calculations, then we can imagine the mass M to be concentrated at the centre O.



Fig. 36.2 Relation between G and g

With a mass m on the Earth's surface, the gravitational force on it is mg, where g is the acceleration due to gravity. But the gravitational force $= GMm/R^2$.

$$\therefore\ mg = \frac{GMm}{R^2}$$

Cancelling m,

Scanned with CamScanner

$$\therefore g = \frac{GM}{R^2} \qquad (1)$$

This relation helps us to calculate M, the mass of the Earth. From (1), $gR^2 = GM$. So

$$M = \frac{gR^2}{G}$$

Using values $g = 9{\cdot}8\,\text{m/s}^2$, $R = 6{\cdot}4 \times 10^6\,\text{m}$, $G = 6{\cdot}7 \times 10^{-11}\,\text{N m}^2/\text{kg}^2$,

$$\therefore M = \frac{9{\cdot}8 \times (6{\cdot}4 \times 10^6)^2}{6{\cdot}7 \times 10^{-11}} = 6 \times 10^{24}\,\text{kg (approx.)}$$

Gravitational field strength or intensity

The gravitational field of the Earth and other planets exists all round them. The *gravitational field strength or intensity* at any point is defined as the *force per kilogram mass* at that point. The unit of field strength or intensity is N/kg.

Suppose g is the acceleration due to gravity at a point in the Earth's field. Then the force on a mass m placed at that point is $F = ma = mg$. So

$$\frac{F}{m} = \text{force per kilogram} = g$$

$$\therefore \text{gravitational field strength or intensity} = g$$

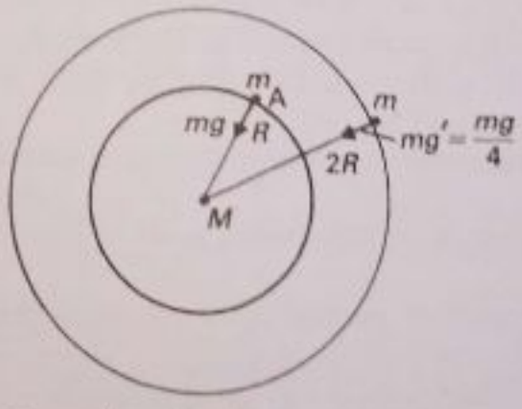


Fig. 36.3 Gravitational field strength or intensity

Near the Earth's surface, as at A in Fig. 36.3, $g = 9{\cdot}8\,\text{m/s}^2$. So here the gravitational field strength or intensity = 9·8 N/kg. The field strength decreases above the Earth's surface. If R is the radius of the Earth, then at a distance $2R$ from the Earth's centre, the field strength reduces to $1/2^2$ or 1/4 of the value at the Earth's surface since $F \propto 1/r^2$ from Newton's law. So the field strength here, $g' = 9{\cdot}8/4 = 2{\cdot}5$ N/kg, approximately. So a satellite of mass 1000 kg, moving in orbit round the Earth at this height, would have a gravitational force on it $= mg' = 1000 \times 2{\cdot}5 = 2500$ N. On the Earth's surface, the satellite would have a gravitational force or weight $= mg = 1000 \times 9{\cdot}8 = 9800$ N.

Gravitational Potential

Moving masses from one point in a gravitational field to a higher point requires *work* because gravitational forces oppose the movement. As explained on page

Scanned with CamScanner

52, if a mass m is raised a small vertical height h from the Earth's surface, then

$$\text{gravitational potential energy change} = mg \times h = mgh$$

The *potential*, V, at a point in a gravitational field is defined as the work done in taking 1 kg or *unit mass* from infinity to that point. Unlike field strength or intensity, which is a vector, potential V is a *scalar*, that is, V has a numerical value but no direction.

The potential at infinity is given the value 0. At a point distance r from the centre of the Earth and outside the Earth, calculation of V shows that

$$V = -\frac{GM}{r} \qquad (1)$$

V is minus or less than zero, because work is done in taking a mass from that point to infinity.

So points in gravitational fields have (a) field strength or intensity values, which are needed to calculate the *force* on masses at these points and (b) potential values, which are needed to calculate the *energy* changes in moving masses between the points.

Because we assume g is constant near the Earth's surface, we can find the energy change here from the formula mgh. If large changes in position are made, however, equation (1) for V must be used to calculate the energy change, as we now see.

Escape velocity

Suppose a rocket of mass m is fired from the Earth's surface S, so that it just escapes from the Earth's gravitational field. The kinetic energy, $\frac{1}{2}mv^2$, needed at S is then equal to the work done in taking the mass from S to infinity. The velocity v is called the *escape velocity*.

On the Earth's surface, distance R from the centre where R is the Earth radius,

$$\text{potential energy of rocket} = m \times -\frac{GM}{R} = -\frac{GMm}{R}$$

To escape, kinetic energy = potential difference from S to infinity

$$\therefore \tfrac{1}{2}mv^2 = 0 - \left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$$

Cancelling m $$\therefore v^2 = \frac{2GM}{R}$$

$$\therefore v = \sqrt{\frac{2GM}{R}}$$

Since $g = GM/R^2$ (p. 597), then $GM = gR^2$

$$\therefore v = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR} \qquad (2)$$

At the Earth's surface, $g = 9{\cdot}8\ \text{m/s}^2$. Also $R = 6{\cdot}4 \times 10^6\ \text{m}$

$$\therefore v = \sqrt{2 \times 9{\cdot}8 \times 6{\cdot}4 \times 10^6} = 11\,000\ \text{m/s (approx.)}$$

So the escape velocity is about 11 km/s, Fig. 36.4a. The rocket would then move out of the Earth's gravitational field to another planet which would attract it.

Scanned with CamScanner

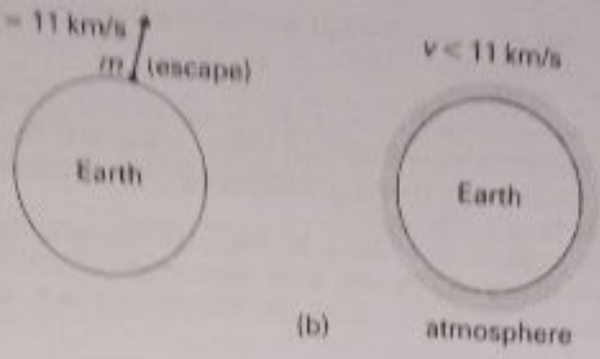


Fig. 36.4 Escape velocity

Earth's atmosphere

The gases such as oxygen, nitrogen and carbon dioxide in the Earth's atmosphere contain molecules with a wide range of speeds. Generally, less dense gases have molecules with an average greater speed than denser gases at the same temperature.

Many molecules of oxygen, nitrogen and carbon dioxide have a smaller speed than the escape velocity, about 11 km/s, Fig. 36.4b. This explains why the Earth has an atmosphere of these gases round it. Less dense gases such as argon also exist in the atmosphere but they are not as plentiful as oxygen and nitrogen because many of their molecules have speeds greater than the escape velocity.

The Moon has a smaller mass and radius than the Earth. So the escape velocity from the Moon's gravitational field is relatively small. For this reason there appears to be no atmosphere round the Moon's surface. Moon astronauts carry oxygen cylinders to breathe when landing on the Moon.

Exercise 36A (Answers, page 644)

(Where required, assume $G = 6{\cdot}7 \times 10^{-11}$ N m^2/kg^2, $g = 9{\cdot}8$ m/s^2, radius of Earth $= 6{\cdot}4 \times 10^6$ m, mass of Earth $= 6{\cdot}0 \times 10^{24}$ kg, mass of Sun $= 2{\cdot}0 \times 10^{30}$ kg, Earth–Sun distance $= 1{\cdot}5 \times 10^{11}$ m.)

1. Choose the best answer from A to E:
The Earth's gravitational field intensity or strength at its surface is about
A 6·8 N/kg B 7·9 N C 8·9 N/kg D 9·8 N E 9·8 N/kg

2. Choose the best answer from A to E:
The Earth has an atmosphere of gas molecules round its surface mainly because
A the mass of the Earth is large
B the surface attracts the molecules
C the molecules' average velocity is greater than the escape velocity
D the molecules' average velocity is less than the escape velocity
E the potential energy of the molecules is large

3. Calculate the force of gravitational attraction between two small masses of (i) 1 kg and 2 kg placed 2 m apart, (ii) the same masses placed 1 m apart.

4. Two equal masses m, placed 2 m apart, have a gravitational attraction of $6{\cdot}7 \times 10^{-9}$ N. What is m?

5. Calculate the force of attraction of the Earth on a 5 kg mass placed on its surface. Assume the Earth is a sphere and its mass acts at its centre.
Show that the result obtained is the same using $F = mg$.

6. A helium nucleus has a mass of $6{\cdot}4 \times 10^{-27}$ kg. Calculate the force of gravitational attraction between two helium nuclei when they are 10^{-6} m apart.

7. Calculate the gravitational attraction of the Sun on the Earth.

8. A mass of 1000 kg is on the Earth's surface. What is its gravitational potential energy due to the Earth? What is the gravitational potential energy when the mass is twice as far away from the centre of the Earth? (*Note* The gravitational potential $V = -GM/r$ is the potential energy per kilogram mass.)

Scanned with CamScanner

9. Calculate the gravitational potential energy of a proton when it is 10^{-6} m from the centre of a helium nucleus. (Mass of proton $= 1{\cdot}6 \times 10^{-27}$ kg, mass of helium nucleus $= 6{\cdot}4 \times 10^{-27}$ kg.)

10. A satellite of mass 1000 kg orbits the Earth at a height of 4×10^7 m above the Earth's centre. Calculate the gravitational force on the satellite by the Earth.

By putting the force equal to mg', calculate g', the acceleration due to gravity or gravitational intensity at this height.

11. Assuming the mass of the Moon is $7{\cdot}4 \times 10^{22}$ kg and its radius is $1{\cdot}7 \times 10^6$ m, calculate the gravitational energy of a 1 kg mass at the Moon's surface.

Calculate the escape velocity v from the Moon's surface, using $\frac{1}{2}mv^2$ = potential energy change.

Electric Fields

In gravitational fields, we discussed forces on masses and their potential energies. In this section we deal with *electric fields*, where electric charges (quantities of electricity, see page 509) have forces on them and potential energies.

One important difference between the two fields is that masses have only attractive forces on them in gravitational fields. In the electric field, however, charges may have forces of repulsion in addition to forces of attraction (see page 510). We shall also see that electric forces between two charge-carrying masses are enormously greater than the gravitational forces between them.

Force between charges

The force F between two charges of amounts q_1 and q_2 is proportional to the product $q_1 \times q_2$ and *inversely*-proportional to r^2, where r is the distance between the charges.

In free space (a vacuum), the force F is calculated from

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \qquad (1)$$

where F is in newtons (N) when q_1 and q_2 are in coulombs (C), r is in metres (m) and ε_0 is a constant called the *permittivity of free space*. The value of ε_0 is about $8{\cdot}9 \times 10^{-12}$ in units $C^2/(N\,m^2)$ from (1).

To a good approximation $1/4\pi\varepsilon_0 = 9 \times 10^9$ numerically. So for estimated values of F, we can write, from (1),

$$F = 9 \times 10^9 \frac{q_1 q_2}{r^2} \qquad (2)$$

In electrolysis, charges carried by ions may exist in water (p. 504). The permittivity of water is about 80 times as great as that of free space and the force between charges in water is therefore about 80 times *less* than in air.

Fundamental law · Force between protons

As we have explained on page 509, there are two kinds of charges, called *positive* and *negative* charges. Normal atoms are neutral because they have equal amounts of positive (+) and negative (−) charges inside them. A fundamental law is:

Like (similar) charges repel. Unlike charges attract

Scanned with CamScanner

So two positive or two negative charges repel but a positive and a negative charge attract each other.

Consider two protons (hydrogen nuclei), each carrying a positive charge $+1{\cdot}6\times10^{-19}$ C and separated in free space a distance of 10^{-10} m. Using $1/4\pi\varepsilon_0 = 9\times10^9$, the force F between the protons is

$$F = \frac{q_1q_2}{4\pi\varepsilon_0 r^2} = \frac{9\times10^9\times(1{\cdot}6\times10^{-19})^2}{(10^{-10})^2}$$

$$= 2\times10^{-8}\,\text{N (approx.)}$$

Each proton has a mass about 2×10^{-27} kg. When we use $F = Gm_1m_2/r^2$ to calculate the gravitational force between the two protons, the result is $F = 3\times10^{-44}$ N. See page 596. The gravitational force is therefore negligible compared with the electrical force and can therefore be omitted in calculations between atomic particles.

Electrical field strength or intensity

Electric fields exist all round electric charges. As shown on page 520, to which the student should refer, the fields can be represented by electric lines of force, which show the direction of the field and their strength or intensity. The greater the density of the lines, the greater is the electric field strength or intensity. If the density is constant in a field, the field is a uniform one.

The *electric field strength or intensity*, E, at a point is defined as the *force per coulomb* on a small charge placed at that point which does not disturb the field. The unit of E is N/C. E is a *vector*. The direction of E is taken to be the direction of movement of a *positive* charge placed at that point.

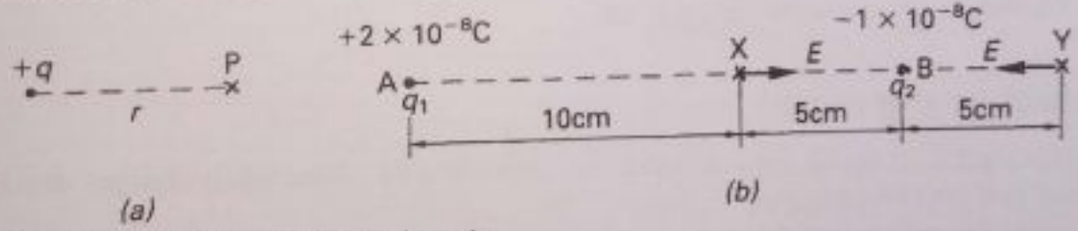


Fig. 36.5 Electric field strength or intensity

Consider the field round an isolated positive point charge q as in Fig. 36.5a. To find the field strength or intensity E at P, distance r from q, suppose a small charge q_1 is placed at P. The force F on q_1 is

$$F = \frac{q_1q}{4\pi\varepsilon_0 r^2}$$

$$\therefore E = \frac{F}{q_1} = \frac{q}{4\pi\varepsilon_0 r^2}$$

Suppose $q = 1\times10^{-8}$ C and $r = 10$ cm $= 0{\cdot}1$ m. Using $1/4\pi\varepsilon_0 = 9\times10^9$, then

$$E = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{9\times10^9\times1\times10^{-8}}{(0{\cdot}1)^2}$$

$$= 9000\,\text{N/C}$$

Since a positive charge at P is repelled by the positive charge $+q$, the direction of E at P is away from q.

Scanned with CamScanner

Example on field strength or intensity

In Fig. 36.5b, a small positive charge q_1 of $+2\times10^{-8}$ C is placed at A at a distance of 15 cm from a small negative charge q_2 of -1×10^{-8} C at B.

Calculate the field strength or intensity E at (i) a point X between A and B where AX = 10 cm, BX = 5 cm and (ii) a point Y distance 5 cm from B on the other side to A. (Assume $1/4\pi\varepsilon_0 = 9\times10^9$ numerically.)

(i) AX = 10 cm = 0·1 m. So at X, due to charge q_1,

$$E_1 = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{9\times10^9\times2\times10^{-8}}{(0{\cdot}1)^2} = 18\,000\ \text{N/C}$$

BX = 5 cm = 0·05 m. So at X, due to charge q_2,

$$E_2 = \frac{q_2}{4\pi\varepsilon_0 r^2} = \frac{9\times10^9\times1\times10^{-8}}{(0{\cdot}05)^2} = 36\,000\ \text{N/C}$$

E_1 is in the direction AX because a positive charge at X is repelled by q_1. E_2 is in the same direction XB because a positive charge at X is attracted by q_2.

So total or resultant intensity $E = E_1 + E_2$

$$= 18\,000 + 36\,000 = 54\,000\ \text{N/C}$$

(ii) AY = 20 cm = 0·2 m. So at Y,

$$E_1 = \frac{9\times10^9\times2\times10^{-8}}{(0{\cdot}2)^2} = 4500\ \text{N/C}$$

BY = 5 cm = 0·05 m. So at Y,

$$E_2 = \frac{9\times10^9\times1\times10^{-8}}{(0{\cdot}05)^2} = 36\,000\ \text{N/C}$$

But E_1 is in the opposite direction to E_2 if we consider a positive charge at Y.

$\therefore$ total or resultant intensity $E = E_2 - E_1 = 36\,000 - 4500 = 31\,500$ N/C

Unlike part (i), E is in the direction YB.

Electric potential

Since forces act on charges in an electric field, *work* is done when charges move from one point to another.

The electric potential V at a point is defined as the *work done per coulomb* in moving a positive charge from infinity to the point. The unit of V is J/C, joules per coulomb, or *volts*, V, since 1 volt is defined as the potential difference between two points when 1 joule per coulomb of work is done in taking a charge from one point to the other (see page 391).

Like gravitational potential, electric potential is a *scalar*, that is, it has magnitude but no direction.

To calculate V due to charges in an electric field requires higher mathematics (see *Advanced Level Physics*, Nelkon and Parker (Heinemann)). For an isolated point positive charge q, the potential at a point distance r from q is

$$V = \frac{q}{4\pi\varepsilon_0 r} \qquad (1)$$

Note that 'r' is in the formula for V but 'r^2' is in the formula for E, the field strength or intensity due to the charge.

As with the gravitational field, the value of V at infinity is given as 0. V is *positive* when q is positive because work is done in moving a positive charge from infinity to points in the field. V is *negative* when q is negative. This time

Scanned with CamScanner

there is an attractive force on a positive charge moved from infinity to points in the field. We say the field does work in this case.

Examples on potential

1. Fig. 36.6 shows a small positive charge q of $+2\times10^{-8}$ C at O. Find the potential (i) at A distance 10 cm (0·1 m) from O, (ii) at B distance 20 cm (0·2 m) from O. What is the potential difference between A and B? (Assume $1/4\pi\varepsilon_0 = 9\times10^9$.)

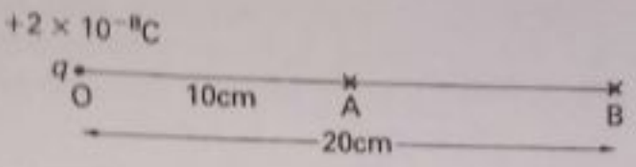


Fig. 36.6 Potential

(i) At A, $$V_1 = \frac{q}{4\pi\varepsilon_0 r} = \frac{9\times10^9\times2\times10^{-8}}{0{\cdot}1} = 1800\text{ V}$$

(ii) At B, $$V_2 = \frac{q}{4\pi\varepsilon_0 r} = \frac{9\times10^9\times2\times10^{-8}}{0{\cdot}2} = 900\text{ V}$$

So potential difference between A and B $= 1800 - 900 = 900$ V

(This means that the work per coulomb in taking a charge from B to A is 900 J/C. So if a charge of 1×10^{-10} C is taken from B to A,

$$\text{work done} = 900\times1\times10^{-10} = 9\times10^{-8}\text{ J})$$

2. In Fig. 36.7, calculate the resultant potential at P due to the positive charge q_1 of $+2\times10^{-8}$ C and the negative charge q_2 of -1×10^{-8} C.
Show approximately the direction at P of the resultant field strength or intensity.

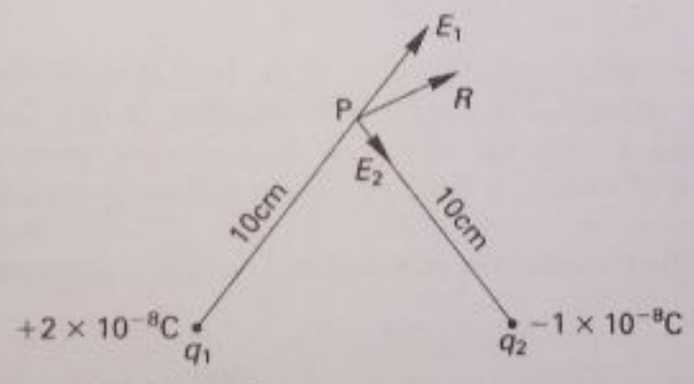


Fig 36.7 Resultant potential and intensity

At P, $$V_1 \text{ due to } q_1 = \frac{q_1}{4\pi\varepsilon_0 r} = \frac{9\times10^9\times2\times10^{-8}}{0{\cdot}1} = 1800\text{ V}$$

and $$V_2 \text{ due to } q_2 = -\frac{9\times10^9\times1\times10^{-8}}{0{\cdot}2} = -900\text{ V}$$

Since potential is a scalar,

resultant potential at P, $V = 1800 - 900 = 900$ V

Field strength is a vector. Due to q_1, E_1 at P is in the direction AP. Due to q_2, E_2 is in the direction PB. From the parallelogram of forces, PR is roughly the direction of the resultant of E_1 and E_2.

Relation between E and V · Potential gradient in fields

We now discuss a relationship between field strength E and potential V which is widely used in electric fields.

Scanned with CamScanner

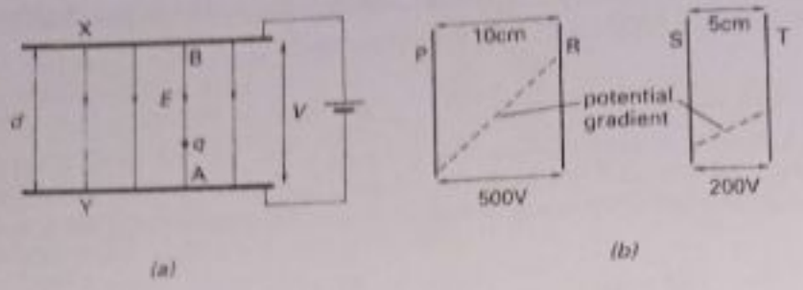


Fig. 36.8 Potential gradient in fields

Suppose a potential difference, p.d., V is applied to two parallel metal plates X and Y by connecting a battery to them, Fig. 36.8a. The field between the plates is fairly uniform as shown by the lines of force, so that the field strength or intensity E is constant in magnitude and direction.

Suppose a positive charge q is moved from A to B between the plates opposite to the direction of E. Then, if d is the distance apart of the plates,

$$\text{work done} = \text{force} \times \text{distance} = Eq \times d$$

Also, $$\text{work done} = V \times q,$$

since V is the work per coulomb in moving a charge from A to B.

$$\therefore Eq \times d = V \times q$$

Cancelling q, $$\therefore E = \frac{V}{d} \qquad (1)$$

The ratio V/d is the *potential gradient* of the field between the plates—it is a measure of the 'change of potential with distance' in the field. The unit of potential gradient is 'volt per metre' or V/m. So the greater the potential gradient in a part of an electric field, the greater will be the electric field strength or intensity there.

In Fig. 36.8b, two parallel plates P and R have a p.d. of 500 V and are 10 cm (0·1 m) apart. So

$$E_1 = \text{potential gradient} = \frac{V}{d} = \frac{500}{0{\cdot}1} = 5000 \text{ V/m} \qquad (2)$$

Similarly, the parallel plates S and T have a p.d. of 200 V and are 5 cm (0·05 m) apart.

So $$E_2 = \frac{V}{d} = \frac{200}{0{\cdot}05} = 4000 \text{ V/m} \qquad (3)$$

From (2) and (3), E_1 is greater than E_2.

Suppose a charge q of $+2 \times 10^{-8}$ C is placed in the field E_1 between the plates P and R. Then

$$\text{force on } q = E_1 q = 5000 \times 2 \times 10^{-8} = 10^{-4} \text{ N}$$

Parallel plates, with a battery connected to them, are often used in experiments where a uniform electric field is required. E is then equal to the potential gradient, V/d.

Scanned with CamScanner

Potential difference and energy gain

The *potential difference* V between two points X and Y in an electric field is the work done per coulomb in moving a charge between X and Y. Since 1 volt is the p.d. for 1 joule per coulomb, when a charge of Q coulombs moves through a p.d. of V volts, then, as on page 392,

$$\text{work done} = \text{energy change}, W = QV \text{ joules}$$

In a cathode ray tube, electrons (negative charges) are attracted from the cathode C to a metal anode A at a high positive potential V relative to C. The force accelerates the electrons to a high speed. On reaching A, if e is the electron charge,

$$\text{work done by field} = eV = \text{kinetic energy gained by electron}$$

So
$$eV = \tfrac{1}{2}mv^2,$$

where v is the speed of the electron when reaching A, assuming its speed at C was zero. This is illustrated in the following example.

Example on energy of electrons

Electrons emitted with negligible speed from a cathode C in an electron tube are accelerated to a metal anode A which has a potential V of +500 V relative to C.

Calculate (a) the energy gained by an electron reaching A, (b) its speed at A, Fig. 36.9. (Assume charge on electron, $e = 1{\cdot}6 \times 10^{-19}$ C, mass of electron $= 10^{-30}$ kg.)

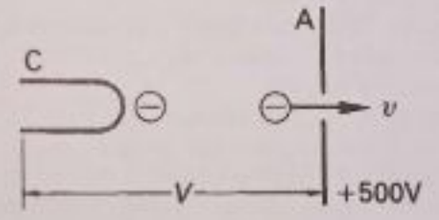


Fig. 36.9 Speed of electrons

(a) Energy gained $= eV = 1{\cdot}6 \times 10^{-19} \times 500 = 8 \times 10^{-17}$ J

(b) $\tfrac{1}{2}mv^2 =$ energy gained $= eV = 8 \times 10^{-17}$

$$\therefore v = \sqrt{\frac{8 \times 10^{-17}}{\frac{1}{2} \times 10^{-30}}} = 1{\cdot}3 \times 10^{7} \text{ m/s}$$

Deflection of charges in electric field

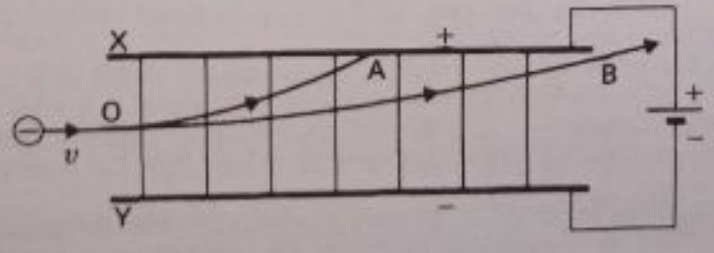


Fig. 36.10 Deflection of charges

In addition to increasing its speed and kinetic energy, charged particles can also be deflected by an electric field.

Fig. 36.10 shows two parallel metal plates X and Y connected to a battery,

Scanned with CamScanner

with X at the higher or positive potential. A beam of electrons, moving with velocity v parallel to the plates, enters the field between X and Y at O, midway between the plates. The force on the electrons, which are negative charges, acts upwards from Y to X. So the beam is deflected along a curved path OA between X and Y. With a higher speed v the beam is deflected less. So it can pass completely through the plates along the path OB.

When the electrons move from O to A, they gain energy eV, where V is the potential difference between O and A.

Summary

Charges in electric fields can gain speed and kinetic energy, or they may be deflected by the field.

Exercise 36B (Answers, page 644)

(Where necessary, assume $\varepsilon_0 = 8{\cdot}9 \times 10^{-12}\,\mathrm{F\,m^{-1}}$ or $1/4\pi\varepsilon_0 = 9 \times 10^9\,\mathrm{m\,F^{-1}}$.)

1. Which of the statements A to E is NOT true?
- A Electric field strength (intensity) is a vector
- B Electric potential is a scalar
- C Electric field intensity is the same as electric potential
- D Potential gradient is equal to field intensity (strength)
- E Electric field intensity or strength has units N/C

2. Which of the statements A to E is true?
When a charge Q coulombs moves through a potential difference of V volts
- A the force on the charge is QV newtons
- B the energy gained is Q/V joules
- C the force on the charge is Q/V newtons
- D the energy gained by the charge is QV joules
- E the force on the charge is V/Q newtons

3. Find the force between two point (small) charges of $+2 \times 10^{-7}$ C and -1×10^{-8} C which are 0·05 m apart in air.
Is this an attractive or repulsive force?

4. Two equal small charges q in air have a force of repulsion between them of $3{\cdot}6 \times 10^{-4}$ N. Their distance apart is 0·01 m. Calculate q.

5. The electric field strength or intensity 0·1 m distant from a point charge is 9×10^4 N/C. What is the magnitude of the charge?

6. Two point charges, each numerically 1×10^{-9} C, are 0·15 m apart in free space. Find the electric field strength or intensity at the point midway between the charges (i) if the two charges are both positive, and (ii) if one charge is positive and one charge is negative.

7. What is the potential at a point 0·06 m from a point charge of $+2 \times 10^{-7}$ C.
What would be the potential if the charge had been -2×10^{-7} C? Explain the difference between the two answers.

8. The potential at a point distance 0·03 m from a small charge q is -1×10^5 V. What is the magnitude and sign of q?

9. Find the resultant potential at a point in free space which is 0·1 m from a small charge of $+1{\cdot}5 \times 10^{-7}$ C and 0·15 m from a small charge of -2×10^{-7} C.

10. An electron of charge $-1{\cdot}6 \times 10^{-19}$ C moves in an electric field between two points which have a potential difference of 500 V. Calculate the energy change of the electron.

11. Two parallel plates are 0·05 m apart in air. The potential difference between the plates is 1000 V. Find the electric field strength or intensity between the plates. Write down the relation you used to calculate the answer.

12. An electron of mass about 10^{-30} kg and charge $-1{\cdot}6 \times 10^{-19}$ C is accelerated in free space to a metal at +200 V relative to the electron emitter or cathode. Calculate the energy gain of the electron.
Assuming the initial speed of the electron was zero, find its speed on reaching the metal.

Scanned with CamScanner

Capacitors

Millions of capacitors are manufactured each year for use in all kinds of electronic apparatus, particularly radio and television. As we explained on page 523, a *capacitor stores charge*. Basically, a capacitor has two metal plates separated by an insulating medium called a *dielectric*. An air capacitor, used in radio receivers for tuning to stations, has air as the dielectric. A mica capacitor has mica as the dielectric. An electrolytic capacitor has a dielectric formed by electrolysis. See page 526.

Fundamental formulae

The capacitance C of a capacitor is a measure of the amount of charge Q it can store when a potential difference (p.d.) V is applied to the plates. On page 523 we explained that a positive charge $+Q$ appears on one plate and a negative charge $-Q$ on the other plate when the p.d. V is applied, Fig. 36.11a. This is due to electrons flowing for a short time. The capacitor is discharged by connecting B and A by a copper wire, Fig. 36.11b. The electrons on B then go back to A and the charges on the plates become zero.

By definition,

$$C = \frac{Q}{V}$$

So

$$Q = CV \quad \text{and} \quad V = \frac{Q}{C}$$

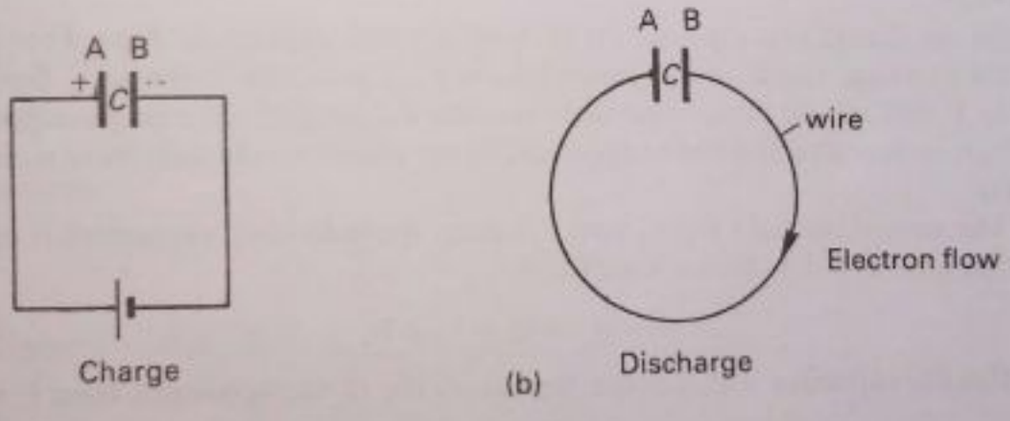


Fig. 36.11 Charge and discharge

When Q is in coulombs (C) and V is in volts (V), then C is in *farads*, F. In practice, C is much less than 1 F, so smaller units for C are used such as the *microfarad*, μF.

$$1\,\mu\text{F} = \text{one-millionth farad} = \frac{1}{10^6}\,\text{F} \quad \text{or} \quad 10^{-6}\,\text{F}$$

Suppose a p.d. of 50 V is connected to a 2 μF capacitor. Since $Q = CV$, the charge stored is

$$Q = \frac{2}{10^6} \times 50 = \frac{100}{10^6} = \frac{1}{10^4} = 10^{-4}\,\text{C}$$

Note that $(100/10^6)$ C can be written as 100 microcoulombs or 100 μC.

Scanned with CamScanner

Capacitors in parallel

Fig. 36.12a shows three capacitors C_1, C_2 and C_3 in *parallel* across a battery of p.d. V. With all the plates connected together electrically at M and at N, the total capacitance C must be the *sum* of the three capacitances. So

$$C = C_1 + C_2 + C_3 \qquad (1)$$

With $C_1 = 1\,\mu F$, $C_2 = 2\,\mu F$ and $C_3 = 3\,\mu F$, then total $C = 1+2+3 = 6\,\mu F$. Note that capacitors in parallel have the same p.d. V across each.

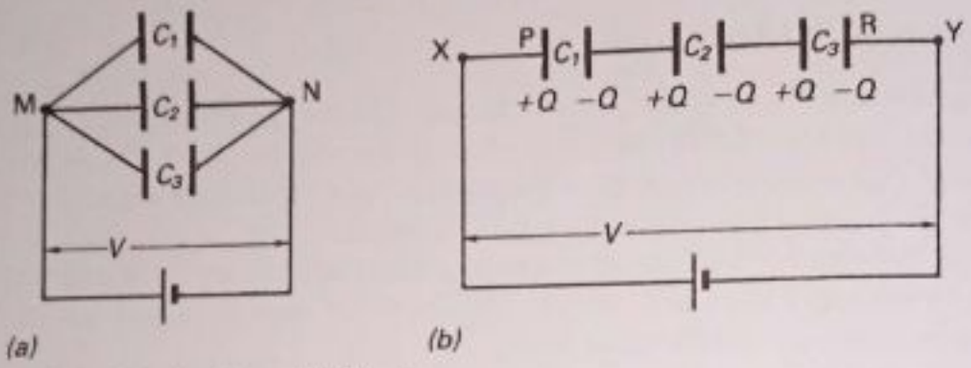


Fig. 36.12 Capacitors in (a) parallel, (b) series

Capacitors in series

Fig. 36.12b shows three capacitors C_1, C_2 and C_3 in *series* across a battery of p.d. V. The battery gives the outside plates P and R charges of $+Q$ and $-Q$ as for a single capacitor. Charges equal to Q are then induced on the other plates, as shown.

So the charge Q is the *same* for each of the series capacitors. Also, if the battery is taken away, and P and R joined by a copper wire, only a charge Q flows from R to P and all the three capacitors become discharged. So a single capacitor C which replaced all the three capacitors in the circuit would only have a charge Q on it.

The sum of the p.d.s V_1, V_2 and V_3 across the individual capacitors is equal to V, the battery p.d. between X and Y. So

$$V = V_1 + V_2 + V_3$$

If C is the capacitor which could replace all the three capacitors, then $V = Q/C$.

So
$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Cancelling Q,
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

If $C_1 = 1\,\mu F$, $C_2 = 2\,\mu F$ and $C_3 = 3\,\mu F$, the total capacitance C is given by

$$\frac{1}{C} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$$

$$\therefore C = \frac{6}{11} = 0{\cdot}55\,\mu F$$

The total capacitances of capacitors in series is always *less* than any of the capacitances.

Scanned with CamScanner

Examples on capacitors

1. A capacitor of 4 μF has a charge of 100 μC. What is the p.d. across the capacitor?

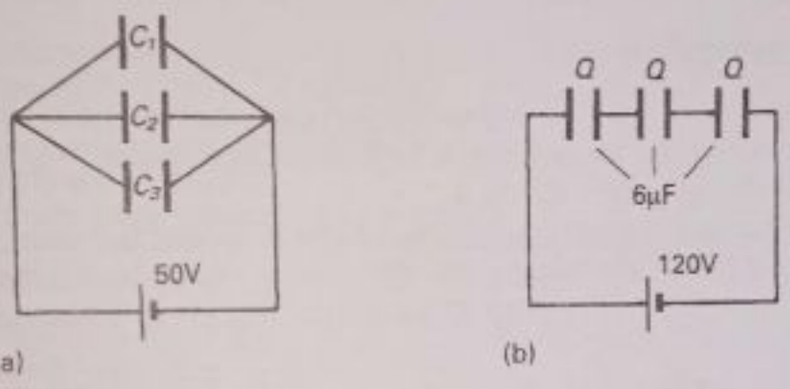
$$C = 4\,\mu\text{F} = \frac{4}{10^6}\,\text{F} \quad \text{and} \quad Q = 100\,\mu\text{C} = \frac{100}{10^6}\,\text{C}$$

$$\therefore V = \frac{Q}{C} = \frac{100/10^6}{4/10^6} = 25\,\text{V}$$

2. Three capacitors, each of 2 μF, are in *parallel* across a 50 V supply, Fig. 36.13a. Calculate (a) the total capacitance, (b) the charge on each capacitor.

(a) Total capacitance $= C_1 + C_2 + C_3 = 2 + 2 + 2 = 6\,\mu\text{F}$
(b) Since 50 V is the p.d. across each capacitor, the charge Q on each is

$$Q = CV = \frac{2}{10^6} \times 50 = \frac{100}{10^6} = 100\,\mu\text{C}$$



(a) (b)

Fig. 36.13 Examples

3. Three capacitors, each of 6 μF, are placed in *series* across a 120 V battery, Fig. 36.18b. Calculate (a) the total capacitance, (b) the charge on each capacitor, (c) the p.d. across each.

(a) In series,

$$\frac{1}{C} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$$

$$\therefore C = \frac{6}{3} = 2\,\mu\text{F}$$

(b) Charge Q on each capacitor = charge Q on total capacitance

$$\therefore Q = CV = \frac{2}{10^6} \times 120 = \frac{240}{10^6} = 240\,\mu\text{C}$$

(c) p.d. across each capacitor $V = \dfrac{Q}{C} = \dfrac{240/10^6}{6/10^6}$

$$= \frac{240}{6} = 40\,\text{V}$$

(alternatively, p.d. across each capacitor = 120 V/3 = 40 V)

Energy in capacitor

A spark can be obtained when the two plates of a charged capacitor are connected by a wire (see page 523). So some heat, light and sound are obtained from a charged capacitor, showing that it has an amount of *energy* stored in it.

The storage of energy can also be shown by charging a large electrolytic

Scanned with CamScanner

capacitor C of 1000 μF by a 12 V battery, and then discharging the capacitor by connecting the plates to the terminals X and Y of a small model motor M, Fig. 36.14a. A small mass W of 10 g is then raised by means of a string tied round the axle motor, which rotates when the capacitor is connected to it.

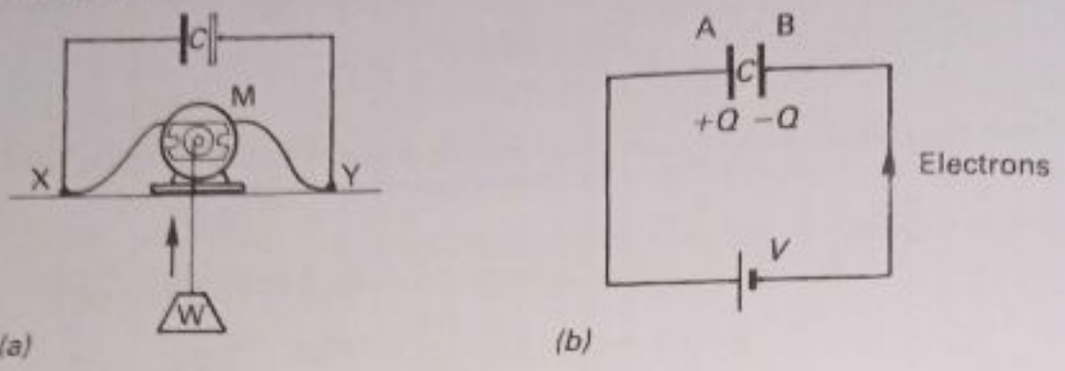


Fig. 36.14 Energy in capacitor

Energy formulae

Suppose a battery of p.d. V is connected to a capacitor with plates A and B Fig. 36.14b. Electrons then move from A to B. Finally, a charge $-Q$ appears on B and an equal charge $+Q$ is left on A.

Before the battery is connected, the p.d. between A and B is zero. The final p.d. between A and B is V, the battery p.d. On average, then, the p.d. between A and B $= \frac{1}{2}(0+V) = V/2$. So the charge Q transferred from A to B has moved through a p.d. of $V/2$.

$$\therefore \text{energy stored in plates} = \text{charge} \times \text{p.d.} = Q \times V/2$$

$$\therefore \text{energy, } W = \tfrac{1}{2}QV \qquad (1)$$

Since $Q = CV$

$$W = \tfrac{1}{2}CV \times V = \tfrac{1}{2}CV^2 \qquad (2)$$

Also, $V = Q/C$

$$\therefore W = \tfrac{1}{2}Q \times Q/C = \tfrac{1}{2}Q^2/C \qquad (3)$$

Any of the three formulae can be used to calculate the energy in a charged capacitor. W is in joules (J) when Q is in coulombs (C), V in volts (V) and C in *farads* (F).

Examples on energy in capacitors

1. (a) The charge on a 4 μF capacitor is 80 μC. Calculate the energy stored.
(b) Two capacitors, each 4 μF, are placed in series across a 100 V battery. Calculate the energy in each capacitor.

(a) $$\text{Energy, } W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2} \times \frac{(80/10^6)^2}{4/10^6}$$

$$= \frac{8}{10^4} = 0{\cdot}0008 \text{ J}$$

(b) p.d. across each capacitor, $V = \frac{1}{2} \times 100 = 50$ V

$$\therefore \text{energy in each, } W = \tfrac{1}{2}CV^2 = \frac{1}{2} \times \frac{4}{10^6} \times 50^2 = 0{\cdot}005 \text{ J}$$

2. A capacitor of 100 μF is charged by a 60 V battery. The capacitor is then discharged through a small electric motor which has a mass of 20 g suspended from the end of a light string wound round the motor axle.

Scanned with CamScanner

Neglecting energy losses, estimate the height which the mass will rise. (Assume $g = 10\,\text{m/s}^2$.)

Energy Energy of capacitor, $W = \frac{1}{2}CV^2 = \frac{1}{2} \times \frac{100}{10^6} \times 60^2 = 0{\cdot}18\,\text{J}$

Height raised Gain in potential energy of mass $= mgh$
Since $m = 20\,\text{g} = 0{\cdot}02\,\text{kg}$, $g = 10\,\text{m/s}^2$ and energy from capacitor $= 0{\cdot}18\,\text{J}$

$$\therefore 0{\cdot}02 \times 10 \times h = 0{\cdot}18$$

$$\therefore h = \frac{0{\cdot}18}{0{\cdot}02 \times 10} = 0{\cdot}9\,\text{m}$$

(*Note* This is only a very rough estimate for h, since energy is lost as heat in the motor wires, for example, when the capacitor sends current through the motor on discharging.)

Exercise 36C (Answers, page 644).

In questions 1 and 2, choose the answer you think is most correct from A, B, C, D or E.

1. A 2 μF capacitor is in series with a 1 μF capacitor across a 100 V battery
- A the charge on each capacitor is different
- B the charge on each capacitor is 300 μC
- C the p.d. across each capacitor is different
- D the total capacitance is 3 μF
- E the p.d. across each capacitor is the same

2. When a battery of p.d. V is connected to a capacitor of capacitance C, a charge Q is stored on the capacitor. The energy stored in the capacitor is
A $\frac{1}{2}QV^2$ B $\frac{1}{2}CV$ C QV D $\frac{1}{2}C/Q^2$ E $\frac{1}{2}CV^2$

3. A capacitor has a charge of 30 μC when a p.d. of 10 V is connected to it. Find (a) the capacitance, (b) the energy in the capacitor.

4. A 2 μF capacitor has a charge of 40 μC. Find (a) the energy stored, (b) the p.d. across the capacitor.

5. In Fig. 36.15a, calculate the total capacitance of the two capacitors shown. What is (a) the charge on each capacitor, (b) the energy in each?

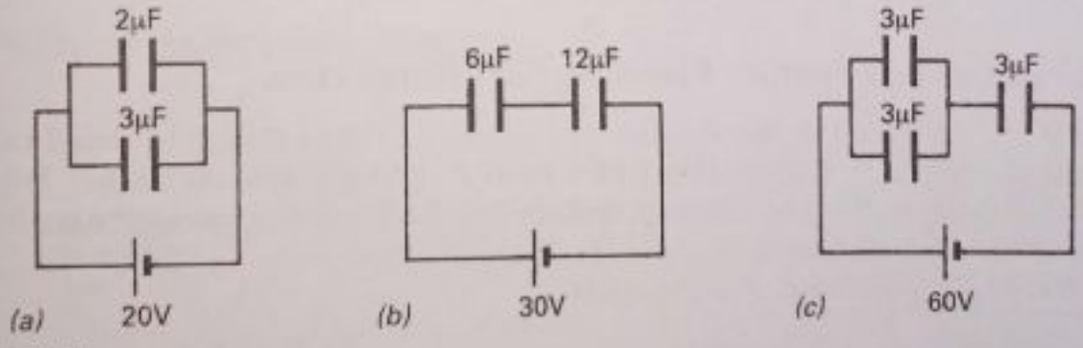


Fig. 36.15

6. In Fig. 36.15b, find (a) the total capacitance of the two series capacitors, (b) the charge on each, (c) the p.d. across each (d) the energy in each capacitor.

7. In Fig. 36.15c, calculate (a) the total capacitance of the circuit, (b) the energy in each capacitor.

8. In Fig. 36.15b, the 30 V battery is removed and a different battery of p.d. V is connected across the two capacitors in series. The charge on each capacitor is then 60 μC. Find (a) the value of V, (b) the total energy stored in the two capacitors.

Magnetic Fields

Force on conductor

Earlier in the book (see page 465), we studied the *force* on a current-carrying conductor or wire placed in a magnetic field.

Scanned with CamScanner

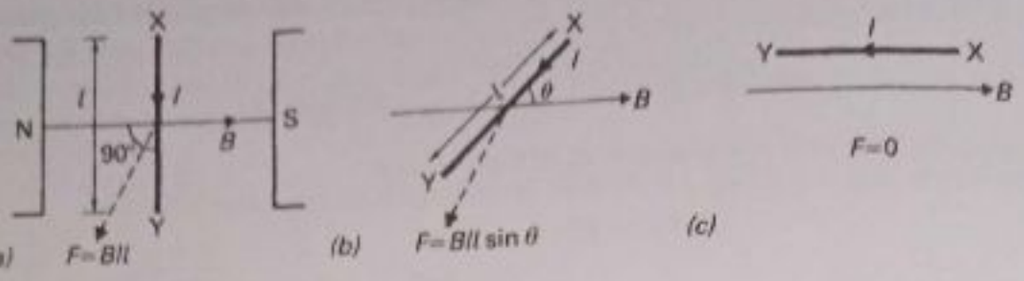


Fig. 36.16 Force on conductor

The force F increases with the current I, the magnetic field strength B and the angle θ between the conductor and B (page 466). When the conductor XY is 90° to B, the force is a maximum, Fig. 36.16a. Less than 90°, Fig. 36.16b, the force is less and when XY is parallel to B, as in Fig. 36.16c, $F = 0$.

Magnetic field strength or intensity B is measured in teslas, symbol T. A very strong magnetic field may have a value such as 0·1 T. A relatively weak magnetic field such as the Earth's magnetic field has a value about 0·00005 T.

Formula for F

With the conductor XY at 90° to B, Fig. 36.16a, the maximum value of F is

$$F = BIl \tag{1}$$

F is in newtons (N) when B is in teslas (T) and l in metres (m). At an angle θ to B as in Fig. 36.16b, the force on XY is

$$F = BIl \sin\theta \tag{2}$$

When θ is 90°, the general formula in (2) changes to (1).

Direction of force · Fleming left hand rule

As stated on page 457, the direction of F is given by the Fleming left hand rule: Hold the first three fingers of the LEFT hand at right angles to each other. Point the forefinger in the field direction and the middle (second) finger in the current direction. Then the *thumb* points in the force direction.

Verify the direction of F in Fig. 36.16.

Force on moving charges in magnetic fields

In a metal conductor such as copper, the current carriers are electrons, which are negative charges. The negative charge on an electron is numerically equal to an amount e about $1{\cdot}6 \times 10^{-19}$ coulombs. When the metal carries a current, the electrons move or drift along the wire with some average velocity v, Fig. 36.17.

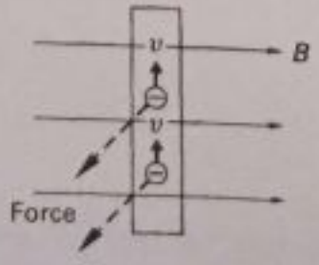


Fig. 36.17 Force on moving electron in field

Scanned with CamScanner

Suppose a uniform magnetic field B is 90° to all the drifting electrons. The force on all the electrons is $F = BIl$. Now $l = vt$, where t is the time for the N electrons in the wire to flow a distance l. Also

$$I = \frac{\text{charge}}{\text{time}} = \frac{Ne}{t}$$

$$\therefore \text{force on } N \text{ electrons} = BIl = B \times \frac{Ne}{t} \times vt$$

$$= N \times Bev$$

$$\therefore \text{force on one electron} = Bev \qquad (1)$$

Since the moving electrons keep hitting the fixed copper ions in the wire XY, v is very small, about a millimetre per second. In a cathode-ray tube, however, where there is a high vacuum, the electrons are free to move and may be accelerated to high speeds such as 10^7 m/s. The force on an electron due to a perpendicular magnetic field can then be high.

Generally, if q is the charge carried by a charged particle moving with velocity v at 90° to a magnetic field of strength B, the force F on it is

$$F = Bqv \qquad (2)$$

If q is a positive charge, it will be deflected by a given field B in the opposite direction to a negative charge moving in the same direction in the field. In both cases the Fleming left hand rule is applied to find the direction of F, remembering that the middle or second finger must point in the direction of conventional current, which is the movement of *positive* charge. A movement of negative charge to the right is equivalent to a movement of positive charge to the left, so the current finger points to the left in applying the Fleming rule.

Path of deflected charges

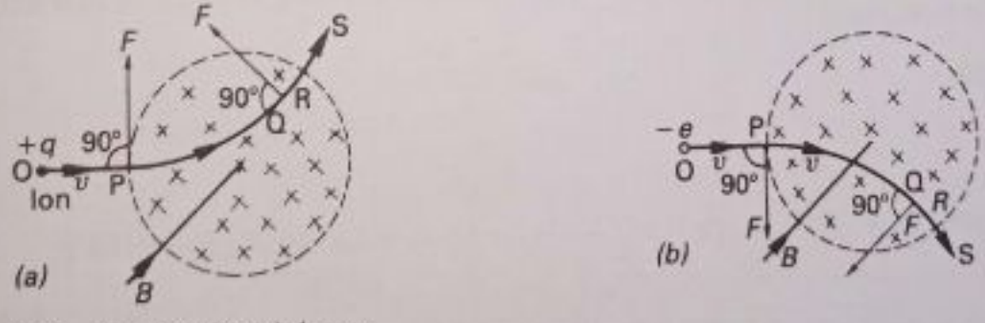


Fig. 36.18 Path of deflected charges

Consider a positive charge $+q$ on an ion which moves in the direction OP with a uniform velocity v, Fig. 36.18a. Suppose it enters a uniform magnetic field of strength B at 90° to v in a vertically downward direction, as shown.

From the Fleming left hand rule, the force F on the charge deflects it upwards at P as it enters the field and $F = Bqv$. So the charge path changes to PQ in the field. Now all the time the charge was moving in the field, the force F was 90° to its direction. So *no work is done by F on the charge*. The kinetic energy of the charge, and therefore its speed v, remains unaltered or constant.

The force F at any point Q is therefore the same value as at P, Bqv. F acts at right angles to the velocity. Now on page 43 on circular motion, we showed that a constant force called a 'centripetal force' acts on a mass moving in a circle.

Scanned with CamScanner

So we deduce that the charged particle moves along a *circular path* PQR in the magnetic field. When it leaves the field, the particle moves along a straight line RS.

Fig. 36.18b shows the circular path of an electron beam entering the magnetic field. The deflection is in the opposite direction to that in Fig. 36.18a since electrons carry negative charges.

Radius of circular path

We can now calculate the radius r of the circular path in Fig. 36.18. From the mechanics for motion in a circle, the centripetal force F is shown to be given by $F = mv^2/r$, where m is the mass of the moving particle, v is the constant speed and r is the radius of the circle. Since $F = Bqv$ in this case,

$$Bqv = \frac{mv^2}{r}$$

Cancelling v and re-arranging,

$$r = \frac{mv}{Bq} \qquad (1)$$

From (1), we see that r is proportional to the speed v of the particle. So the higher the speed, the greater is the radius of the circular path.

From (1), for a given speed v, the radius r is proportional to m/q, which is the *mass–charge ratio* of the charged particle. This relationship is used in the mass spectrometer, an electrical machine which measures the relative masses of fast-moving ions. The machine has a strong magnetic field which deflects the ions in circular paths whose radii are proportional to the mass–charge ratio.

Examples

1. An electron beam is accelerated to a speed of 2×10^7 m/s and enters a perpendicular field of strength $B = 0{\cdot}001$ T (10^{-3} T). The charge–mass ratio, e/m, for an electron is about $1{\cdot}8 \times 10^{11}$ C/kg. Calculate the radius of the circular path of the deflected beam.

From $$Bev = \frac{mv^2}{r}$$

$$r = \frac{mv}{Be}$$

Now $e/m = 1{\cdot}8 \times 10^{11}$, so $m/e = 1/(1{\cdot}8 \times 10^{11})$. Also $v = 2 \times 10^7$ and $B = 0{\cdot}001$ T.

$$\therefore r = \frac{m}{e} \times \frac{v}{B} = \frac{1}{1{\cdot}8 \times 10^{11}} \times \frac{2 \times 10^7}{0{\cdot}001}$$

$$= 0{\cdot}11 \text{ m} = 11 \text{ cm}$$

2. A beam of protons, accelerated to a speed of 10^6 m/s, is deflected in a circular path of radius r when it enters a perpendicular magnetic field B.
(a) Calculate B if $r = 0{\cdot}125$ m. (b) In a similar experiment with another beam of protons, the radius r is 1·25 m. Find the new speed v of this beam.

(Assume proton mass $= 2 \times 10^{-27}$ kg, proton charge $= 1{\cdot}6 \times 10^{-19}$ C)

(a) $F = Bqv = \dfrac{mv^2}{r}$

So $$B = \frac{mv}{qr} = \frac{2 \times 10^{-27} \times 10^6}{1{\cdot}6 \times 10^{-19} \times 0{\cdot}125}$$

$$= 0{\cdot}1 \text{ T}$$

Scanned with CamScanner

(b) Since $r = mv/Bq$, then $v \propto r$ in this case.

So $$\frac{\text{new speed } v}{\text{speed } 10^6} = \frac{1{\cdot}25}{0{\cdot}125} = 10$$

$$\therefore v = 10 \times 10^6 = 10^7 \text{ m/s}$$

Exercise 36D (Answers, page 644)

(Where necessary, assume charge on electron $e = -1{\cdot}6 \times 10^{-19}$ C, mass of electron, $m_e = 9 \times 10^{-31}$ kg and charge–mass ratio of electron, $e/m = 1{\cdot}8 \times 10^{11}$ C/kg.)

In questions 1, 2 and 3, give the answer you think is most correct from A, B, C, D or E.

1. When a beam of electrons enters a uniform magnetic field B perpendicular to the beam

A the direction of the beam is unchanged
B the force on all the electrons is Bev
C the beam is deflected in a circular path
D Fleming's right hand rule is applied to find the direction of deflection
E the beam is deflected along the direction of B

2. A beam of positive ions, each of charge $+q$, moving with velocity v enters a uniform magnetic field B.

A If B is parallel to the beam, the force on each ion is Bqv
B The greater v, the smaller is the force on the beam
C When B is perpendicular to the beam, there is no deflection
D The radius of the deflected beam path is found from $Bqv = mv^2/r$
E The radius of the deflected path does not depend on the mass of the ion

3. An electron beam moving with uniform velocity v is deflected in a curved path when it enters a uniform magnetic field perpendicular to its direction.

A Along the curved path, the magnitude of the force is different
B The energy of the electrons is constant along the curved path
C The energy of the electrons varies along the curved path
D The speed of the electrons change along the curved path
E The speed is a maximum at the end of the path

4. In Fig. 36.19a, a horizontal electron beam PQ, moving with velocity of 10^7 m/s enters a uniform field B of 2×10^{-4} T which acts vertically downwards.

(a) Calculate the force on each electron, (b) draw a sketch showing how the beam is deflected in the field and the direction it travels when it passes out of the field.

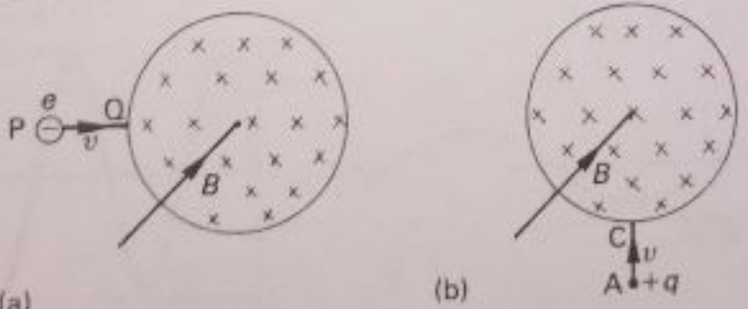


Fig. 36.19 (a) (b)

5. In Fig. 36.19b, a beam AC of *positive* ions, charge $+q$, moving vertically with a uniform velocity 10^6 m/s, enters a uniform magnetic field B of 10^{-3} T which acts horizontally into the paper. The force on each ion is $3{\cdot}2 \times 10^{-16}$ N.

(a) Calculate q. (b) Draw a sketch showing the deflected beam and the direction it travels after coming out from the field.

6. An electron beam moving with velocity $2{\cdot}0 \times 10^6$ m/s enters a perpendicular field B of 10^{-4} T.

(a) Using force $Bev = mv^2/r$, calculate the radius of the deflected beam path. (b) Draw a sketch showing the force at two positions of its deflected path. (c) Explain why the speed of the beam stays constant in the curved path.

7. A beam of positive ions (positive charges) moving with a velocity of $1{\cdot}2 \times 10^5$ m/s enters a uniform magnetic field B of 0·1 T. The radius of the deflected beam is 0·25 m.

Using $Bqv = mv^2/r$, calculate (a) the ratio m/q (mass–charge ratio) of the ions, (b) the mass m of the ions if q is $1{\cdot}6 \times 10^{-19}$ C.

Scanned with CamScanner

37. Alternating Current (A.C.) Circuits

In previous chapters we discussed direct current (d.c.) circuits. In this chapter we shall deal with alternating current (a.c.) circuits which have alternating voltages (a.c. voltages) in them. A knowledge of a.c. circuits is essential in radio and television, for example.

In place of 'resistance' in d.c. circuits, we need 'reactance' and 'impedance' in a.c. circuits. As we shall show, formulae in a.c. circuits are similar to those in d.c. circuits, although there are differences in the circuits.

Sine wave a.c. voltage · Peak voltage · Frequency

The simplest a.c. voltage, from a simple dynamo (page 486), has a sine waveform, Fig. 37.1a or b. The voltage E at a time t can be written as

$$E = E_0 \sin \omega t,$$

where E_0 is the *maximum* or *peak voltage* value and $\omega = 2\pi f$, where f is the frequency of the voltage or number of cycles per second (see p. 486). Using V for voltage, the a.c. voltage can also be written as

$$V = V_0 \sin \omega t,$$

where V_0 is the peak voltage.

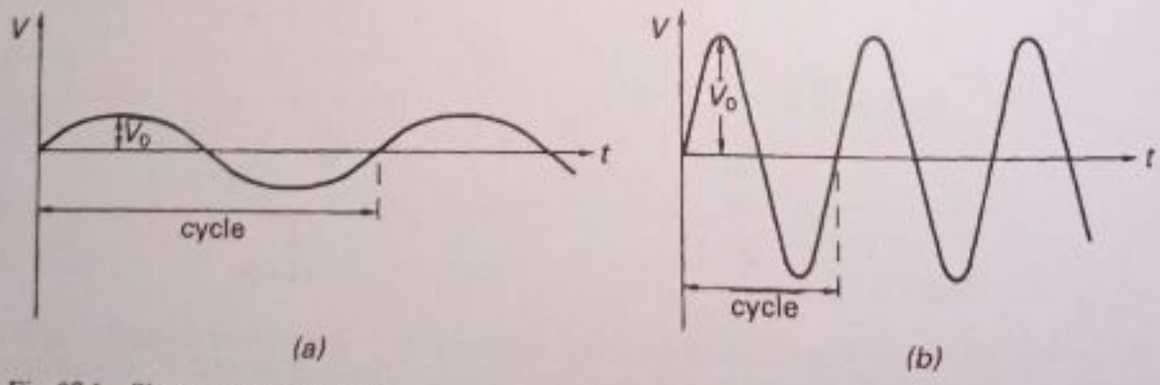


Fig. 37.1 Sine wave a.c. voltage

In Fig. 37.1a, the peak (maximum) voltage V_0 is less than in Fig. 37.1b. The frequency f has the unit Hz, so if $f = 50$ Hz, or 50 cycles per second, the period T or time for one cycle is 1/50 s. With a frequency f of 1000 Hz in a radio receiver, the period $T = 1/1000$ s.

Example

An a.c. voltage across a resistor in a radio circuit in volts is given by $V = 2 \sin 1600\pi t$. What is (a) the peak voltage, (b) the frequency?

Scanned with CamScanner

(a) Peak voltage = 2 V

(b) Comparing the a.c. voltage with $V = V_0 \sin \omega t$, then

$$\omega t = 2\pi f t = 1600\pi t$$

So

$$f = \frac{1600}{2} = 800 \text{ Hz}$$

Root-mean-square value

It is difficult to measure peak voltages of an a.c. voltage. The commercial value of an a.c. voltage is taken as the *root-mean-square* (*r.m.s.*) value of the a.c. voltage over a cycle. We shall use V_{rms} for the root-mean-square value.

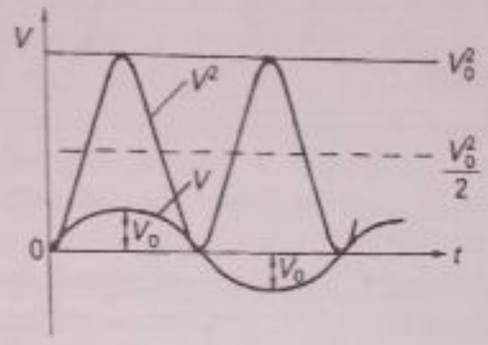


Fig. 37.2 Root-mean-square (r.m.s.) voltage

The r.m.s. voltage may be defined as the steady or direct voltage V_{dc} which produces the same heating effect per second in a given resistor. As shown on page 427, the heat per second in a given resistor is proportional to the *square* of the voltage. So, from the definition,

$$V_{dc}^2 = \text{average value of } V^2 \text{ over a cycle}$$

The variation of V^2 for a sine wave cycle is shown in Fig. 37.2. Its average value is $V_0^2/2$, where V_0 is the peak voltage because all the V^2 values vary symmetrically from zero to V_0^2 about the half-way line. So

$$V_{dc}^2 = \frac{V_0^2}{2}$$

$$\therefore V_{dc} = \text{r.m.s. value} = \frac{V_0^2}{2} = \frac{1}{\sqrt{2}} V_0 \qquad (1)$$

Since $1/\sqrt{2} = 0{\cdot}71$ (approx.), we see that

$$\text{r.m.s. voltage} = 0{\cdot}71 \times \text{peak voltage}$$

Also, from (1),

$$\text{peak voltage} = \sqrt{2} \times \text{r.m.s. voltage} \qquad (2)$$

$$= 1{\cdot}41 \times \text{r.m.s. voltage}$$

With a commercial power supply of 240 V r.m.s.,

$$\text{peak voltage} = \sqrt{2} \times 240 \text{ V} = 340 \text{ V (approx.)}$$

So an electrical machine which would burn out at 300 V should *not* be used on a voltage supply of 240 V r.m.s.

Similar numerical results to (1) and (2) are obtained for alternating currents.

Scanned with CamScanner

So $$I_{rms} = \frac{1}{\sqrt{2}} I_0 \text{ (peak value)}$$

and $$I_0 = \sqrt{2}\, I_{rms}$$

A.C. voltage and resistance *R*



Fig. 37.3 Resistance in a.c. circuit

Suppose an a.c. voltage V_{rms} of frequency f is connected to a resistor R, Fig. 37.3a. At any instant, $I = V/R$.

Fig. 37.3b shows how V and I vary. Firstly, the peak value of I is obtained at the peak value of V. Secondly, I and V are *in phase* (or in step with each other). To show in a diagram that I and V are in phase, vectors OA and OB are drawn parallel to each other to represent the peak values of V and I respectively, as in Fig. 37.3c.

So using peak values I_0 and V_0,

$$\therefore I_0 = \frac{V_0}{R}$$

Now $I_{rms} = I_0/\sqrt{2}$ and $V_{rms} = V_0/\sqrt{2}$. So we can write

$$I = \frac{V}{R}.$$

where I and V are both r.m.s. values. So, as in a d.c. circuit, we can use formulae

$$V = IR \quad \text{and} \quad R = V/I$$

Examples

1. A voltage supply of 12 V r.m.s., $f = 60$ Hz, is connected to a 6 Ω resistor. Calculate (a) the r.m.s. value of the current, (b) the peak value of current.

(a) $$I = \frac{V}{R} = \frac{12}{6} = 2 \text{ A r.m.s.}$$

The current is the r.m.s. value because 12 V was the r.m.s. voltage.

(b) Peak current = $\sqrt{2} \times$ r.m.s. current = $1{\cdot}41 \times 2 = 2{\cdot}8$ A (approx.)

2. A current of 3 mA (0·003 A) r.m.s. flows in a 2 kΩ (2000 Ω) resistor in a radio circuit. Find the r.m.s. and peak voltages across the resistor.

$$V = IR = 0{\cdot}003 \times 2000 = 6 \text{ V r.m.s.}$$

Peak voltage = $\sqrt{2} \times 6 \text{ V} = 1{\cdot}41 \times 6 = 8{\cdot}5$ V (approx.)

Scanned with CamScanner

A.C. voltage and inductance, *L*

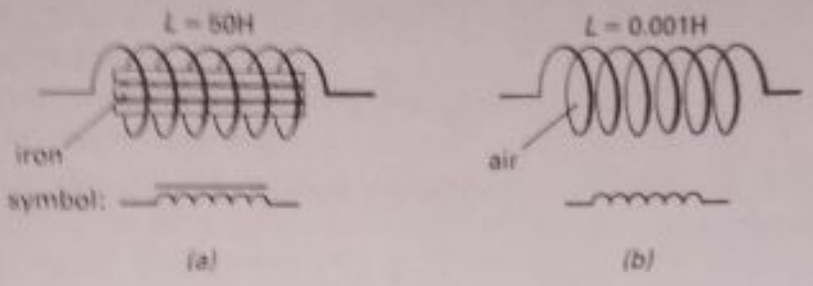


Fig. 37.4 Iron-cored and air-cored inductors

Radio and other a.c. circuits have one or more of the components: *resistors*, *inductors*, *capacitors*.

A coil of wire is an inductor in an a.c. circuit. When an alternating (varying) current flows in a coil such as Fig. 37.4a or b, an induced e.m.f. or voltage is produced in the coil which always *opposes* the current change, from Lenz's law (see page 483). The *inductance*, symbol L, is a measure of the opposition to the varying current.

L is measured in *henrys*, H. For a coil wound round soft-iron, the induced voltage is high and so L is high, for example, 50 H, Fig. 37.4a. Without the iron, that is, with an air core, the value of L is low, for example, 0·001 H (1 mH or 1 millihenry).

Reactance of inductor

The opposition *in ohms* of an inductor to the flow of alternating current is called its *reactance*, symbol X_L. The value of X_L is calculated from the formula

$$X_L = \omega L = 2\pi f L \text{ ohms} \qquad (1)$$

So X_L is similar to a 'resistance'. If a coil of $L = 4$ H is used in a radio receiver a.c. circuit of frequency 50 Hz, then

$$\text{reactance of coil, } X_L = 2\pi f L = 2\pi \times 50 \times 4 = 1250\,\Omega \text{ (approx.)}$$

If the same inductor is used in a circuit of frequency 1000 Hz, then

$$\text{reactance } X_L = 2\pi f L = 2\pi \times 1000 \times 4 = 25\,000\,\Omega \text{ (approx.)}$$

So the same inductor now has a reactance 20 times bigger than before. This is because, from (1), $X_L \propto f$ for a given inductance value L, Fig. 37.5a. Also, for a given frequency, $X_L \propto L$, Fig. 37.5b.

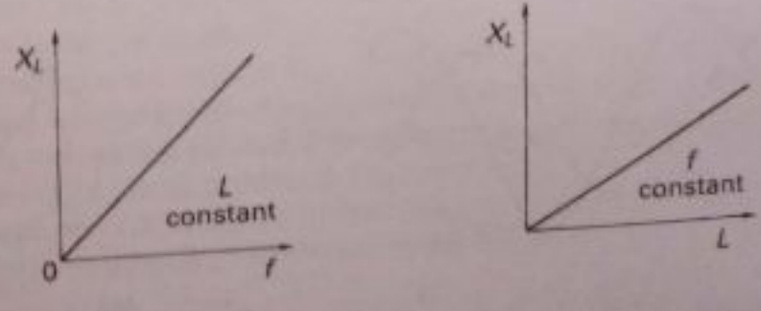


Fig. 37.5 How reactance X_L varies with f and with L

Scanned with CamScanner

A.C. circuit with reactance

Resistance R is defined by $R = V/I$ (see page 395). Similarly, reactance X_L is defined by

$$X_L = \frac{V}{I}, \tag{1}$$

where V and I are both r.m.s. voltages (or both peak voltages). So in an a.c. circuit, for an inductor,

$$I = \frac{V}{X_L} \tag{2}$$

and

$$V = IX_L \tag{3}$$

These formulae are similar to those in d.c. circuits. As we shall see later, however, I and V are not in phase in an inductor.

Examples on L

1. A supply of 240 V r.m.s., $f = 50$ Hz, is connected to an inductor of 2 H. Calculate (a) the reactance of the inductor, (b) the r.m.s. value of current.

(a) $X_L = 2\pi f L = 2\pi \times 50 \times 2 = 628\,\Omega$

(b) $$I = \frac{V}{X_L} = \frac{240}{628} = 0{\cdot}4 \text{ A r.m.s. (approx.)}$$

2. In a radio circuit, a current of 2 mA r.m.s., $f = 1000$ Hz, flows through an inductor L of 0·5 H. Calculate the r.m.s. and peak values of the voltage across L.

$$\text{Reactance } X_L = 2\pi f L = 2\pi \times 1000 \times 0{\cdot}5 = 3140\,\Omega$$

$$\therefore V = IX_L = 0{\cdot}002 \times 3140 = 6{\cdot}28 \text{ V r.m.s.}$$

So $$\text{peak voltage} = \sqrt{2} \times \text{r.m.s.} = 1{\cdot}41 \times 6{\cdot}28 = 8{\cdot}9 \text{ V (approx.)}$$

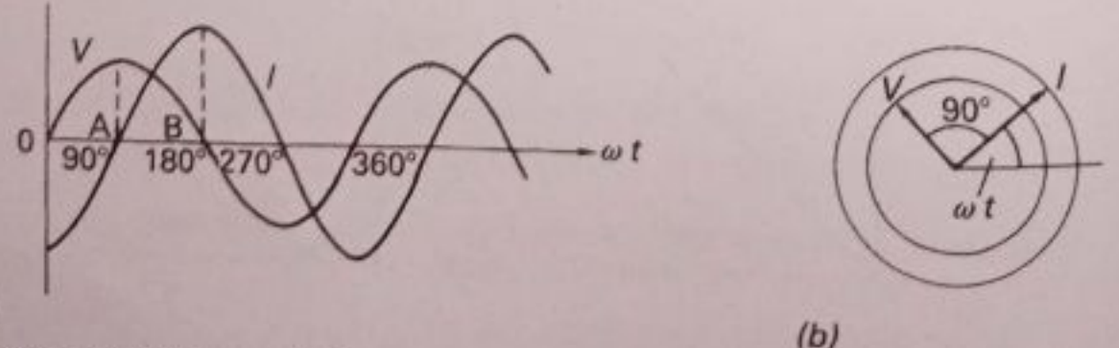
3. To find the inductance L of an inductor, a voltage of 6 V r.m.s., $f = 50$ Hz, is connected to the coil. The current flowing is then 3·0 mA r.m.s. Calculate L.

$$X_L = \frac{V}{I} = \frac{6}{0{\cdot}003} = 2000\,\Omega$$

$$\therefore 2\pi \times 50 \times L = 2000$$

$$\therefore L = \frac{2000}{2\pi \times 50} = 6{\cdot}4 \text{ H (approx.)}$$

Phase difference with inductance



(a) (b)

Fig. 37.6 Phase difference with L

Scanned with CamScanner

When an a.c. voltage V is connected to an inductor L, the current I varies as shown in Fig. 37.6a. The voltage V is out of phase (not in step) with the current I. V reaches its peak at a time corresponding to A but I reaches its peak at a later time corresponding to B.

Sine curves can be produced from vectors rotating round a circle (see p. 585). In Fig. 37.6b, the voltage vector V and current vector I are rotating round their respective circles at the same frequency. The radii of the two circles are proportional to the peak values of V and I respectively but V is always *leading* (ahead of) I by 90°. This corresponds to V and I in Fig. 37.6a as at A and B. Here the angle of rotation of the radii in Fig. 37.6b are shown on the horizontal axis as values of ωt as on page 586.

So, with an inductor L,

V leads by 90° on I

The vectors representing V and I for an inductor are shown in Fig. 37.7a. With a resistor R, however, V and I are in phase (p. 618). So they are represented by the vectors in Fig. 37.7b.

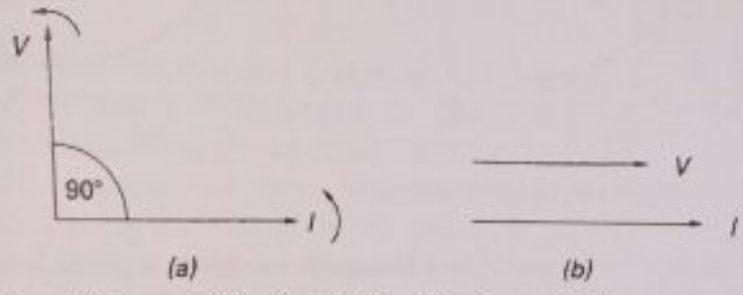


Fig. 37.7 (a) Phase difference of 90° with L (b) V and I in phase with R

A.C. and capacitor *C*

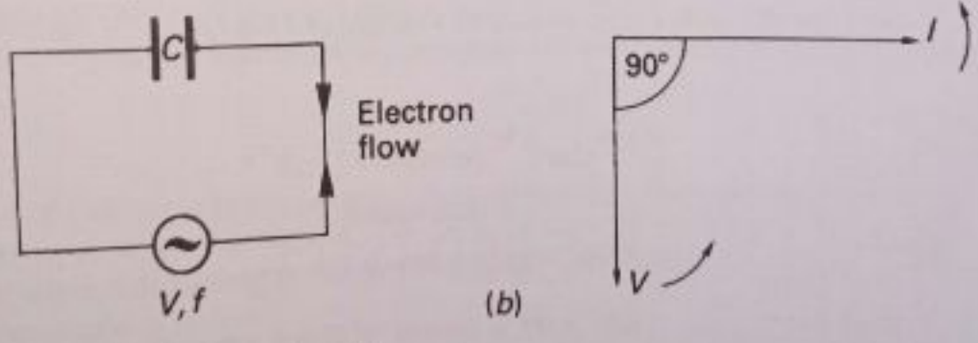


Fig. 37.8 A.c. voltage and capacitance C

We now consider the capacitor in an a.c. circuit, Fig. 37.8a. A capacitor stores charge (p. 607). Its capacitance C is defined by $C = Q/V$ where Q is the charge stored and V is the p.d. applied.

When an alternating voltage V_{rms} of frequency f is connected to a capacitor C, the varying voltage keeps charging the plates by different amounts because V varies. So an alternating current, due to electrons flowing in opposite directions in the connecting wires, is produced. This time V *lags* on I by 90°, Fig. 37.8b.

The opposition in ohms to the alternating current by C is called the *reactance* of the capacitor, symbol X_C. Reactance is defined by

$$X_C = \frac{V}{I}, \qquad (1)$$

Scanned with CamScanner

where V and I are both r.m.s. (or both peak) values. So

$$I = \frac{V}{X_C} \quad \text{and} \quad V = IX_C \tag{2}$$

The reactance X_C depends on C and on the frequency f. The formula for X_C is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \text{ ohms} \tag{3}$$

where C is in *farads* (F) and f is in Hz. From (3), we see that $X_C \propto 1/f$ for a given capacitor, so X_C decreases as f increases, as shown in Fig. 37.9a.

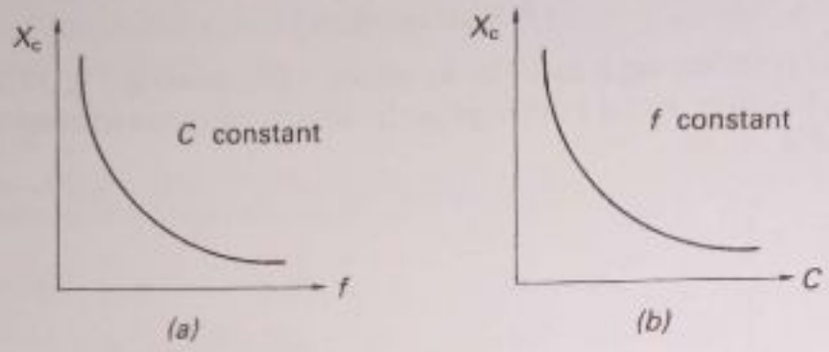


Fig. 37.9 Variation of reactance with (a) f and (b) C

Also, when C increases, the reactance becomes smaller at a given frequency f, as shown in Fig. 37.9b.

Examples

1. A capacitor of 2 μF in a radio circuit has a current of 4 mA r.m.s., $f = 1000$ Hz, in its circuit. Find (a) the reactance of the capacitor, (b) the voltage across it.

(a) $$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1000 \times 2 \times 10^{-6}}$$
$$= 80\,\Omega \text{ (approx.)}$$

(b) $$V = IX_C = 0{\cdot}004 \times 80 = 0{\cdot}32 \text{ V r.m.s.}$$

2. A lamp lights up normally with a current of 0·5 A r.m.s. Omitting the lamp resistance, calculate (a) the capacitance C and then (b) the inductance L needed to make the lamp light up normally, when a supply of 240 V r.m.s., $f = 60$ Hz, is connected to the lamp in series (a) with C and then (b) with L.

$$\text{Reactance needed} = \frac{V}{I} = \frac{240}{0{\cdot}5} = 480\,\Omega$$

(a) Then $$X_C = 480 = \frac{1}{2\pi \times 60 \times C}$$

So $$C = \frac{1}{480 \times 2\pi \times 60} \text{ F}$$
$$= 5{\cdot}5 \times 10^{-6} \text{ F} = 5{\cdot}5\,\mu\text{F}$$

(b) With L, $$X_L = 480 = 2\pi \times 60 \times L$$

Scanned with CamScanner

So $$L = \frac{480}{2\pi \times 60} = 1{\cdot}3\,\text{H (approx.)}$$

Series circuits · Impedance of *L* and *R*

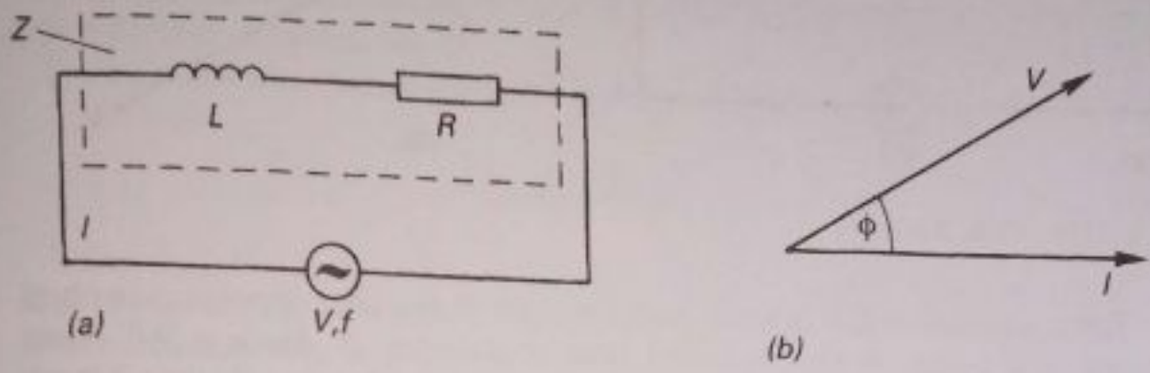


Fig. 37.10 Series circuit of L and R

Fig. 37.10a shows an inductor L in series with a resistor R, and an a.c. voltage of V_{rms} and frequency f connected to them. Together, L and R have an opposition in ohms in the circuit called their *impedance*, symbol Z.

Unlike d.c. circuits, we must add vectorially the reactance X_L to the resistance R. Taking account of the 90° phase difference between the current I and voltage V for L and the in-phase current I and voltage V for R (see p. 621), the formula for their impedance Z is

$$Z = \sqrt{R^2 + X_L{}^2} \qquad (1)$$

Also, the current I in this circuit differs in phase with the applied voltage V by an angle ϕ given by (see Fig. 37.10(b))

$$\tan\phi = \frac{X_L}{R} \qquad (2)$$

Since X_L varies with frequency f, the impedance Z will vary with f, from (1). For example, when f is high, X_L ($= 2\pi f L$) is high. So X_L may be much bigger than R when calculating Z. When f is low, the reverse may be true. So Z varies with frequency.

Example on series circuit

A voltage of 40 V r.m.s., $f = 100$ Hz, is connected to a coil of inductance of 2 H and negligible resistance in series with a resistor R of 1000 Ω.

Calculate (a) the circuit impedance, (b) the current flowing, (c) the voltage across L and across R.

(a) $X_L = 2\pi f L = 2\pi \times 100 \times 2 = 1256\,\Omega$

$\therefore Z = \sqrt{R^2 + X_L{}^2} = \sqrt{1000^2 + 1256^2} = 1605\,\Omega$

(b) $\therefore I = \dfrac{V}{Z} = \dfrac{40}{1605} = 0{\cdot}025\,\text{A (25 mA)}$

(c) $V_L = IX_L = 0{\cdot}025 \times 1256 = 31{\cdot}4\,\text{V}$

$V_R = IR = 0{\cdot}025 \times 1000 = 25\,\text{V}$

Scanned with CamScanner

Impedance of *C* and *R* in series

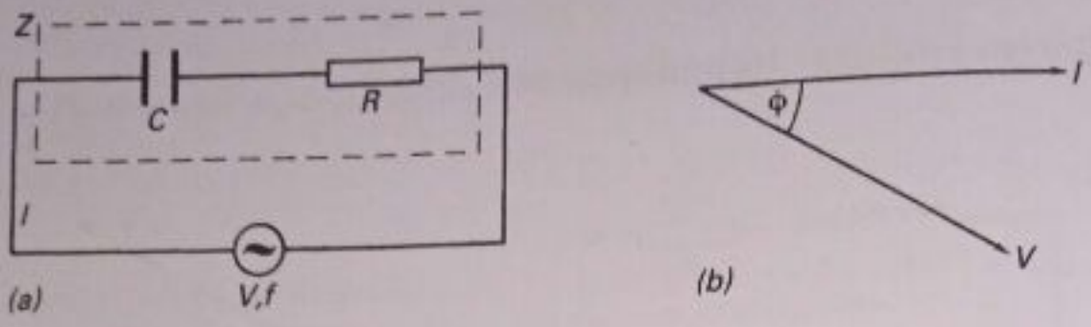


Fig. 37.11 *C* and *R* in series

When a capacitor C is in series with a resistor R, the total opposition in ohms to an alternating current is called their impedance, Z. Since a 90° phase difference is also the case between V and I for C (see p. 621), we calculate Z from

$$Z = \sqrt{R^2 + X_C{}^2} \qquad (1)$$

This is a similar relation for Z to the L and R in series case. Also, the current I flowing in the circuit in Fig. 37.11a differs in phase from the current I flowing by an angle ϕ (Fig. 37.11b) calculated from

$$\tan \phi = \frac{X_C}{R} \qquad (2)$$

Unlike the L–R series circuit, V *lags* on I in this case.

Example on C *and* R *in series*

A 1 μF capacitor is in series with a resistor of 4000 Ω. A voltage of 10 V r.m.s., $f = 50$ Hz, is connected to them.

Calculate (a) the current flowing, (b) the voltage across the capacitor.

(a)
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 1 \times 10^{-6}}$$
$$= 3180\,\Omega$$

$$\therefore \text{impedance } Z = \sqrt{R^2 + X_C{}^2} = \sqrt{4000^2 + 3180^2} = 5110\,\Omega$$

$$\therefore I = \frac{V}{Z} = \frac{10}{5110} = 00196\,\text{A} = 1{\cdot}96 \quad \text{A}$$

(b)
$$V_C = IX_C = 0{\cdot}00196 \times 3180 = 6{\cdot}2\,\text{V r.m.s.}$$

Series *L*, *C*, *R* circuit

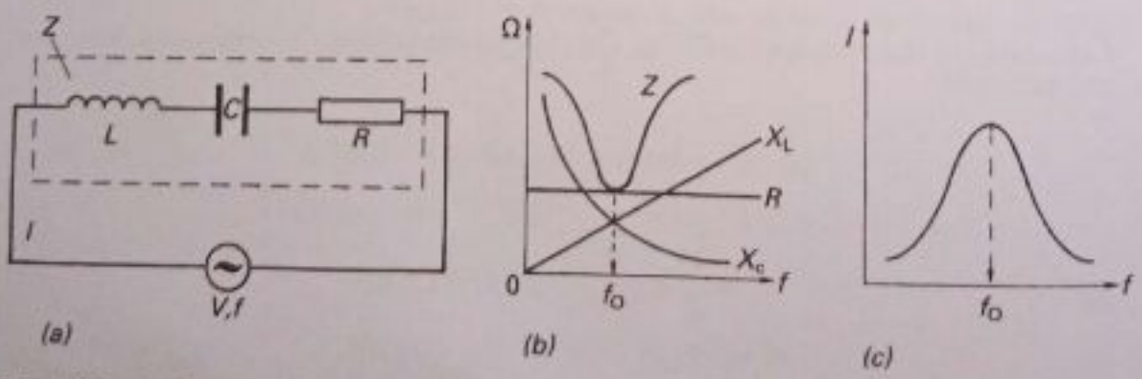


Fig. 37.12 *L*, *C*, *R* series circuit and resonance

Scanned with CamScanner

In radio receivers, a voltage V from a transmitting station is applied to a circuit having an inductor, capacitor and resistor all in series, Fig. 37.12a. In this case, the total impedance Z of L, C, R is calculated from

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{1}$$

Note that the two reactances X_L and X_C must be subtracted *before* squaring their difference in calculating Z.

The current I and voltage V are out of phase here by an angle ϕ given by

$$\tan\phi = \frac{X_L - X_C}{R} \tag{2}$$

Using the individual graphs for the variation of X_L and X_C, and the graph of R, as in Fig. 37.12b, the variation of Z with frequency shows a *minimum* value at some particular frequency f_0, as shown. This occurs, from (1), when $X_L = X_C$ at this frequency. So the minimum value of $Z = R$, the resistance in the circuit.

The current I in the circuit varies in the opposite way to Z. As shown in Fig. 37.12c, I is a *maximum* for the frequency f_0 when $X_L = X_C$. This is used in 'tuning' radio receivers to transmitting stations. If the frequency of the station is f_0, a loud sound is heard since the current flowing is then a maximum.

Examples

1. In an a.c. circuit, a current of 2 mA r.m.s., $f = 1000$ Hz, flows in a series arrangement of an inductor 0·01 H, a capacitor 6·0 μF and a resistor of 10 Ω.
Calculate the voltage across (a) the whole circuit, (b) the inductor.

(a) Impedance $Z = \sqrt{R^2 + (X_L - X_c)^2}$

Now $X_L = 2\pi f L = 2\pi \times 1000 \times 0{\cdot}01 = 62{\cdot}8\ \Omega$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 1000 \times 6 \times 10^{-6}} = 26{\cdot}5\ \Omega$$

$$\therefore Z = \sqrt{10^2 + (62{\cdot}8 - 26{\cdot}5)^2} = 37{\cdot}6\ \Omega$$

$$\therefore V = IZ = 0{\cdot}002 \times 37{\cdot}6 = 0{\cdot}075\ \text{V r.m.s.}$$

(b) $V = IX_L = 0{\cdot}002 \times 62{\cdot}8 = 0{\cdot}13\ \text{V r.m.s.}$

2. In the L, C, R series circuit in example 1, calculate the frequency f_0 which would produce the smallest impedance Z or maximum current in the circuit.

The smallest Z or maximum current I is produced when $X_L = X_C$.

Then $$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

Simplifying $$f_0^2 = \frac{1}{4\pi^2 LC}$$

$$\therefore f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi \times \sqrt{0{\cdot}01 \times 6 \times 10^{-6}}}$$

$$= 650\ \text{Hz}$$

Scanned with CamScanner

Power in a.c. circuits

We now consider the *power* dissipated in a.c. circuits. Here we may have single components L, C or R, or an arrangement of them in series or parallel.

Since power $P = IV$, we can find the average power dissipated in an a.c. circuit by plotting a graph of P from the two individual graphs for I and V. Fig. 37.13a shows the results for a *resistor* R. Here I and V are always in phase. So the average power over a cycle, $P = I_0V_0/2$, where I_0 and V_0 are peak values. But

$$I_0 = \sqrt{2}I, \text{ where } I \text{ is the r.m.s. value}$$

and

$$V_0 = \sqrt{2}V, \text{ where } V \text{ is the r.m.s. value}$$

So

$$P = I_0V_0/2 = 2IV/2 = IV$$

Since $V = IR$,

$$P = I^2R = V^2/R$$

So the same formula $P = IV = I^2R = V^2/R$ can be used for a.c. and d.c. circuits. With the a.c. circuit, remember that I and V are r.m.s. values.

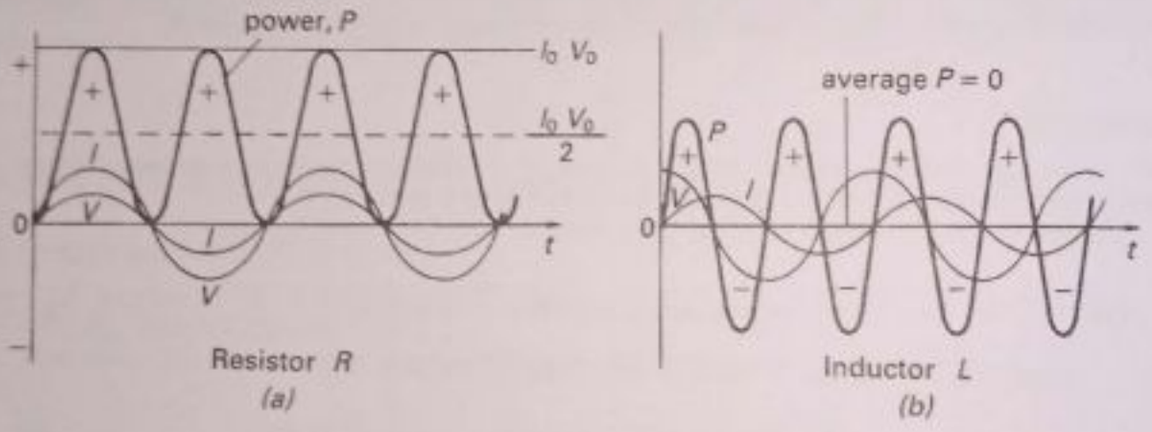


Fig. 37.13 Power in R, L and C circuits

With an *inductor* L, I and V are 90° out of phase (p. 621). Plotting I and V as in Fig. 37.13b, and then P, we see the power graph varies symmetrically above and below the time-axis, unlike the case of the resistor in Fig. 37.13a. So the average value of P over a cycle is *zero*. The result is explained by the fact that P is positive when energy is stored in the magnetic field of the coil and negative when the energy is given back to the voltage supply on the next part of the cycle. See Fig. 37.13b.

The same result $P = 0$ is obtained for a *capacitor* C because I and V are also 90° out of phase in this case (p. 621). Here P is positive when energy is stored in the electric field between the capacitor plates and negative when the energy is given back to the voltage supply.

Summary

Power is dissipated in a resistor R in an a.c. circuit ($P = IV = I^2R = V^2/R$) but no power is dissipated in an inductor L or capacitor C over a cycle.

Scanned with CamScanner

Examples on power

1. A heater resistor of 100 Ω has an a.c. of 2·82 A peak flowing through it. Calculate the power dissipated.

$$I_{rms} = \frac{1}{\sqrt{2}} \times I_{peak} = \frac{1}{1{\cdot}41} \times 2{\cdot}82 = 2{\cdot}0\,\text{A}$$

$$\therefore \text{power in } R = I^2R = 2^2 \times 100 = 400\,\text{W}$$

2. A current of 0·2 A r.m.s. flows in a series circuit containing a resistor R of 50 Ω, an inductor L of 2 H and a capacitor C of 0·1 μF. Calculate the total power absorbed in the circuit.

Power is only dissipated in R, not in L or C.

So total power dissipated $= I^2R = 0{\cdot}2^2 \times 50 = 2\,\text{W}$

Summary of a.c. circuits

Single components

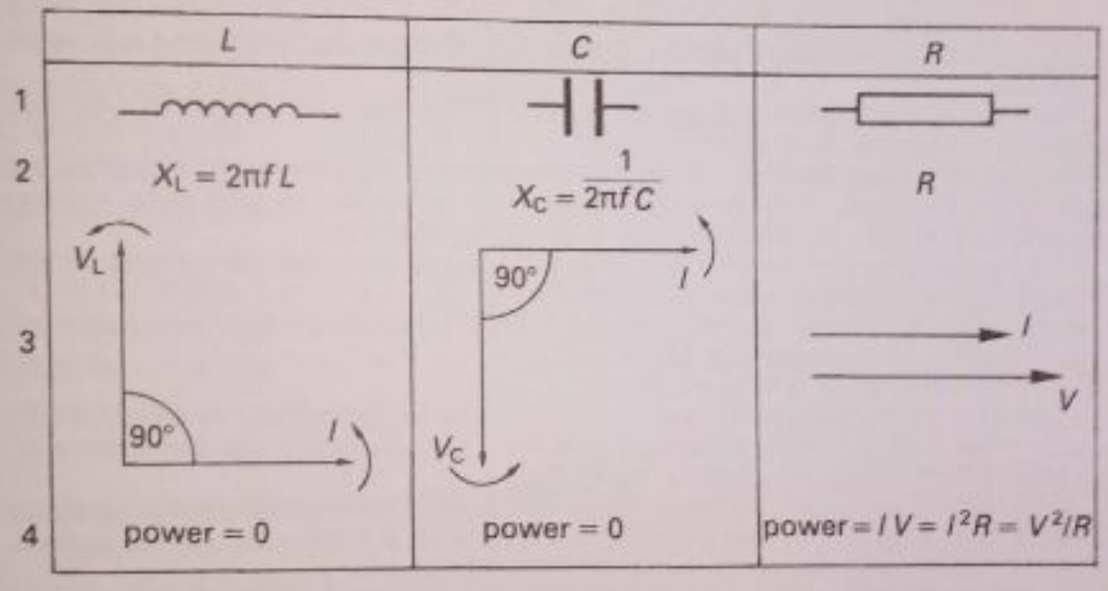


Fig. 37.14

Series circuits

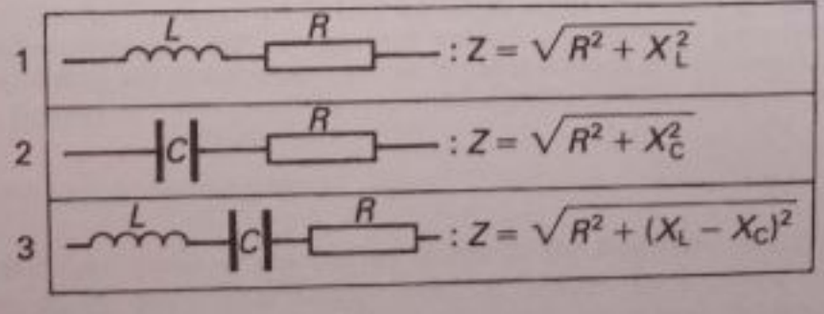


Fig. 37.15

Scanned with CamScanner

Exercise 37 (Answers, page 644)

In questions 1 and 2, choose the most correct answer from A, B, C, D, or E.

1. Fig. 37.16 shows how the alternating current I and the voltage V vary in a circuit with a single component.

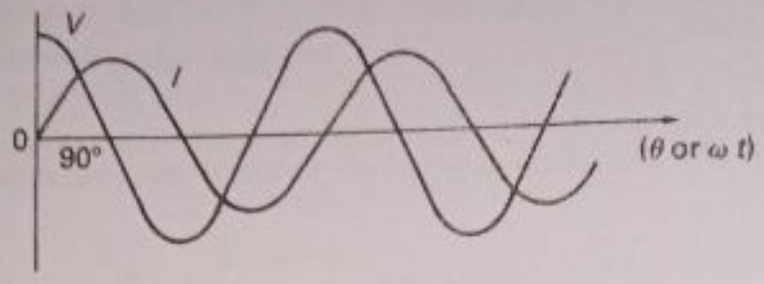


Fig. 37.16

A V lags on I by 90°
B V is in phase with I so there is a resistor in the circuit
C V leads on I so there is a capacitor or an inductor in the circuit
D V leads on I by 90°
E V is in phase with I

2. The power dissipated in an a.c. circuit is given by

A I^2R for a resistor R
B I^2X_L for an inductor L
C IV for *capacitor* C
D IV for C in series with R
E I^2Z for L in series with R

3. An a.c. supply produces a current of peak value 0·50 A. Calculate the r.m.s. value of current.

4. The power dissipated in a resistor of 100 Ω is 36 W. Calculate (i) the r.m.s. current flowing, (ii) the peak value of the current.

5. An alternating current of 0·1 A r.m.s. and frequency 50 Hz flows in a circuit with a capacitor of 1 μF in it. Find (i) the reactance of the capacitor, (ii) the r.m.s. voltage across the capacitor, (iii) the peak voltage across the capacitor.

With the same current flowing, the frequency decreases. Does the voltage across the capacitor increase, decrease or stay the same?

6. An a.c. supply of 5 V r.m.s. and frequency 150 Hz is connected to (i) an inductor of 2 H and then to (ii) a capacitor of 50 μF.

Find the current in r.m.s. in each case.

7. In an a.c. circuit where the frequency is 60 Hz, the voltage across an inductor coil is 12 V r.m.s. and the current flowing is 0·1 A r.m.s. Calculate the inductance L, assuming the resistance of the coil is negligible.

8. An inductor L of 2 H and a resistor R of 1000 Ω are in series across an a.c. voltage of 100 V, frequency 60 Hz. Calculate (i) the impedance of the circuit, (ii) the current in r.m.s. flowing, (iii) the r.m.s. voltage across the inductor.

9. An a.c. voltage of 12 V r.m.s. and frequency 100 Hz is connected across a capacitor of 1 μF in series with a resistor of 1000 Ω.

Calculate (i) the impedance of the circuit, (ii) the r.m.s. current flowing, (iii) the voltage across the capacitor.

What power is dissipated in the whole circuit?

10. An inductor of 0·012 H, a capacitor of 8 μF and a resistor of 100 Ω are all connected in series. An a.c. voltage of 5 V r.m.s. and frequency 200 Hz is connected to the arrangement.

Calculate (a) the reactance of the capacitor and of the inductor, (b) the impedance of the whole circuit, (c) the r.m.s. current flowing, (d) the frequency which would make the reactance of the capacitor equal to the reactance of the inductor, and the value of the circuit impedance in this case.

Scanned with CamScanner

38. Energy Levels in Atoms · Quanta · Photoelectricity · Wave-Particle Duality

Energy levels in atoms · Quanta

Energy levels

Hydrogen, the simplest atom, has one electron moving round its positive nucleus in a circular orbit (see page 570). The attractive force on the electron by the nucleus provides the centripetal force needed for circular motion (p. 43).

In 1911, Bohr studied the energy of the moving electron. From experimental results of the radiation obtained from energetic atoms, he assumed that:

1. The energy E of the electron could only have definite values E_0, E_1, E_2 and so on, which are called the *energy levels* of the electron.
2. The energy of the electron could only change from one energy level to another but not to any value between these levels. For example, the energy of the electron could increase from E_0 to E_1, or decrease from E_2 to a lower level E_1 or E_0.

This is similar to a person gaining or losing energy by going only from one level to another in a building but forbidden to occupy any level between. So inside the atom, an electron has well-defined states of energy. These separated energy values can now be discussed.

Hydrogen energy levels

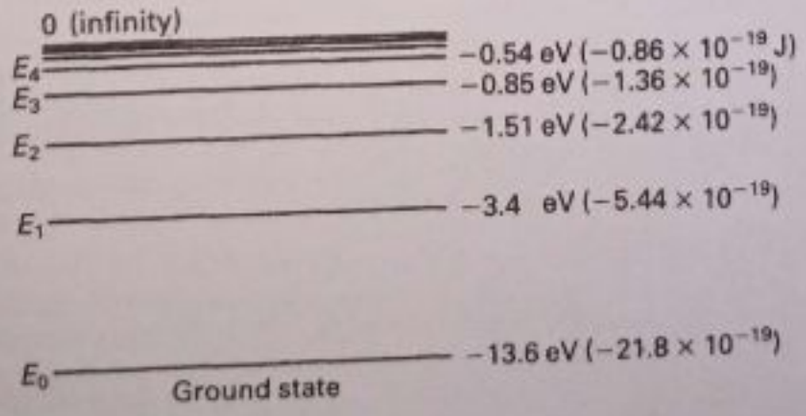


Fig. 38.1 Some hydrogen energy levels

Fig. 38.1 shows some of the energy levels of the hydrogen atom on Bohr's calculation. E_0 is the lowest energy level or *ground state* of the electron. It corresponds to the normal state of the atom. All the values can be calculated from the formula E_0/n^2, where $n = 1(E_0)$, $n = 2(E_1)$, $n = 3(E_2)$ and so on. The

Scanned with CamScanner

values *increase* from the ground state to a conventional value 0, when the electron is just removed from the atom and becomes free. So negative values of energy correspond to the energy values of the electron inside the atom or its bound state.

Energy is usually in joules, J. In atomic theory, a unit called the *electronvolt*, symbol eV, is used, as shown in Fig. 38.1.

$$1\,\text{eV} = 1{\cdot}6 \times 10^{-19}\,\text{J}$$

The ground state or lowest level of the electron in the hydrogen atom is $-13{\cdot}6\,\text{eV}$ or $-21{\cdot}8 \times 10^{-19}\,\text{J}$. Other atoms will have different energy levels.

Emission of electromagnetic radiation

Many years before the Bohr theory, Planck had shown that heat or electromagnetic radiation was emitted from hot objects in units which he called *quanta*. So radiation was not continuous in energy value but 'granular' or in separate units.

The unit or quantum of energy in electromagnetic radiation of frequency f is hf, where h is a constant known as the *Planck constant*. The value of h is about $6{\cdot}6 \times 10^{-34}\,\text{J s}$. So with radiation of frequency $10^{15}\,\text{Hz}$,

$$\text{quantum of energy} = hf = 6{\cdot}6 \times 10^{-34} \times 10^{15} = 6{\cdot}6 \times 10^{-19}\,\text{J}$$

So the radiation consists of millions of quanta of energy units $6{\cdot}6 \times 10^{-19}\,\text{J}$.

When an atom such as hydrogen gains energy by heating, for example, the electron moves from the ground state, energy E_0, to a higher energy level such as E_2. The electron is then unstable and so it may fall back from the energy level E_2 to a lower level such as E_1 or E_0. Bohr suggested that the excess energy, $E_2 - E_0$ say, was released in the form of *electromagnetic radiation of frequency f*, where

$$\text{energy change } E = E_2 - E_0 = hf \qquad (1)$$

Emission spectra of hydrogen

The formula in (1), $E = hf$, can be used to calculate the frequency and wavelengths in the hydrogen spectrum.

Suppose the atom is excited (or given energy) to an energy level E_1 ($-5{\cdot}4 \times 10^{-19}\,\text{J}$) and then falls to the ground state E_0 ($-21{\cdot}8 \times 10^{-19}\,\text{J}$). From the Bohr theory, the energy change $E = hf$, where f is the frequency of the electromagnetic radiation emitted. So.

$$E_1 - E_0 = hf$$

$$\therefore (-5{\cdot}4 \times 10^{-19}) - (-21{\cdot}8 \times 10^{-19}) = 6{\cdot}6 \times 10^{-34} \times f$$

$$\therefore 16{\cdot}4 \times 10^{-19} = 6{\cdot}6 \times 10^{-34} \times f$$

$$\therefore f = \frac{16{\cdot}4 \times 10^{-19}}{6{\cdot}6 \times 10^{-34}} = 2{\cdot}5 \times 10^{15}\,\text{Hz}$$

The wavelength λ of the radiation in free space or a vacuum is always found from the relation $c = f\lambda$, where c is the speed of electromagnetic waves in free space, $3{\cdot}0 \times 10^8\,\text{m/s}$ (p. 345). So

$$\lambda = \frac{c}{f} = \frac{3{\cdot}0 \times 10^8}{2{\cdot}5 \times 10^{15}} = 1{\cdot}2 \times 10^{-7}\,\text{m}$$

Scanned with CamScanner

This wavelength is in the ultraviolet part of the electromagnetic wave spectrum (see p. 345).

If the atom is excited to a higher energy level such as E_2 and falls to the lower energy level E_1, a new radiation is emitted whose frequency f is given by

$$E_2 - E_1 = hf$$

From Fig. 38.1, $(-2{\cdot}4 \times 10^{-19}) - (-5{\cdot}4 \times 10^{-19}) = 6{\cdot}6 \times 10^{-34} \times f$

So $$f = \frac{3{\cdot}0 \times 10^{-19}}{6{\cdot}6 \times 10^{-34}} = 4{\cdot}5 \times 10^{14}\ Hz$$

$$\therefore \text{wavelength} = \frac{c}{f} = \frac{3{\cdot}0 \times 10^{8}}{4{\cdot}5 \times 10^{14}} = 6{\cdot}7 \times 10^{-7}\ \text{m}$$

This wavelength is in the visible spectrum of electromagnetic waves (p. 345).

Relation of frequency and wavelength to *E*

As we have seen, the energy change $E = hf$. So

$$f \propto E \qquad (1)$$

The greater the energy change, the greater will be the frequency f of the emitted radiation. From the spacing in the energy level diagram in Fig. 38.1, we therefore see that f is greater for the energy change E_1 to E_0 than for the energy change E_2 to E_1.

Since the wavelength $\lambda = c/f = ch/E$, from $E = hf$, we see that λ *decreases* when the energy change E *increases*. This is why the short ultraviolet wavelengths are obtained for high energy changes. In the visible spectrum, blue wavelengths are shorter than red wavelengths. So the energy change for blue wavelengths is greater than for red wavelengths.

Absorption spectra

The wavelengths emitted from heated atoms are emission spectra. Here an electron falls from a high to a lower energy level as we have seen.

Atoms can also absorb energy from the radiation emitted from heated atoms. A heated tungsten lamp filament, for example, emits a continuous band of visible wavelengths varying from red to violet. When this radiation is incident on a glowing yellow sodium lamp at a lower temperature than the tungsten filament, the sodium atoms *absorb* radiation and electrons are raised to a higher energy level. When they fall to a lower energy level, the radiation emitted spreads in different directions. The incident tungsten radiation has therefore lost some energy corresponding to the yellow radiation from the sodium atoms. A dark line then appears in the visible spectrum of the tungsten. The wavelength of the dark line is that of the yellow radiation from the sodium atoms.

The dark lines in the emission spectrum of a heated substance due to absorption are called *absorption spectra*. The wavelengths of the dark lines are characteristic of the absorbing atoms. Fraunhofer found a number of fine dark lines in the Sun's visible spectrum. They were produced by vaporised elements in the outer cooler regions of the Sun, through which the Sun's radiation passed. By measuring the wavelengths of the dark lines, the elements sodium and iron are known to exist in the Sun.

Scanned with CamScanner

Exercise 38A (Answers, page 644)

In questions 1 and 2, choose the most correct answer from A, B, C, D or E.

1. Fig. 38.2 shows some energy levels in the mercury atom.

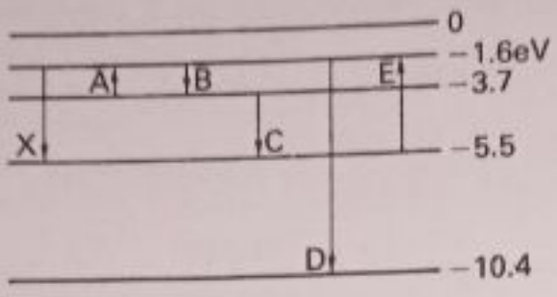


Fig. 38.2

The energy change X produces a red line in the visible emission spectrum. Which energy change is most likely to produce a line in the ultraviolet emission spectrum if ultraviolet light has a shorter wavelength than visible light?

2. Fig. 38.3 shows some energy levels in the mercury atom, where level 1 is the ground state or level.

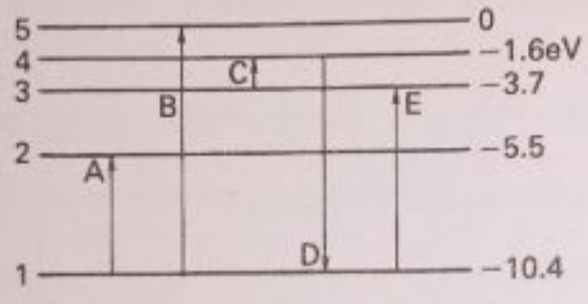


Fig. 38.3

Which energy change represents the *ionisation* of the atom?

3. A photon has a wavelength $5{\cdot}0 \times 10^{-7}$ m. Calculate (i) its frequency, (ii) its energy in joules, (iii) its energy in eV if 1 eV = $1{\cdot}6 \times 10^{-19}$ J.

4. The energy of a photon is 1·6 eV. What is (i) its energy in joules, (ii) its frequency, (iii) its wavelength in metres?

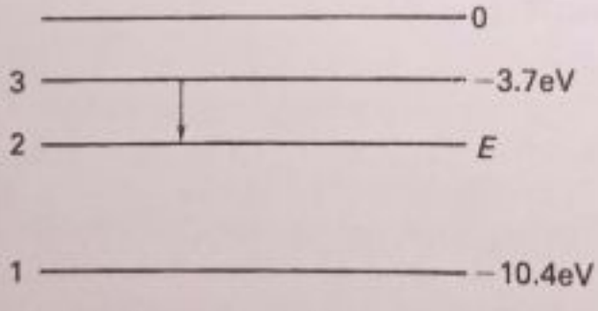


Fig. 38.4

5. In Fig. 38.4, an electron falls from energy level 3 to level 2 as shown. It then emits a photon of wavelength $6{\cdot}875 \times 10^{-7}$ m. Calculate (a) the energy change, (b) the energy level 2 in eV.

6. Fig. 38.5 shows some of the energy levels in the hydrogen atom. E_0 is the ground state.

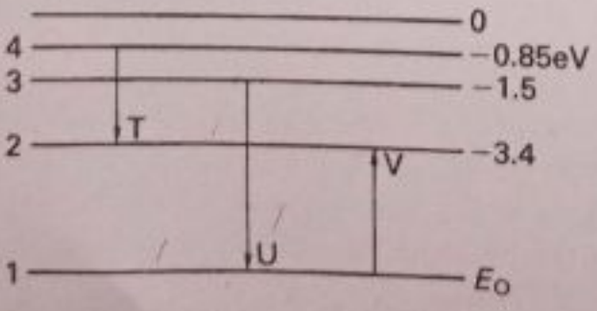


Fig. 38.5

Scanned with CamScanner

Calculate the wavelength of the photon emitted when an electron falls from energy level 4 to 2, as shown by T.

7. When an electron makes the energy change U, Fig. 38.5, it emits a photon of wavelength $1{\cdot}02 \times 10^{-7}$ m. Calculate the energy E_0 of level 1.

8. (a) in Fig. 38.5, what is the ionisation energy of the hydrogen atom, using your value of E_0 from **7**. (b) What energy change in Fig. 38.5 would produce an absorption spectrum line in the hydrogen emission spectrum? (Answer T, U or V?)

Photoelectricity

Photoelectrons · Threshold wavelength

In 1902, Lenard and other scientists found that electrons were emitted from a metal plate when it was illuminated by light. The subject is called *photoelectricity* and the emitted electrons are often called *photoelectrons*. A well-cleaned zinc plate emits electrons when illuminated by ultraviolet light. In Fig. 38.6, a cleaned zinc plate Z is connected to the cap of a gold-leaf electroscope (p. 511).

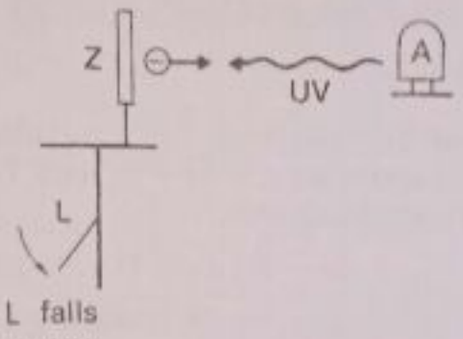


Fig. 38.6 Emission of photoelectrons

Z is first charged *negatively* by induction (p. 514) and the leaf opens. When an ultraviolet lamp A is brought near to Z in a darkened room, the leaf L is seen to fall. The negatively-charged zinc Z loses its charge by the emission of electrons (negative charges) which are repelled away from Z.

Lenard found that the maximum kinetic energy of the emitted electrons depended *only* on the frequency or wavelength of the incident light and not on the intensity (power) of the light beam. Also, above a certain wavelength called the *threshold wavelength*, no electrons were emitted even though the incident light beam was very strong.

Work function · Failure of wave theory of light

An electron, a negative charge, on or near the surface of a metal has attracting positive charges of many nuclei of atoms behind it. Some energy is therefore needed to pull an electron out of the metal.

The minimum energy needed to pull out an electron is called the *work function* of the metal, symbol w_0. The size of the work function varies with different metals. For example, $w_0 = 2{\cdot}0\,\text{eV}$ or $3{\cdot}2 \times 10^{-19}$ J for sodium and $1{\cdot}35\,\text{eV}$ or $2{\cdot}2 \times 10^{-19}$ J for caesium.

On the wave theory of light, the waves from a lamp in front of the metal spread out and fall on the metal surface continuously. After a short time, the energy falling on the surface from a powerful lamp should reach the work function value and electrons should then be emitted. But if the light is above the threshold wavelength, experiment shows that no electrons are emitted. So the wave theory of light can *not* explain the photoelectric effect.

Scanned with CamScanner

Einstein's photoelectric theory ·The photon

In 1905, Einstein used a particle theory of light to explain the experimental results in photoelectricity. He suggested that light could be considered as 'packets' of energy, like particles, and these are called *photons*. As Planck had done in heat radiation (p. 630), Einstein said that the unit or quantum of energy in a light beam was given by

$$E = hf,$$

where f is the light frequency and h is the Planck constant, $6{\cdot}6 \times 10^{-34}$ J s.

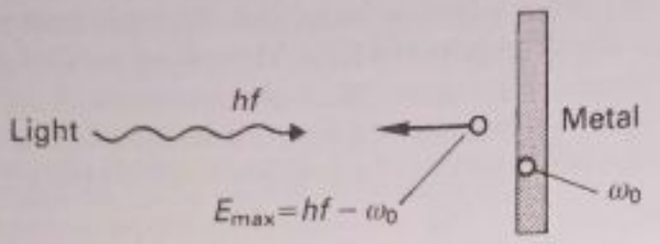


Fig. 38.7 Einstein photoelectric theory

Suppose a monochromatic light beam has a wavelength $4{\cdot}5 \times 10^{-7}$ m. The frequency f is given by c/λ and in air, $c = 3{\cdot}0 \times 10^8$ m/s. Using the values of c and h, the photon energy E is calculated from

$$E = hf = \frac{hc}{\lambda} = \frac{6{\cdot}6 \times 10^{-34} \times 3 \times 10^8}{4{\cdot}5 \times 10^{-7}}$$

$$= 4{\cdot}4 \times 10^{-19}\,\text{J}$$

From $1\,\text{eV} = 1{\cdot}6 \times 10^{-19}$ J,

$$E = \frac{4{\cdot}4 \times 10^{-19}}{1{\cdot}6 \times 10^{-19}} = 2{\cdot}75\,\text{eV}$$

With the metal potassium, the work function w_0 is 1·9 eV. So the photon energy is greater than w_0. As shown in Fig. 38.7, an electron is therefore emitted from the metal. From the Einstein theory, no electrons are emitted if the incident photon energy hf is *less* than w_0.

We now see that the threshold frequency f_0 for a metal is given by

$$hf_0 = w_0$$

So for $w_0 = 1{\cdot}9\,\text{eV}$, $\quad f_0 = \dfrac{w_0}{h} = \dfrac{1{\cdot}9 \times 1{\cdot}6 \times 10^{-19}}{6{\cdot}6 \times 10^{-34}} = 4{\cdot}6 \times 10^{14}\,\text{Hz}$

$$\therefore \text{ threshold wavelength } \lambda_0 = \frac{c}{f} = \frac{3 \times 10^8}{4{\cdot}6 \times 10^{14}} = 6{\cdot}5 \times 10^{-7}\,\text{m}$$

Wavelengths *shorter* than this value would produce photoelectrons.

Einstein photoelectron equation

When photoelectrons are emitted from a metal, those electrons just below the surface make collisions with atoms before they come out. So they have smaller kinetic energy than electrons initially on the metal surface. So generally, the emitted electrons have different energies and speeds.

Those electrons emitted with maximum energy, E_{max}, are those which

Scanned with CamScanner

needed the least energy to pull them out of the metal. Since w_0 is the least energy to pull an electron from the metal,

$$E_{max} = hf - w_0 \quad (1)$$

This is the *Einstein photoelectric equation*. If m is the mass of an electron and v_{max} is the maximum speed of the emitted electrons, then $E_{max} = \frac{1}{2}mv_{max}^2$.

Examples on photoelectricity

1. Radiation of frequency $8{\cdot}0 \times 10^{14}$ Hz is incident on a metal surface. The maximum energy of the emitted electrons is $1{\cdot}7 \times 10^{-19}$ J.
Calculate (a) the work function, (b) the threshold frequency. (Assume $h = 6{\cdot}6 \times 10^{-34}$ J s.)

(a) From the Einstein equation, $E_{max} = hf - w_0$

$$\therefore w_0 = hf - E_{max}$$
$$= (6{\cdot}6 \times 10^{-34} \times 8 \times 10^{14}) - 1{\cdot}7 \times 10^{-19}$$
$$= 5{\cdot}28 \times 10^{-19} - 1{\cdot}7 \times 10^{-19}$$
$$= 3{\cdot}58 \times 10^{-19}\ \text{J}$$

(b) If f_0 is the threshold frequency, $hf_0 = w_0$

$$\therefore f_0 = \frac{w_0}{h} = \frac{3{\cdot}58 \times 10^{-19}}{6{\cdot}6 \times 10^{-34}} = 5{\cdot}4 \times 10^{14}\ \text{Hz}$$

2. In Example 1, calculate the wavelength of incident light which would emit electrons with maximum kinetic energy $2{\cdot}0 \times 10^{-19}$ J.

From the Einstein equation, $E_{max} = hf - w_0$

$$\therefore hf = E_{max} + w_0$$

Now $E_{max} = 2{\cdot}0 \times 10^{-19}$ J and $w_0 = 3{\cdot}58 \times 10^{-19}$ J from example 1.

Adding E_{max} and w_0, $\therefore hf = 5{\cdot}58 \times 10^{-19}$

$$\therefore f = \frac{5{\cdot}58 \times 10^{-19}}{6{\cdot}6 \times 10^{-34}} = 8{\cdot}45 \times 10^{14}\ \text{Hz}$$

$$\text{wavelength } \lambda = \frac{c}{f} = \frac{3 \times 10^8}{8{\cdot}45 \times 10^{14}} = 3{\cdot}6 \times 10^{-7}\ \text{m}$$

Stopping potential

In 1911, Millikan carried out an experiment to verify the Einstein photoelectric equation, $E_{max} = hf - w_0$.

The basic principle of the experiment is illustrated in Fig. 38.8a. Light of known frequency f is incident on a metal C inside a glass tube containing a vacuum.

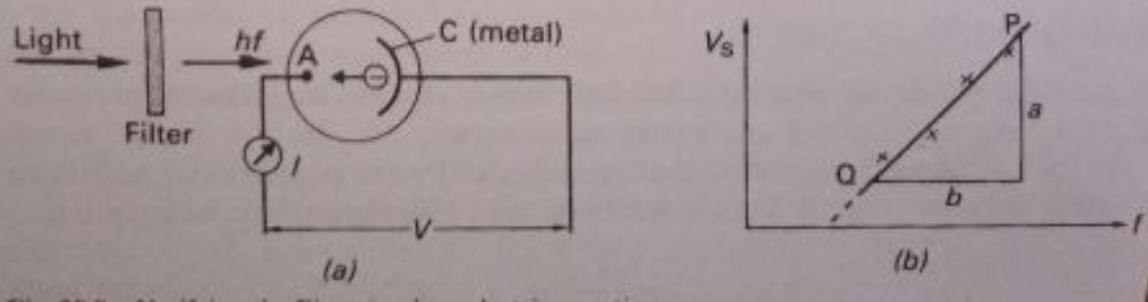


Fig. 38.8 Verifying the Einstein photoelectric equation

Scanned with CamScanner

Opposite C is a metal collector A of the emitted electrons. Those electrons reaching A produce an electric current I recorded on the meter in the circuit.

By means of a battery (not shown), A can be given a varying *negative* potential relative to C from zero. Electrons emitted from C then use up energy reaching A because A *opposes* the movement of these negative charges towards it.

As the potential of A becomes more and more negative, at one potential the current I becomes zero. This is called the *stopping potential*—it is the negative potential of A which just stops all the electrons reaching it.

In this case, the maximum kinetic energy E_{max} is used up in moving from C to A. Now the energy change of a charge Q moving through a p.d. V is QV (p. 605). So if e is the electron charge and V_s is the stopping potential, then

$$eV_s = E_{max} = hf - w_0 \qquad (1)$$

So

$$V_s = \frac{h}{e}f - \frac{w_0}{e} \qquad (2)$$

In an experiment, measurements of the stopping potential V_s are made for different values of f, the frequency of the incident light beam. Results show that a straight line PQ is obtained, Fig. 38.8b, when V_s is plotted against f. When the gradient a/b is measured, its value is h/e from equation (2). So

$$h = (\text{gradient } a/b) \times e$$

Knowing the value for e, Millikan calculated the value for h from this relation and found the result to be near the value $6{\cdot}6 \times 10^{-34}$ J s. This verified the Einstein equation.

Example on stopping potential

Calculate the stopping potential when light of wavelength $4{\cdot}5 \times 10^{-7}$ m is incident on a metal of work function 1·8 eV.
(Assume $h = 6{\cdot}6 \times 10^{-34}$ J s, $e = 1{\cdot}6 \times 10^{-19}$ C, $c = 3 \times 10^{8}$ m/s, 1 eV = $1{\cdot}6 \times 10^{-19}$ J.)

The stopping potential V_s is calculated from

$$eV_s = hf - w_0$$

Now

$$hf = \frac{hc}{\lambda} = \frac{6{\cdot}6 \times 10^{-34} \times 3 \times 10^{8}}{4{\cdot}5 \times 10^{-7}} = 4{\cdot}4 \times 10^{-19} \text{ J}$$

and

$$w_0 = 1{\cdot}8 \times 1{\cdot}6 \times 10^{-19} = 2{\cdot}88 \times 10^{-19} \text{ J}$$

$$\therefore eV_s = hf - w_0 = (4{\cdot}4 - 2{\cdot}88) \times 10^{-19} = 1{\cdot}52 \times 10^{-19}$$

$$\therefore V_s = \frac{1{\cdot}52 \times 10^{-19}}{1{\cdot}6 \times 10^{-19}} = 0{\cdot}95 \text{ V}$$

Wave-particle duality

In photoelectricity, we have seen that light waves can act as particles (photons). The converse is also true. Particles can act as waves.

In 1925, de Broglie suggested that particles had wave properties. De Broglie stated that the wavelength λ of the waves carried with the particle was given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}, \qquad (1)$$

Scanned with CamScanner

where mv or p is the momentum of the particle of mass m and velocity v, and h is the Planck constant.

Some years later, fast-moving electrons were shown to produce *diffraction rings* when they were incident on very thin gold film. The ring pattern was similar to those made by X-rays when diffracted by a crystal. This showed that electrons (particles) could also behave as waves. By varying the speed v of the electron beam and measuring λ from the radii of the rings formed, the de Broglie formula in (1) was shown to be true.

The formula for λ in (1) applies to all sizes of moving particles. For an electron of mass m about 10^{-30} kg and moving with a velocity v of 10^7 m/s,

$$\lambda = \frac{h}{mv} = \frac{6{\cdot}6 \times 10^{-34}}{10^{-30} \times 10^7} = 6{\cdot}6 \times 10^{-11}\,m,$$

using $h = 6{\cdot}6 \times 10^{-34}$ J s. This wavelength is comparable with that of an X-ray wavelength. So as experiment showed, these electrons can be diffracted by thin metal foil like X-rays. Protons and neutrons, which are heavier than electrons, are diffracted much less than electrons moving with the same speed. Slow neutrons, however, are used today in diffraction experiments.

In the case of a ball of mass 0·2 kg moving with a velocity of 20 m/s, however,

$$\lambda = \frac{h}{mv} = \frac{6{\cdot}6 \times 10^{-34}}{0{\cdot}2 \times 20} = 1{\cdot}7 \times 10^{-34}\,\text{m}$$

This extremely small wavelength shows that the ball behaves like a particle and not like a wave. So massive objects do not show wave properties.

Wave-particle

So particles such as electrons can have wave properties. Waves such as X-rays can have particle properties. For example, Compton showed in an experiment that the conservation of momentum and energy were obeyed when X-rays were incident on an electron.

The wave-particle duality, the double nature of waves and particles, appears to be general in nature. Depending on the experimental conditions, particle *or* wave properties can be used to explain the result of the experiment. In photoelectricity, for example, light waves are considered as particles (photons). Particles such as high-speed electrons or slow-speed neutrons may be treated as waves.

Heisenberg Uncertainty Principle

With large objects such as a moving football or a moving car, their momentum at a particular position, or their energy at a particular time, can both be measured with an accuracy which depends on the instruments used.

In the world of atomic particles such as electrons, however, Heisenberg recognised that there was always an uncertainty or inaccuracy in these measurements additional to those of the instruments. Electrons, for example, behave as waves which spread out, leading to uncertainty in measurement of their positions. If the electron position, x, is measured to a high degree of accuracy, there is a relatively low degree of accuracy in the simultaneous measurement of the momentum, p, according to Heisenberg. We can never measure both x and p simultaneously to a high degree of accuracy.

Scanned with CamScanner

The Heisenberg *Uncertainty Principle* states that if Δx is the uncertainty in measuring x, and Δp is the uncertainty in measuring p, then their product is equal to or greater than h, the Planck constant. So

$$\Delta x \,.\, \Delta p \geqslant h \qquad (1)$$

Since $\Delta p \geqslant h/\Delta x$, we see that when Δx is small (small uncertainty), then Δp is large (great uncertainty). The converse is also true. When Δx is large (great uncertainty), then Δp is small (small uncertainty).

The Uncertainty Principle also holds for simultaneous measurements of the energy E of an object at a particular time t. If the uncertainty in E is ΔE and the uncertainty in t is Δt, then

$$\Delta E \,.\, \Delta t \geqslant h \qquad (2)$$

So the greater the accuracy in measuring the time, the smaller will be the accuracy in measuring the energy.

Example on Uncertainty Principle

The uncertainty in the position x of an electron moving through a cathode-ray tube is 10^{-9} m.

What is the uncertainty in the simultaneous measurement of the velocity v of the electron, assuming its mass is 10^{-30} kg? (Assume $h = 6{\cdot}6 \times 10^{-34}$ J s.)

We have $\Delta x = 10^{-9}$ m, and $\Delta p = \Delta(mv) = m\,\Delta v = 10^{-30}\,\Delta v$

From the Uncertainty Principle,

$$\Delta x \,.\, \Delta p \geqslant h$$

$$\therefore 10^{-9} \times 10^{-30} \Delta v \geqslant 6{\cdot}6 \times 10^{-34}$$

$$\therefore \Delta v \geqslant \frac{6{\cdot}6 \times 10^{-34}}{10^{-9} \times 10^{-30}} \geqslant 6{\cdot}6 \times 10^{5} \text{ m/s}$$

So if the measured velocity is $1{\cdot}0 \times 10^{7}$ m/s, its velocity v may be $(1{\cdot}0 \times 10^{7} \pm 7 \times 10^{5})$ m/s, using the uncertainty value Δv.

Exercise 38B (Answers, page 644)

(Where necessary, assume $h = 6{\cdot}6 \times 10^{-34}$ J s, $e = 1{\cdot}6 \times 10^{-19}$ C, $c = 3{\cdot}0 \times 10^{8}$ m/s, 1 eV $= 1{\cdot}6 \times 10^{-19}$ J.)

1. Which of the statements A to E is NOT true?
   Photoelectrons are always emitted from the surface of an illuminated metal.
   - A by energetic incident photons
   - B by photons of energy greater than the work function
   - C by light of wavelength smaller than the threshold value
   - D when hf is greater than w_0 (work function)
   - E when hc/λ is greater than w_0

2. Which of the statements A to E is true?
   The wave theory of light does not explain photoelectricity because
   - A light waves do not carry energy
   - B particles can not be emitted by waves
   - C the wavelength of light waves is too short
   - D there is a threshold wavelength for electron emission
   - E the frequency of the light waves is high

3. Calculate the photon energies in joules in (a) red light of wavelength $7{\cdot}0 \times 10^{-7}$ m, (b) ultraviolet light of wavelength $2{\cdot}0 \times 10^{-7}$ m, (c) X-rays of wavelength $3{\cdot}0 \times 10^{-10}$ m.
4. What is the wavelength in photons of energy 3·0 eV?
5. The work function of a metal is 2·7 eV. Calculate (i) the threshold frequency for the metal, (ii) the threshold wavelength.

Scanned with CamScanner

6. Calculate the frequency in radiation of wavelength $4{\cdot}0 \times 10^{-7}$ m.
For a metal of work function 1·4 eV, find the maximum energy of emitted electrons when illuminated by this radiation.

7. The work function of the metal potassium is 2·0 eV. Calculate the maximum energy of electrons emitted from its surface when illuminated by radiation of wavelength $3{\cdot}5 \times 10^{-7}$ m.

8. In question **7**, find the stopping potential V_s needed to prevent all the emitted electrons reaching a collecting electrode, using your result for the maximum energy of the electrons.

9. The maximum energy of electrons emitted from a metal surface is $3{\cdot}2 \times 10^{-19}$ J when the metal is illuminated by light of wavelength $4{\cdot}0 \times 10^{-7}$ m.
Calculate (i) the photon energy in the incident light, (ii) the work function of the metal in eV.

10. Light of frequency $6{\cdot}0 \times 10^{14}$ Hz is incident on a metal surface. The maximum energy of the photoelectrons is $2{\cdot}4 \times 10^{-19}$ J.
Calculate (i) the work function of the metal, (ii) the frequency of the radiation which would produce photoelectrons with a maximum energy of $3{\cdot}4 \times 10^{-19}$ J, (iii) the stopping potential for these electrons.

11. Write down the Einstein photoelectron equation.
In an experiment to verify the equation, the following results for f, the frequency of the incident light, and V_s, the stopping potential, were obtained:

f ($\times 10^{14}$ Hz)	6·0	7·0	8·0	9·0
V_s (volts)	0·65	1·00	1·44	1·80

(i) Draw a graph of V_s against f. (ii) Find the gradient of the best straight line you can draw through the four points. (iii) Using your gradient result, work out h from the equation h = gradient $\times e$, where e is $1{\cdot}6 \times 10^{-19}$ C.

12. Fig. 38.9 shows the results of photoelectric experiments. V is the potential difference between a collecting electrode and the metal from which photoelectrons are emitted towards the collector. V can be made positive or negative (repulsion of electrons) as shown. I is the current produced by those electrons reaching the collector. The light frequency used is the same for X, Y and Z.

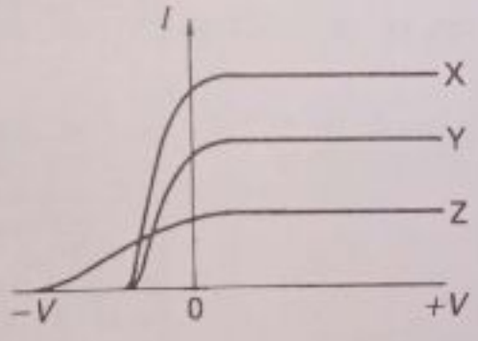


Fig. 38.9 Example

(i) Is the incident light beam on the metal greater in intensity for X or for Y? Explain your answer.
(ii) Does X, Y and Z use the same metal in their respective experiments? Explain your answer.
(iii) Is the work function of the metal used in the experiment for Z less than, equal to, or greater than the work function of the metal used in the experiment for X? Explain your answer.

13. In visible spectra, energy changes of the electron of the order 2 eV are obtained, using $E = hf$ for a wavelength 6×10^{-7} m, where E is the energy change. What energy change of the electron is needed to produce X-ray spectra, where the wavelengths are of the order 6×10^{-10} m say?

14. Choose the answer you think is most correct from A, B, C, D or E.
The Uncertainty Principle can be most usefully applied to measurements on
A the momentum of a train at a particular position
B the energy of a satellite at a particular time in space
C the energy of an electron in a cathode-ray tube at a particular time
D the momentum of a football as it just reaches the goal position
E the momentum of an electron at a particular velocity

Scanned with CamScanner

Answers to Exercises

Exercise 2 (page 33)

1. 100 m
2. $262\frac{1}{2}$ m, 25 m/s
3. 10 m/s^2, 20 m
4. 2 s, 4 s
5. (a) 3 m/s^2 (b) 300 m
6. (a) 2 s (b) 30 m
8. 1275 m, 3 m/s^2
9. (a) No (b) Yes
10. (a) 2 Hz (b) (i) decreased (ii) same (iii) same
12. 10 m/s or 36 km/h
13. 6 km
15. 3 s
16. 10·3 m/s
17. (a) 45 m (b) 30 m/s
18. 60° to bank, 23 s
19. 200 m/s
20. 267 m, 80 s
21. 5·4 s, 39·6 m, 54 m

22. C **23.** D **24.** B **25.** E **26.** E. **27.** A **28.** D **29.** C

Exercise 3 (page 61)

1. (a) (i) force (ii) energy (iii) power (b) force
3. 1000 N
4. 0·6 m/s^2, 67·5 m
5. 480 J
6. 1440 J
7. 2500 J, 2500 J
8. 7500 W, 500 N
9. 1200 J, 6×10^4 N
10. m/s^2, N; energy (work), power, mass, momentum
11. (a) 2 N s (b) 4 J
12. (a) 1·4 N s (b) 1 J
13. 30 W
15. 72 J
16. (a) 5 s (b) 700 m (c) 65 s; 3000 N
18. 160 N, $1\frac{1}{15}$ or 1·1 m/s^2
19. (ii) 27 m
20. (a) 10 kg m/s (b) 4 kg m/s (d) 25 J (e) A: 4 J, B: 6 J (f) 15 J
21. (a) 13 300 N s (b) 89 000 J, 333 N
22. (a) 3·9 s (b) 38·7 m/s (c) 10 000 N
24. 2·94 N
25. 2·4 kg m/s, 2·4 N, 71·0 N

26. B **27.** E **28.** E **29.** D **30.** (i) A, E (ii) B, D (iii) C (iv) A, E

Exercise 4 (page 78)

1. (a) 10 N m, 40 N m (b) 0·6 m (c) (i) 200 N m (ii) 5 R N m (iii) 40 N
2. (a) N (b) 1200 N
4. (a) 45 g (b) 0·75 N
5. 0·625 N, 0·875 N
8. 4 kg
9. 90 g
10. 50 N, 35 J
11. 2·9 cm from end of shorter rod
12. 100 cm from A, 60 N
14. 43 cm from A, 30 N
15. 4800 km
16. (a) 72° (b) 63°

18. D **19.** C **20.** D

Exercise 5 (page 97)

1. (a) 8 (b) 10 (c) 2000 J, 1600 J; 80% (d) (i) 25 N (ii) 50 N
2. (a) 4 (b) 2·4 (c) 60%
3. (b) 1000 J (c) 0·48 (d) (i) 4 (ii) 125 N (iii) 1250 J
4. (a) 2·5 m (b) 125 J (c) 25 J
5. 80%
6. 1·25 N m, 5 N
7. (a) 62·5% (b) 600 J
8. 70, 176, 40%
9. (a) $187\frac{1}{2}$ N (b) 1125 J
10. 754 N
11. 12 N
12. 50 N, 37°N of E, 3 m/s^2
13. 235 N
14. 5·2 N, 10·4 N
15. B, D, E

16. D. **17.** E **18.** D

Exercise 6 (page 116)

1. 6500 g, 20 cm^3
2. 4250 g
3. 7·5 cm
4. 1·07
6. 60%
7. 5 cm^3
8. 250 g, 30 cm^3, 292 g
9. (a) 2·0 N, 200 g, 200 cm^3 (b) 100 cm^3, 0·5 g/cm^3 (d) 108 kg
10. (b) 212 N
11. (a) 510 cm^3 (b) 340 cm^3 (c) 1·53 N
12. 0·45 g/cm^3

Scanned with CamScanner

13. (a) 150 cm^3 (b) 4 g/cm^3 (c) 4
14. (a) 150 g (b) 1·42 N
15. (b) metal: 8·3 g/cm^3, oil: 0·67 g/cm^3
16. 20 600 kg
17. (a) 120 kg (b) 0·07 kg/m^3
18. 1 cm^2
19. (a) $1{\cdot}33 \times 10^3$ N/m^2 (b) 0·67 g/cm^3; 10 000 N, $13\frac{1}{3}$ cm (c) 800 g (d) 896 g
20. 12·5 cm^3, 0·8 g/cm^3
21. 800 kg/m^3
22. 50 cm^3
23. D **24.** C

Exercise 7 (page 138)

1. (a) 50 N/m^2 (b) 20 N (c) 100 000 N/m^2 (d) 10 litres
2. 6×10^5 N/m^2
3. each level rises 2 cm
4. 10 cm
5. 3200 N/m^2, 16 000 N/m^2
6. 1000 m
7. (a) 12 740 N/m^2
8. 16 cm
10. (a) 12 cm mercury (b) 6·3 cm^3
11. 23 cm
12. (a) 9000 N (b) $1\frac{2}{3}$ cm
13. 0·2 N/mm^2
14. 738 mm
15. D **16.** B

Exercise 8 (page 164)

1. (a) 3 N (b) 2 N (c) D **8.** 8×10^{-9} m **13.** 0·5 N/cm **15.** 5×10^{-8} kg
17. D **18.** B **19.** C **20.** E

Exercise 9 (page 173)

1. (a) 25°C (b) 88 **4.** 20°C **8.** (a) 250°C

Exercise 10 (page 186)

3. $1{\cdot}2 \times 10^{-5}$/K
5. 48 cm
6. (b) 0·062 cm
7. (a) 0°C, 4°C
9. 0·2 cm
10. 389·3 cm
11. C **12.** A **13.** D **14.** C **15.** A

Exercise 11 (page 199)

1. (a) 120 cm^3 (b) $3{\cdot}6 \times 10^5$ N/m^2 (c) 4·141, 1·21 g/l
2. 636°C
3. 125 cm^3
4. 75 cm^3
5. −73°C
7. fraction = $\frac{19}{20}$
9. (a) 40°C (b) −40°C
10. (a) 71 cm Hg (b) 0·0038/K
11. 40·7 g/l
12. C **13.** D **14.** A **15.** D

Exercise 12 (page 212)

1. 20 000 J
2. 420 g or 0·42 kg
3. (a) 30 000 J (b) 0·5 J/gK (500 J/kg K)
4. 12·6 kJ/g
5. $22{\cdot}4 \times 10^6$ J/m^3
6. 105 J/K
7. (a) 10 500 J/min (b) 262·5 J/K
8. 2·4°C
9. 80 000 J
10. 0·1°C
13. 72°C
14. 0·23°C
15. 206 500 J, 3200 J
16. C **17.** D **18.** C **19.** B

Exercise 13 (page 232)

1. (a) 660 000 J (b) 1320 J
2. 320 000 J/kg
3. (a) 4×10^5 J (b) 2×10^5 J
5. 2040 J
6. 20 g
7. 4·4 g
8. 26 kW
9. 27·3 J/g (27 300 J/kg)
11. 1178 J, 1336 J; 4·9 A
18. No (2 h 55 min)
19. B **20.** C **21.** D **22.** D **23.** A

Exercise 14 (page 248)

6. (a) 1312·5 J/min (b) 328 J/g
13. C **14.** B **15.** C **16.** D

Scanned with CamScanner

Exercise 15 (page 257)

5. 1·125 m
6. 50 cm
7. (c) 11·4 cm
9. 20 mm, 20 mm
10. D **11.** C **12.** A

Exercise 16 (page 266)

3. 2·9°
8. 180 cm
9. 100°, 80°, 100°
10. C **11.** B **12.** C **13.** A **14.** D

Exercise 17 (page 278)

3. (a) 26·7 cm (b) 3
5. $18\frac{2}{3}$ cm, 75 cm
6. 2 cm
7. 30 cm behind mirror, 6 cm
8. 15 cm from mirror
9. $13\frac{1}{3}$ cm from mirror, $6\frac{2}{3}$ cm to mirror
11. 333·3 mm from mirror

Exercise 18 (page 297)

5. 1·8 m
10. $1{\cdot}78 \times 10^5$ cm^2
12. 4/3
14. 48·6°
15. B **16.** D **17.** B **18.** D

Exercise 19 (page 310)

1. (b) 30 cm, 2 (c) (i) 20 cm
2. (a) virtual (b) 7·5 cm
3. (b) 20 cm, 60 cm
4. 15 cm
6. (a) $6\frac{2}{3}$ cm (b) $13\frac{1}{3}$ cm
7. 12 cm, 2 cm high, virtual
8. (b) 2, virtual
9. (b) 7·5 cm from lens, virtual
10. 1·4 million km
11. 12 cm from lens, 1·6 cm
12. A: +20 cm, B: +10 cm
13. 10 cm; converging; (b)
14. A **15.** D **16.** C **17.** A

Exercise 20 (page 326)

4. converging (a) 28·8 cm
(b) 7·5 m at least
5. (a) 0·5 (b) 5 (c) 50 cm
7. 0·25 cm nearer film
9. 5
12. $3\frac{1}{8}$ m from lens, $f = 3$ m
13. 6·25 cm
14. D **15.** E **16.** C

Exercise 22 (page 364)

1. 300 m, 3×10^5 km/s
2. (a) 20 cm (b) 2 cm (c) 40 Hz
3. (a) B (b) A (c) D
5. 1·55
7. (a) 336 m/s (b) 480 Hz
10. (a) A violet, B red
(b) C ultraviolet, D infrared
(c) E gamma rays, F X-rays (d) E, F; D
11. (a) 5·5 m (b) $1\frac{1}{15}$ s (c) 4λ
12. 1650 Hz
13. 22°57′, 51°16′
14. (b) 21°6′ (c) $1{\cdot}2 \times 10^{-4}$ cm
16. E **17.** B **18.** C **19.** C

Exercise 23 (page 385)

4. (a) 82·5 cm (b) 41·25 cm
9. 384 Hz
10. (a) 450 Hz (b) 346 Hz
12. D **13.** B **14.** A **15.** C

Exercise 24 (page 405)

1. (a) 0·4 A, 5 V (b) 4 Ω; 2 A, 1 A, 3 A
(c) 2 Ω (d) 2:1
2. 750 Ω
3. $x = 2$A, $r = 6{\cdot}4\,\Omega$
4. 0·5 A (a) 0·56 A (b) 0·093 A
5. (a) $1\frac{1}{6}$, $\frac{1}{2}$, $\frac{2}{3}$ A (b) 2 V
7. (a) 0·6 V (b) 1 A (c) 2·5 A
8. (a) 13 Ω (b) 0·25 V; 0·60 V
9. 600 Ω; 0·002 A, 0·375 V
10. $6\frac{1}{8}\,\Omega$; 6 A, 24 V, $1\frac{5}{7}\,\Omega$
11. (a) 0·00267 A, 1·33 V (b) 0·004 A, 1 V
12. 4 Ω
13. 1·19 m
14. 0·84 Ω
15. 0·20 Ω, 65·40 m
16. 1:2
17. C **18.** E **19.** C **20.** B **21.** C

Scanned with CamScanner

Exercise 25 (page 422)

1. (a) 2 V (b) 1 V (c) 1·5 V
5. (a) 0·3 A, 2·4 V (b) 2 Ω (c) (i) 0·2 A (ii) 3 V (d) 0·5 A
6. 1·2 V
7. 0·2 A
8. 60 Ω, 0·40 A
9. (a) 0·2 A (b) 1·1 V
10. 0·66 A, 3·3 V
11. $R = 4·3\,\Omega$, 0·5 Ω
12. (a) $2E$, $2r$, (b) E, $r/2$ (c) 1·5 V (d) 4Ω
13. (a) 1 A (b) 2 A (c) 2 V; zero
14. 4 Ω

Exercise 26 (page 436)

1. (a) 4 Ω (b) 2160 J
2. (a) 0·4 A (b) 10 p
3. 9·4 A, 36 p
5. 7·8 min
6. 2·8 Ω
7. (a) (ii) 60 J (iii) 5 A (b) 800 cm
8. (a) 62·5 Ω (b) $3\frac{1}{2}$ p
10. (a) $\frac{1}{2}$ (b) $\frac{2}{1}$
12. (a) 12·5 A (b) 2800 s
13. 4/1
14. 2·8 kW, 11·7 A
15. (a) 2550 J/K (b) 0·9 p
16. 16 A, 2·4 A
17. (a) 9·6 V (b) 1·4 Ω
18. 19·2 Ω (a) 12 A (b) 115·2 W
19. E **20.** C **21.** B

Exercise 27 (page 452)

13. E **14.** B **15.** C

Exercise 28 (page 463)

13. D **14.** E **15.** A

Exercise 29 (page 475)

2. (b) shunt, 5/499 Ω
3. (d) 2 Ω (e) 1250 W (f) 50 W
4. (b) $1\frac{1}{19}$ Ω (i) 0·32 A (ii) 61·4 J
5. (i) 2 mA (ii) 4 mA (iii) 1·998 A (iv) 2 A
8. (a) 80 mV
9. 8·4 V, 9·3 V
11. 95 Ω series
14. D **15.** A **16.** C **17.** C

Exercise 30 (page 494)

1. (b) (i) 12 V (ii) 4 A (iii) 43·2 W (iv) 0
5. (b) 1250 W; decreased
9. (a) 91% (b) 7·9 p
13. (a) thicker (b) 160 (c) eddy current losses (d) burn out (e) increase
14. 400/1, 40 A
15. B **16.** E **17.** A

Exercise 31 (page 507)

2. 10 h, 4 p
3. 12 000 J
4. 16 p
5. 3·4 g
6. 1·43 A
7. 35·9 min
8. 1·12 litre
9. $1·118 \times 10^{-6}$ kg/C
10. 0·2 A low
11. C **12.** E **13.** C

Exercise 32 (page 529)

15. B **16** E

Exercise 33 (page 554)

9. E **10.** D **11.** A **12.** B **13.** B

Exercise 34 (page 579)

1. n: 0, m; H: $+e$, m; α: $+2e$, $4m$
2. p: 3; e: 3; n: 4
4. (b) New atomic no. 16
5. β-particles
8. 16 per second
10. isotopes: H, deuterium
11. 8 s
12. New atoms: at. no. 84, mass no. 216; then at. no. 82, mass no. 212
13. 50/min
14. (a) $\frac{1}{2}$ (b) $\frac{1}{4}$
15. C **16.** D **17.** B
18. D **19.** B, E **20.** A

Exercise 35A (page 584)

1. C
2. (i) 2 s (ii) 20 m (iii) 139 m
3. (i) 1500 m (ii) 34·6 s
4. 40 m/s, 139 m
5. (a) 45° (b) 250 m
6. (i) 73 m/s (ii) 12·6 s

Scanned with CamScanner

Exercise 35B (page 594)

1. D 2. D 3. E
4. (i) 2π (6·3) (ii) 1·6 m/s^2 (iii) 0·25 m/s (iv) 0·22 m/s
5. (i) π (3·1) (ii) 0·1 m/s^2 (iii) $4{\cdot}9 \times 10^{-5}$ J (iv) $4{\cdot}1 \times 10^{-5}$ J
6. 200 N/m (i) 0·31 s (ii) 0·0016 J (iii) 0·073 J
7. (i) 0·375 m (ii) 1·2 s
9. 2·4 s. (i) 0·33 m/s^2 (ii) 0·12 m/s (iii) 0·0017 J
10. 0·2 m (i) 0·9 s (ii) 0·625 J (iii) 0·021 J

Exercise 36A (page 599)

1. E 2. D
3. (i) $3{\cdot}4 \times 10^{-11}$ N (ii) $1{\cdot}3 \times 10^{-10}$ N
4. 20 kg
5. 49 N
6. $2{\cdot}7 \times 10^{-51}$ N
7. $3{\cdot}6 \times 10^{22}$ N
8. $-6{\cdot}3 \times 10^{10}$ J, $-3{\cdot}1 \times 10^{10}$ J
9. $-6{\cdot}9 \times 10^{-58}$ J
10. 250 N, 0·25 m/s^2
11. $-2{\cdot}9 \times 10^{6}$ J, $2{\cdot}4 \times 10^{3}$ m/s

Exercise 36B (page 606)

1. C
2. D
3. $7{\cdot}2 \times 10^{-3}$ N, attractive
4. 2×10^{-9} C
5. 1×10^{-7} C
6. (i) 0 (ii) $3{\cdot}2 \times 10^{6}$ N/C
7. $+3{\cdot}0 \times 10^{4}$ V, $-3{\cdot}0 \times 10^{4}$ V
8. $-3{\cdot}3 \times 10^{-7}$ C
9. 1500 V
10. 8×10^{-17} J
11. 2×10^{4} N/C
12. $3{\cdot}2 \times 10^{-17}$ J, 8×10^{6} m/s

Exercise 36C (page 611)

1. C 2. E
3. (a) 3 μF (b) $1{\cdot}5 \times 10^{-4}$ J
4. (a) 4×10^{-4} J (b) 20 V
5. 5 μF. (a) 40 μC, 60 μC (b) 4×10^{-4} J, 6×10^{-4} J
6. (a) 4 μF (b) 120 μC (c) 20 V, 10 V (d) $1{\cdot}2 \times 10^{-3}$ J, 6×10^{-4} J
7. (a) 2 μF (b) 6×10^{-4} J, 6×10^{-4} J, $2{\cdot}4 \times 10^{-3}$ J
8. (a) 15 V (b) $4{\cdot}5 \times 10^{-4}$ J

Exercise 36D (page 615)

1. C 2. D 3. B
4. (a) $3{\cdot}2 \times 10^{-16}$ N
5. (a) $3{\cdot}2 \times 10^{-19}$ C
6. (a) 0·11 m
7. (a) $2{\cdot}1 \times 10^{-7}$ kg/C (b) $3{\cdot}4 \times 10^{-26}$ kg

Exercise 37 (page 628)

1. D 2. A
3. 0·35 A
4. (i) 0·60 A (ii) 0·85 A
5. (i) 3200 Ω (ii) 320 V (iii) 450 V; increases
6. (i) $2{\cdot}7 \times 10^{-3}$ A (ii) 0·24 A
7. 0·32 H
8. (i) 1300 Ω (ii) 0·08 A (iii) 60 V
9. (i) 1880 Ω (ii) 6·4 mA (iii) 0·2 V; 0·041 W
10. (a) 99 Ω, 15 Ω (b) 130 Ω (c) 510 Hz, 100 Ω

Exercise 38A (page 632)

1. D 2. B
3. (i) 6×10^{14} Hz (ii) 4×10^{-19} J (iii) 2·5 eV
4. (i) $2{\cdot}6 \times 10^{-19}$ J (ii) $3{\cdot}9 \times 10^{14}$ Hz (iii) $7{\cdot}7 \times 10^{-7}$ m
5. (a) 1·8 eV (b) −5·5 eV
6. $4{\cdot}9 \times 10^{-7}$ m
7. −13·6 eV
8. (a) −13·6 eV (b) V

Exercise 38B (page 638)

1. A 2. D
3. (a) $2{\cdot}8 \times 10^{-19}$ J (b) $9{\cdot}9 \times 10^{-19}$ J (c) $6{\cdot}6 \times 10^{-16}$ J
4. $4{\cdot}1 \times 10^{-7}$ m
5. (i) $6{\cdot}5 \times 10^{14}$ Hz (ii) $4{\cdot}6 \times 10^{-7}$ m
6. $7{\cdot}5 \times 10^{14}$ Hz, $2{\cdot}7 \times 10^{-19}$ J
7. $2{\cdot}5 \times 10^{-19}$ J
8. 1·5 V
9. (i) $4{\cdot}95 \times 10^{-19}$ J (ii) 1·1 eV
10. (i) $1{\cdot}56 \times 10^{-19}$ (ii) $7{\cdot}5 \times 10^{14}$ Hz (iii) 2·1 V
11. 6×10^{-34} J s (about)
12. (i) X (ii) No—only X and Y (iii) less
13. 2000 eV
14. C

Scanned with CamScanner

Multiple-choice Questions

These two model papers each have forty questions covering all branches of the subject. Read the directions in each section carefully before answering the questions.

Paper 1

(Time: $1\frac{1}{4}$ hours) (Answers on page 649)

Section I

Directions The group of questions below consists of five lettered headings followed by a list of numbered questions. For each numbered question select the heading which is most closely related to it. Each heading may be used once, more than once, or not at all.

Questions 1–4
The graphs A to E in Fig. 1 are velocity–time graphs. Which is the most suitable graph for the following cases:

1. A ball thrown vertically upwards and returning to the ground.
2. An ice puck pushed forward on a smooth horizontal surface.
3. An object released from an aeroplane at a great height.
4. A lift moving between two floors.

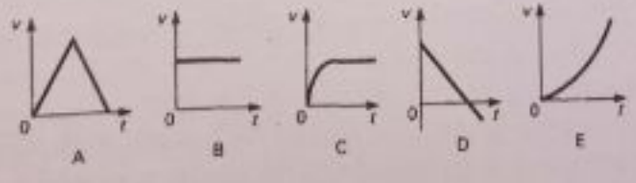


Fig. 1

Questions 5–8
Five electrical instruments or machines are listed below:
A carbon microphone, B telephone receiver, C moving-coil galvanometer, D simple motor, E dynamo.
Which of these

5. uses the attractive force on soft iron
6. works by electromagnetic induction
7. uses a commutator
8. does not rely on the magnetic effect of current for its working.

Scanned with CamScanner

Section II

Directions Each of the questions or incomplete statements in this section is followed by five suggested answers. Select the *best* answer in each case.

Questions 9–20

9. When released from a height a ball falls 5 m in 1 sec. In 4 sec from release it falls

A 400 m, B 100 m, C 80 m, D 40 m, E 10 m.

10. An object of mass 2 kg falls from a height of 20 m above the ground. If $g = 10\,m/s^2$, the loss of potential energy just before the mass strikes the ground is

A 20 J, B 40 J, C 200 J, D 400 J, E 800 J.

11. A rectangular solid box has dimensions 2 m by 1 m by 0·5 m and weighs 100 N. When the box is placed on flat ground the maximum pressure exerted is

A 100 N, B 200 N/m^2, C 100 N/m^2, D 50 N/m^2, E 200 N.

12. A rough measure of the size of a molecule can be obtained by

A measuring the height to which water rises in a narrow capillary tube.
B finding the speed with which bromine vapour spreads in air.
C observing Brownian motion of smoke particles.
D measuring the area of the circle into which a small oil drop spreads on water.
E measuring the friction force between the molecules of a solid.

13. A 50 W heating coil is used for 2 min to heat a metal block of 0·5 kg and specific heat capacity 1000 J/kg K. The temperature rise is

A 6°C, B 12°C, C 15°C, D 24°C, E 25°C.

14. A closed flask contains air at 1 atmosphere and 27°C. When the flask is heated to 127°C, the pressure becomes

A 4·7 atm, B 4 atm, C $3\frac{1}{3}$ atm, D $1\frac{1}{3}$ atm, E $1\frac{1}{6}$ atm.

15. Colours are produced when white light passes through a glass prism because

A the light waves interfere, B the glass colours the light, C different colours travel at different speeds in glass, D the different colours are filtered, E diffraction of light occurs.

16. Light waves differ from sound waves because

A light is an electromagnetic wave.
B light waves are long and sound waves are short.
C interference is obtained with light waves but not with sound waves.
D the speed of light is independent of the medium in which it travels.
E sound waves do not travel in water.

17. A wire X is half the diameter and half the length of a wire Y of similar material. The resistance of X to that of Y is

A 8:1, B 4:1, C 2:1, D 1:1, E 1:4.

18. A 120 W, 240 V lamp has a resistance when working of

A 480 Ω, B 120 Ω, C 60 Ω, D 20 Ω, E 2 Ω.

19. When a downward current flows in a straight vertical conductor, the direction of its magnetic field at a point due north of the wire is

A upward, B north, C south, D west, E east.

20. The mass of an atom depends on the number of

A neutrons, B electrons, C protons, D neutrons plus protons, E α-particles.

Scanned with CamScanner

Section III

Directions For each of the questions below, *one* or *more* of the responses given are correct. Decide which of the responses is (are) correct. Then choose

A if 1 and 2 and 3 are correct.
B if 1 and 2 only are correct.
C if 2 and 3 only are correct.
D if 1 only is correct.
E if 3 only is correct.

Summary of directions

A	B	C	D	E
1, 2, 3, only	1, 2 only	2, 3 only	1 only	3 only

Questions 21–32

21. Mercury, and not water, is used in a barometer because
1. Mercury has a high density.
2. Mercury has a low vapour pressure.
3. Water is a poor conductor.

22. Fig. 2 shows five successive positions of two balls falling under gravity, one dropping vertically and the other simultaneously projected forward. The diagram shows

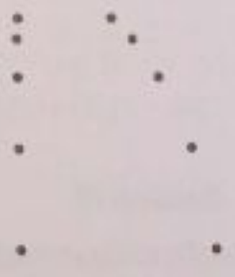


Fig. 2

1. The vertical acceleration is non-uniform.
2. The velocity in a horizontal direction is constant.
3. The vertical motion is that of constant acceleration.

23. In any machine
1. The velocity ratio is usually greater than the mechanical advantage.
2. The efficiency = mechanical advantage/velocity ratio.
3. The velocity ratio is independent of friction.

24. The following phenomena show that molecules are in rapid motion
1. A small oil drop spreads into a large circular patch on water.
2. Particles of smoke are seen to move haphazardly in air.
3. Bromine gas spreads quickly into a vacuum.

25. Wet washing can dry quickly when hung on a line outside the house
1. if the barometric pressure is low.
2. because maximum surface area is exposed to the air.
3. on a windy day.

26. A concave-type mirror is used for
1. a shaving mirror.
2. an astronomical large telescope.
3. a dentist's mirror.

Scanned with CamScanner

27. The accommodation of the eye is
1. due to a change in the size of the pupil.
2. produced by movement of the retina.
3. produced by ciliary muscles.

28. In connection with waves,
1. X-rays and light rays are both electromagnetic waves.
2. sound waves and X-rays have similar wavelengths.
3. sound waves cannot be diffracted.

29. In a moving coil galvanometer
1. the turning effect on the coil is proportional to the current squared.
2. the radial magnetic field produces a uniform scale.
3. the turning effect on the coil is proportional to the current.

30. When an electric current flows through a copper voltameter with copper electrodes,
1. the concentration of the solution is constant.
2. the anode loses copper.
3. the copper deposited on the cathode is due to copper ions.

31. When a vertical electron beam passes through a uniform horizontal magnetic field
1. the beam is deflected into a circular path.
2. a horizontal force acts on the beam.
3. a vertical force acts on the beam.

32. When a uranium nucleus $^{235}_{92}U$ emits an α-particle $^{4}_{2}He$, the nucleus remaining has
1. an atomic number of 90.
2. an atomic mass of 231.
3. an atomic mass of 235.

Section IV

Directions These groups of questions deal with practical situations. Each situation is followed by questions with five suggested answers. Select the *best* answer for each question.

Questions 33–36

A ball of mass 0·5 kg, released from a height of 20 m above the ground, rebounds to a height of 5 m. Assuming $g = 10\,m/s^2$ and the ball is in contact with the ground for 0·1 s.

33. the velocity just before striking the ground is
A 40 m/s, B 20 m/s, C 15 m/s, D 10 m/s, E 5 m/s.

34. the force on the ball as it falls is
A 20 N, B 10 N, C 7·5 N, D 5 N, E 0·5 N.

35. the force acting on the ball on impact with the ground is
A 250 N, B 200 N, C 150 N, D 20 N, E 10 N.

36. the loss of energy due to impact is
A 5 J, B 10 J, C 25 J, D 50 J, E 75 J.

Questions 37–40

In Fig. 3, a diode valve X is in parallel with a resistor Y. A voltage supply Z, with terminals H and K, is connected across the arrangement. Under suitable conditions, X and Y each conducts a *medium* current when a battery or an a.c.

Scanned with CamScanner

voltage is applied. If the two medium currents flow in the same direction in one part of the circuit, a *high* current is obtained there; if they flow in the opposite direction, *zero* current is obtained.

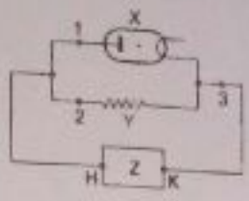


Fig. 3

With the voltage supply that given in questions 37–40, write down the magnitudes of the currents each time at the places 1, 2 and 3 in the circuit of Fig. 3. Give your answer as *high* or *medium* or *zero*.

37. A battery connected with its positive pole to H and its negative pole to K.
38. A battery connected with its negative pole to H and its positive pole to K.
39. An a.c. voltage connected between H and K.
40. A battery, in series with an a.c. voltage of lower maximum p.d. than the p.d. of the battery, connected between H and K; the negative pole of the battery is on the same side as H and the positive pole is on the same side as K.

Answers to multiple-choice paper 1

1. D **2.** B **3.** C **4.** A **5.** B **6.** E **7.** D **8.** A **9.** C **10.** D
11. B **12.** D **13.** B **14.** D **15.** C **16.** A **17.** C **18.** A **19.** E **20.** D
21. B **22.** C **23.** A **24.** C **25.** C **26.** A **27.** E **28.** D **29.** C **30.** A
31. D **32.** B **33.** B **34.** D **35.** C **36.** E **37.** (1)M (2)M (3)H
38. (1)0 (2)M (3)M **39.** (1)M (2)M (3)H **40.** (1)0 (2)M (3)M

Paper 2

(Time: $1\frac{1}{4}$ hours) (Answers on page 654)

Section I

Directions The group of questions below consists of five lettered headings followed by a list of numbered questions. For each numbered question select the heading which is most closely related to it. Each heading may be used once, more than once, or not at all.

Questions 1–4
Fig. 4 shows five graphs of quantities in heat, A, B, C, D and E.

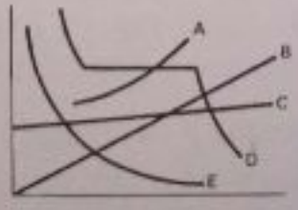


Fig. 4

Which of these best represents the variation of
1. the volume of a fixed mass of gas with temperature in °C, the pressure being constant.

Scanned with CamScanner

2. the volume of a fixed mass of gas with pressure, the temperature being constant.
3. the boiling point of a liquid with external pressure.
4. the temperature of a pure liquid with time before and after freezing.

Questions 5–8
The following are five electrical conductors or components:
A copper wire, B manganin wire, C nichrome wire, D diode valve, E carbon lamp filament.
Which of these
5. has a fairly constant resistance as its temperature rises and does not oxidise at high temperatures.
6. decreases in resistance as its temperature rises.
7. increases in resistance as its temperature rises.
8. has a high or low resistance depending on the direction of the potential difference across it.

Section II

Directions Each of the questions or incomplete statements in this section is followed by five suggested answers. Select the *best* answer in each case.

Questions 9–20
9. The acceleration of a moving object may be found from
A the area under its velocity–time graph.
B the slope of the velocity–time graph.
C the area under its distance–time graph.
D the slope of the distance–time graph.
E the slope at the peak of its distance–time graph.
10. If the solid cube X is just tilted about O by the force F in Fig. 5, the weight W of the cube is
A 1 N, B 5 N, C 10 N, D 15 N, E 20 N.

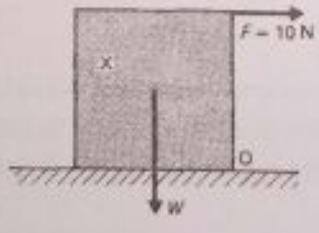


Fig. 5

11. A force of 10 N, acting continuously, increases the kinetic energy of an object from 20 J to 60 J. The distance moved by the object is then
A 400 m, B 200 m, C 60 m, D 20 m, E 4 m.
12. The specific latent heat of steam is 2300 J/g and the specific heat capacity of water is 4·2 J/g K. When 2 g of steam at 100°C condenses to water at 40°C, the heat given up is
A 5104 J, B 4936 J, C 4636 J, D 4684 J, E 4600 J.
13. When a gas is compressed at constant temperature, the gas molecules
A move faster so that the pressure is increased.
B move at the same speed so that the pressure is unchanged.
C gain kinetic energy.
D increase slightly in size.

Scanned with CamScanner

E make more impacts per second on the walls of the container.

14. If an object is 30 cm from a concave mirror of focal length 15 cm, the image will be

A upright, B the same size as the object, C diminished,
D virtual, E twice the object size.

15. When total internal reflection is just about to occur at an air-water boundary as the incident angle is increased from zero, the refracted ray then

A travels in the water at 90° to the normal.
B travels in air along the normal.
C makes the critical angle with the normal in air.
D travels in the air at 90° to the normal.
E makes an angle of 90° with the incident ray.

16. A vibrating string has a tension of 40 N and produces a note of 200 Hz when plucked in the middle. When the length of string is unaltered and the tension is increased to 160 N, the frequency becomes

A 1600 Hz, B 800 Hz, C 400 Hz, D 200 Hz, E 25 Hz.

17. A parallel arrangement of 3 and 6 ohms is placed in series with an 8 ohm resistor. If a p.d. of 30 V is connected across the whole circuit, the current in the 3 ohm resistor is

A 15 A, B 10 A, C 3 A, D 2 A, E 1 A.

18. In an iron-cored transformer, the number of secondary turns is 100 and of primary turns is 2000. When 240 V a.c. is connected from the mains, the output is

A 240 V, B 12 V, C 100 V, D 20 V, E 4800 V.

19. A horizontal electron beam passes between two parallel horizontal plates X and Y with X above Y. When a high tension battery is connected with its negative pole to X and its positive pole to Y, the beam is deflected

A upwards, B sideways, C downwards, D upwards at 45° to the horizontal, E downwards at 45° to the horizontal.

20. A nucleus of an element X which is $^{202}_{84}X$ emits an α-particle and then a β-particle. The final nucleus formed has an atomic number of

A 83, B 82, C 198, D 200, E 80.

Section III

Directions For each of the questions below, one or more of the responses given are correct. Decide which of the responses is (are) correct. Then choose

A if 1, 2 and 3 are correct.
B if 1 and 2 only are correct.
C if 2 and 3 only are correct.
D if 1 only is correct.
E if 3 only is correct.

Directions summarised

A	B	C	D	E
1, 2, 3	1, 2 only	2, 3 only	1 only	3 only

Questions 21–32

21. A vertical string, suspended from a fixed point, has a small mass swinging to and fro at the other end.

1. The potential energy of the mass is a minimum in the middle of the swing.
2. Its kinetic energy is a maximum in the middle of the swing.
3. The sum of the potential and kinetic energies is constant throughout the swing.

Scanned with CamScanner

22. Archimedes' Principle can be used to support or deduce the following statements:
1. A vacuum is needed at the top of a mercury barometer.
2. Hydrometers have narrow stems to make them more sensitive.
3. The relative density of a solid is the ratio of the weight in air to the apparent loss in weight in water.

23. The pressure exerted at a place X below a liquid
1. does not depend on the magnitude of the area placed at X.
2. depends on the liquid density.
3. depends on the direction of the force at X.

24. A rough measure of the size of a molecule can be found by
1. measuring the radius of the circle into which a known small volume of oil spreads on water.
2. measuring the mass of hydrogen in a 1 litre container.
3. using a microscope to measure the diameter of sand grains.

25. When a mass of liquid is heated in a flask
1. its density normally decreases.
2. the liquid level is usually seen to fall at first.
3. the kinetic energy of its molecules increases.

26. When an object is placed 20 cm from a converging lens of focal length 15 cm,
1. a virtual image is obtained.
2. the magnification is 3.
3. an inverted image is formed.

27. When water waves travel to a shallower region,
1. the frequency changes.
2. the wavelength is unchanged.
3. the velocity decreases.

28. A 60 W and a 120 W domestic lamp are used on the 240 V mains.
1. The filament of the 60 W lamp has a higher resistance than that of the 120 W lamp.
2. The total current through the mains is 1 ampere.
3. The p.d. across each lamp is 120 V.

29. A coil of wire, joined to a galvanometer, is moved quickly away from a magnet pointing along its axis.
1. There is a change in the flux (lines of force) linking the coil.
2. An induced e.m.f. is obtained.
3. The magnitude of the induced current is increased if the coil and magnet are moved together.

30. A horizontal straight wire is held above and at right angles to a magnetic compass needle. When a current is passed into the wire,
1. the magnetic field due to the current is perpendicular to the needle.
2. the needle is not deflected.
3. the magnetic field due to the current is parallel to the needle.

31. In radioactivity,
1. α-particles have relatively a very small mass compared to an electron.
2. γ-rays are not very penetrating.
3. β-particles have relatively a very small mass compared to an atom.

32. In a diode valve
1. the current is carried by electrons.
2. the current is carried by negative charges.
3. the valve conducts when the anode is positive relative to the cathode.

Scanned with CamScanner

Section IV

Directions These groups of questions deal with a practical situation. Each situation is followed by questions with five suggested answers. Select the *best* answer for each question.

Questions 33–36

In Fig. 6, a hydraulic press P is used to raise a load of 10000 N. A force F of 25 N is applied at the end of a lever pivoted at O to just raise the load.

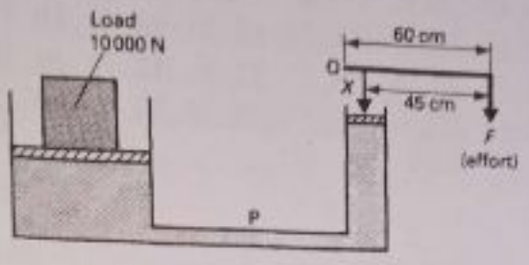


Fig. 6

33. The force X applied to the press is then
A 1500 N, B 1125 N, C 100 N, D $33\frac{1}{3}$ N, E $13\frac{1}{3}$ N.

34. If the load rises 0·1 cm when X moves down 20 cm, the work obtained is
A 20000 J, B 10000 J, C 2000 J, D 10 J, E 5 J.

35. The work done by the force X in moving the load 0·1 cm is
A 20 J, B 25 J, C 75 J, D 80 J, E 100 J.

36. Excluding the lever, the efficiency of the press is
A 90%, B 80%, C 75%, D 60%, E 50%.

Questions 37–40

In Fig. 7, RT is a conductor loosely suspended from pivots at X and Y. X and Y are joined to terminals at L and M.

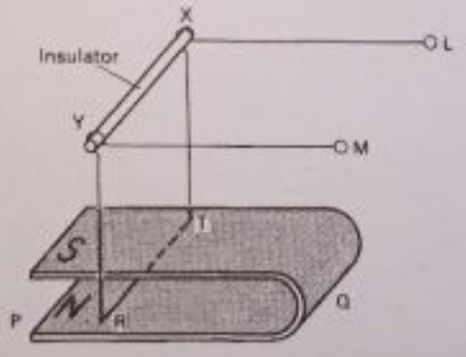


Fig. 7

37. When a battery is joined with its positive pole to L and its negative pole to M, then RT
A remains stationary, B moves upwards towards YX,
C moves towards Q, D moves towards P.

38. The magnet is moved to the right so that RT is between the poles N and S. When the battery is re-connected to L and M, the force on RT
A remains zero, B increases, C decreases, D reverses in direction.

39. The battery is now disconnected from L and M and a galvanometer is joined between the two terminals. If RT is now pushed towards Q, the galvanometer shows the induced current is

Scanned with CamScanner

A zero, B in the direction RT, C in the direction PQ,
D in the direction TR.

40. When RT is made to swing to-and-fro between P and Q, the induced current is
A alternating, B zero, C in the direction PQ, D in the direction QP.

Answers to multiple-choice paper 2

1. C	2. E	3. A	4. D	5. C	6. E	7. A	8. D	9. B	10. E
11. E	12. A	13. E	14. B	15. D	16. C	17. D	18. B	19. C	20. A
21. A	22. C	23. B	24. D	25. A	26. C	27. E	28. D	29. B	30. C
31. E	32. A	33. C	34. D	35. A	36. E	37. D	38. B	39. D	40. A

Scanned with CamScanner

Revision Questions

(Answers, page 659)

The following short questions cover basic points in the syllabus.

Where necessary, assume $g = 10\,m/s^2$ or $10\,N/kg$ and the density of water $= 1000\,kg/m^3$ or $1\,g/cm^3$.

1. A ball of mass 0·2 kg is thrown vertically upwards and reaches the top of its motion in 2 s. Calculate (a) its initial velocity, (b) the maximum height, (c) its initial momentum and kinetic energy.

2. An electrically-operated hoist raises a load of 3000 N through a vertical distance of 40 m, at a constant rate, in 100 s. Calculate the power output from the hoist. The hoist has an efficiency of 60 % and operates from a 440 V mains supply What current is taken from the mains? (O and C)

3. In Fig. 8, weights of 2 N and 4 N are placed at the ends of a light beam resting on supports at A and B. Calculate the reactions at A and B. What additional weight at C will *just* tilt the beam about B?

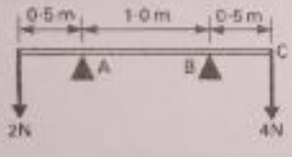


Fig. 8

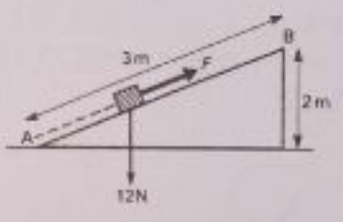


Fig. 9

4. In Fig. 9, a weight of 12 N is pulled steadily up a rough plane from the bottom A to the top B. The frictional force along the plane is 2 N. Calculate (a) the component of the 12 N weight down the plane, (b) the magnitude of the force F pulling the weight up the plane, (c) the work done in moving the weight from A to B, (d) the efficiency of the inclined plane as a machine.

5. A rectangular raft 4 m by 3 m stands 0·5 m above the water surface. Find the mass placed on the raft which would *just* submerge it.

6. An air bubble has a volume of $1\,cm^3$ at the bottom of a pool of water 10 m deep. The volume increases to $2\,cm^3$ when it reaches the surface. What height of water can the atmospheric pressure just support?

7. Give *one* piece of evidence which shows that forces of attraction exist between molecules and *one* piece of evidence which shows that forces of repulsion also exist. (O and C)

8. Using molecular theory, explain the following (a) gas pressure, (b) water spreads on a clean glass surface, (c) Brownian motion in a liquid.

9. An aluminium block of mass 0·5 kg is heated electrically for 10 min by a 12 W

Scanned with CamScanner

heater inside it. Assuming no heat loss, calculate the temperature rise of the block. (Specific heat capacity of aluminium = 1000 J/kg K.)

10. The gas pressure in a closed tank falls from $1{\cdot}50 \times 10^5$ N/m^2 to $1{\cdot}40 \times 10^5$ N/m^2 when the temperature changes from 27°C to t°C. Calculate t.

11. Explain the following: (a) a polystyrene packing round a metal instrument feels warm but the metal feels cool, (b) a glass flask is often heated on a wire gauze, (c) the double-walled glass vessel inside a vacuum (thermos) flask contains a vacuum and its internal walls are silvered.

12. By drawing ray diagrams in each case, show how each of the following images are formed by using a plane mirror or a curved mirror: (a) a real image of the same size as the object, (b) an upright image smaller than the object, (c) a virtual image of the same size as the object, (d) an upright magnified image of the object.

13. Draw a ray diagram showing how a converging lens is used as a magnifying glass.

A converging lens 4 cm from the object produces a magnified upright image five times the object height. Find the focal length of the lens.

14. Show in a diagram how a fairly pure spectrum of white light can be obtained using a prism and a lens. In your diagram label the colours obtained.

15. Flint glass has a refractive index of 1·6. Calculate the critical angle with air for this glass. Draw a ray diagram to illustrate the meaning of 'critical angle'.

16. Why are radio waves similar to light waves but not to sound waves? A radio station has a wavelength of 1500 metres. What is its frequency in kilohertz? (velocity of electromagnetic waves in air = 300 000 km/s).

17. Fig. 10 shows plane waves refracted from a medium A to medium B with the relative dimensions shown. (a) In which medium is the wave speed greater? Give a reason for your answer. (b) If the wave speed in A is 30 m/s, calculate the wave speed in B.

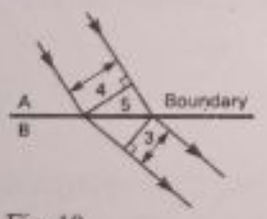


Fig. 10

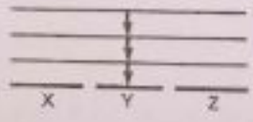


Fig. 11

18. In Fig. 11, water waves are incident on two narrow openings between the barriers X, Y and Y, Z. (a) What is the appearance of the water surface beyond XYZ? Explain your answer briefly. (b) If the barrier Y was removed, leaving X and Z, draw a sketch of the wave appearance beyond XZ and explain briefly your sketch.

19. A sonometer wire of length 40 cm between two bridges produces a note of 512 Hz when plucked in the middle. (a) How would you set the wire into resonance using a tuning fork and how would you demonstrate the resonance? (b) What length of wire would produce a note of 256 Hz with the same tension?

20. You are provided with a long coiled metal spring, the coils being of constant diameter. Describe how you would use it to demonstrate (a) transverse waves, (b) longitudinal waves. Explain how the motions of the coils differ in the two cases. (L)

21. Explain why (a) an unmagnetised piece of iron is attracted to either pole of a bar magnet, (b) a ring of iron may be used as a magnetic shield. (N)

Scanned with CamScanner

22. In Fig. 12, calculate (a) the current in one 8 Ω resistor if the battery has an e.m.f. of 6 V and internal resistance 2 Ω, (b) the terminal p.d. at the battery.

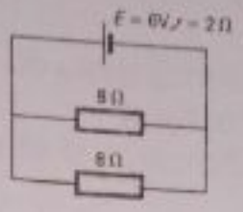


Fig. 12

23. Describe and explain the action of a *carbon microphone*.

24. With the aid of a diagram, explain why the coil of a moving-coil meter (a) is deflected when a current flows, (b) then comes to rest, (c) moves in the air-gap between curved magnet poles and a cylindrical soft-iron core.

25. A commercial (soft-iron) step-down transformer has a primary coil of 2000 turns and produces 12 V from a 240 V supply. (a) Find the number of turns in the secondary, (b) calculate the primary current if the secondary current is 1 A, (c) give *one* reason why the transformer is not 100 % efficient.

26. (a) An insulated plate B is brought near to the metal cap A of a negatively-charged electroscope (Fig. 13). Explain what happens to the divergence of the leaf. (b) If B is now earthed, what happens to the divergence? Explain your answer. (c) How does the *capacitance* between A and B change from case (a) to case (b)?

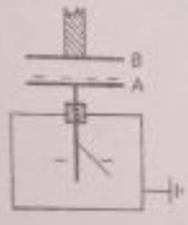


Fig. 13

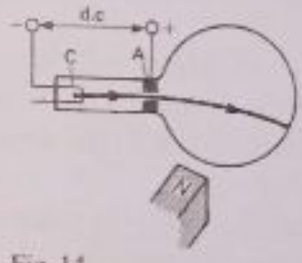


Fig. 14

27. In Fig. 14, electrons are emitted from a hot cathode C and a high +ve d.c. p.d. is connected between C and A.

Explain (a) why the electron beam travels in a straight line from C to A, (b) why the beam is deflected in a circular path when a strong magnet is brought up sideways to the beam as shown, (c) why this deflection shows that electrons carry −ve charges.

28. (a) What are α-particles, β-particles, γ-rays? How are they produced in radioactivity?

(b) A radon nucleus $^{222}_{86}Rn$ emits an α-particle and then a β-particle. What are the mass (nucleon) number and atomic (proton) number of the final nucleus formed?

(c) A radioactive source has a half-life of 6 days. How long will it take for the count-rate to fall from 160 per minute to 20 per minute?

29. (a) State *two* ways of increasing the brightness of the electron beam in a cathode ray oscilloscope. Explain your answer.

(b) Two complete stationary waves are obtained on the screen of an oscilloscope when an a.c. voltage of frequency 200 Hz is connected to the Y-plates. What procedure is necessary to obtain four complete stationary waves on the screen? Explain your answer.

Scanned with CamScanner

30. (a) Draw the characteristic (I–V) graph for (i) a length of constantan wire, (ii) a diode valve, showing −ve voltage values on your graph.

(b) How would you obtain a *constant* current in a circuit by using a diode valve? Explain your answer.

(c) Draw a circuit diagram showing how a diode valve acts as a *rectifier*.

31. Which of the following are vectors?

A Weight, B Mass, C Energy, D Momentum, E Mechanical advantage

32. Mercury is used in barometers because it has

A High expansivity, B Good conductivity, C High density,
D Low heat capacity, E High boiling point

33. A gas at 27°C is heated at constant pressure so that its volume is doubled. The new temperature is

A 5400 K, B 600 K, C 270 K, D 60°C, E 54°C.

34. Which of the following is necessary for total internal reflection to occur at an air–glass boundary?

A Light must travel from air to glass,
B The angle of incidence in glass must be less than the critical angle,
C Light must travel from glass to air,
D The angle of incidence in glass must be greater than the critical angle,
E The angle of incidence in air must be greater than the critical angle.

35. A converging (convex) lens has a focal length of 20 cm. The lens forms a real inverted image of the same size as the object when the object distance from the lens is

A 40 cm, B 30 cm, C 20 cm, D 10 cm, E 5 cm.

36. Which of the following is a longitudinal wave?

A Radio wave, B Light wave, C Sound wave, D Water wave,
E Infra-red wave.

37. In an electric motor, the rotation can be reversed by

A Increasing the current,
B Decreasing the current,
C Reversing both the current and the magnetic field,
D Reversing the current,
E Using springs.

38. When working normally, a 60 W – 240 V lamp has a filament resistance in ohms of

A 1440, B 960, C 340, D 144, E 4.

39. When an electron beam X is passed into a uniform magnetic field Y, the beam is deflected into a circular path because

A X is 90° to the direction of Y,
B X is parallel to the direction of Y,
C There is attraction between X and Y,
D There is repulsion between X and Y,
E Energy is released.

40. When a beam X of electrons and a beam Y of γ-rays enter a magnetic field perpendicular to the beams

A X is deflected parallel to the field,
B X is undeflected and Y is deflected,
C X is deflected perpendicular to the field,
D X and Y are both deflected perpendicular to the field,
E Y is deflected perpendicular to the field.

Scanned with CamScanner

Answers to Revision Questions

1. (a) 20 m/s (b) 20 m (c) 4 kg m/s, 40 J
2. 1200 W, 4·5 A
3. 1 N, 5 N; 2 N
4. (a) 8 N (b) 10 N (c) 30 J (d) 80%
5. 6000 kg
6. 10 m
9. 14·4 K(°C)
10. 7 K(°C)
13. 5 cm
15. 39°
16. 200 kHz
17. (a) B (b) 40 m/s
19. 80 cm
22. (a) 0·5 A (b) 4 V
25. (a) 100 (b) 0·05 A
28. (b) 218, 85 (c) 18 days

Multiple Choice

31. A, D **32.** C **33.** B **34.** D **35.** A
36. C **37.** D **38.** B **39.** A **40.** C

Scanned with CamScanner

Index

Aberration, spherical, 270
Absolute temperature, 169, 191
Absolute zero, 170, 190
Absorption, 244, 563
Absorption spectra, 333, 631
Acceleration, 15–20
  and force, 39
  and mass, 39
  due to gravity, 39
  in circle, 42
Accommodation (eye), 314
Accumulators, 411
Action (force), 46
Action at points, 515
Adhesion, 162
Air column, vibrations in, 382
Alcohol thermometer, 170
Alloy, 403, 443
Alpha-particle, 556, 561
Alternating current, 486, 616
  circuits, 618–25
Alternating
  generator, 486
  voltage, 486
Altimeter, 132
Ammeter, 390, 470
Ammeter
  , hot wire, 430
  , moving coil, 468
  , moving iron, 462
Ampere
  balance, 473
  , the, 390, 473
Amplifier, 551
Amplitude, wave, 340
Aneroid barometer, 131
Anode (discharge tube), 532
  (electrolysis), 496
  (valve), 537
Anomalous expansion of water, 185
Antinode, 374
Aperture
  , camera, 316
  , narrow, 353
  , wide, 353
Apparent depth, 282, 286
Apparent expansivity, 183
Aqueous humour, 313
Archimedes' Principle, 106
Aristotle, 2
Armature, 472
Arrhenius theory, 503
Astronaut, 41, 43
Astronomical telescope, 322
Atmosphere, 127
Atmospheric pressure, 130
Atmospheric refraction, 295
Atom, 142
Atomic
  clock, 6
  energy, 575
  nucleus, 569
  number, 569
  structure, 568
Attraction
  , charges, 510
  , magnetic, 440
Attractive molecular forces, 146, 159
Audio-frequency, 490
Aurora borealis, 535
AVO meter, 469
Axis, principal, 269, 303

Back e.m.f., 410
Background radiation, 560
Balance
  , common, 9, 40
  , spring, 9, 40
Balloon, 110
Bar, 131
Bardeen, 550
Barometer
  , aneroid, 131
  , mercury, 128
Base (transistor), 551
Battery, 408–14
Beams of light, 251
Becquerel, 556
Bell, electric, 549
Beta-particle, 556, 562
Bicycle
  dynamo, 480
  pump, 136
Bifocal lens, 316
Bimetal strip, 178
Binding energy, 575
Binocular vision, 313
Binoculars, 324
Black surface, 243
Blanket, electric, 179
Board of Trade unit, 428
Bohr theory, 629
Boiling, 169, 229
Boiling and pressure, 229
Bourdon gauge, 135
Boyle's law, 134, 196
Bragg, 546
Brake, hydraulic, 125
Brattain, 550
Breeze
  , land, 241
  , sea, 241
Bridges, 65, 69
Bridges, equilibrium of, 69
Brownian motion, 148
Buoyancy in gases, 110
  in liquids, 109
Burner, temperature, 206
Burning glass, 301
Brushes, motor, 472

Cable, 490
Caesium clock, 6
Calibration of ammeter, 501
  of thermometer, 168, 172
Calipers, 7
Caloric, 201
Calorie, 201
Calorific value, 201
Calorimeter, 205
  experiments, 205–6
  , fuel, 205
  heat capacity, 205
Camera
  lens, 316
  , pin-hole, 252
Capacitance, 524
Capacitor, 523, 607
  , electrolytic, 527,
  , parallel, 608
  , series, 608
  , variable air, 526
Capillarity, 161
Car
  battery, 414
  brakes, 125
  headlamp, 270
Carbon microphone, 460
Cathode
  , cold, 532
  , discharge tube, 532
  , electrolysis, 496
  , hot, 533
  rays, 533
  valve, 537
Cathode-ray tube, 541
Cell
  , Daniell, 410
  , dry, 410

Scanned with CamScanner

Cell, Leclanché, 410
, primary, 411
, secondary, 411
, simple, 409
Cells in parallel, 418
in series, 417
Cellulose acetate, 510
Celsius (Centigrade) scale, 168
Centimetre, 6
Centre of curvature, 269
Centre of gravity, 73
of mass, 72
Centrifuge, 43
Centripetal force, 42
Chadwick, 569
Chain reaction, 575
Change of state, 214
Characteristics, diode, 538
Characteristics, sound, 370
Charge
carriers, 389, 612
, electric, 509
Charge on conductor, 514
Charging an accumulator, 413
a capacitor, 523
Charles' law, 190
Chemical balance, 9, 40
Ciliary muscles, 313
Circuit, closed, 415
, open, 415
, parallel, 399
, ring, 434
, series, 398
Circular coil, 456
motion, 42
Clinical thermometer, 170
Clock rule, 456
Closed pipe, 382
Cloud chamber, 557
Coherent sources, 358
waves, 358
Cohesion, 162
Cold cathode, 532
Collector (transistor), 550
Colour, additive, 333
Colour
code, 432
, complementary, 333
disc, 330
dispersion of, 328
lights, 329, 330
object, 333
pigments, 334
, primary, 330
television, 334
triangle, 333
Common-emitter circuit, 551
Commutator
, motor, 472
slip rings, 487
split-ring, 487
Components, 30, 50, 93
Compression
of gas, 197
sound wave, 369
Concave mirror, 269
images, 273
Conduction of electricity, 389
of heat, 234
Conductivity
, electrical, 403
, thermal, 234
Conductor of electricity, 389, 510
of heat, 234
Conservation
of energy, 4, 57
of linear momentum, 49
Constant-volume gas thermometer, 171
Constructive interference, 357
Convection, 238
heating system, 240
in gas, 240
in liquid, 238
Convector, 242
Convergent beam, 251
Converging lens, 301
Conversion of temperature, 170
Convex mirror, 268, 275
Cooker, pressure, 230
Cooling
by evaporation, 222
curve, 217
Copper voltameter, 496
Core
, iron, 458, 461
, laminated, 488
Corkscrew rule, 455
Cornea, 313
Coulomb, 389
Critical angle, 291
Crystals, 503
Cubic expansion, 183
of gas, 189
of liquid, 183
of solid, 183
Cubic expansivity, 183, 190
Current
amplifier, 551
balance, 473
, electric, 388
, measurement of, 468, 473
meter, 390
Curvature, centre of, 269
Curved mirrors, 268
Cycle of wave, 369, 487

Damp course, 162
Davy lamp, 235
Decay, radioactive, 566
Deceleration, 16
Declination, 450
Defects of vision, 315
Demagnetisation, 442
Density, 100, 107
bottle, 102
of air, 103
of electric charge, 514
of gases, 104
of liquids, 102, 107
of solids, 100, 107
of water, 100, 183
, relative, 101
Destructive interference, 357
Detection, radio, 552
Detergents, 161
Deuterium, 573
Deuterons, 573
Dewar flask, 246
Dew point, 226
Diameter of molecule, 143
Dielectric constant, 526
Diffraction grating, 355
Diffraction
, light, 355
, microwave, 354
, sound, 353
, water waves, 353
Diffuse reflection, 260
Diffusion
cloud chamber, 557
Diffusion of molecules
, gas, 147
, liquid, 146
, solid, 146
Diode, 537
characteristic, 538
, junction, 549
, rectification, 540
Dip, magnetic, 451
Directly-heated valve, 537
Discharge tube, 532
Discharging
, accumulator, 413
, capacitor, 523
Dispersion, 328
Displacement, 15
curves (sound), 369
graphs, 24
Distance–time graphs, 21, 24
Distribution
of a.c. power, 491
of charge, 514
Divergent beam, 251
Diverging lens, 307
Divided touch, 441
Domains, magnetic, 449
Domestic boiler, 240
Driving mirror, 275
Drops, shape of, 162
Dry cell, 410
Dynamics, 13
Dynamo
, a.c., 485
, bicycle, 480
, d.c., 487

Earthing, 432
Earth potential (electrical), 432, 521
(gravitational), 52
Earth's magnetic field, 450
Echo, 348, 373
Eclipse
, annular, 256
, partial, 255
, total, 255
Eddy currents, 484
Efficiency
, machine, 82
Effort (force), 81
Einstein's mass–energy law, 575
photoelectric equation, 634

Scanned with CamScanner

Elastic limit, 151
Elasticity, 150
Electric
  bell, 459
  deflection, 533
  fields, 519, 600
  intensity, 519
  lamp, 426
Electric motor, 472
Electrical
  energy, 424
  heating, 428
  installation, 432
  laws, 428
Electricity
  , current, 388
  , quantity of, 389, 500
  , static, 509
Electrochemical experiment, 499
Electrolysis, 496
  , Faraday's laws of, 500–2
  of copper sulphate, 496
  of water, 496
  , theory of, 503
Electrolyte, 496
Electrolytic capacitor, 498, 527
Electromagnet, 458
Electromagnetic induction, 478
Electromagnetic waves, 345, 349, 630
Electromotive force, 409, 425
Electron, 389, 510, 533–44
  beam, 535
  charge, 510, 536
  deflection, 533–5, 605, 613
  gun, 541
Electronics, 537
Electrophorus, 517
Electroplating, 497
Electroscope, 511
  and induction, 514
  and p.d., 521
  and resistance, 510
  , charging, 511
Electrostatic induction, 513
Element, electric fire, 403
Emission
  , electron, 533
  , heat, 244
Emitter, transistor, 550
Energy, 2, 51
  , chemical, 3, 409
  , conservation of, 4, 57
  conversions, 2, 57
  , electrical, 391, 424
  , heat, 201
  , kinetic, 53
  , levels, 629
  , mechanical, 51–7
  , nuclear, 575
  , potential, 52
  , sound, 371
Engine cooling, 239
Engine, heat, 208
Equations of motion, 19
Equilibrium
  of forces, 67, 92
  , neutral, 76
  , stable, 76
  , unstable, 76
Escape velocity, 598
Evaporation, 221
Expansion and force, 175
Expansion, 175
  of gases, 188
  of liquids, 182
  of metals, 176, 180
  of solids, 175
  of water, 183
Exposure
  meter, 318
  time, 318
Eye, 313

Falling body, 28, 31
Far point, 315
Farad, 524
Faraday, 478
  constant, 503
  induction, 478, 483
  , laws of electrolysis, 500, 502
Ferrite, 443
Fibre optics, 293
Field
  , electric, 519, 601
  , magnetic, 443, 611
Filament, lamp, 426
Film, oil, 143
Fine-beam tube, 535
Fish field of view, 289
Fission, 575
Fixed points, thermometer, 168
Fleming's
  left-hand rule, 467, 612
  right-hand rule, 482
Floating objects, 109
Flotation, principle of, 108
Fluid
  friction, 155
  upthrust, 104
Fluorescent screen, 536
f-number, 317
Focal length
  of lens, 303
  of mirrors, 269, 271
Focus
  , principal, 269, 303
  , secondary, 303
  , virtual, 303
Force, 36, 45, 64
  and acceleration, 37
  and mass, 39
  between molecules, 146, 159
  , component of, 50, 93
  , molecular, 146, 159
  , moments of, 65
  of expansion, 175
  of friction, 42, 153
  on conductors, 465
  , parallel, 68
  , resultant of, 91
Forced vibrations, 346
Four-stroke cycle, 208
Fraunhofer lines, 333
Free fall for *g*, 20
Freezing, 218
Frequency, 344
  of sound waves, 370
Friction, 41, 153
  , fluid, 155
  , kinetic (dynamic), 154
  , static, 154
Fuel
  calorimeter, 205
  cell, 412
Fundamental
  of pipes, 382
  of strings, 378
Furnace, solar, 261
Fuse, electrical, 433
Fused plug, 435
Fusion (heat), 214
  , nuclear, 577

g, 18, 20
Galileo, 56
Galvani, 408
Galvanometer, mirror, 263, 390, 469
Gamma rays, 345, 563
Gas laws, 195
  Boyle's, 134
  Charles', 190
Gas
  pressure, 133, 192
  thermometer, 171
Gases
  , density of, 104
  in motion, 147, 196
  , kinetic theory of, 196
Geiger
  and Marsden, 568
  –Müller tube, 558
Generator, 485, 517
Germanium crystal, 547
Gilbert, 440
Glass expansion, 183
Gold-leaf electroscope, 511
Gram, 6
Graphs, 10, 22, 24–7
Gravitational intensity, 597
  potential, 597
Gravity, 20, 39, 595
  , acceleration of, 20, 40, 596
  , centre of, 73
Greenhouse, 245
Grid system (electrical), 491

Half-life period, 566
Harmonics, 379
Health hazards, radioactive, 565
Heat
  and electricity, 203, 428
  capacity, 202
    , specific, 202
  engines, 208
  , mechanical energy and, 207
  , nature of, 201

Scanned with CamScanner

Heat, quantity of, 201, 202
, transfer of, 204, 234
units, 201
Heating
, electrical, 203, 428
, hot water, 240
of room, 246
Heavy hydrogen, 573
Height of c.g., 76
Helium, 562
High-voltage transmission, 432, 490
Hoffmann voltameter, 497
Hole, semiconductor, 547
Hooke's law, 151
Hope's experiment, 183
Horizontal
component, 93
motion, 30
Horse-power, 428
Hot wire ammeter, 430
Hydraulic press, 124
Hydroelectric energy, 52
Hydrogen
atom, 570, 629
, polarisation by, 410
Hydrometer, 113
Hygrometer, 227
Hypsometer, 169

Ice, 110, 184
Iceberg, 109, 217
Ice-point, 168
Ignition
point, gas, 236
Images
in curved mirrors, 274, 275
in lenses, 305–8
in plane mirrors, 263
, multiple, 265
Impedance, 623
Impulse, 46
Inclined plane, 86
Induced
charge, 513
current, 479
e.m.f., 483
magnetism, 446
Induction
coil, 481
, electromagnetic, 478
, electrostatic, 513
, magnetic, 446
Inertia, 37
Inertial mass, 37
Infra-red rays, 245, 335
Insulators
, electrical, 389, 547
, heat, 238
Intensity, sound, 390
Interaction, fields, 466
Interference
in light, 358
in microwaves, 361
in sound, 362
in water, 356
Internal resistance, 415
Invar, 178
Inverse-square law, 564

Ion, 503
Ionising radiation, 556
Iris, 313
Iron
filings, 444
, magnetic property, 442
Isobar (atom), 571
Isotope, 573

Jack, car, 89
Jet aeroplane, 47, 209
Joule, the, 50, 201
Joule's heat laws, 428
Junction diode, 549
forward and reverse bias, 549

Kaleidoscope, 265
Keeper, magnet, 450
Kelvin scale, 169, 191
Kilogram, 4
Kilowatt, 58, 447
-hour, 428
Kinetic
energy, 53
Kinetic theory,
evaporation, 222
gases, 196

Lake, freezing of, 185
Lamp
, electric, 426
, fluorescent, 426
Land breeze, 241
Latent heat, 214
of fusion, specific, 214
of vaporisation, specific, 218
Lateral inversion, 264
Laws of motion, 36, 45, 46
Layer battery, 410
Leclanché cell, 410
Lens camera, 316
Lenses
, converging, 301
, diverging, 301
Lenz's law, 483
Leslie cube, 243
Levers, classes of, 71
Light
, colours, 328
, deviation, 263
, diffraction of, 355
, interference of, 358
-meter, 318
rays, 251
, reflection of, 259
, refraction of, 280
, speed of, 345
waves, 351
, white, 328
Lighting circuit, 434
Lightning, 516
Lightning conductor, 516
Linear
expansivity, 180
magnification, 276, 306
motion, 19
Lines of force
(current), 455–8
, electric, 520
, magnetic, 443
Liquid
, buoyancy in, 109
conductivity, 236
, density, 102, 107
, expansion of, 182
pressure, 121
state, 142
Litre, 7
Live (line) a.c., 432
Load
on machines, 81
on wire, 151
Loaded tube, 592
Long sight, 315
Long waves, radio, 345
Longitudinal waves, 343
, progressive, 343
, stationary, 374
Loudness of sound, 371
Loudspeaker, moving-coil, 473
Lower fixed point, 168
Lubrication, 157
Luminous
energy, 3
object, 251

Machines, 81–90
Magdeburg hemispheres, 128
Magnet
, bar, 440
, horse-shoe, 441
, permanent, 441, 458
, temporary, 441, 458
Magnetic
alloys, 442
deflection, 534, 613
domains, 449
effect of current, 454
fields, 443, 611
flux, 443, 483
induction, 446
relay, 460
screening, 448
tape, 439
Magnets, making of, 440
Magnification
, angular, 320
of images (lenses), 305–8
(mirrors), 274, 276
Magnifying glass, 306
Maltese cross, 537
Manometer, mercury, 133
Manometer, water, 133
Mass, 9, 40
number, 569
spring oscillation, 589
Maxwell's corkscrew rule, 455
Mechanical
advantage, 81
energy, 51–7
and heat, 207
Megohm, 395
Melting-point, 216
, effect of pressure, 217
Mercury thermometer, 168
Metals, 150, 234, 389, 510

Scanned with CamScanner

Metre, 4
  bridge, 404
Mica capacitor, 526
Microfarad, 524
Micrometer gauge, 8
Microphone, carbon, 460
Microprocessor, 531
Microscope
  , compound, 320
  , simple, 306, 320
Microwaves, 349, 351, 353
Milliammeter, 470
Mirage, 294
Mirror
  , dentist's, 274
  , driving, 275
  formulae, 277
  , parabolic, 269
  , plane, 259
  , shaving, 268
  , spherical, 268
  , thick glass, 292
Mirror galvanometer, 263, 469
Mixtures, method of, 206
Molecular
  forces, 146
  theory, 146, 196
Molecules
  in gas, 147, 196
  in liquid, 146, 159
  , size of, 142, 145
  in solid, 146, 152
  , velocity of, 197
  , vibration of, 146
Moments, 65
  , principle of, 67
Momentum, 45
  change, 47
Momentum, conservation of, 49
Monomolecular layer, 143
Moon's
  attraction, 40
  eclipse, 256
Motion
  , equations of, 19
  in circle, 42
  , laws of, 36, 45, 46
  of molecules, 145–50
Motor
  , electric, 472
  force, 465
Moving-coil
  galvanometer, 469
  instrument, 462
  loudspeaker, 473
Moving-iron ammeter, 462
Multimeter, 469
Multiple images, 292
Mu-metal, 443, 446

N-type semiconductor, 549
Near point, 315
Negative electricity (charge), 509
Nerve, optic, 313
Neutral
  equilibrium, 76
  points, 446
  wire (a.c.), 432
Neutron, 569
New Cartesian, 277
Newton
  , Sir Isaac, 36, 329
  , the, 39
Newton's laws of motion, 36, 45, 46
Nichrome, 389, 403
Nodes, 374
  in pipes, 382
  in strings, 374, 378
Normal
  ray, 262
  vision, 315
North pole, geographic, 450
Nuclear
  collision, 572
  energy, 575
  fission, 575
  fusion, 577
  mass, 569
  reaction, 570
  reactor, 576
Nucleons, 569
Nucleus, 569

Octave, 373
Oersted, 454
Ohm, 395
Ohm-meter, 469
Ohm's law, 394
Oil film, 143
Optical
  fibre, 293
  instruments, 316–24
Orbits, 44
Organ, pipe, 381
Oscillations, 23
Oscilloscope, cathode ray, 541–4
Osmosis, 144
  , reverse, 145
Overtones, 372

P-type semiconductor, 548
Paints, 334
Palomar telescope, 268
Parabolic mirror, 269
Parallax, 263
Parallel
  beam, 251, 269
  cells, 418
  circuits, 398
  forces, 68
Parallelogram
  of forces, 92
  of velocities, 29
Part-immersion, 282
Pascal, the, 131
Pendulum
  , energy and, 56
  , g by, 23
Penetration power, 564
Penumbra, 255
Periscope, 262, 292
Perrin tube, 535
Persistence of vision, 314
Petrol engine, 208
Phase difference, 621, 623
Photoelectricity, 633
Photon, 634
Pigments, 334
Pinhole camera, 252
Pipe, vibrations in, 387
Pitch, 370
Plane
  , inclined, 86
  mirrors, 259
  wave, 342
Planets, 44
Plastic stage, 151
Plimsoll line, 112
Plug
  , fused, 435
  , sparking, 209, 481
p–n junction, 549
Points, action of, 515
Polarisation of cell, 410
  of light, 363
Pole
  , magnetic, 440
Polythene, 510
Positive electricity (charge), 509
Potential
  (electrical), 521, 602
  (gravitational), 52, 55, 597
  difference, 391, 605
  energy, 52
  of conductor, 522
  of electroscope, 521
Potentiometer, 420
Power
  , electrical, 427, 626
  , lens, 303
  , mechanical, 57
Pressure, 119, 192
  and depth, 122
  atmosphere, 130
  coefficient, 194
  cooker, 230
  formula, 123
  , hydraulic, 125
  , laws of, 195
  , molecular theory of gas, 196
  , sound wave, 369
  transmission, 124
Primary cells, 408–10
Principal axis, 269
Principle of Reversability, 269
Prism
  binoculars, 324
  , refraction by, 280, 287
  , refractive index of, 293
  , right-angled, 292
  total reflecting, 292
Progressive wave, 343
Projectile motion, 582
Proof-plane, 514
Properties
  of gases, 147, 188
  of liquids, 142, 182
  of solids, 145, 175
  of waves, 341–62
Proton, 569

Scanned with CamScanner

Pulleys, 83
Pump, bicycle, 136

Quality (sound), 372
Quantity of heat, 202
Quantum, energy, 630

Radar, 544
Radiation, 242, 630
, infra-red, 245, 345
, solar, 335
, ultra-violet, 335
, visible, 335
Radiators, 244
Radio resonance, 347
Radio telescope, 323
Radio waves, 345
Radioactivity, 556
Radioisotope, 573
Radius of curvature, 269, 271
Rarefaction, sound wave, 369
Ratemeter, 606
Rays, 251
Reactance, 619, 621
Reaction, 46, 65
, nuclear, 570
Reactor, nuclear, 576
Real is positive, 277
Receiver, 552
Rectangular coil, 467
Rectification, 540
Rectifier
, metal, 540
, semiconductor, 550
, valve, 540
Rectilinear propagation, 252
Reflecting telescope, 268, 323
Reflection
, internal, 289
, laws of, 262
, mirror, 259
of waves, 347–9
Refracted rays, 288
Refraction
, laws of, 284
of light, 287, 351
of microwaves, 350
of water waves, 349
Refractive index, 285, 352
Refrigerator, 223
Regelation, 218
Relative density, 101
humidity, 227
Relay, magnetic, 460
Repulsion
, charges, 510
, magnetic poles, 440
, molecular, 152
Resistance, 395
, measurement of, 396, 404
Resistivity, 402
Resistor, 395
in parallel, 398
in series, 398
Resonance
, electrical, 347, 625
, mechanical, 346
, sound, 346, 380
tube, 380
Resultant, 91
Retina, 313
Reversibility, light, 269
Rheostat, 395
Richardson, 533
Ring circuit, 434
Ripple tank, 341
Rivet, 177
Rocket engine, 210
Roentgen, 545
Root-mean-square, 616
Rutherford, Lord, 557, 569

Salt mixture, 183
Satellite, 40, 44
Saturated vapour, 225
pressure, 225
Saturation
current, 539
magnetic, 449
Scalar, 28
Scale of temperature, 169
Scaler, 559
Scattering
by nuclei, 568
of light, 260
Scientific method, 2
Screening, electrical, 520
Screening, magnetic, 448
Screw, 89
Screw jack, 89
Sea breezes, 241
Searchlight, 270
Secondary cell, 411
Semiconductor, 389, 547
n-type, 549
p-type, 548
rectifier, 550
Series cells, 417
Series circuits, 398
Shadows, 254
Shaving mirror, 268, 274
Short sight, 315
Shunt resistor, 470
SI units, 4
Sign convention, 277, 308
Silicon, 547
Silver plating, 497
Silvery surface, 243
Simple cell, 409
harmonic motion, 584
Siphon, 133
Siren, 370
Skate, ice, 218
Slide projector, 319
Slinky coil, 343
Snell's law, 284
Soap film, 159
Solar
furnace, 261
radiation, 335
Solenoid, 457
Solid state detector, 559
Solidifying, 217
Solids, properties of, 142
Sonometer, 375
Sound, 368
amplitude, 369
energy, 371
intensity, 371
quality, 372
reflection, 348
resonance, 346, 380
speed, 370, 373
waves, 369
South pole
, geographic, 450
, magnetic, 450
Spark plug, 209, 481
Specific heat capacity, 202–7
Spectacles, 315
Spectra, emission, 332, 630
Spectrum
by grating, 356
by prism, 328
, pure, 331
Speed, 13
of electromagnetic waves, 345
Speed
of light, 345
of sound, 344
of water waves, 344
Speedometer, 484
Spherical wave, 342
Spring and constant, 152
balance, 9, 40, 150
extension, 40, 150
Stability, 75
Standard pressure, 130
Standard temperature, 195
State, change of, 214, 218
States of matter, 142
Static
electricity, 509, 522
friction, 154
Stationary waves
in coil, 374
in pipe, 382
in strings, 374
Steam
, latent heat of, 219
point, 168
Steel, properties of, 443
Stopping potential, 636
Straight conductor, 455
Streamline flow, 157
Stretching of metals, 150
String, vibrations in, 374–80
Stroboscope, 342
Submarine, 112
Sun
, eclipses of, 257
, radiation from, 335
Superconductivity, 403
Surface tension, 157

Telephone earpiece, 461
Telescope
, astronomical, 322
, radio, 323
, reflecting, 323
, refracting, 322
Television
camera tube, 334
, colour, 334
Temperature, 168, 190
and resistance, 397

Scanned with CamScanner

Temperature of flame, 206
  scales, 169
Tension, 65
  , surface, 158
Terminal p.d., 414
Terminal velocity, 42, 155
Terrestrial magnetism, 450
Therm, 201
Thermal
  capacity, 202
  conduction, 234
  expansion, 175
Thermals, 242
Thermionic emission, 533
Thermocouple, 172
Thermodynamic scale, 169
Thermoelectric thermometer, 172
Thermometer
  , alcohol, 170
  , clinical, 170
  , gas, 171
  , max and min, 171
  , mercury-in-glass, 168
  , thermoelectric, 172
Thermonuclear fusion, 578
Thermopile, 243
Thermos flask, 246
Thermostat
  , electric, 179
  , gas, 179
Thomson, Sir J. J., 533
Ticker-timer, 37
Ticonal, 443
Timbre, 372
Time-base, 543
Torricelli, 129
Total internal reflection, 289
Totally reflecting prism, 292
Transformer, 488
Transistor, 550
  amplification, 551
Transmission of pressure, 124
Transmutation, 570
Transverse waves, 343
Triangle of forces, 92
Tritium, 573
Tuning-fork, 371
Turbulent flow, 158
Tyre, motor, 154

Ultra-violet rays, 335, 345
Uncertainty Principle, 637
Uniform
  acceleration, 16
  velocity, 15
Units
  , electrical, 390, 395
  , heat, 201
  , mechanical, 39, 45, 50
  , SI, 4
Unsaturated vapour, 224
Unstable, 76
Upper fixed point, 168
Upthrust
  in gas, 110
  in liquid, 104
Uranium, 556, 575

Vacuum, 129
  flask, 246
Valve, diode, 537
Van Allen belts, 535
Van de Graaff generator, 518
Vapour, 224
  pressure, 224
Variation (declination), 450
Vectors, 28
  , parallelogram of, 29, 92
  , triangle of, 29
Velocity, 14, 344
  of light, 345
  of radio waves, 345
  of sound, 344, 373
  of water waves, 344
  ratio, 81
Velocity–time graph, 25–7
Verniers, 7
Vertical
  component, 31, 93
  motion, 18, 30
Vibrating strings, 374–80
Vibration of pendulum, 23
Vibrational energy, 146
Vibrations, 23, 145, 346, 589
Virtual images, 263, 274, 275, 306
Viscosity, 155
Vision, 313–6
Visual angle, 320
Vitreous humour, 313

Volt, 391
Voltage
  , a.c., 486, 616
  , d.c., 488
Voltaic cell, 408
Voltameter, 496
Voltmeter, 392, 470
Volume, 7
  change (ice-water), 110, 184
  measurement, 101, 103
Volume expansion
  of gas, 189
  of liquid, 182
  of solid, 183

Water
  , density of, 183
  , expansivity of, 183
  vapour, 226
  voltameter, 497
  waves, 341
Watt, 58, 425
Wave theory of light, 351
Wavelength, 344
  of light, 345, 359
  of microwaves, 349
  of radio waves, 345
  of sound, 344
  of water waves, 344
Waves, 341–62, 636
Weather, 131
Weight, 9, 40
Weightlessness, 44
Wet-and-dry bulb hygrometer, 227
Wetting, 162
Wheatstone bridge, 404
Wilson cloud chamber, 557
Work, 50
  function, 633

X-ray tube, 546
X-rays
  , diffraction of, 546
  , nature of, 546
  , production of, 546
  , properties of, 345, 546

Yield point, 151
Young's experiment, 371
  modulus, 153

Scanned with CamScanner

Principles of Physics for Senior Secondary Schools

The key new feature of this edition is a section on *Further Topics*, intended to help students with the harder or more advanced parts of the new Senior Secondary School syllabus.

Multiple-choice questions have been added to the many exercises to bring these in line with the structure of Senior Secondary examination.

As always, the text is richly illustrated with very helpful diagrams and photographs.

Suggestions for practical works are placed at appropriate positions in the text, backed by a tinted panel that makes them easy to find.

ISBN (Nigeria) 978 139 461 7

LearnAfrica

ISBN 0-582-09245-0

9 780582 092457

Scanned with CamScanner